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Overview of control approaches and algorithms for distributed space systems

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Content

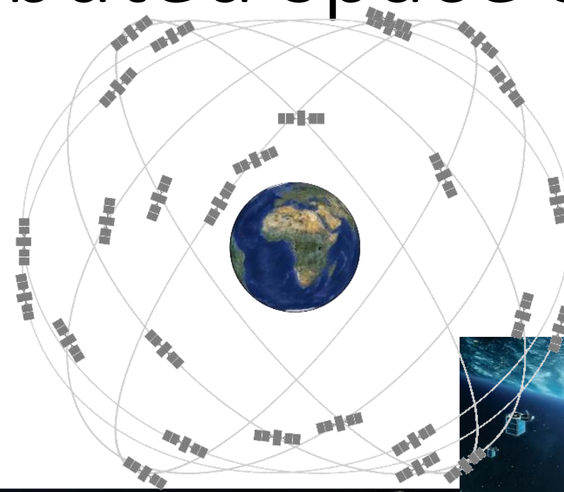
- Introduction
- Distributed space systems control approaches
- Fuelless satellite formation flying control and algorithms
- Conclusion

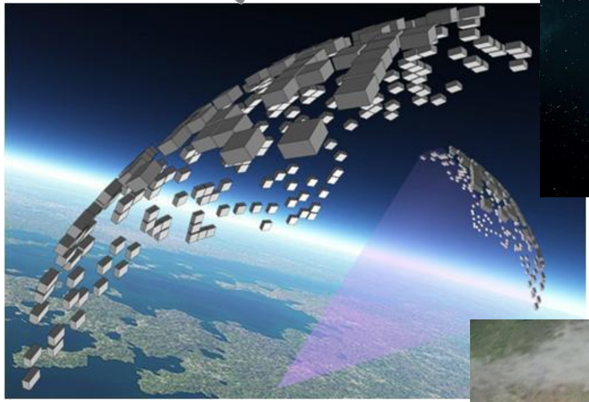
What is distributed system?

- A space system consisting of multiple space elements that can communicate, coordinate and interact in order to achieve a common goal.
  - Concurrency of elements
  - Tolerance for failure of individual systems
  - Scalability and flexibility in design and deployment of system

Definitions for distributed space systems

- Constellation: similar trajectories without control for relative position; coordination from a control center.
- Formation: closed-loop control on-board in order to preserve topology in the group and to control relative distances
- Cluster: distributed heterogeneous system of satellites to achieve in cooperation a joint objective.
- Swarm: a group of similar (homogenous) vehicles cooperating to achieve a joint goal without fixed positions; Each member determines and controls relative positions in relations to others.

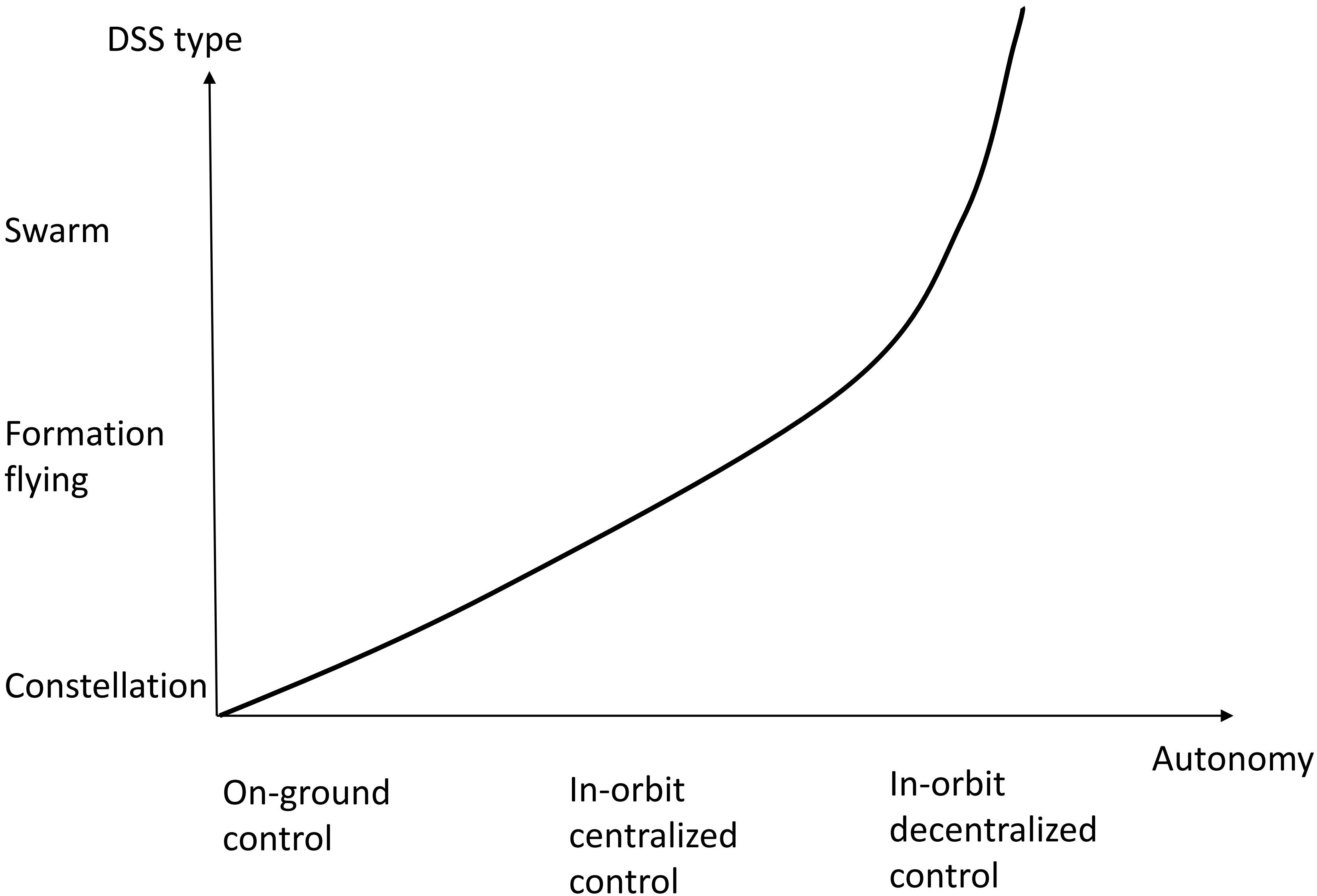




Main parameters of distributed SS

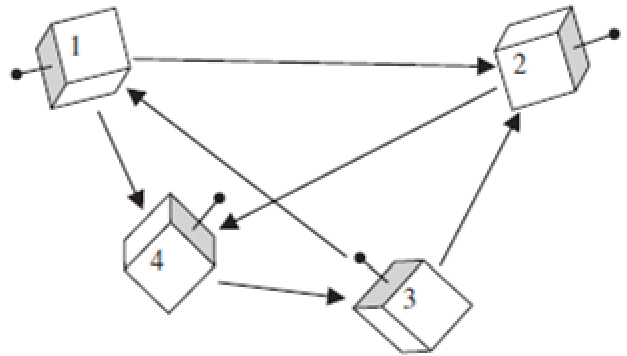
- A number of satellites
- A degree of autonomy
- Communication links between satellites
- Relative trajectory types

Autonomy in relative control



Communication

- The communication is information exchange or just measuring of relative pose
- There could be directed or mutual communication
- If det(A)≠0, the formation is decentralized
- If det(A)=0, the formation is of leader-follower type, communication is cycled



$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Natural distributed systems

School of fishes

Flock of birds

Swarm of bees

Herd of animals

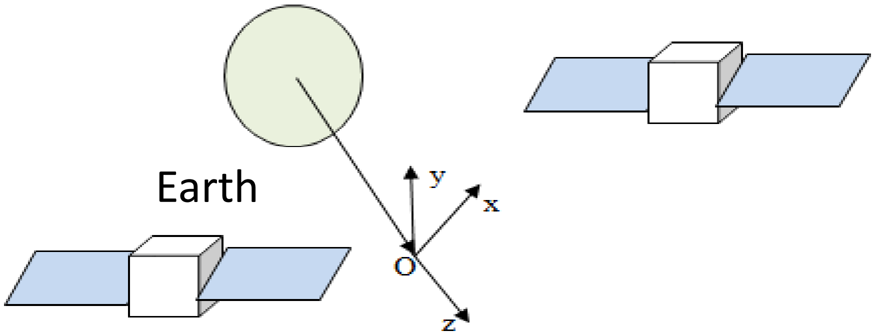
Satellite formation flying features

- A small number of satellites
- Centralized control:
  - Mother-daughter relationship: mother knows the best for her children and command them
  - Leader-follower relationship: leader moves everywhere it wants, the followers pursue it
- Communication with all the group members
- Motion along predefined trajectories

Equations of relative motion: linear model, near circular orbit

On the first stage of control algorithms investigation Clohessy-Wiltshire model is used:

$$\begin{cases} \ddot{x} + 2\omega\dot{z} = 0 \\ \ddot{y} + \omega^2 y = 0 \\ \ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0 \end{cases}$$



