Attitude system design:
Balancing real satellite and analysis complexity

Magnetic control example

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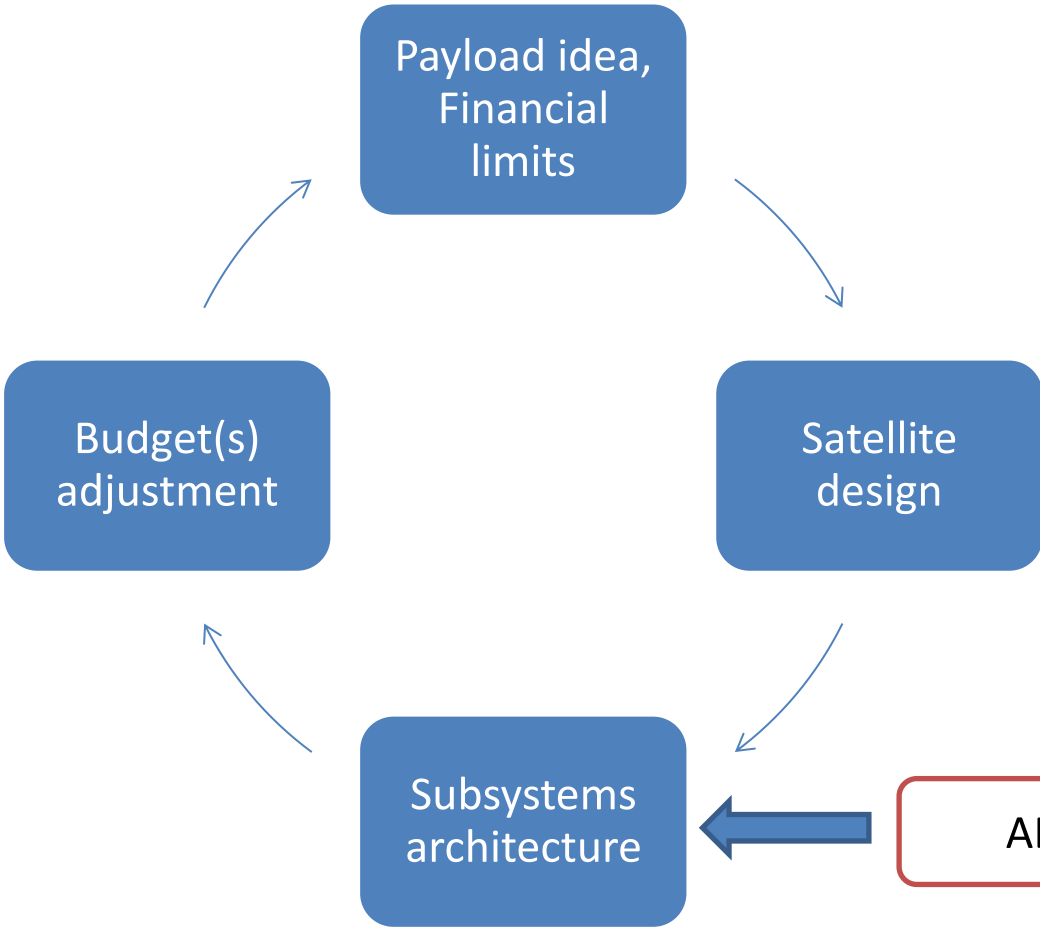
Contents

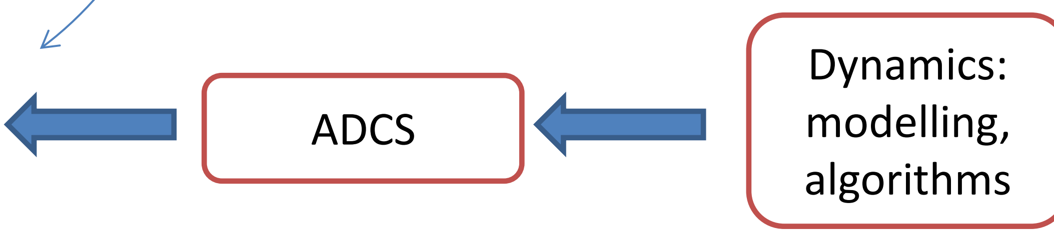
- Simplifying decisions
  - Buy or develop ADCS
  - Control hardware
  - Control algorithms
  - Analysis method
  - Environment models (geomagnetic field)
- Application example (dynamics analysis)

Dynamics place in mission design



Dynamics: mathematical aspects of one of the subsystems.

Do it have profound influence on the whole mission?



Buy or develop?

Why bother about AOCS among other subsystems?

- Dynamical problems (both angular and orbital) seem negligible in overall mission structure.
- *Maybe buy popular solutions?*

ADCS easily consume third of space and energy budget available

- *Maybe spend more effort on ADCS?*
- Free some mission resources for improvement of other subsystems and payload.

Definitely develop!

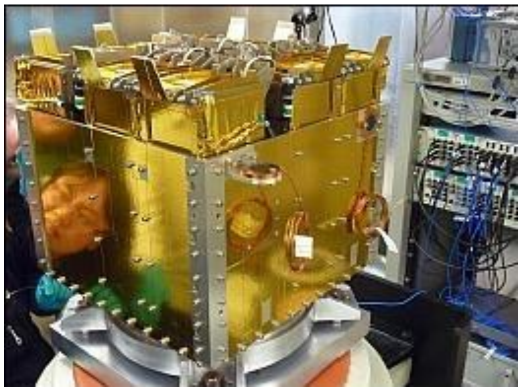
Buy AOCS	Develop architecture, build/buy components
• Fast • Reliable • Expensive	• Slow • Initially prone to faults • Initially expensive (education) • *Optimized for a mission* • Long-term investment in skilled personnel and overall group expertise • Interesting!

