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The methods of design and modelling of distributed satellite systems

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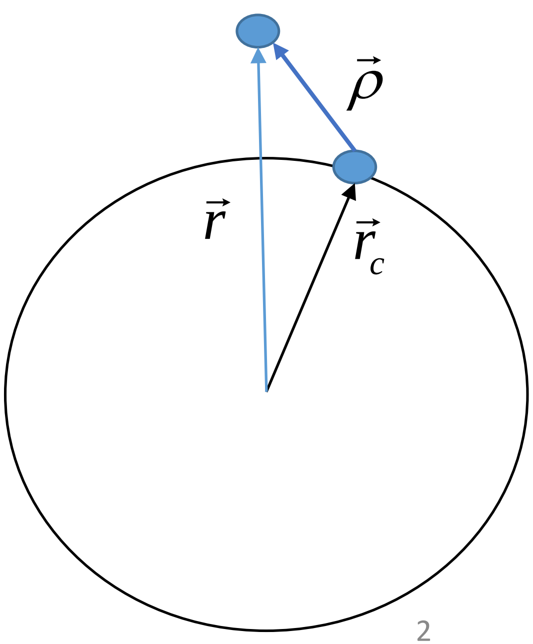
Model of Motion

- Two satellites are close to each other
- Study relative motion

Newton's gravity: $\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$

ω is an angular velocity of first satellite



Relative motion is

$$\ddot{\vec{\rho}} + 2\vec{\omega}\times\dot{\vec{\rho}} + \dot{\vec{\omega}}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \ddot{\vec{r}}_c = -\left(\frac{\mu}{r^3}\vec{r} - \frac{\mu}{r_c^3}\vec{r}_c\right)$$

Hill-Clohessy-Wiltshire Equations

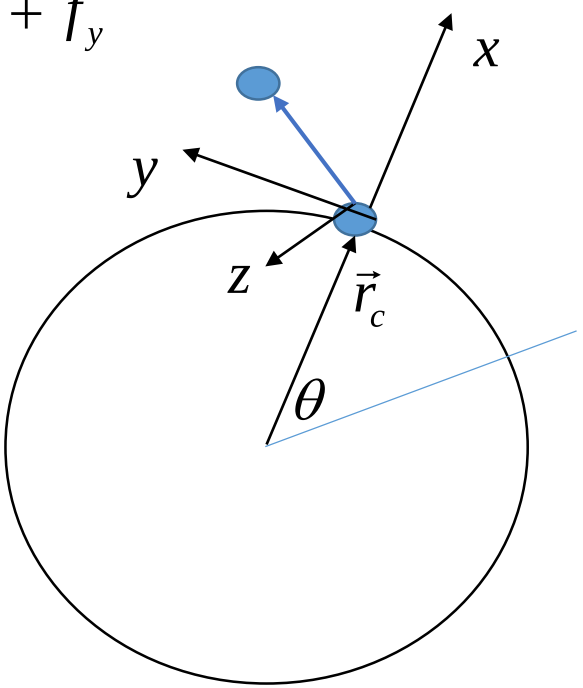
$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} + f_x$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + f_y$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + f_z$$

$$\ddot{r}_c = r_c\dot{\theta}^2 - \frac{\mu}{r_c^2}\left[1 + f_r\right]$$

$$\ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c}\left[1 + f_\theta\right]$$



HCW Equations

Pure keplerian motion

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} + \cancel{f_x}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \cancel{f_y}$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \cancel{f_z}$$

$$\ddot{r}_c = r_c\dot{\theta}^2 - \frac{\mu}{r_c^2}\left[1 + \cancel{f_r}\right]$$

$$\ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c}\left[1 + \cancel{f_\theta}\right]$$

