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# The methods of design and modelling of distributed satellite systems

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# Model of Motion

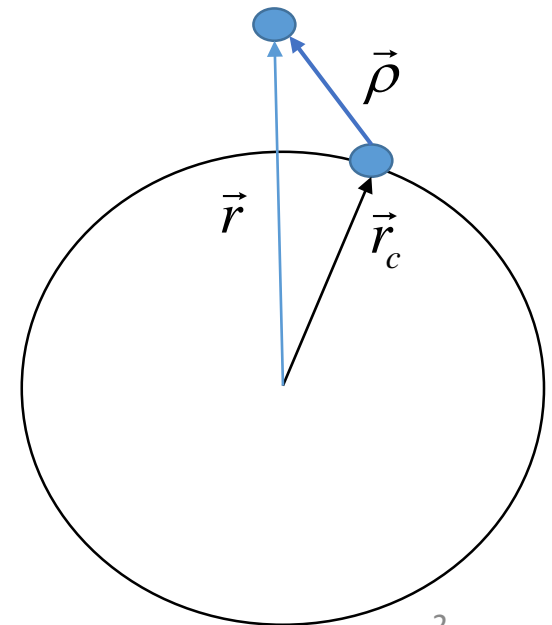
- Two satellites are close to each other
- Study relative motion

Newton's gravity:  $\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$

$\omega$  is an angular velocity of first satellite

Relative motion is

$$\ddot{\vec{\rho}} + 2\vec{\omega} \times \dot{\vec{\rho}} + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \ddot{\vec{r}}_c = -\left( \frac{\mu}{r^3} \vec{r} - \frac{\mu}{r_c^3} \vec{r}_c \right)$$



# Hill-Clohessy-Wiltshire Equations

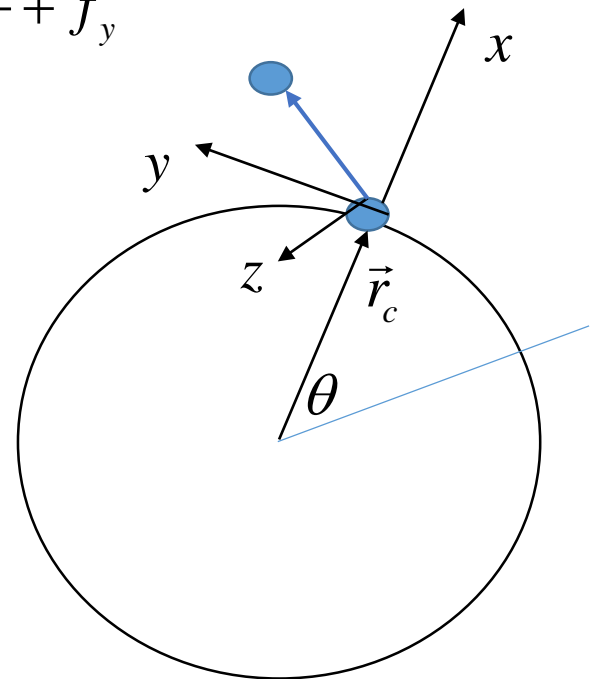
$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} + f_x$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + f_y$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + f_z$$

$$\ddot{r}_c = r_c \dot{\theta}^2 - \frac{\mu}{r_c^2} [1 + f_r]$$

$$\ddot{\theta} = -\frac{2\dot{r}_c \dot{\theta}}{r_c} [1 + f_\theta]$$



# HCW Equations

## Pure keplerian motion

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} + \cancel{f_x}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \cancel{f_y}$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \cancel{f_z}$$

$$\ddot{r}_c = r_c \dot{\theta}^2 - \frac{\mu}{r_c^2} \left[ 1 + \cancel{f_r} \right]$$

$$\ddot{\theta} = -\frac{2\dot{r}_c \dot{\theta}}{r_c} \left[ 1 + \cancel{f_\theta} \right]$$

# HCW Equations

## Circular reference orbit

$$\ddot{x} - 2\dot{\theta}\dot{y} - \cancel{\ddot{y}} - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \cancel{\ddot{x}} - \dot{\theta}^2 y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}}$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}}$$

$$\ddot{r}_c = r_c \dot{\theta}^2 - \frac{\mu}{r_c^2} = 0 \quad \Rightarrow \quad \dot{\theta}^2 = \frac{\mu}{r_c^3} = n^2$$

$$\ddot{\theta} = -\frac{2\dot{r}_c \dot{\theta}}{r_c} = 0 \quad \Rightarrow \quad r_c = \text{const}$$