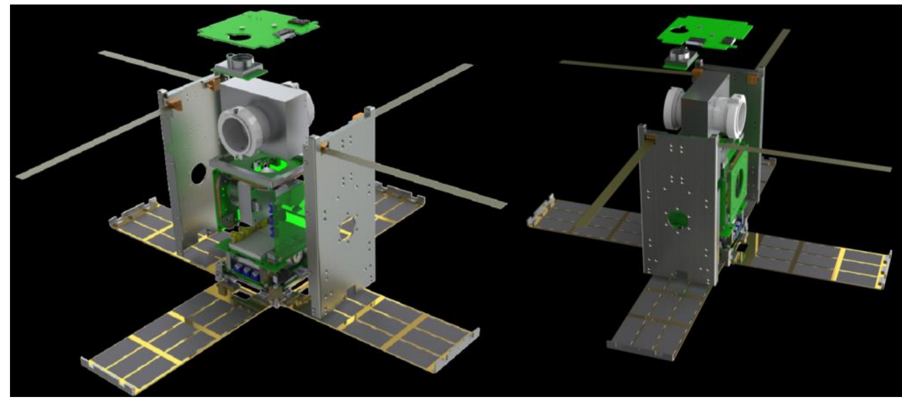
Choosing hardware and algorithms

Wheels, jets, GG boom	Magnetorquers
• Well studied and proved • No or mitigated control authority issues • Good accuracy and time response • Expensive, bulky etc.	• Slow and inaccurate • Subject to underactuation • Compact, cheap, reliable • Interesting!

$$\frac{\text{Magnetorquers + some other actuators}}{\text{Both problems and benefits are balanced}}$$

Example: CXBN-2 satellite





- Cosmic X-Ray Background in the 30-50 keV range
- 10/1 degrees attitude knowledge for primary/sec mission
- Possibly even celestial sphere coverage, maximizing overall science data
- Sensor can withstand bright sources, however loosing data

Simplifying ADCS

- Possibly even celestial sphere coverage
- Different control schemes provide necessary result

Three-axis → **Three-axis Gravitational** → **Spin stabilization** → **“Free” flying**

Three-axis
- Wheels, MTQ
- Common algorithms

Three-axis Gravitational
- GG-Boom, MTQ
- Common algorithms

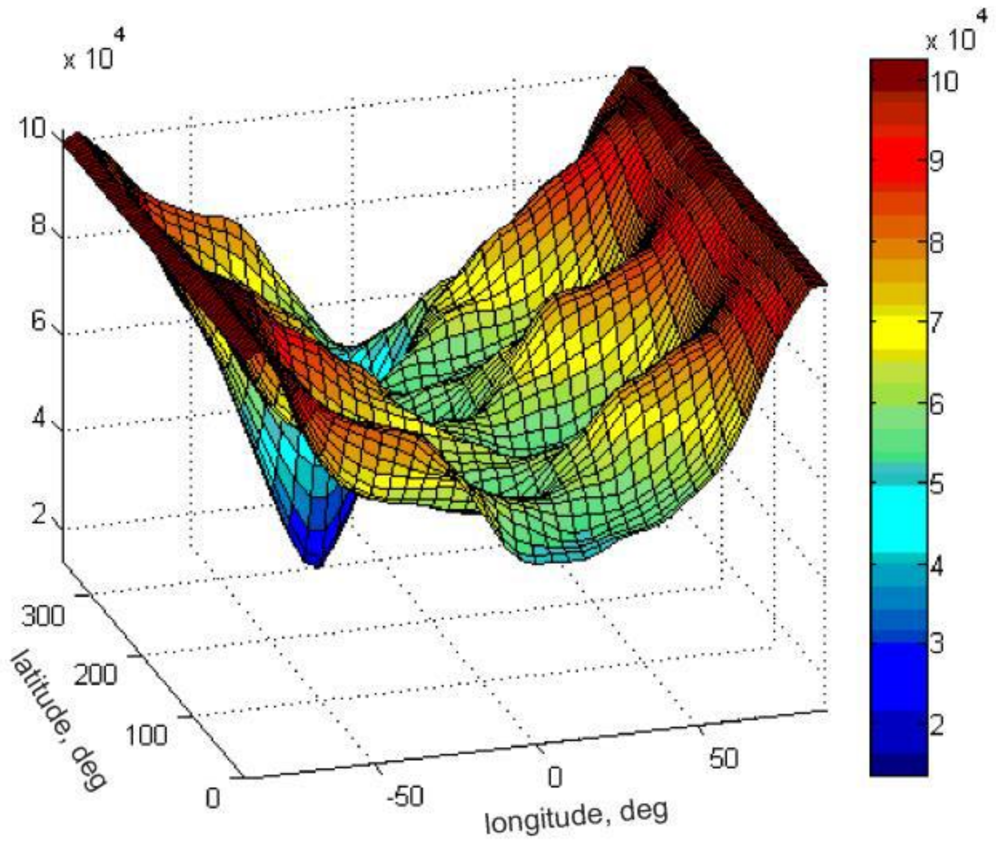
Spin stabilization
- MTQ
- Common algorithms
- Complex control cycle

