Mathematical model for a satellite with hinged flexible elements

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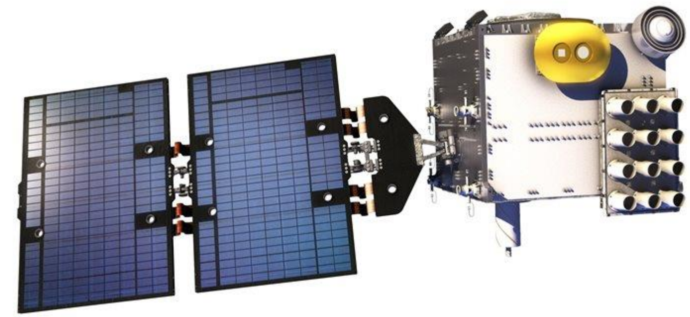
Main topics

- Introduction
- Problem definition
- Satellite with antenna and hinged solar panel
- Satellite with hinged solar panel
- Numerical modeling

Angular motion models

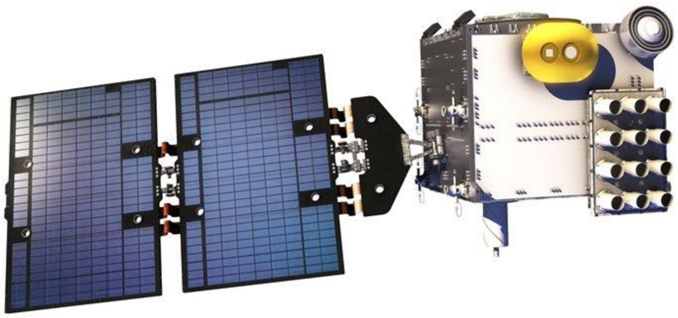
- Solar panels
- Antennas
- Scientific equipment

- Ground precise model
  usually non-linear
- On-board simplified model
  usually linear