# HCW Equations

## Linearizing

$$\begin{aligned}\ddot{x} - 2n\dot{y} - n^2x &= -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} = \\ &= -\frac{\mu r_c \left(1 + \frac{x}{r_c}\right)}{r_c^3 \left[\left(1 + \frac{x}{r_c}\right)^2 + \left(\frac{y}{r_c}\right)^2 + \left(\frac{z}{r_c}\right)^2\right]^{3/2}} + \frac{\mu}{r_c^2}\end{aligned}$$

# HCW Equations

Linearizing

$$(1+x)^\alpha = 1 + \alpha x$$

$$\begin{aligned} & \left(1 + \frac{x}{r_c}\right) \cdot \frac{1}{\left[\left(1 + \frac{x}{r_c}\right)^2 + \left(\frac{y}{r_c}\right)^2 + \left(\frac{z}{r_c}\right)^2\right]^{3/2}} = \\ & = \left(1 + \frac{x}{r_c}\right) \cdot \frac{1}{\left[1 + \frac{2x}{r_c}\right]^{3/2}} = \left(1 + \frac{x}{r_c}\right) \cdot \left(1 + \left(-\frac{3}{2}\right) \cdot \frac{2x}{r_c}\right) = \\ & = 1 + \frac{x}{r_c} - \frac{3x}{r_c} = 1 - \frac{2x}{r_c} \end{aligned}$$

# HCW Equations

$$\begin{aligned} \ddot{x} - 2n\dot{y} - n^2 x &= -\frac{\mu(r_c + x)}{\left[ (r_c + x)^2 + y^2 + z^2 \right]^{3/2}} + \frac{\mu}{r_c^2} = \\ &= -\frac{\mu r_c \left( 1 + \frac{x}{r_c} \right)}{r_c^3 \left[ \left( 1 + \frac{x}{r_c} \right)^2 + \left( \frac{y}{r_c} \right)^2 + \left( \frac{z}{r_c} \right)^2 \right]^{3/2}} + \frac{\mu}{r_c^2} \\ &= -\frac{\mu}{r_c^2} \cdot \left( 1 - \frac{2x}{r_c} \right) + \frac{\mu}{r_c^2} = \frac{2\mu x}{r_c^3}, \end{aligned}$$

$$n^2 = \frac{\mu}{r_c^3},$$

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0$$



# HCW Equations

Assumptions:

- Keplerian motion
- Circular reference orbit
- Ratio of relative distance to orbit radius is small

$$\ddot{x} - 2n\dot{y} - 3n^2x = 0$$

$$\ddot{y} + 2n\dot{x} = 0$$

$$\ddot{z} + n^2z = 0$$

$$x = 2(2x_0 + \dot{y}_0 / n) - (3x_0 + 2\dot{y}_0 / n)\cos nt + (\dot{x}_0 / n)\sin nt$$

$$y = (y_0 - 2\dot{x}_0 / n) - 3(2x_0 + \dot{y}_0 / n)nt + (2\dot{x}_0 / n)\cos nt + 2(3x_0 + 2\dot{y}_0 / n)\sin nt$$