“Free” flying
- MTQ
- Simple algorithms
- Only speed control

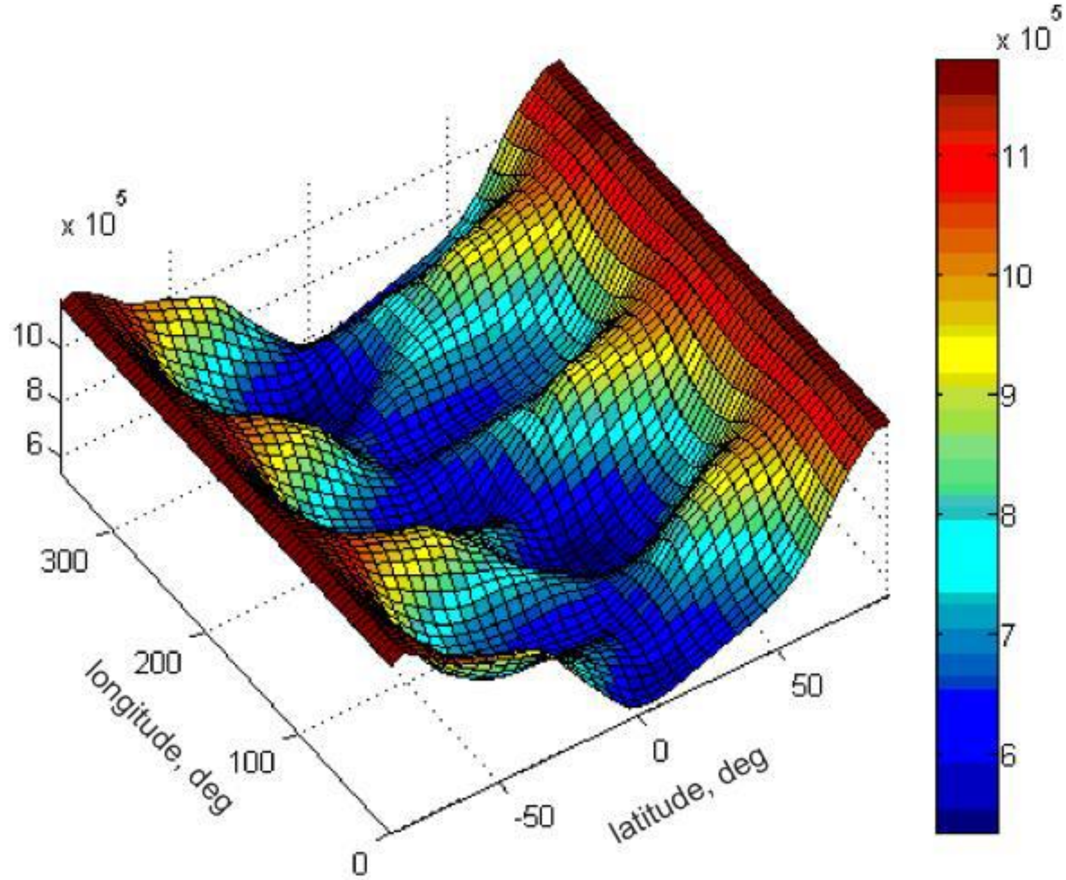
Control laws

- Spin stabilized
  - Nutation damping $\mathbf{m}_{nut} = -k_{nut}\left(\frac{d\mathbf{B}}{dt}\mathbf{e}_3\right)\mathbf{e}_3$
  - Spinning $\mathbf{m}_{spin} = k_{spin}\left(B_2, -B_1, 0\right)^T$
  - Reorientation $\mathbf{m}_{or} = \left(0, 0, k_{or}\left(\Delta\mathbf{L}\cdot\left[\mathbf{e}_3 \times \mathbf{B}\right]\right)\right)^T$
- "Free" flying, speed control $\mathbf{m} = \pm k_{damp}\frac{d\mathbf{B}}{dt}$

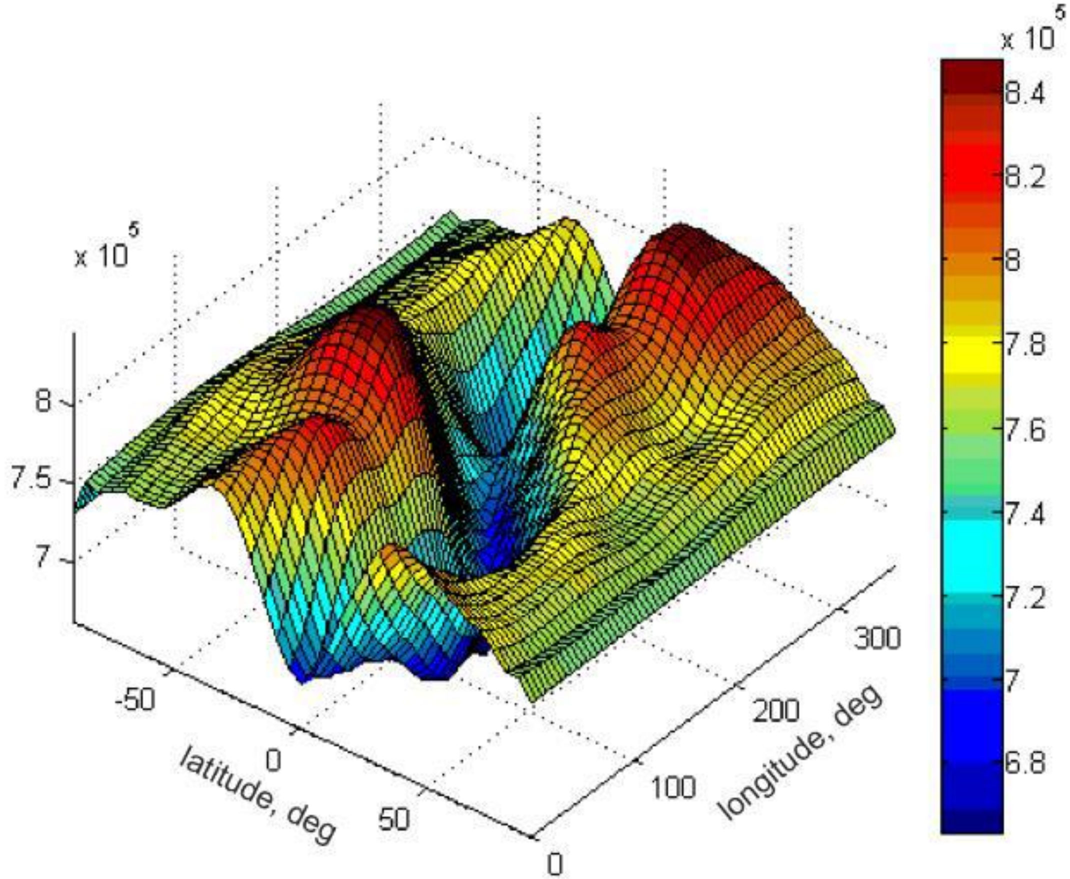
Continuous rotation
One month scientific data



Continuous rotation
One year scientific data



Free flying
One year scientific data



Control schemes data comparison

	Overall sets/year	Min. sets	Max. sets	Dipole moment, Am^2
Spin stabilization	31.346.066 (100.3%)	534.320 (80.6%)	1.177.844 (139.0%)	0.05
SS with Earth avoiding	31.875.472 (102.1%)	266.819 (40.2%)	1.106.402 (130.6%)	0.15
SS, Earth avoiding, charge	32.244.917 (103.3%)	641.783 (96.8%)	1.207.403 (142.5%)	0.15
Free flying	31.229.476 (100%)	663.068 (100%)	847.258 (100%)	0.05

Overall comparison

Spin stabilization	Free flying
• More overall data • Better polar regions coverage	• More even data • Simple control cycle • Better attitude knowledge • Less power consumption