Scheme of motion

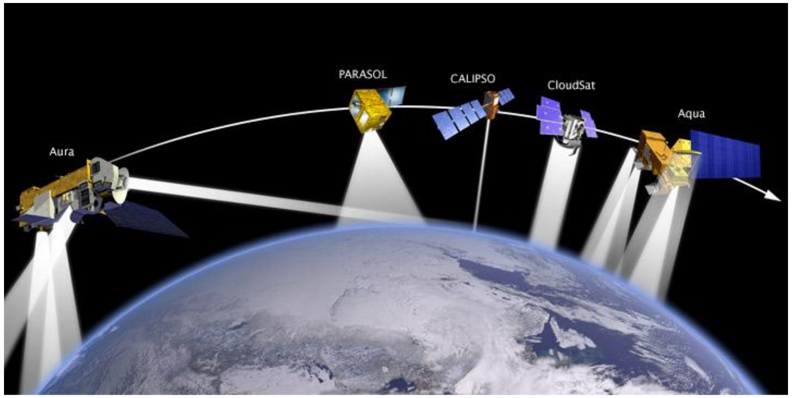
Solution is :

$$\begin{cases} x = -3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t + C_4 \\ y = C_5\sin\omega t + C_6\cos\omega t \\ z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t \end{cases}$$

$-3C_1\omega t$ - Relative drift

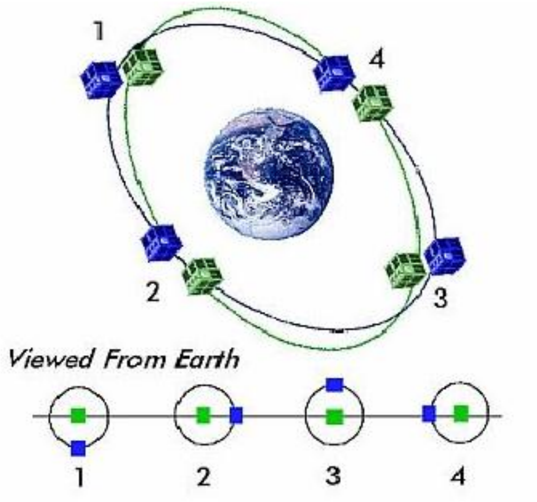
Formation flying specific relative trajectories

- Train formation
- Relative circular orbit
- Projected circular orbit
- Docking trajectories



A-train formation flying

KIKU-7 mission



CanSat4&5 mission

Satellite swarm features

- A large number of satellites
- Decentralized control
- Communication with limited number of group member
- Motion along occasional trajectories:
    - Random but bounded relative trajectories

Swarm control objectives

- Collision avoidance
  - When the relative distance d_{ij} is less then fixed threshold R_{av} the collision maneuver is performed
- Alignment
  - The satellites tent to align to its neighbors $R_{av} < d_{ij} < R_{al}$
- Attraction
  - Each satellite try to be closer to far members $R_{al} < d_{ij} < R_{att}$

Artificial potential control approach

- Collision avoidance

$$U_{ij}^{rep} = -C_{rep} e^{-\frac{d_{ij}}{R_{rep}}}$$

- Alignment

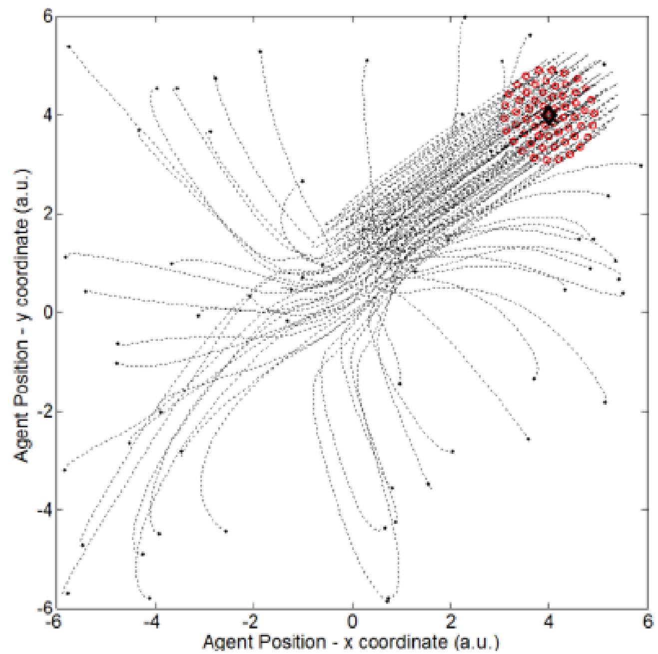
$$\mathbf{d}_i = \sum_{j,\, j \neq i} C_{al} \left(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij} \right) e^{-\frac{d_{ij}}{R_{al}}} \mathbf{r}_{ij}$$

- Attraction

$$U_{ij}^{at} = -C_{at} e^{-\frac{d_{ij}}{R_{at}}}$$

Equations of motion

$$m_i \mathbf{r}_i = -\nabla_i U(\mathbf{r}_i) + \mathbf{d}_i$$



M. Sabatini, G. B. Palmerini and P. Gasbarri. Control Laws for Defective Swarming Systems// Advances in the Astronautical Sciences, Second IAA DyCoss'2014, V. 153. p. 132-153.

Linear quadratic regulator application

- Collision avoidance

$$\mathbf{u}^{rep} = \sum_{j,\, j\neq i} \mathbf{u}_{ij}^{rep}, when\ d_{ij} < R_{rep}$$

- Alignment

$$\mathbf{u}_i^{al} = \sum_{j,\, j\neq i} \mathbf{u}_{ij}^{al}, when\ R_{al} < d_{ij} < R_{at},$$

$$\mathbf{x}_i^d = [-\frac{\dot{y}}{2\omega_0}\ 0\ 0\ 0\ 0\ 0]$$

- Attraction

$$\mathbf{u}^{rep} = \sum_{j,\, j\neq i} \mathbf{u}_{ij}^{rep}, when\ d_{ij} < R_{rep}$$

Equations of motion

$$\dot{\mathbf{x}}_i = \mathrm{A}\mathbf{x}_i + \mathrm{B}\mathbf{u}_i,$$

Feedback control is

$$\mathbf{u}_i = -\mathrm{R}^{-1} B^T \mathrm{P}\mathbf{e}_i,$$

where $\mathbf{e}_i = \mathbf{x}_i - \mathbf{x}_i^d$,

matrix P is the solution

of Riccati equation

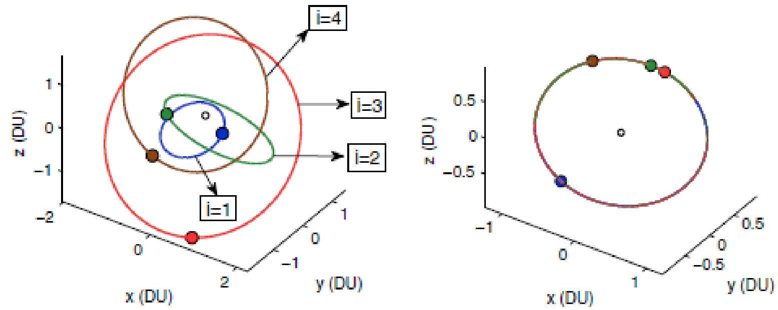
$$\mathrm{Q} - \mathrm{PBR}^{-1}\mathrm{B}^T\mathrm{P} + \mathrm{PA} + \mathrm{A}^T\mathrm{P} = 0.$$

M. Sabatini, G. B. Palmerini. Collective control of spacecraft swarms for space exploration// Celest Mech Dyn Astr (2009) 105:229–244

Swarm consensus control

- Convergence to a common orbital plane
  - The error function:

$$\xi_i = \sum_{j=1}^{n} a_{ij}\left(1-\mathbf{n}_i^T\mathbf{n}_j\right)$$

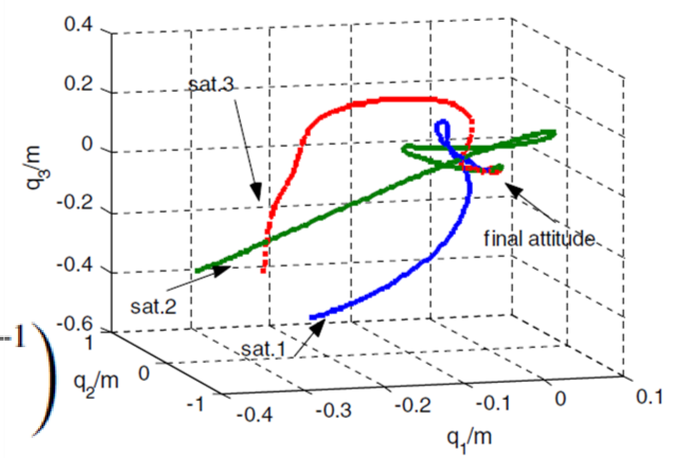


a) Initial orbits
b) Final converged orbits

Thakur D., Hernandez S., Akella M.R. Spacecraft swarm finite-thrust cooperative control for common orbit convergence // J. Guid. Control. Dyn. 2015. Vol. 38, № 3. P. 478–487.