HCW Equations

Circular reference orbit

$$\ddot{x} - 2\dot{\theta}\dot{y} - \cancel{\ddot{\theta}y} - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \cancel{\ddot{\theta}x} - \dot{\theta}^2 y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}}$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}}$$

$$\ddot{r}_c = r_c\dot{\theta}^2 - \frac{\mu}{r_c^2} = 0 \qquad \Rightarrow \qquad \dot{\theta}^2 = \frac{\mu}{r_c^3} = n^2$$

$$\ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c} = 0 \qquad \Rightarrow \qquad r_c = \text{const}$$

HCW Equations

Linearizing

$$\ddot{x} - 2n\dot{y} - n^2 x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} =$$

$$= -\frac{\mu r_c\left(1 + \dfrac{x}{r_c}\right)}{r_c^3\left[\left(1 + \dfrac{x}{r_c}\right)^2 + \left(\dfrac{y}{r_c}\right)^2 + \left(\dfrac{z}{r_c}\right)^2\right]^{3/2}} + \frac{\mu}{r_c^2}$$

HCW Equations

Linearizing

$$(1+x)^{\alpha} = 1+\alpha x$$

$$\left(1+\frac{x}{r_c}\right)\cdot\frac{1}{\left[\left(1+\frac{x}{r_c}\right)^2+\left(\frac{y}{r_c}\right)^2+\left(\frac{z}{r_c}\right)^2\right]^{3/2}}=$$

$$=\left(1+\frac{x}{r_c}\right)\cdot\frac{1}{\left[1+\frac{2x}{r_c}\right]^{3/2}}=\left(1+\frac{x}{r_c}\right)\cdot\left(1+\left(-\frac{3}{2}\right)\cdot\frac{2x}{r_c}\right)=$$

$$=1+\frac{x}{r_c}-\frac{3x}{r_c}=1-\frac{2x}{r_c}$$

HCW Equations

$$\ddot{x} - 2n\dot{y} - n^2 x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} =$$

$$= -\frac{\mu r_c \left(1 + \frac{x}{r_c}\right)}{r_c^3 \left[\left(1 + \frac{x}{r_c}\right)^2 + \left(\frac{y}{r_c}\right)^2 + \left(\frac{z}{r_c}\right)^2\right]^{3/2}} + \frac{\mu}{r_c^2}$$

$$= -\frac{\mu}{r_c^2} \cdot \left(1 - \frac{2x}{r_c}\right) + \frac{\mu}{r_c^2} = \frac{2\mu x}{r_c^3},$$

$$n^2 = \frac{\mu}{r_c^3},$$

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0$$

HCW Equations

Assumptions:

- Keplerian motion
- Circular reference orbit
- Ratio of relative distance to orbit radius is small

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0$$

$$\ddot{y} + 2n\dot{x} = 0$$

$$\ddot{z} + n^2 z = 0$$

$$x = 2(2x_0 + \dot{y}_0 / n) - (3x_0 + 2\dot{y}_0 / n)\cos nt + (\dot{x}_0 / n)\sin nt$$

$$y = (y_0 - 2\dot{x}_0 / n) - 3(2x_0 + \dot{y}_0 / n)nt + (2\dot{x}_0 / n)\cos nt + 2(3x_0 + 2\dot{y}_0 / n)\sin nt$$

$$z = z_0 \cos nt + (\dot{z}_0 / n)\sin nt$$

HCW Equations

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0$$
$$\ddot{y} + 2n\dot{x} = 0$$
$$\ddot{z} + n^2 z = 0$$

$$x = 2C - A\cos nt + B\sin nt$$
$$y = D - 3Cnt + 2B\cos nt + 2A\sin nt$$
$$z = E\cos nt + F\sin nt$$