$$z = z_0 \cos nt + (\dot{z}_0 / n)\sin nt$$

# HCW Equations

$$\ddot{x} - 2n\dot{y} - 3n^2x = 0$$

$$\ddot{y} + 2n\dot{x} = 0$$

$$\ddot{z} + n^2z = 0$$

$$x = 2C - A\cos nt + B\sin nt$$

$$y = D - 3Cnt + 2B\cos nt + 2A\sin nt$$

$$z = E\cos nt + F\sin nt$$

$$x = 2C + \bar{A}\sin(nt + \alpha)$$

$$y = D - 3Cnt + 2\bar{A}\cos(nt + \alpha)$$

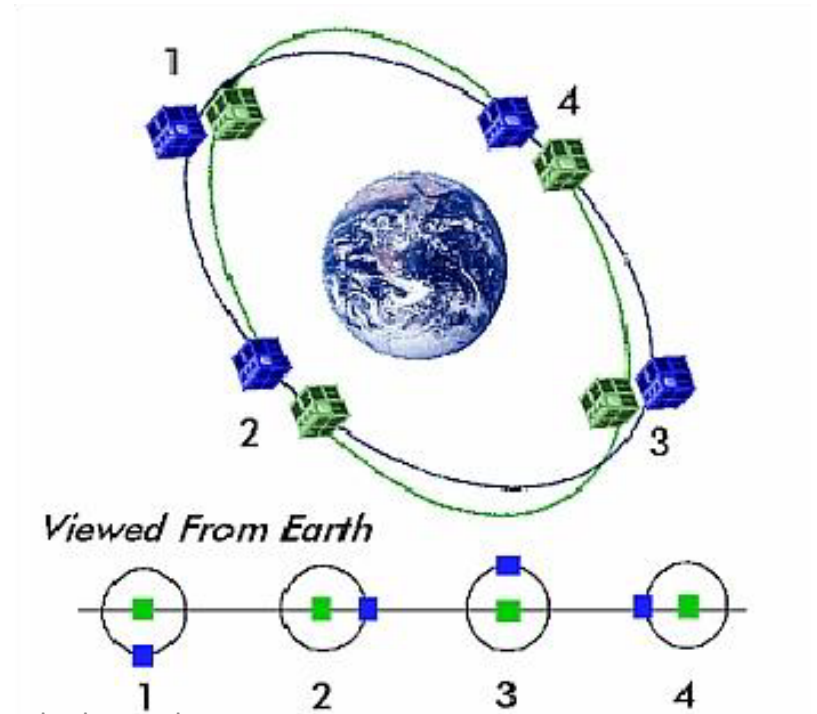
$$z = \bar{B}\cos(nt + \beta)$$

# HCW Equations

## Periodic solutions

- Leader-Follower,  $x = z = 0$ ,  $y = \text{const}$
- Circle,  $\alpha = \beta$ ,  $\bar{B} = \sqrt{3}\bar{A}$ ,  $x^2 + y^2 + z^2 = 4\bar{A}^2$
- Projected Circular Orbit,

$$\bar{A} = \bar{B} / 2, \quad \alpha = \beta, \quad y^2 + z^2 = \bar{B}^2$$

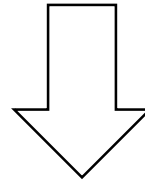


# More equations

- Drop the third order, not the second
- Schweighart-Sedwick equations include  $J_2$
- Schaub and Alfriend  $J_2$ -invariant relative orbits
- Fasano and D'Erico model for large formations
- Elliptic reference orbits: Tshauer-Hempel and Lawden equations
- many more...

# Missions

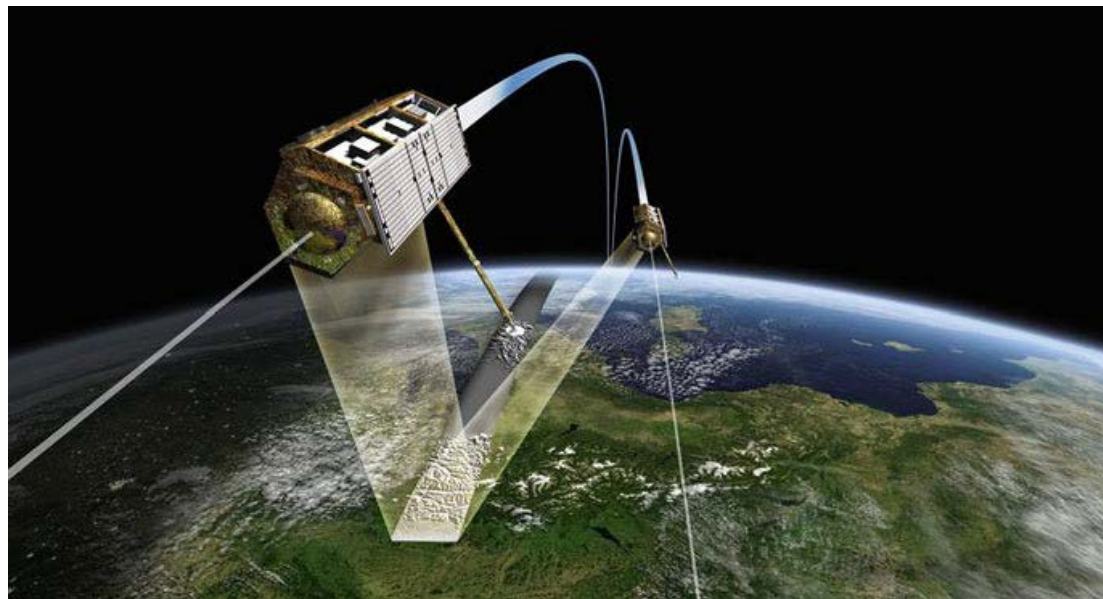
- 1965 Gemini 6A and Gemini 7
  - Stayed close one to another for app. 5 h
- 1997 autonomous FF mission ETS-7
  - Rendezvous docking