Choosing analysis method

- Numerical analysis
  - \+ Comprehensive satellite and environment models
  - \+ Exceptional accuracy
  - ± Time consuming, rewarded with a long lasting tool
  - – Unique result
- Analytical solution
  - \+ General result, satellite behavior prediction
  - ± Time consuming, rewarded with a tool and publications
  - – Simplified and restricted satellite and environmental models
  - – Bad accuracy
  - – Higher qualification necessary

System state tyranny

- System in arbitrary motion is often analyzed using numerical methods
- Simplifying assumptions are governed by system motion peculiarities

Transient motion

- *Multiple scales method*
- Van der Pole method
- Characteristic exponents

Arbitrary motion

- *Numerical analysis*

Steady-state motion

- *Poincare method*
- *Linearization*

Dynamics simplification steps

- Convenient equations of motion
- *Osculating variables, Euler angles with proper rotation*

- Simple, but authentic environment models
- *Averaged or dipole geomagnetic field*

- Assumptions and analysis method
- *Multiple time scales for fast rotation*

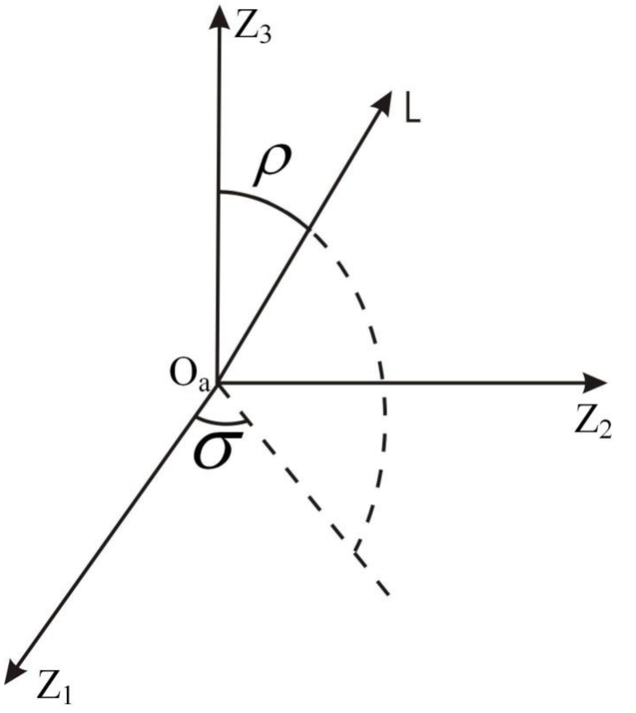
- Solution in explicit form
- *Different parameters influence on satellite behavior*

First analysis example: angular velocity detumbling

- Fast initial rotation
  - Multiple time scales method
  - Angular momentum changes slowly
  - Satellite attitude changes rapidly
  - Evolution of angular momentum obtained by averaging equations of motion
- Axisymmetrical satellite
  - Simple averaging over fast variables

Osculating variables

- Convenient for the transient motion analysis
- The rate of angular velocity is characterized using only one variable – magnitude of angular momentum
- Z_i define any inertial reference frame



Equations of motion

$$\frac{dL}{dt} = M_3, \qquad \frac{d\rho}{dt} = \frac{1}{L}M_1, \qquad \frac{d\sigma}{dt} = \frac{1}{L\sin\rho}M_2,$$

$$\frac{d\theta}{dt} = \frac{1}{L}\left(M_2\cos\psi - M_1\sin\psi\right),$$

$$\frac{d\varphi}{dt} = L\cos\theta\left(\frac{1}{C} - \frac{1}{A}\right) + \frac{1}{L\sin\theta}\left(M_{1L}\cos\psi + M_{2L}\sin\psi\right),$$

$$\frac{d\psi}{dt} = \frac{L}{A} - \frac{1}{L}M_{1L}\cos\psi\,\mathrm{ctg}\,\theta - \frac{1}{L}M_{2L}\left(\mathrm{ctg}\,\rho + \sin\psi\,\mathrm{ctg}\,\theta\right),$$

where M_1, M_2, M_3 are components of the torque in the frame associated with angular momentum

Control torque averaging

- Damping control torque is

$$\mathbf{M}_L = \begin{pmatrix} \omega_{3L} B_{1L} B_{3L} - \omega_{1L} B_{3L}^2 - \omega_{1L} B_{2L}^2 + \omega_{2L} B_{1L} B_{2L} \\ \omega_{1L} B_{1L} B_{2L} - \omega_{2L} B_{1L}^2 - \omega_{2L} B_{3L}^2 + \omega_{3L} B_{2L} B_{3L} \\ \omega_{2L} B_{2L} B_{3L} - \omega_{3L} B_{2L}^2 - \omega_{3L} B_{1L}^2 + \omega_{1L} B_{1L} B_{3L} \end{pmatrix}$$

- Averaging involves dimensionless geomagnetic induction vector components,

$$B_{ij} = \frac{1}{2\pi} \int_0^{2\pi} B_i B_j du,$$

Geomagnetic field models

- Gauss decomposition (IGRF, WMM)

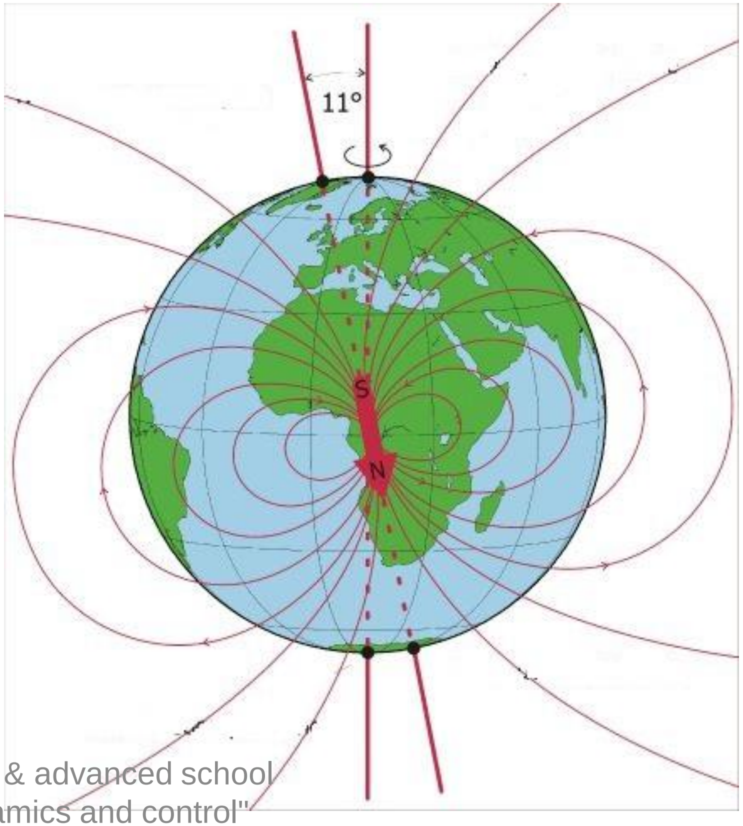
$$\mathbf{B} = \mu_0 \nabla V,\ V = -R\sum_{i=1}^{k}\left(\frac{R}{r}\right)^{i+1}\sum_{n=0}^{m}\left(g_n^m(t)\cos m\lambda_0 + h_n^m(t)\sin m\lambda_0\right)P_n^m(\cos\vartheta_0)$$