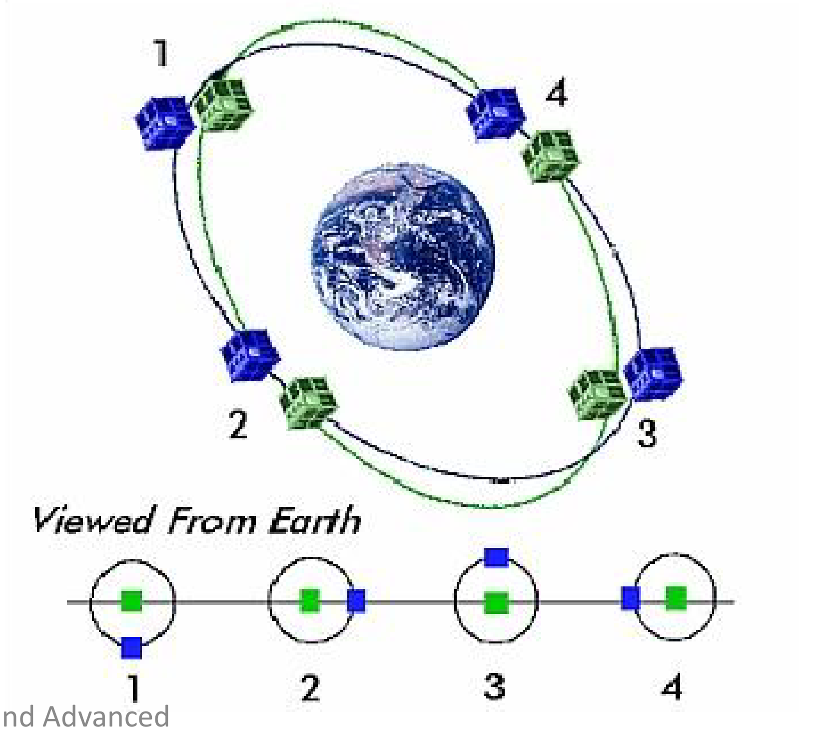
$$x = 2C + \bar{A}\sin(nt + \alpha)$$
$$y = D - 3Cnt + 2\bar{A}\cos(nt + \alpha)$$
$$z = \bar{B}\cos(nt + \beta)$$

HCW Equations

Periodic solutions

- Leader-Follower, $x = z = 0, \quad y = \text{const}$
- Circle, $\alpha = \beta,\ \overline{B} = \sqrt{3}\overline{A}, \quad x^2 + y^2 + z^2 = 4\overline{A}^2$
- Projected Circular Orbit,

$$\overline{A} = \overline{B}/2,\ \ \alpha = \beta,\ \ y^2 + z^2 = \overline{B}^2$$



More equations

- Drop the third order, not the second
- Schweighart-Sedwick equations include J2
- Schaub and Alfriend J2-invariant relative orbits
- Fasano and D’Erico model for large formations
- Elliptic reference orbits: Tshauner-Hempel and Lawden equations
- many more…

Missions

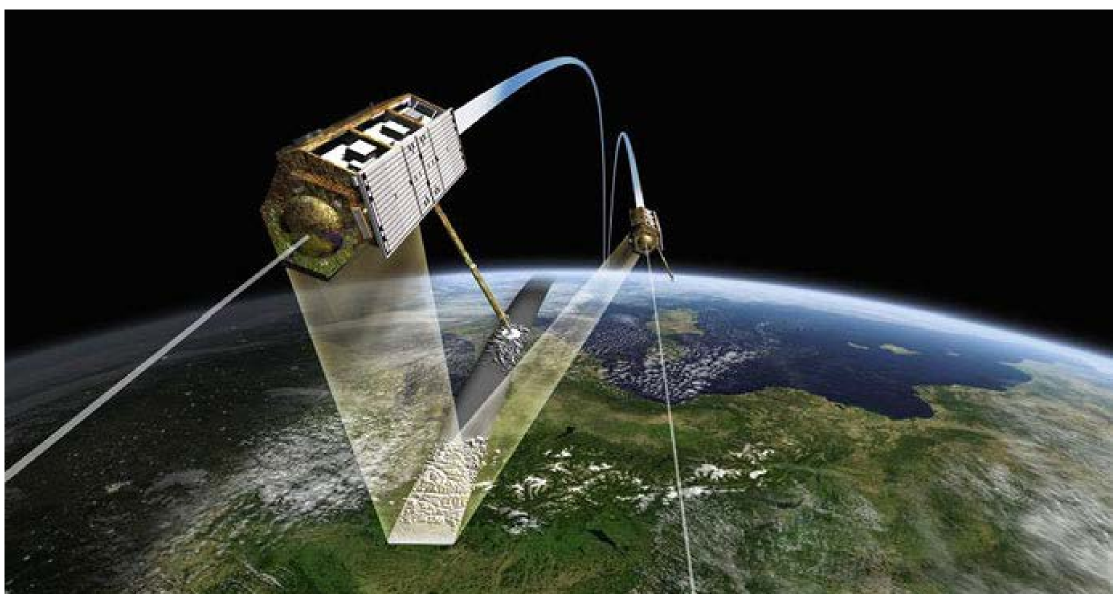
- 1965 Gemini 6A and Gemini 7
  - Stayed close one to another for app. 5 h
- 1997 autonomous FF mission ETS-7
  - Rendezvous docking
- 2014 CanX-4&5
  - Submeter relative tracker accuracy