- Attitude synchronization
  - Non-linear control law:

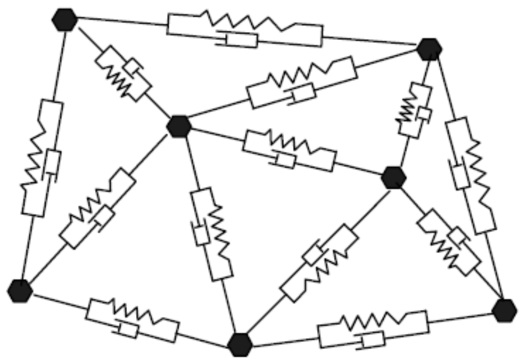
$$\boldsymbol{\tau}_i = \boldsymbol{\omega}_i^{\times}\boldsymbol{J}_i\boldsymbol{\omega}_i + \boldsymbol{J}_i\Bigg(-\boldsymbol{Q}_i^{-1}\dot{\boldsymbol{Q}}_i\boldsymbol{\omega}_i - \boldsymbol{Q}_i^{-1}k_1 \times \Bigg\{(\boldsymbol{Q}_i\boldsymbol{\omega}_i)^p + k_2^p\Bigg[\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_i-\boldsymbol{q}_j) + b_i(\boldsymbol{q}_i-\boldsymbol{q}_d)\Bigg]\Bigg\}^{2/p-1}\Bigg)$$



Zhou J., Hu Q., Friswell M.I. Decentralized Finite Time Attitude Synchronization Control of Satellite Formation Flying // J. Guid. Control. Dyn. 2013. Vol. 36, № 1. P. 185–195.

Virtual structure control approach

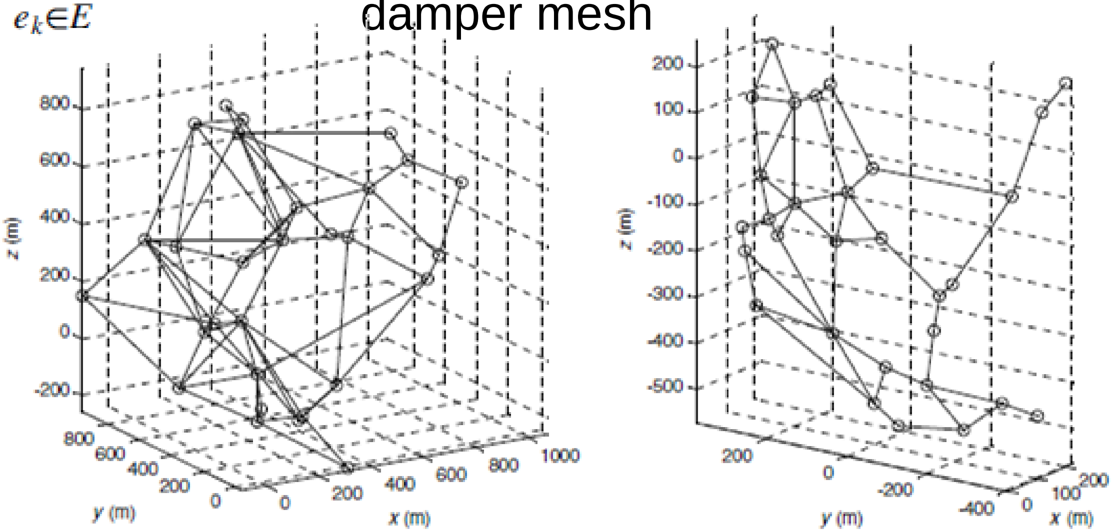
- Imitation the satellite system by a solid structure model
- Control law



$$\boldsymbol{u}_i = -\sum_{e_k \in E} k_s d_{ik}(\boldsymbol{p}_k - \boldsymbol{p}_k^d) - \sum_{e_k \in E} k_d d_{ik} \dot{\boldsymbol{p}}_k$$

Point masses connected by a spring-damper mesh

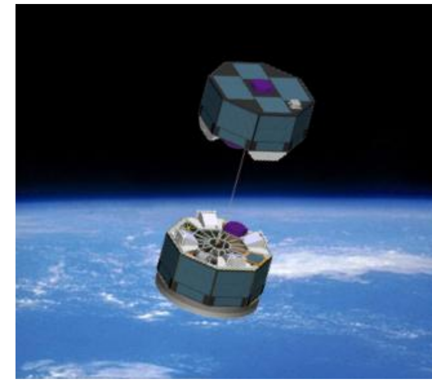
Chen Q. et al. Virtual Spring-Damper Mesh-Based Formation Control for Spacecraft Swarms in Potential Fields // J. Guid. Control. Dyn. 2015. Vol. 38, № 3. P. 539–546.

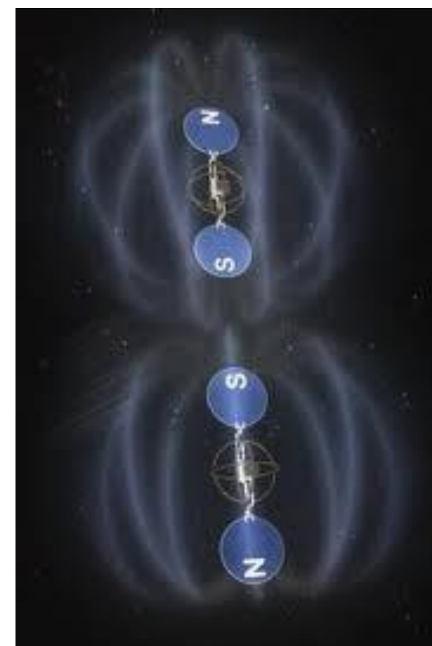


a) Initial time (t = 0 s) b) Steady state (t = 8000 s)

Fuelless FF Control Concepts

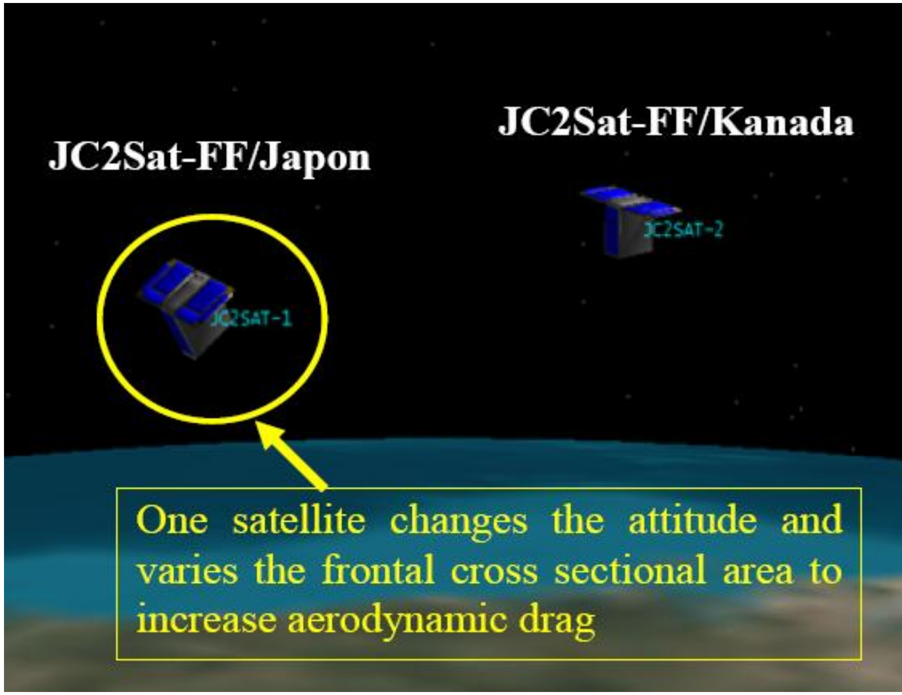
- Tethered systems
- Aerodynamic drag
- Electro-magnetic interaction
- Solar pressure
- Momentum exchange





Aerodynamic drug based control

- Features:
  - *Low Earth Orbit*
  - *Satellites with variable cross section area*
- Shortcomings:
  - *Short lifetime*
  - *Reaction wheel saturation during attitude control*