- Inclined dipole

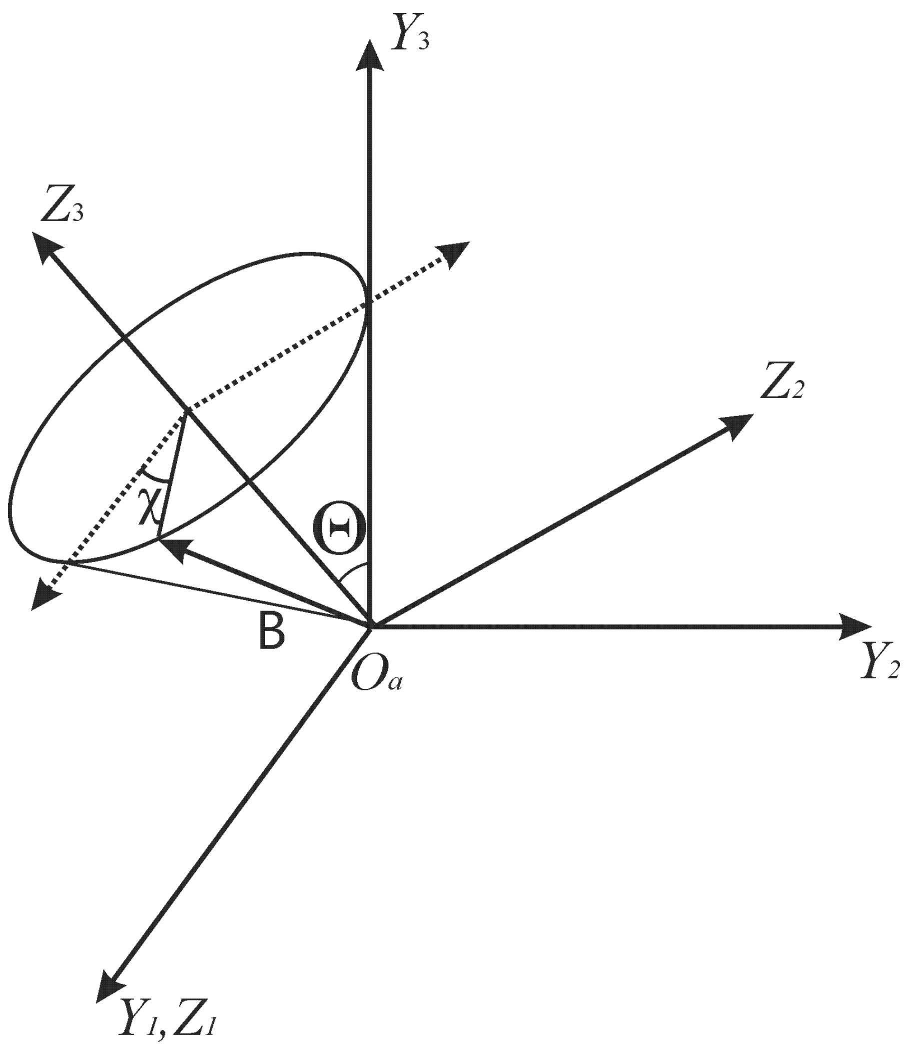
$$\mathbf{B} = \frac{\mu_e}{r^5}\left(\mathbf{k}\mathbf{r}^2 - 3(\mathbf{k}\mathbf{r})\mathbf{r}\right)$$

- Right dipole

$$\mathbf{B} = \frac{\mu_e}{r^3}\begin{pmatrix} -1.5\sin 2u \sin i \\ -3\sin^2 u \sin i + \sin i \\ \cos i \end{pmatrix}$$



Averaged geomagnetic field model



Geomagnetic induction vector evenly rotates on the cone with half opening angle given by

$$\mathrm{tg}\,\Theta = \frac{3\sin 2i}{2\left(1 - 3\sin^2 i + \sqrt{1 + 3\sin^2 i}\right)}$$

$$\mathbf{B} = B_0 \begin{pmatrix} \sin\Theta \sin 2u \\ \sin\Theta \cos 2u \\ \cos\Theta \end{pmatrix}$$

Averaged model result

$$\frac{dl}{du} = -\varepsilon l\left[2p + (1-3p)\sin^2\rho\right]\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\rho}{du} = \varepsilon(3p-1)\sin\rho\cos\rho\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\sigma}{du} = 0,$$

$$\frac{d\theta}{du} = \frac{1}{2}\varepsilon\left(1-\frac{C}{A}\right)\left[2(1-p) + (3p-1)\sin^2\rho\right]\sin\theta\cos\theta.$$

- Full set of autonomous first integrals can be found

Dipole model result

$$\frac{dl}{du} = -\varepsilon l\left[\frac{20}{9}a + \sin^2\rho\left(c - a\cos^2\sigma - \frac{11}{9}a\sin^2\sigma\right) + 2d\cos^2\rho\sin\sigma\cos\sigma\right]\times$$
$$\times\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\rho}{du} = \varepsilon\left[\left(\frac{11}{9}a\sin^2\sigma + a\cos^2\sigma - c\right)\sin\rho\cos\rho - d\sin\sigma\cos 2\rho\right]\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\sigma}{du} = \varepsilon\left[\frac{2}{9}a\sin\sigma\cos\sigma + d\cos\sigma\,\mathrm{ctg}\,\rho\right]\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\theta}{du} = \varepsilon\lambda\left[\frac{20}{9}a + c\left(1 + \cos^2\rho\right) + a\sin^2\rho\left(\cos^2\sigma + \frac{11}{9}\sin^2\sigma\right) + 2d\sin\rho\cos\rho\sin\sigma\right]\times$$
$$\times\sin\theta\cos\theta.$$