TanDEM-X

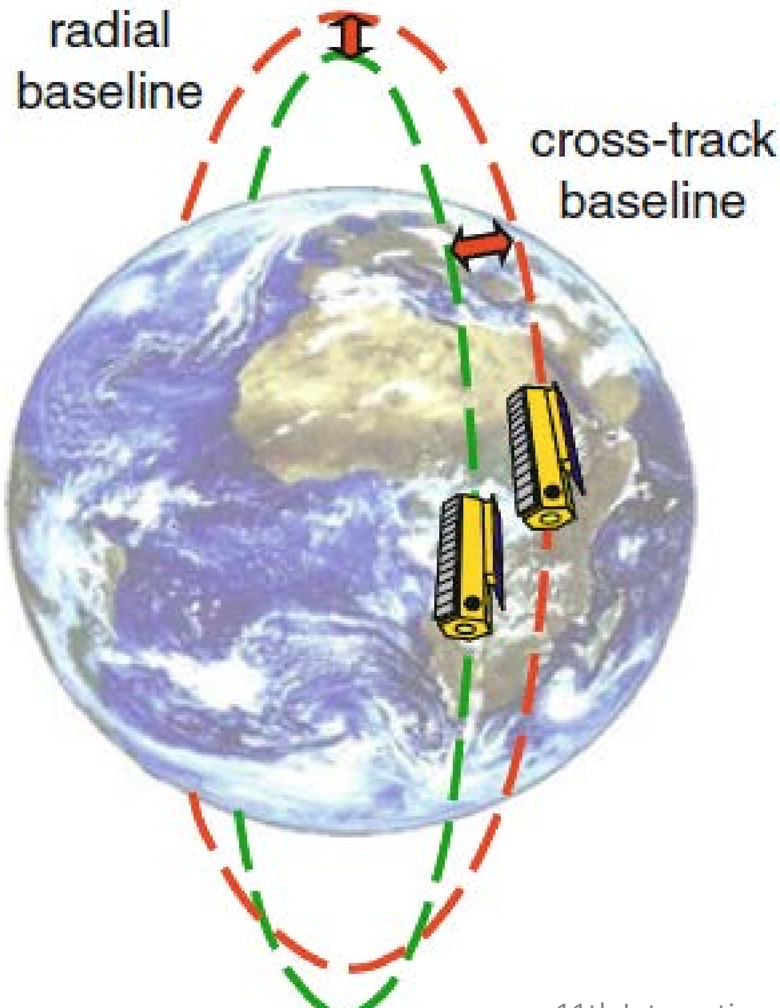
- Earth observation and mapping
- 2 sats, each with a synthetic aperture radar –