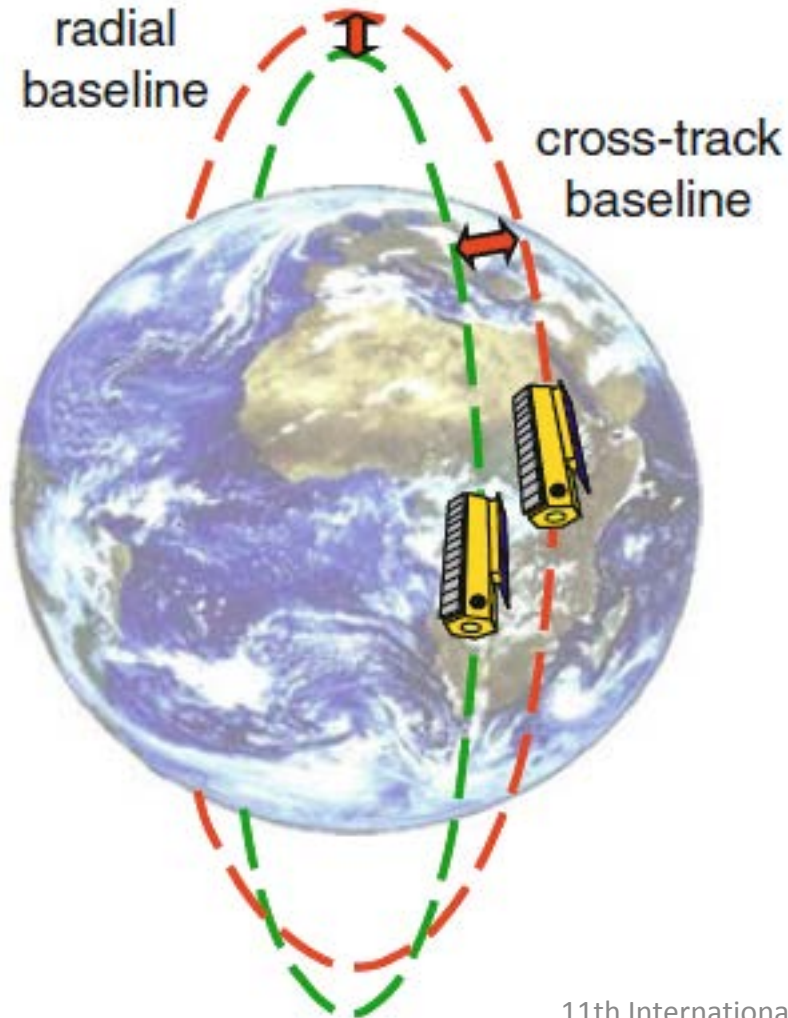
- 2014 CanX-4&5
  - Submeter relative tracker accuracy

# TanDEM-X

- Earth observation and mapping
- 2 sats, each with a synthetic aperture radar – 12 m horizontal, 2 m vertical resolution
- Launched in June 2010