Dipole model result

- One more B_{ij} term

- One more term in equations

- Two first integrals can be found

- Solution to equations of motion is unavailable

Solution in explicit form: spherical satellite damping

torque value

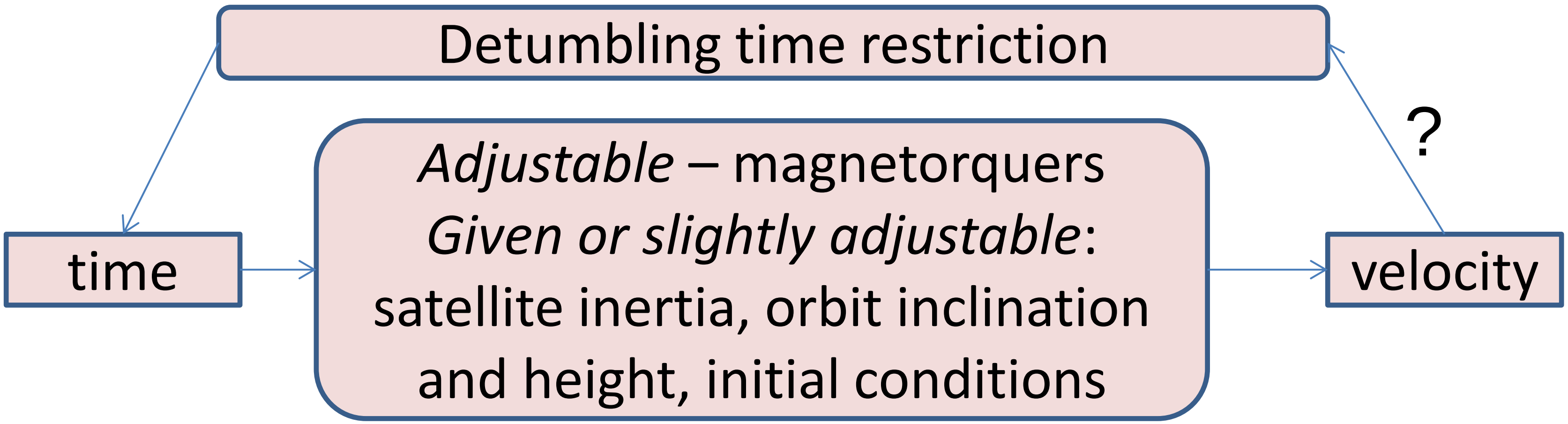
geomagnetic model parameter

$$l = \exp\left[-2\varepsilon p u + \frac{1}{2}\ln\left(\frac{1+\exp\left(2\varepsilon\left(3p-1\right)u + c_0\right)}{1+\exp c_0} \right) \right]$$

angular momentum

argument of latitude (time)

Simple parameters adjustment



Numerical simulation – verification, more accurate result after parameters are roughly adjusted

Second analysis example: planar motion with magnet

- Satellite moves on polar orbit
- Permanent magnet should point the satellite along geomagnetic field
- Linearized equation of planar motion:
  - Averaged field

$$\ddot{\beta} + \beta\left(\lambda^2 + \varepsilon\cos 2u\right) = -\varepsilon/2\sin 2u$$

control torque gravitational (disturbing) torque

  - Dipole model (near node)

$$\ddot{\beta} + \beta\left(3\lambda^2/2 + \left(2\varepsilon - \lambda^2\right)\Big/2\cos 2u\right) = \left(\lambda^2 - \varepsilon\right)\Big/2\sin 2u$$

  - Inclined dipole: no planar motion

Unstable area

- Averaged field

$$1-\frac{\varepsilon}{2\lambda^2}+\frac{7}{32}\left(\frac{\varepsilon}{\lambda^2}\right)^2+\ldots\le\lambda^2\le1+\frac{\varepsilon}{2\lambda^2}+\frac{7}{32}\left(\frac{\varepsilon}{\lambda^2}\right)^2+\ldots$$

  - No area if gravitational torque is zeroed: $\varepsilon=0$
  - Quite sensible and easily interpreted result

- Dipole model

$$1-\frac{2\varepsilon-\lambda^2}{6\lambda^2}+\frac{7}{32}\left(\frac{2\varepsilon-\lambda^2}{3\lambda^2}\right)^2+\ldots\le\lambda^2\le1+\frac{2\varepsilon-\lambda^2}{6\lambda^2}+\frac{7}{32}\left(\frac{2\varepsilon-\lambda^2}{3\lambda^2}\right)^2+\ldots$$

  - No area if gravitational torque is small enough:

$$\left|A-B\right|\omega_0^2\big/mB_0\le1/6$$

  - More general result, excessive strict assumption

Third analysis example: three axis magnetic control

- The dipole moment (PD-controller inspired)

$$\mathbf{m} = \mathbf{B} \times \left(-k_\omega \boldsymbol{\omega} - k_a \mathbf{S} \right), \qquad \mathbf{S} = \left(a_{23} - a_{32},\, a_{31} - a_{13},\, a_{12} - a_{21} \right)^T$$

- Control and gravitational torques are taken into account
- Circular orbit
- *Dipole* geomagnetic field

Linearized equations of motion

$$\frac{d\omega_1}{du} = -K_\omega \frac{B_0^2}{A\omega_0^2}\left[\left(B_2^2 + B_3^2\right)\omega_1 - B_1B_2\omega_2 - B_1B_3\omega_3\right] -$$

$$-2k_a \frac{B_0^2}{A\omega_0^2}\left[-B_1B_2\varphi - B_1B_3\theta + \left(B_2^2 + B_3^2\right)\psi\right] + \omega_2 + \frac{B-C}{A}\left(\omega_2 + \psi\right),$$

$$\frac{d\omega_2}{du} = -K_\omega \frac{B_0^2}{B\omega_0^2}\left[-B_1B_2\omega_1 + \left(B_1^2 + B_3^2\right)\omega_2 - B_2B_3\omega_3\right] -$$