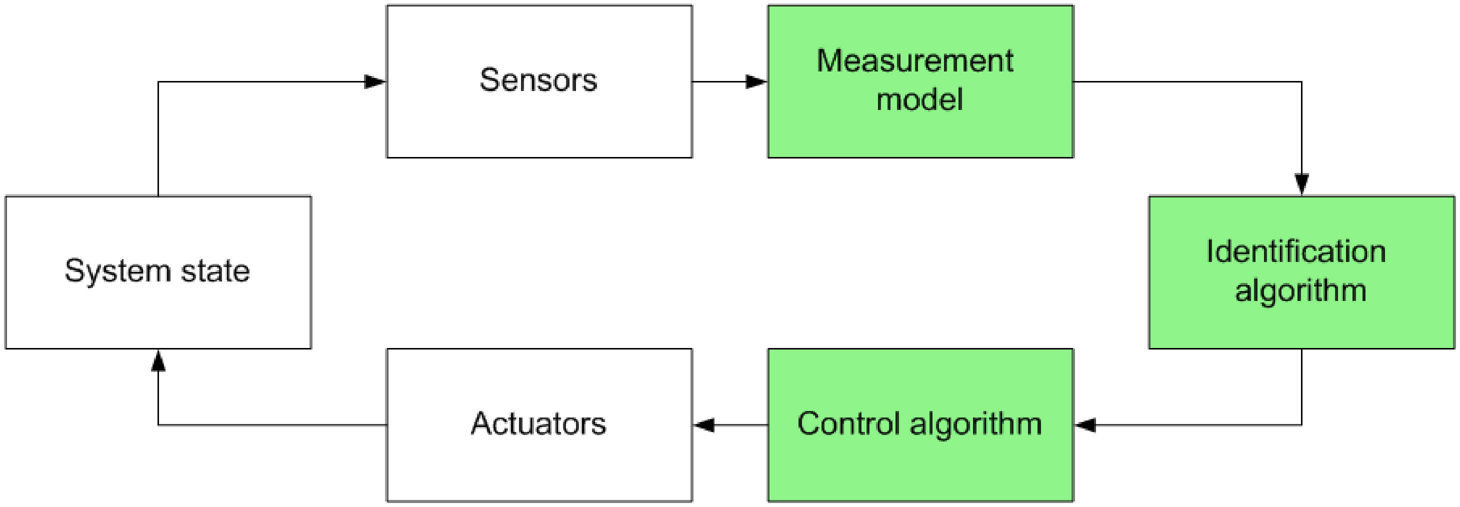


Angular motion models

- Solar panels
- Antennas
- Scientific equipment



- Ground precise model
- On-board simplified model



Equations derivation approaches

Satellite with flexible elements has almost infinite degrees of freedom

Usage of partial differential equations

Pros

- Precise
- Rather simple form

Cons

- Difficult to solve numerically
- Difficult to use for algorithms synthesis

Usage of normal modes

Pros

- Easy to analyze
- Simple to solve numerically

Cons

- Precision varies

Equations derivation approaches

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Usage of normal modes

Pros

- Easy to analyze
- Simple to solve numerically

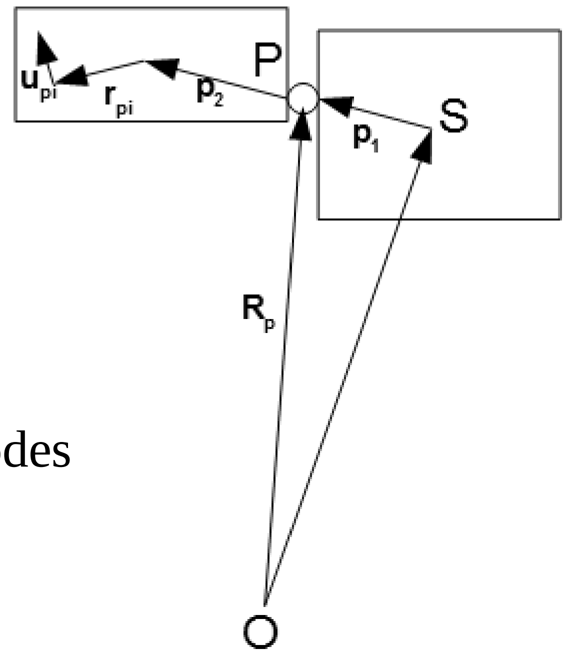
Cons

- Precision varies

Modes with low frequencies and high amplitudes are left

- The most part of oscillation energy
- Slow damping

Normal (Eigen) modes



Degrees of freedom
3 for center of mass motion
3 for angular motion
N for flexible deformation (normal modes)

Deformations

$$\mathbf{u}_{pi} = \mathbf{A}_{pi}(\mathbf{r}_{pi})\mathbf{q}(t),\ \mathbf{A}_{pi}(\mathbf{r}_{pi}) - 3\times N - \text{forms of modes}$$

$$\mathbf{q}(t) \qquad - \text{amplitudes}$$

Langrangian equations or general dynamics equation can be used

$$\sum_i \left(m_i \ddot{\mathbf{R}}_i - \mathbf{F}_i \right) \delta \mathbf{R}_i = 0$$

Problem definition

Several flexible parts
At least one of the rotating w.r.t. to the bus

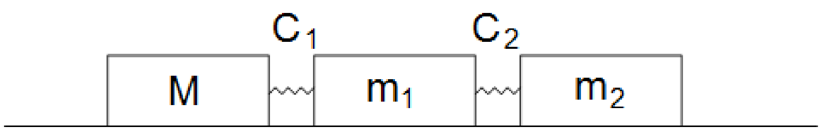
↓

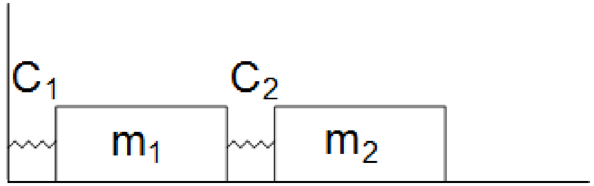
Normal modes of the whole satellite are varying