12 m horizontal, 2 m vertical resolution

- Launched in June 2010

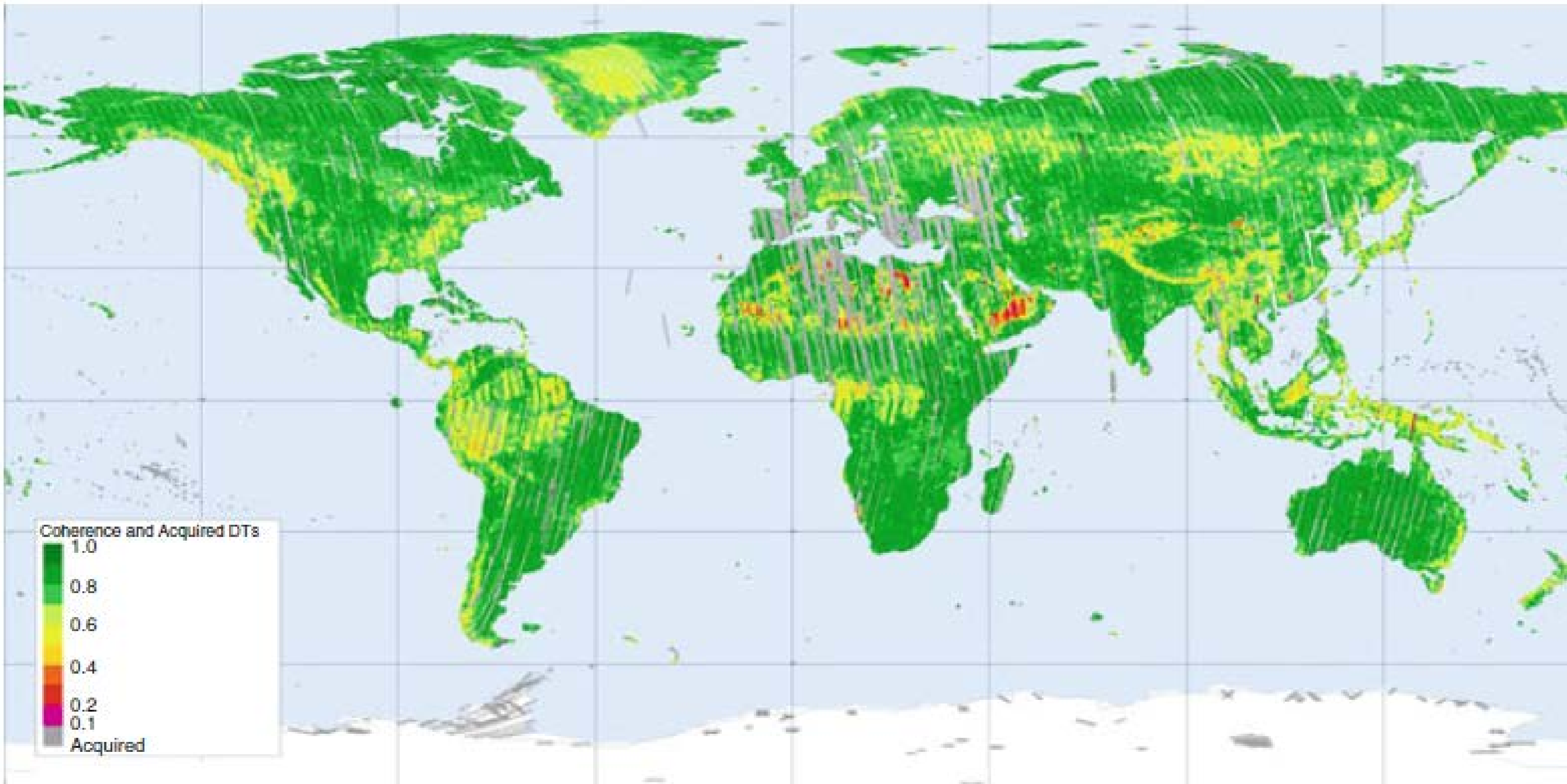


TanDEM-X