JC2Sat Mission

LQR-based control algorithm

- Aerodynamic drug force

$$\mathbf{f}_i = -\frac{1}{m}\rho V^2 S\{(1-\varepsilon)(\mathbf{e}_V,\mathbf{n}_i)\mathbf{e}_V + 2\varepsilon(\mathbf{e}_V,\mathbf{n}_i)^2\mathbf{n}_i + (1-\varepsilon)\frac{\upsilon}{V}(\mathbf{e}_V,\mathbf{n}_i)\mathbf{n}_i\}^*,$$

$$\mathbf{n} = (\cos\alpha\cos\beta;\sin\beta;\sin\alpha\cos\beta).$$

- Linear-quadratic regulator

$$\dot{\mathbf{x}} = \mathrm{A}\mathbf{x} + \mathbf{b}u,$$

Minimising cost function

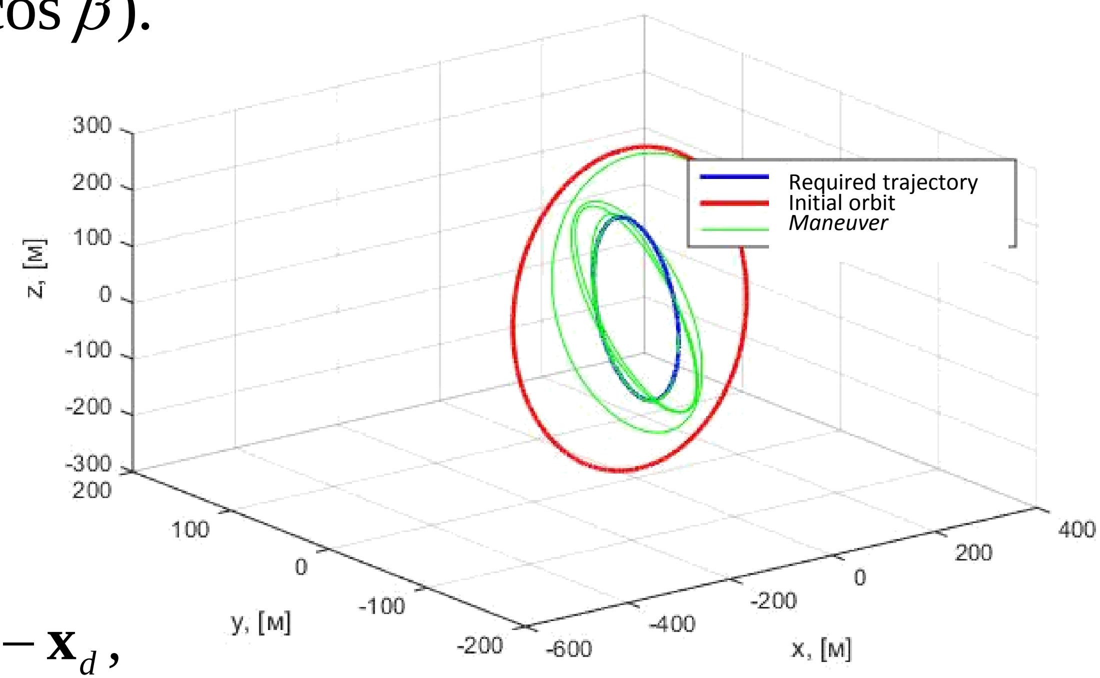
$$J = \int_{\tau}^{\infty}(\mathbf{e}^T \mathrm{Q}\mathbf{e} + \mathbf{u}^T \mathrm{R}\mathbf{u})dt,$$

Feedback control is

$$\mathbf{u} = -\mathrm{R}^{-1}\mathbf{b}^T \mathrm{P}\mathbf{e}, \quad \text{where } \mathbf{e} = \mathbf{x} - \mathbf{x}_d,$$

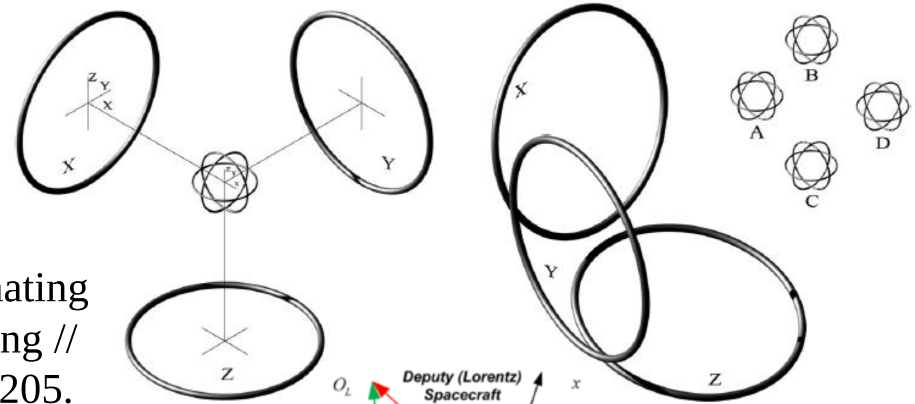
matrix P is the solution of Riccati equation

$$\mathrm{Q} - \mathrm{PBR}^{-1}\mathrm{B}^T\mathrm{P} + \mathrm{PA} + \mathrm{A}^T\mathrm{P} = 0.$$



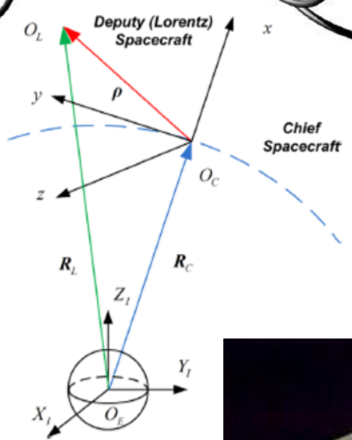
Relative trajectories during the maneuver

Electro-magnetic interaction based control



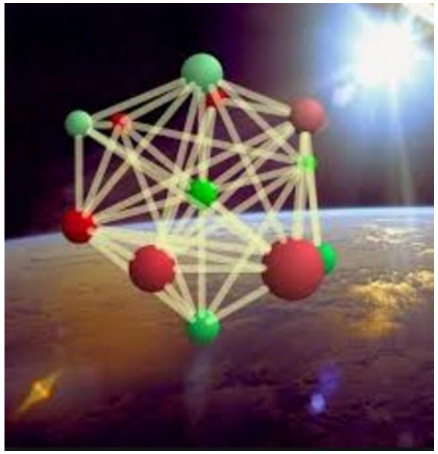
- Magnetic interaction

Youngquist R.C., Nurge M.A., Starr S.O. Alternating magnetic field forces for satellite formation flying // Acta Astronaut. Elsevier, 2013. Vol. 84. P. 197–205.



- Lorenz force of charged satellite

Peck M.A. et al. Spacecrat Formation Flying Using Lorentz Forces // J. Br. Interplanet. Soc. 2007. Vol. 60. P. 263–267.

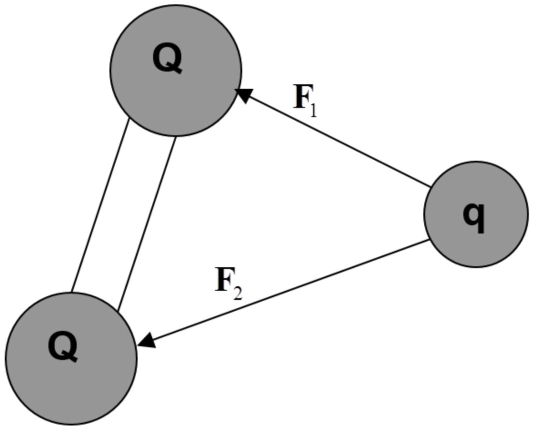


- Coulomb force interaction

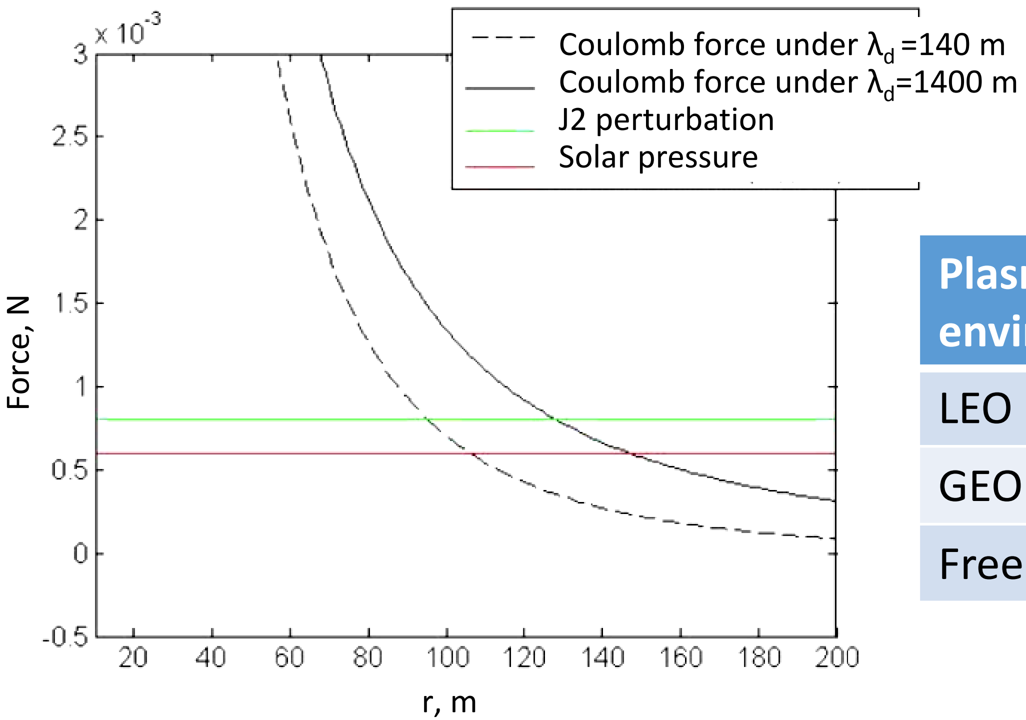
Schaub H. et al. Challenges and Prospects of Coulomb Spacecraft Formation Control of the Astronautical Sciences // J. Astronaut. Sci. 2004. Vol. 52. P. 169–193.