↓

Using the normal modes for each flexible element

Small dimension example





$$L_1 = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m_1(\dot{x}+\dot{y}_1)^2 + \frac{1}{2}m_2(\dot{x}+\dot{y}_1+\dot{y}_2)^2 - \frac{1}{2}C_1y_1^2 - \frac{1}{2}C_2y_2^2$$

$$L_1 = \frac{1}{2}\begin{pmatrix}\dot{x} & \dot{\mathbf{y}}\end{pmatrix}\mathbf{A}_1\begin{pmatrix}\dot{x}\\ \dot{\mathbf{y}}\end{pmatrix} + \frac{1}{2}\mathbf{y}^T\mathbf{C}\mathbf{y}$$

$$L_1 = \frac{1}{2}(M+m_1+m_2)\dot{x}^2 + \dot{x}\begin{pmatrix}m_1 & m_2\end{pmatrix}\mathbf{U}\dot{\boldsymbol{\theta}} + \frac{1}{2}\dot{\boldsymbol{\theta}}^2 - \frac{1}{2}\boldsymbol{\theta}^T\boldsymbol{\Lambda}\boldsymbol{\theta}$$

$$L_2 = \frac{1}{2}m_1\dot{y}_1^2 + \frac{1}{2}m_2(\dot{y}_1+y_2)^2 - \frac{1}{2}C_1y_1^2 - \frac{1}{2}C_2y_2^2$$

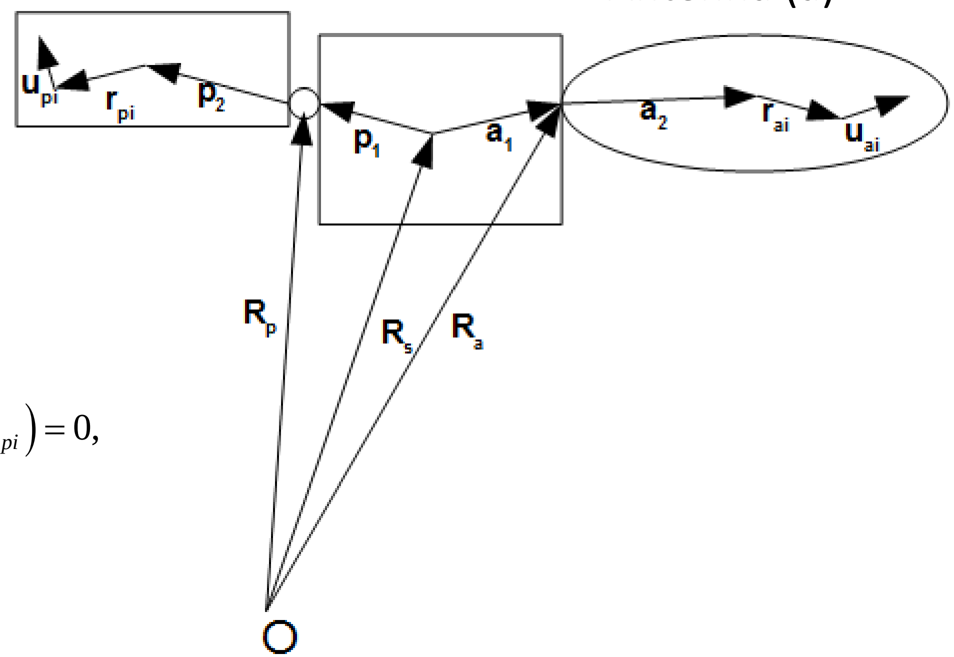
$$L_2 = \frac{1}{2}\dot{\mathbf{y}}^T\mathbf{A}_2\dot{\mathbf{y}} + \frac{1}{2}\mathbf{y}^T\mathbf{C}\mathbf{y}$$

$$L_2 = \frac{1}{2}\dot{\boldsymbol{\theta}}^2 - \frac{1}{2}\boldsymbol{\theta}^T\boldsymbol{\Lambda}\boldsymbol{\theta},\ \ \mathbf{y} = \mathbf{U}\boldsymbol{\theta}$$

$$\boldsymbol{\theta} - \text{normal coordinates},\ \mathbf{I}_{2x2} = \mathbf{U}^T\mathbf{A}_2\mathbf{U},\ \boldsymbol{\Lambda} = \mathbf{U}^T\mathbf{C}\mathbf{U}$$

$$\mathbf{A}_1 = \begin{pmatrix} M+m_1+m_2 & \begin{pmatrix}m_1 & m_2\end{pmatrix}\mathbf{U} \\ \mathbf{U}^T\begin{pmatrix}m_1 & m_2\end{pmatrix}^T & \mathbf{I}_{2x2}\end{pmatrix}$$