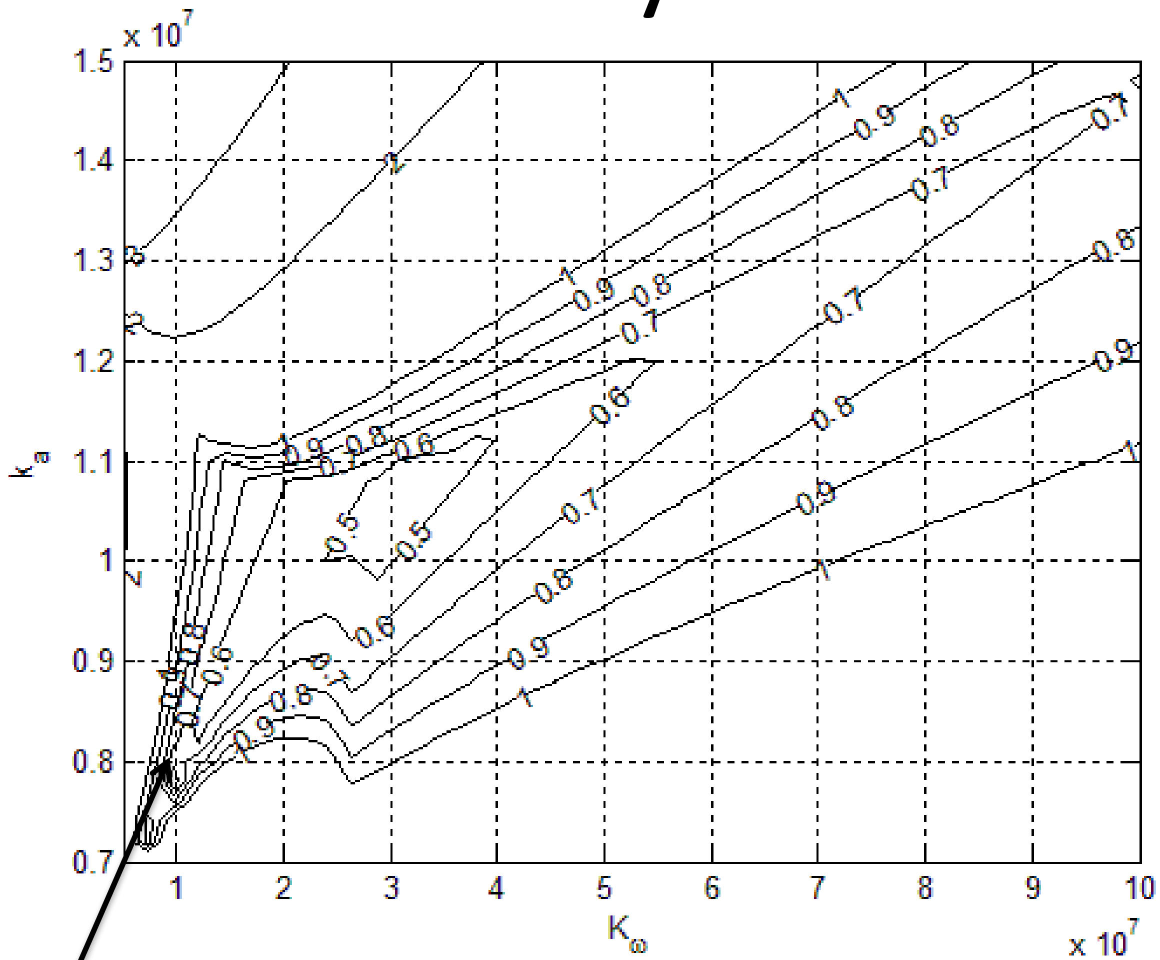
$$-2k_a \frac{B_0^2}{B\omega_0^2}\left[\left(B_1^2 + B_3^2\right)\varphi - B_2B_3\theta - B_1B_2\psi\right] - \omega_1 + \frac{C-A}{B}\left(\omega_1 - 4\varphi\right),$$

$$\frac{d\omega_3}{du} = -K_\omega \frac{B_0^2}{C\omega_0^2}\left[-B_1B_3\omega_1 - B_2B_3\omega_2 + \left(B_1^2 + B_2^2\right)\omega_3\right] -$$

$$-2k_a \frac{B_0^2}{C\omega_0^2}\left[-B_2B_3\varphi + \left(B_1^2 + B_2^2\right)\theta - B_1B_3\psi\right] + 3\frac{A-B}{C}\theta,$$

$$\frac{d\varphi}{du} = \omega_2, \qquad \frac{d\theta}{du} = \omega_3, \qquad \frac{d\psi}{du} = \omega_1.$$