Coulomb force based control algorithm

- Features:
  - *The charging device is required*
  - *Small relative distances*
  - *Charges are eliminating by plasma*



$$f_{12} = k_c \frac{r_{12}}{r_{12}^3} q_1 q_2 e^{-\frac{r_{12}}{\lambda_d}}$$



Plasma environment	$\lambda_{d\ min}$, m	$\lambda_{d\ max}$, m
LEO	0.02	0.4
GEO	142	1496
Free space	7.4	24

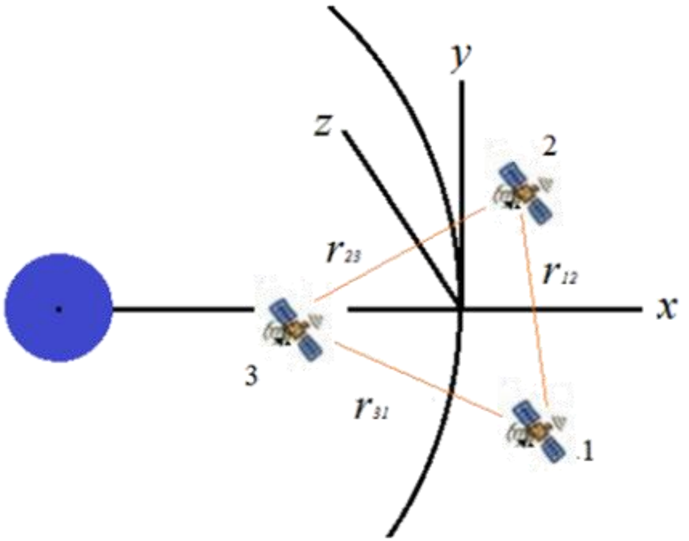
Equations of motion for three satellites

In the orbital reference frame

$$\ddot{\boldsymbol{r}}_1 + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_1 + 3\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r}_1 = \frac{1}{m_1}\frac{\boldsymbol{r}_{12}}{r_{12}}\cdot\frac{\alpha_3}{r_{12}^2} - \frac{1}{m_1}\frac{\boldsymbol{r}_{31}}{r_{31}}\cdot\frac{\alpha_2}{r_{31}^2}$$

$$\ddot{\boldsymbol{r}}_2 + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_2 + 3\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r}_2 = -\frac{1}{m_2}\frac{\boldsymbol{r}_{12}}{r_{12}}\cdot\frac{\alpha_3}{r_{12}^2} + \frac{1}{m_2}\frac{\boldsymbol{r}_{23}}{r_{23}}\cdot\frac{\alpha_1}{r_{23}^2}$$

$$\ddot{\boldsymbol{r}}_3 + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_3 + 3\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r}_3 = \frac{1}{m_3}\frac{\boldsymbol{r}_{31}}{r_{31}}\cdot\frac{\alpha_2}{r_{31}^2} - \frac{1}{m_3}\frac{\boldsymbol{r}_{23}}{r_{23}}\cdot\frac{\alpha_1}{r_{23}^2}$$



where

$$\alpha_i(\boldsymbol{r}_{jk},\dot{\boldsymbol{r}}_{jk}) = k_c q_j(\boldsymbol{r}_{jk},\dot{\boldsymbol{r}}_{jk}) q_k(\boldsymbol{r}_{jk},\dot{\boldsymbol{r}}_{jk}),$$

Sliding-mode control

- Lyapunov-candidate function

$$V=\frac{1}{2}\dot{r}_{12}^2+\frac{1}{2}\dot{r}_{23}^2+\frac{1}{2}\dot{r}_{31}^2+\frac{1}{2}k_1(r_{12}-a_1)^2+\frac{1}{2}k_2(r_{23}-a_2)^2+\frac{1}{2}k_3(r_{31}-a_3)^2,$$

- Its derivative

$$\dot{V}=\dot{r}_{12}(\ddot{r}_{12}+k_1(r_{12}-a_1))+\dot{r}_{23}(\ddot{r}_{23}+k_2(r_{23}-a_2))+\dot{r}_{31}(\ddot{r}_{31}+k_3(r_{31}-a_3)).$$

- For negative sign should be:

$$\ddot{r}_{12}(\alpha_3)+g_1\dot{r}_{12}+k_1(r_{12}-a_1)=0,$$
$$\ddot{r}_{23}(\alpha_1)+g_2\dot{r}_{23}+k_2(r_{23}-a_2)=0,$$
$$\ddot{r}_{31}(\alpha_2)+g_3\dot{r}_{31}+k_3(r_{31}-a_3)=0.$$

Control algorithm

- The solution of equations is:

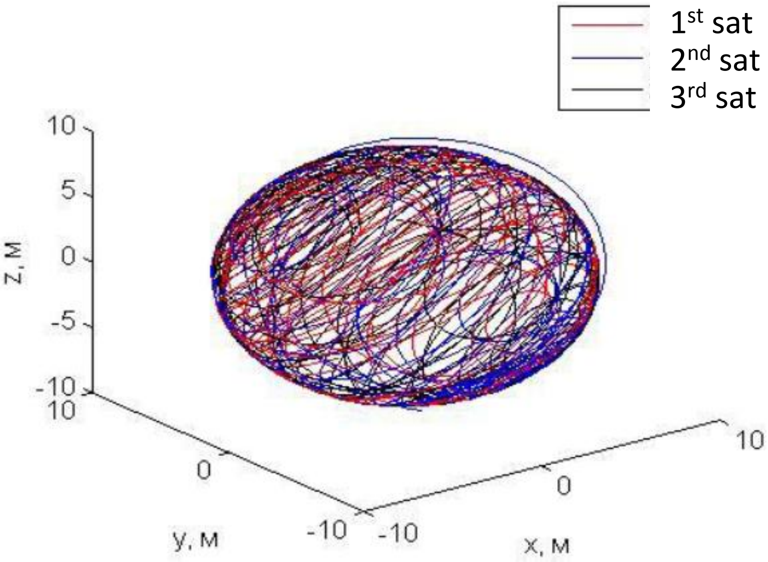
$$\boldsymbol{\alpha} = \boldsymbol{A}^{-1} \cdot \boldsymbol{b},$$

- It could not be always performed by the charges.
- So, trying to minimize the function

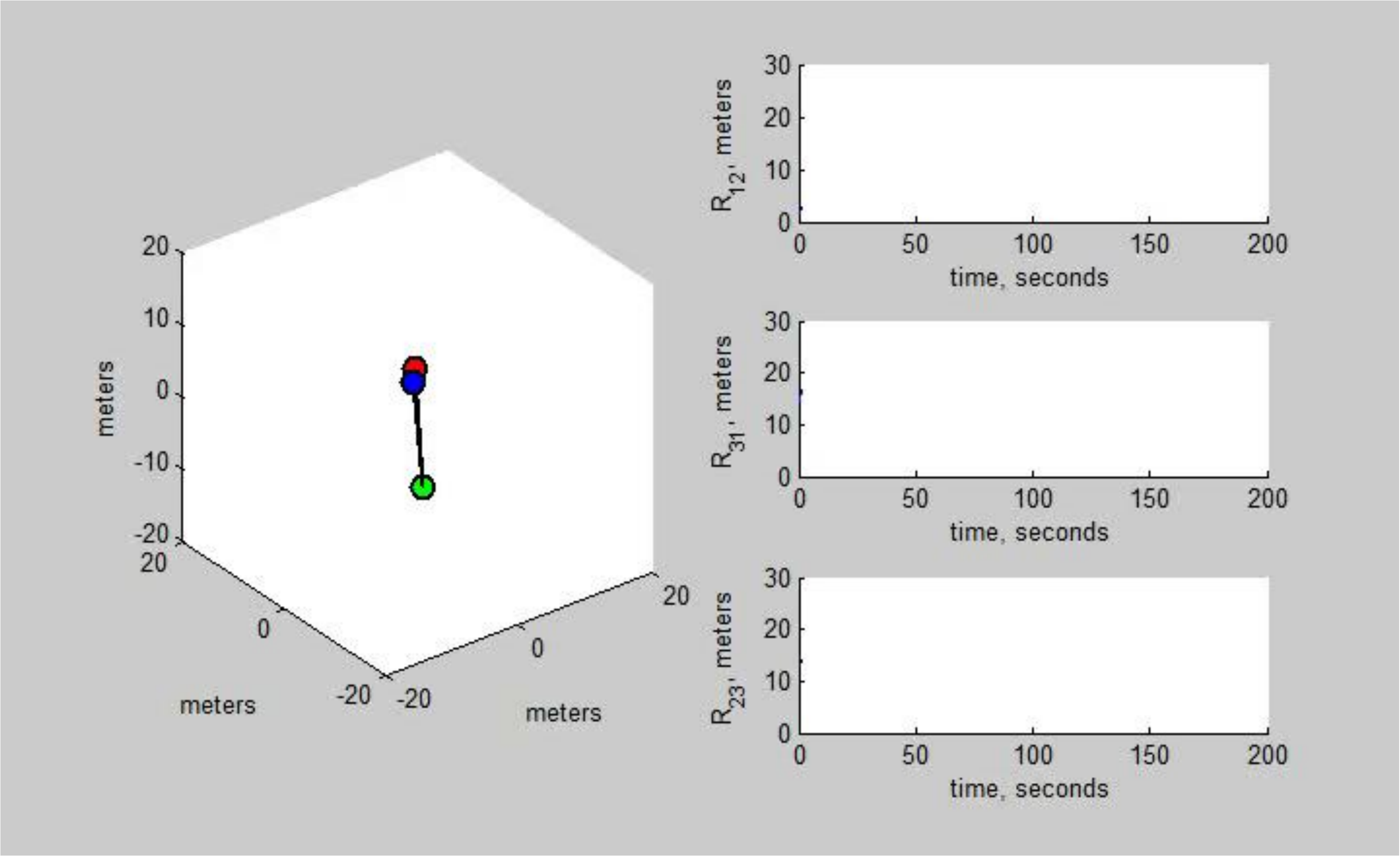
$$\Phi = (q_1 q_2 - \alpha_3)^2 + (q_2 q_3 - \alpha_1)^2 + (q_1 q_3 - \alpha_2)^2 \rightarrow \min$$