# TanDEM-X

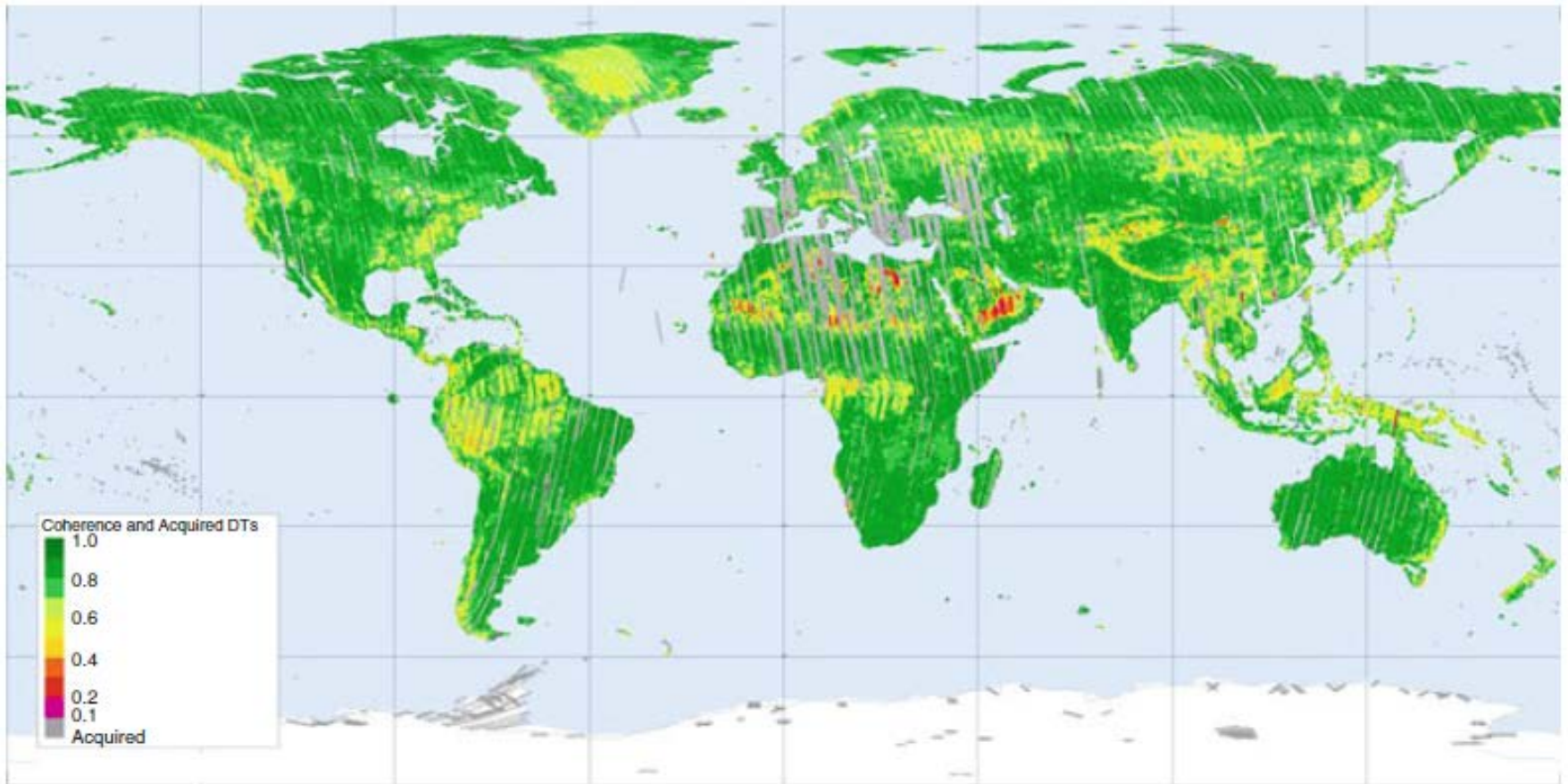


Helix satellite formation  
for TanDEM-X

Sun-synchronous orbit

Modified HCW  
equations used for  
modelling

# TanDEM-X



Global coherence map of TanDEM-X data takes acquired between December 2010 and December 2011



# GRACE - Gravity Recovery And Climate Experiment

The diagram illustrates the GRACE mission architecture. Two GRACE satellites are shown in orbit around Earth, maintaining a precise distance between them. They are connected to GPS satellites (L1 & L2) via a 24 & 32 GHz Crosslink. The satellites also communicate with ground stations on Earth via S-Band TT&C. Ground stations are located at Poker Flat, Spitzbergen, Neustrelitz, and Weilheim. Data is processed at the Raw Data Centre (DLR-DFD) and the Science Data System (CSR/JPL/GFZ). Mission Control is located at DLR-GSOC.