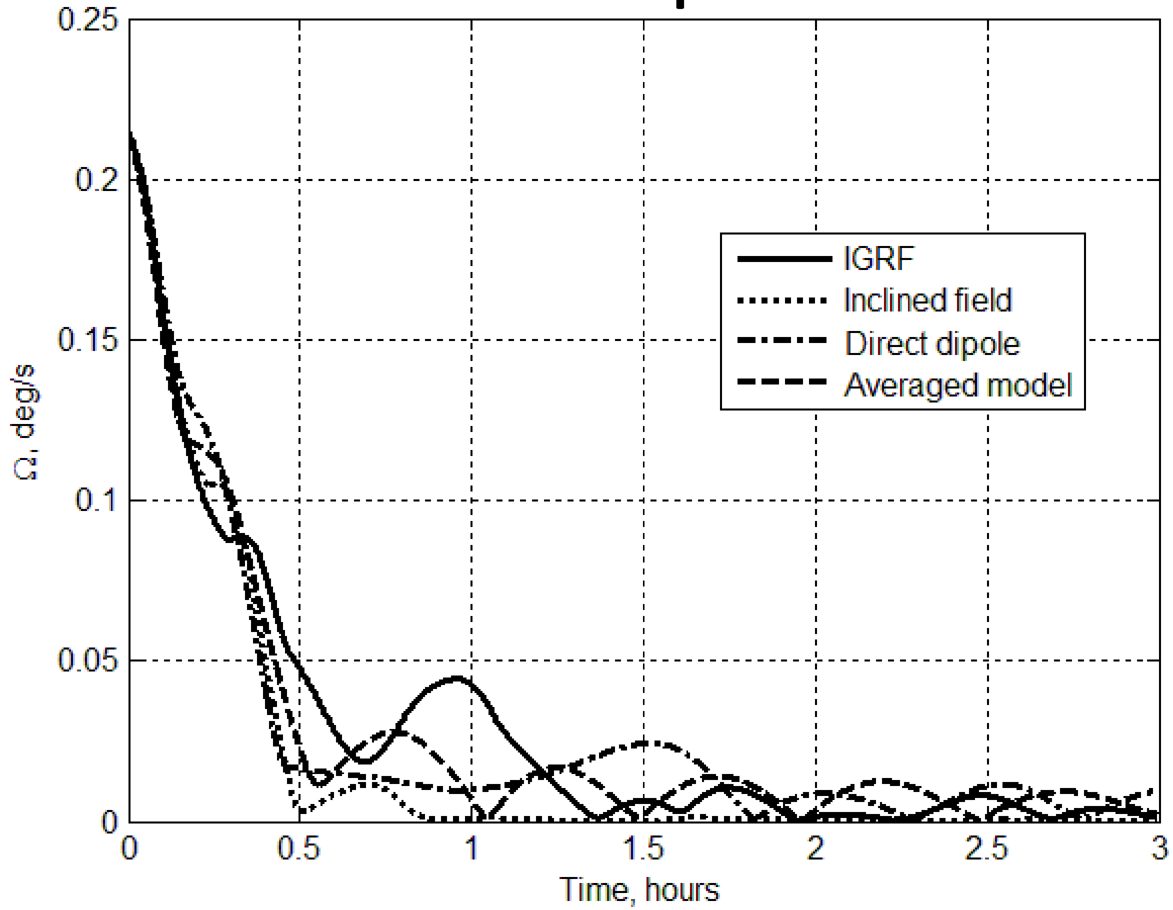
Stability area



Manually adjusted parameters $k_\omega = \frac{1 \cdot 10^7}{\omega_0} \frac{N \cdot m}{T^2}, \; k_a = 8 \cdot 10^6 \frac{N \cdot m}{T^2}$

Different models in simulation

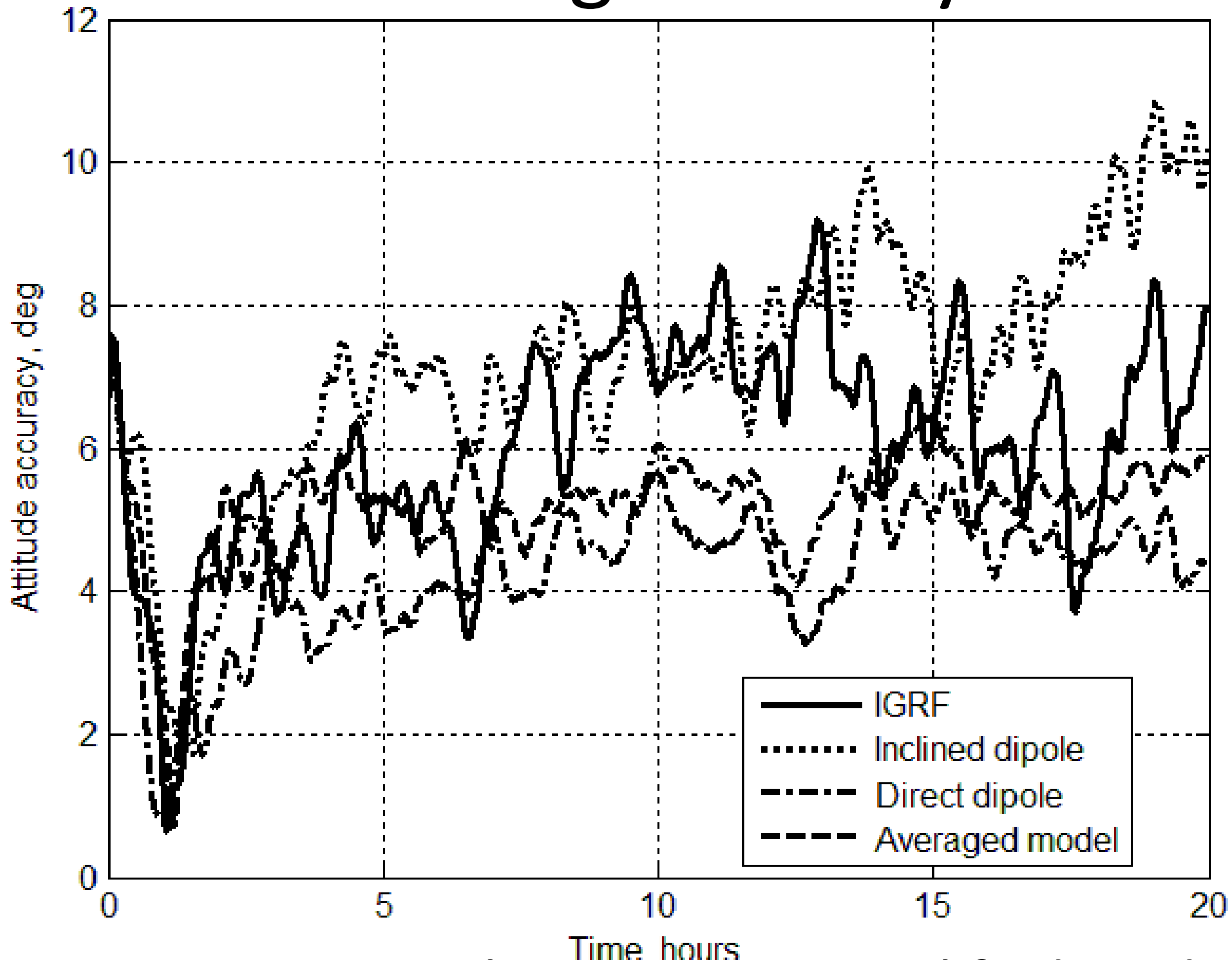
Time response



IGRF model 147.7 s (128.5 s model itself), inclined field 33.8 s, direct dipole 15 s, averaged model 14.8 s.

Different models in simulation

Pointing accuracy



Even accuracy analysis permits simplified models

Conclusion

- ADCS has significant effect on satellite design
- Analytical results prove to be convenient tool in a mission design process
- Even obvious missions benefit from some dynamical effort
- A number of simplifying assumptions suitable for the analysis method can lead to a very convenient equations