- Get four solutions:

$$\begin{pmatrix} 0 \\ \pm\sqrt{-\dfrac{\alpha_1\alpha_2}{\alpha_3}} \\ \mp\sqrt{-\dfrac{\alpha_3\alpha_1}{\alpha_2}} \end{pmatrix}, \begin{pmatrix} \pm\sqrt{-\dfrac{\alpha_1\alpha_2}{\alpha_3}} \\ 0 \\ \mp\sqrt{-\dfrac{\alpha_3\alpha_2}{\alpha_1}} \end{pmatrix}, \begin{pmatrix} \pm\sqrt{-\dfrac{\alpha_3\alpha_1}{\alpha_2}} \\ \mp\sqrt{-\dfrac{\alpha_3\alpha_2}{\alpha_1}} \\ 0 \end{pmatrix}, \begin{pmatrix} \pm\sqrt{\dfrac{\alpha_3\alpha_2}{\alpha_1}} \\ \pm\sqrt{\dfrac{\alpha_3\alpha_1}{\alpha_2}} \\ \pm\sqrt{\dfrac{\alpha_1\alpha_2}{\alpha_3}} \end{pmatrix}$$



Algorithm simulation

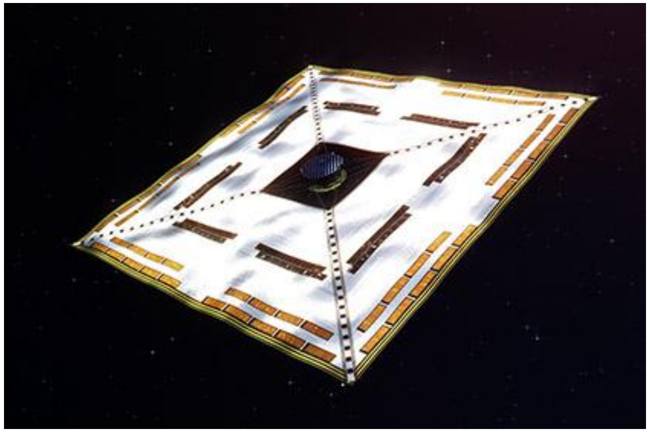


Solar radiation pressure based control

- Solar sail with fixed orientation

Smirnov G.V., Ovchinnikov M.Y., Guerman A.D. Use of solar radiation pressure to maintain a spatial satellite formation // Acta Astronaut. 2007. Vol. 61, № 7-8. P. 724–728.

- Solar sail with variable reflection

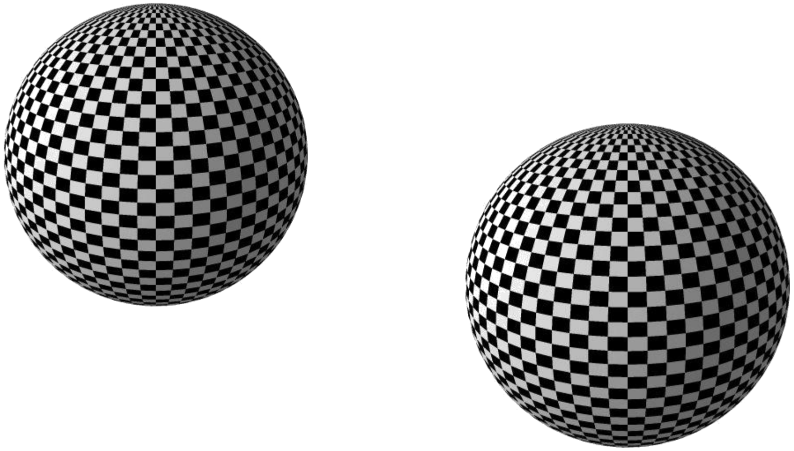


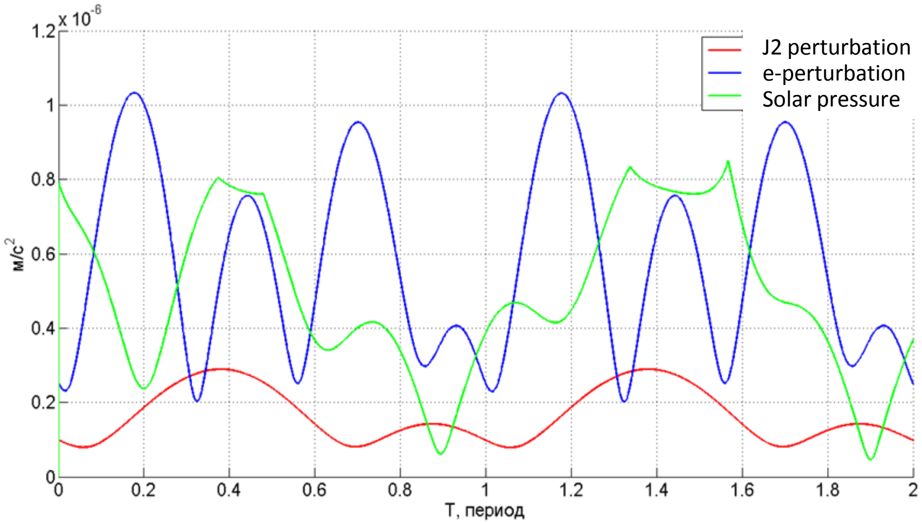
Mori O. et al. First Solar Power Sail Demonstration by IKAROS // Trans. Japan Soc. Aeronaut. Sp. Sci. Aerosp. Technol. Japan. 2010. Vol. 8, № ists27. P. To_4_25 – To_4_31.

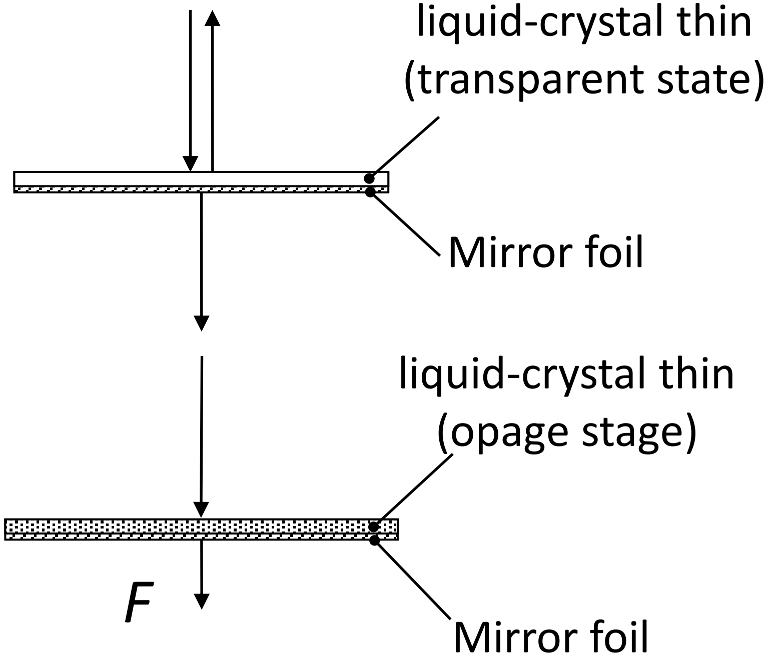
Solar radiation pressure based control

We consider:

- *Spherical satellites*
- *Variable reflection on “pixel” surface*
- *Nearcircular orbits*