## GRACE Mission

**Science Goals**  
High resolution, mean & time variable gravity field mapping for Earth System Science applications.

**Mission Systems**

**Instruments**

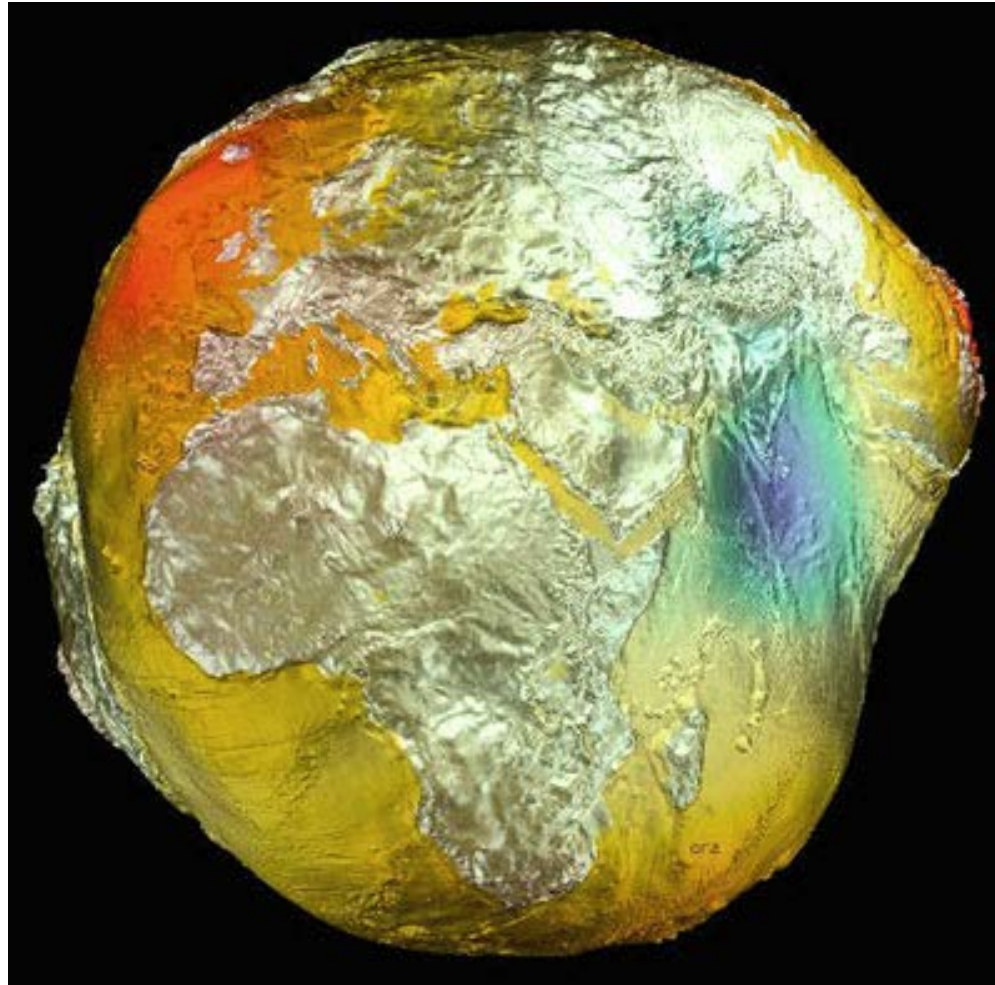
- KBR (JPL/SSL)
- ACC (ONERA)
- SCA (DTU)
- GPS (JPL)

**Satellite** (JPL/DSS)  
**Launcher** (DLR/Eurokot)  
**Operations** (DLR/GSOC)  
**Science** (CSR/JPL/GFZ)

**Orbit**

- Launch: March 2002
- Altitude: 485 km
- Inclination : 89 deg
- Eccentricity: ~0.001
- Lifetime: 5 years
- Non-Repeat Ground Track
- Earth Pointed, 3-Axis Stable

# GRACE



The Potsdam Gravity potato

# Gravity missions

- Lageos 1 (1976) – J2, J3 coefficients
- Lageos 2 (1992) – more coefficients
- CHAMP (2000) – mapping gravity
- GRACE (2002) – detecting the temporal variations of the Earth gravity field
- GOCE (2009) – map of the static Earth gravity field using the technique of the gradiometry
- NGGM – Next Generation Gravity Mission

# PRISMA

in-orbit test-bed for  
guidance, navigation, and  
control algorithms,  
relative navigation  
sensors, and propulsion  
systems