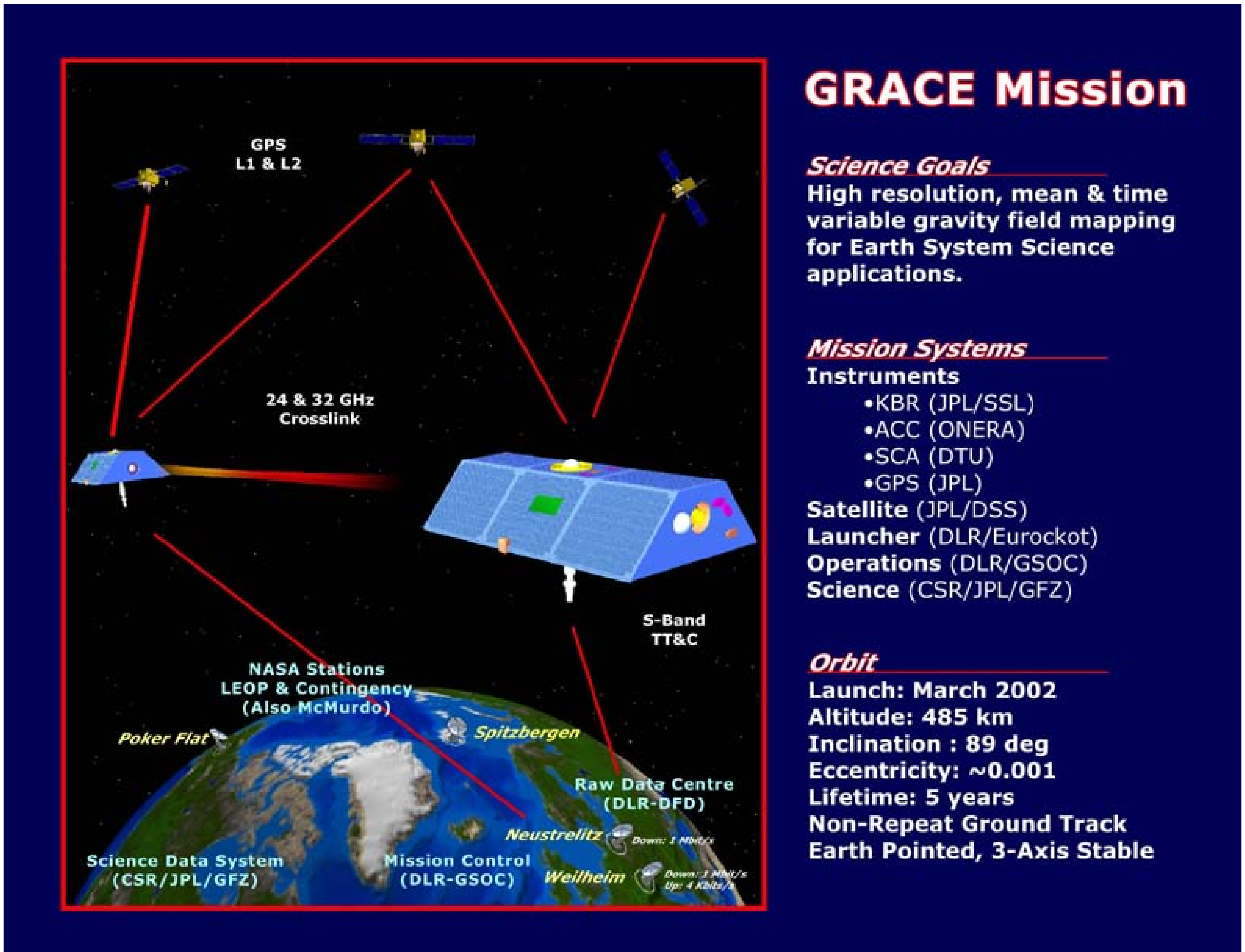
- Helix satellite formation for TanDEM-X
- Sun-synchronous orbit
- Modified HCW equations used for modelling