PD-controller-based control algorithm

- Motion equations:

$$\begin{cases} \dot{\boldsymbol{\rho}} = \mathbf{v}, \\ \dot{\mathbf{v}} = \mathbf{f}(\boldsymbol{\rho}, \mathbf{v}) + \mathbf{u}. \end{cases}$$

- PD-regulator:

$$\mathbf{u} = -k_\rho(\boldsymbol{\rho} - \boldsymbol{\rho}_{ref}) - k_v(\mathbf{v} - \mathbf{v}_{ref}) + \dot{\mathbf{v}}_{ref} - \mathbf{f}$$

where

$$k_\rho,\ k_v = \text{const} > 0, \text{ choosen that } k_v = \frac{k_\rho^2}{4};$$

Solar pressure model

- The solar pressure force:

$$\mathbf{F} = -P_c \left(\int_{S^+} (1-k)(\mathbf{s},\mathbf{n})\mathbf{s}\,dS + 2\int_{S^+} k\mathbf{n}(\mathbf{s},\mathbf{n})^2\,dS \right)$$

- The reflection function:

$$k(\varphi,\theta) = g(\varphi)\cdot h(\theta),$$

where

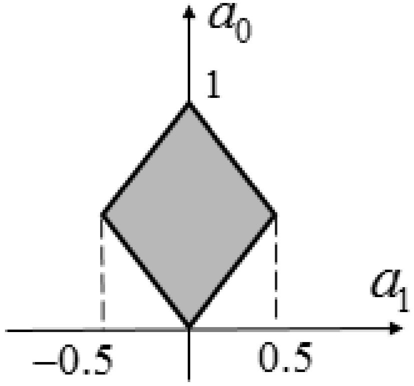
$$g(\varphi) = a_1\cos(\varphi+\alpha)+a_0,\ h(\theta) = \frac{1}{2}+\frac{1}{2}\sin 4\theta.$$

$a_0, a_1, \alpha -$ Variable control parameters

Restrictions are: $0 \le k \le 1.$

$$0 < \left(a_1\cos(\varphi+\alpha)+a_0\right)\left(\frac{1}{2}+\frac{1}{2}\sin 4\theta\right) \le 1$$

$$0 \le a_1\cos(\varphi+\alpha)+a_0 \le 1$$

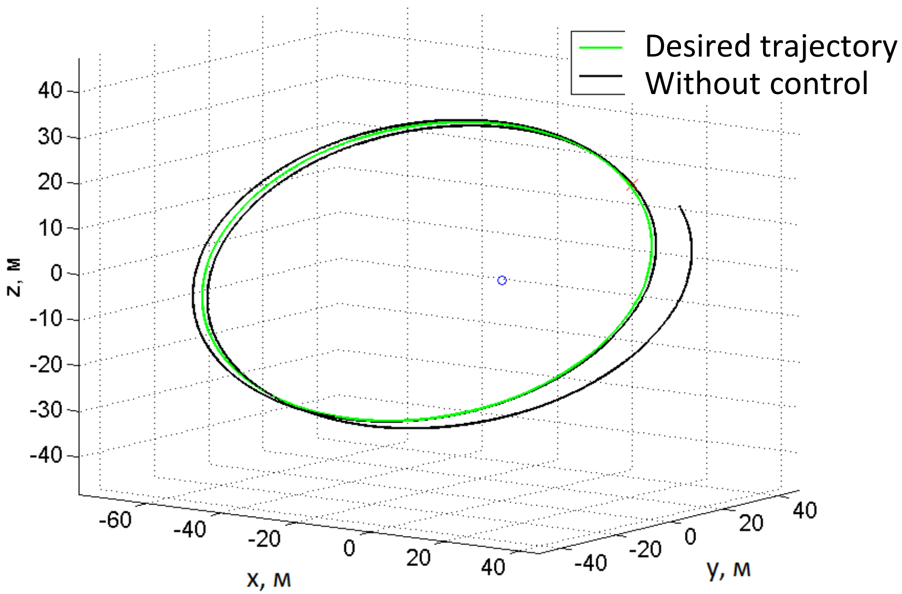


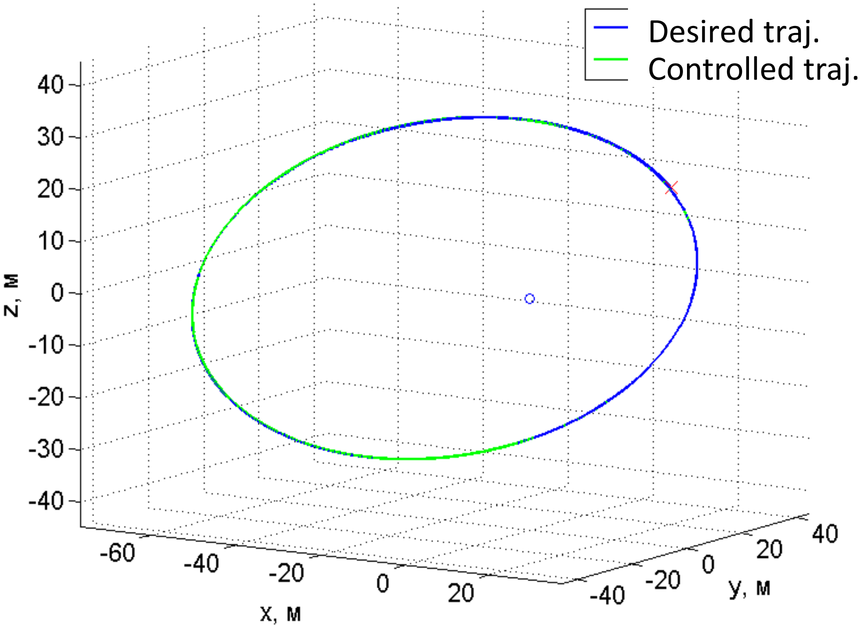
Numerical example

$$F_1 = -\frac{\pi^2}{32} R_d^2 P_c a_1 \cos\alpha, \qquad \alpha = \mathrm{arctg}\left(-\frac{F_2}{F_1}\right),$$

$$F_2 = \frac{\pi^2}{32} R_d^2 P_c a_1 \sin\alpha, \qquad \Rightarrow \qquad a_1 = -\frac{32}{\pi^2 P_c R_d^2} \cdot \frac{F_1}{\cos\alpha},$$

$$F_3 = -\frac{P_c \pi^2 R_d^2}{4} a_0 + P_c \pi (R_p^2 - R_d^2). \qquad a_0 = -\frac{4}{\pi^2 P_c R_d^2} F_3 + \frac{4}{\pi R_d^2}\left(R_p^2 - R_d^2\right).$$





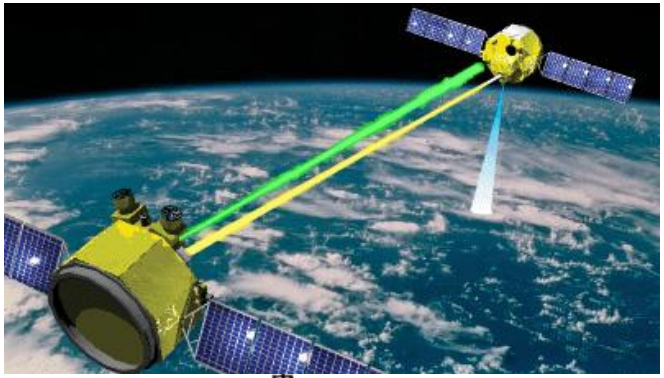
The momentum exchange-based control

- The momentum from lasers for repulsive force

Y. K. Bae. A contamination-free ultrahigh precision formation flying method for micro-, nano-, and pico-satellites with nanometer accuracy. In Space Technology and Applications International Forum-Staif 2006, volume 813, pages 1213–1223, 2006.

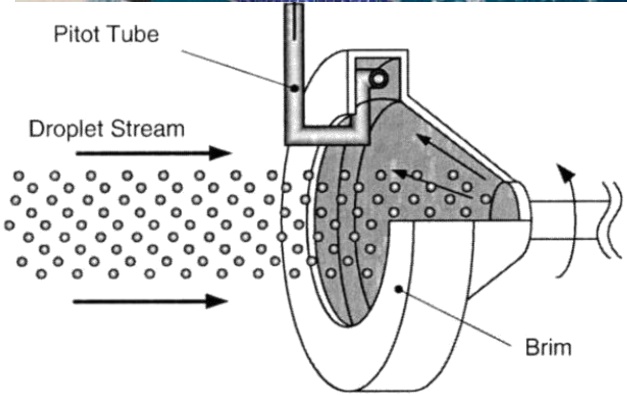
- Continuous stream of mass travelling between the satellites

S. G. Tragesser. Static formations using momentum exchange between satellites. Journal of guidance, control, and dynamics, 32(4):1277 – 1286, 2009.



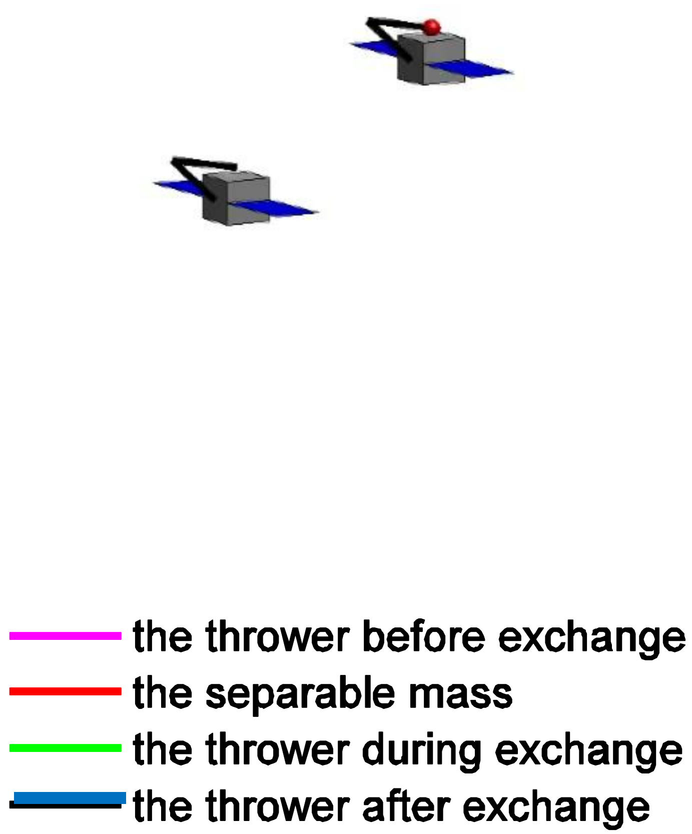
- Liquid droplet streams exchange

T. Joslyn and A. Ketsdever. Constant momentum exchange between microspacecraft using liquid droplet thrusters. In 46th joint Propulsion Conference, volume 6966, pages 25–28, 2010.



Single mass exchange control concept

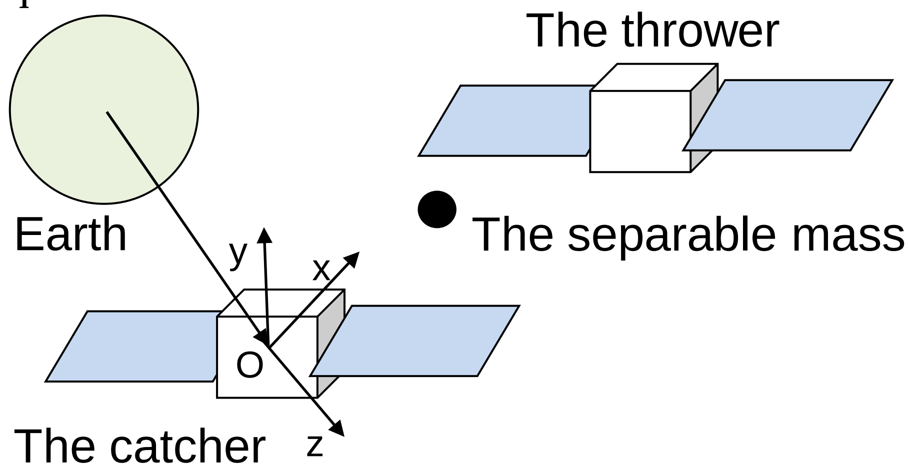
- At command the single mass separates from the satellite
- The separated mass moves to the other satellite and impacts it absolutely inelastically
- After the whole mass transfer the resulting relative trajectory changes in adjustable way



The Problem Formulation

Boundary problem:

What is the initial relative velocity of the mass required to hit the thrower?



Initial conditions:

$$x_0 = x(t_0),\ y_0 = y(t_0),\ z_0 = z(t_0),$$

The final position:

$$x_1 = x\left(t_1\right) = 0,\ y_1 = y\left(t_1\right) = 0,\ z_1 = z\left(t_1\right) = 0.$$

Hill - Clohessy - Wiltshire equations:

$$\ddot{x} + 2\omega\dot{z} = 0,$$

$$\ddot{y} + \omega^2 y = 0,$$

$$\ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0$$

The exact solution:

$$x = C_4 - 3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t,$$

$$y = C_5\sin\omega t + C_6\cos\omega t,$$

$$z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t$$

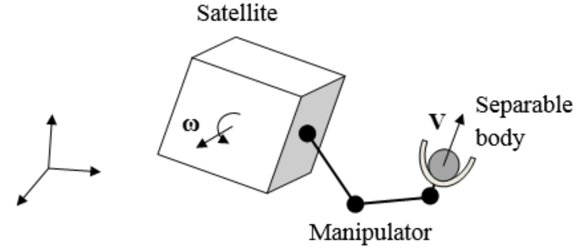
$$C_1 = 2z(t_0) + \frac{\dot{x}(t_0)}{\omega}, C_2 = \frac{\dot{z}(t_0)}{\omega},$$

$$C_3 = -3z(t_0) - \frac{2\dot{x}(t_0)}{\omega}, C_4 = x(t_0) - \frac{2\dot{z}(t_0)}{\omega},$$

$$C_5 = \frac{\dot{y}(t_0)}{\omega}, C_6 = y(t_0).$$

The Analytical Problem Solution

Throwing mass relative velocity:



$$\delta\dot{x} = -\dot{x}_0 - 2z_0\omega + \frac{1}{\Delta}[x_0\omega\sin u + 2z_0\omega(\cos u - 1)],$$

$$\delta\dot{y} = -\dot{y}_0 - y_0\omega\frac{\cos u}{\sin u},$$

$$\delta\dot{z} = -\dot{z}_0 - \frac{1}{\Delta}[2x_0\omega(1-\cos u) + z_0\omega(3u\cos u - 4\sin u)],$$

where $u = \omega(t_m - t_e)$, $\Delta = 3u\sin u - 8(1-\cos u)$.

The resulting thrower satellite velocity after mass throwing

$$\mathbf{v}_t = \mathbf{v}_{t,0} - \frac{m}{M}\delta\mathbf{v}.$$

The resulting *catcher* **satellite velocity after mass catching**

$$\mathbf{v}_c(t_m) = \frac{m}{M+m}\mathbf{v}_s(t_m).$$

For instance, the final relative trajectory $\tilde{\mathrm{C}}_1$ constant:

$$\tilde{C}_1 = \left(2z_0 + \frac{\dot{x}_0}{\omega}\right) + \frac{k(k+2)}{(k+1)^2}\cdot\frac{x_0\cos s - 2z_0\sin s}{8\sin s - 6s\cos s}.$$

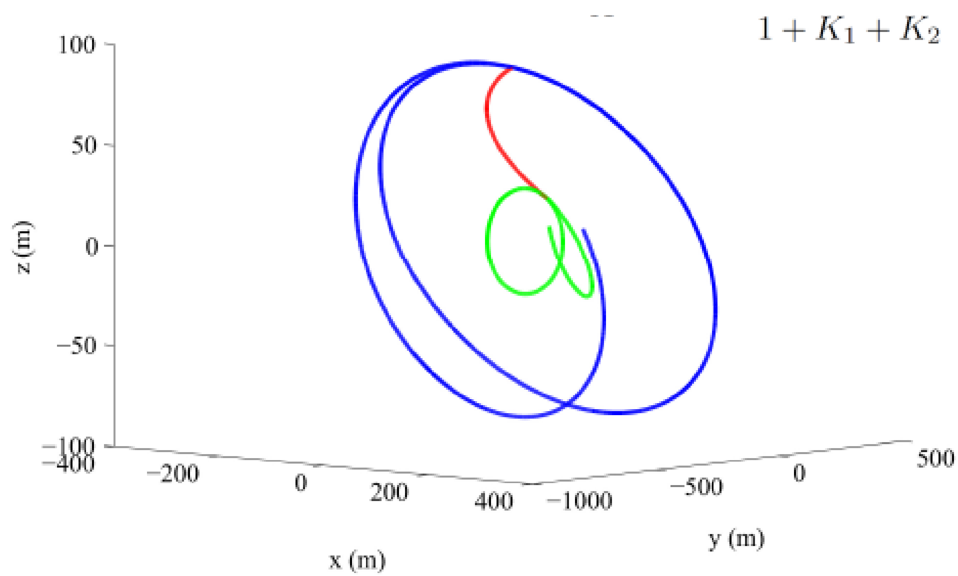
Swarm construction objective: eliminate the relative drift

- Consider three satellites with relative drifts
- The mass exchange aims to set the drifts of first two satellites (C_c and C_t) equal to the third one C_r
- It is possible if the inequality holds (Kc, Kt – mass ratios)

$$A - B \le C_r \le A + B$$

$$A = \frac{C_c(K_c - 3) + C_t(K_t + 1)}{1 + K_c + K_t}$$

$$B = \frac{2}{1 + K_c + K_t}\sqrt{(2C_c - z_{c,0})^2 + \frac{\dot{z}_{c,0}^2}{\omega^2}}$$



Conclusion

- The swarm of the satellites is a new paradigm in space systems
- The fuelless control approaches are fitting small satellite restrictions, they are smart but challenging
- We should allow for the distributed system to be autonomous and self-organizing, but we must be watchful