TanDEM-X

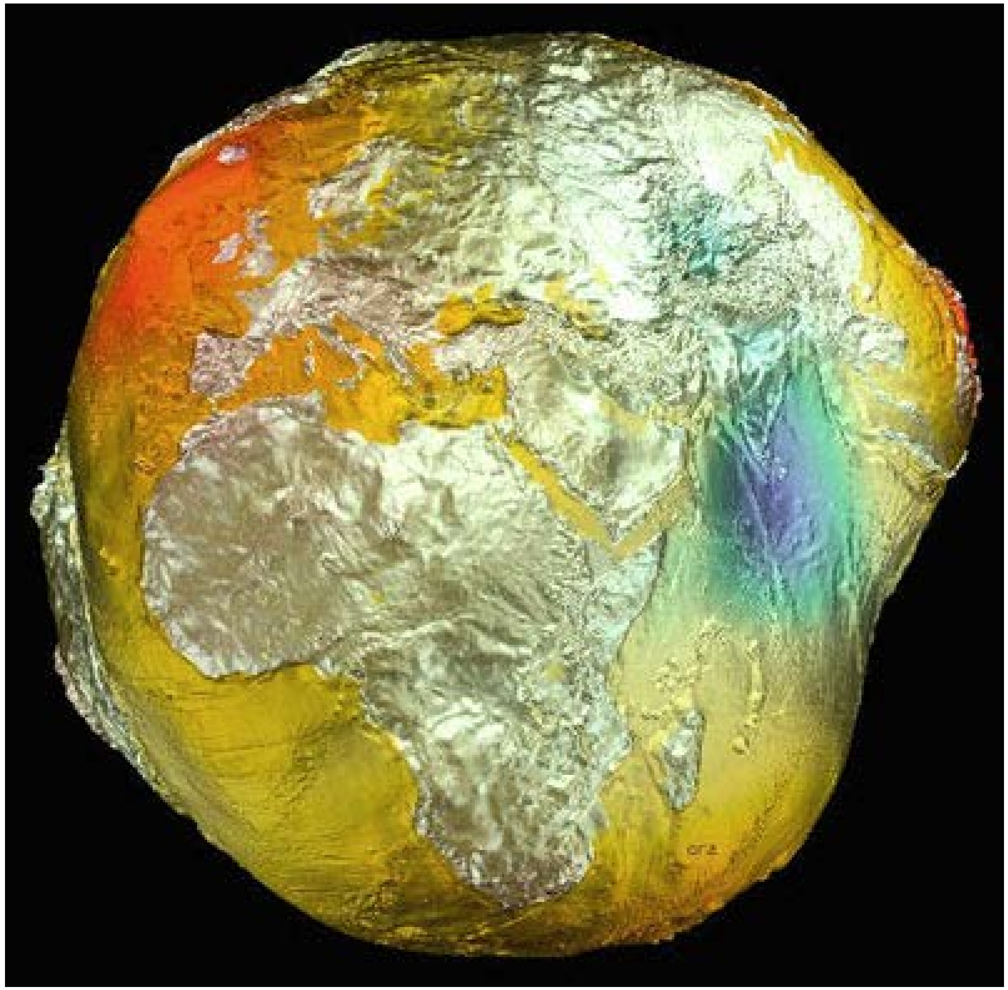


Global coherence map of TanDEM-X data takes acquired between December 2010 and December 2011

GRACE - Gravity Recovery And Climate Experiment



GRACE



The Potsdam Gravity potato

Gravity missions

- Lageos 1 (1976) – J2, J3 coefficients
- Lageos 2 (1992) – more coefficients
- CHAMP (2000) – mapping gravity
- GRACE (2002) – detecting the temporal variations of the Earth gravity field
- GOCE (2009) – map of the static Earth gravity field using the technique of the gradiometry
- NGGM – Next Generation Gravity Mission

PRISMA

in-orbit test-bed for guidance, navigation, and control algorithms, relative navigation sensors, and propulsion systems