3 part satellite



Satellite
Rigid bus
Fixed flexible antenna
Hinged flexible solar panel

$$\sum_i \left(m_{si}\ddot{\mathbf{R}}_{si} - \mathbf{F}_{si} \right) + \sum_i \left(m_{ai}\ddot{\mathbf{R}}_{ai} - \mathbf{F}_{ai} \right) + \sum_i \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi} \right) = 0,$$

$$\sum_i \mathbf{e}^T \left(\left(\mathbf{p}_2 + \mathbf{r}_{pi} + \mathbf{u}_{pi} \right) \times \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi} \right) \right) = M_e,$$

$$\sum_i \mathbf{A}_{ai}^T \left(m_{ai}\ddot{\mathbf{R}}_{ai} - \mathbf{F}_{ai} \right) = 0,$$

$$\sum_i \mathbf{A}_{pi}^T \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi} \right) = 0,$$

$$\sum_i \mathbf{r}_{si} \times \left(m_{si}\ddot{\mathbf{R}}_{si} - \mathbf{F}_{si} \right) + \sum_i \left(\mathbf{a}_1 + \mathbf{r}_{ai} + \mathbf{u}_{ai} \right) \times \left(m_{ai}\ddot{\mathbf{R}}_{ai} - \mathbf{F}_{ai} \right) + \sum_i \left(\mathbf{p}_1 + \mathbf{r}_{pi} + \mathbf{u}_{pi} \right) \times \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi} \right) = 0.$$

+ kinematic equations

3 part satellite

$$\mathbf{J}\dot{\boldsymbol{\omega}}+\mathbf{S}_{\omega\varphi}\mathbf{e}\dot{\psi}+\mathbf{S}_{\omega a}\ddot{\mathbf{q}}_a+\mathbf{S}_{\omega p}\ddot{\mathbf{q}}_p+\boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega}+2m_a\mathbf{a}_1\times\boldsymbol{\omega}\times\dot{\tilde{\mathbf{a}}}_2+2\sum_i\left(\mathbf{r}_{ai}+\mathbf{u}_{ai}\right)\times m_{ai}\boldsymbol{\omega}\times\dot{\mathbf{u}}_{ai}+2\sum_i\left(\mathbf{r}_{pi}+\mathbf{u}_{pi}\right)\times m_{pi}\boldsymbol{\omega}_2\times\dot{\mathbf{u}}_{pi}+$$

$$+\tilde{\mathbf{J}}_p\left(\boldsymbol{\omega}\times\mathbf{e}\psi\right)+\boldsymbol{\omega}\times\tilde{\mathbf{J}}_p\mathbf{e}\psi+\mathbf{e}\psi\times\tilde{\mathbf{J}}_p\boldsymbol{\omega}+\mathbf{e}\psi\times\tilde{\mathbf{J}}_p\mathbf{e}\psi-2\frac{1}{m}\left(m_a\mathbf{a}+m_a\tilde{\mathbf{a}}_2+m_p\mathbf{p}+m_p\tilde{\mathbf{p}}_2\right)\times m_a\boldsymbol{\omega}\times\dot{\tilde{\mathbf{a}}}_2+$$

$$+\left(m_p\mathbf{p}_1-\frac{1}{m}\left(m_a\mathbf{a}+m_a\tilde{\mathbf{a}}_2+m_p\mathbf{p}+m_p\tilde{\mathbf{p}}_2\right)\right)\times\left(2m_p\boldsymbol{\omega}\times\mathbf{e}\psi\times\left(\mathbf{p}_2+\tilde{\mathbf{p}}_2\right)+m_p\mathbf{e}\psi\times\mathbf{e}\psi\times\left(\mathbf{p}_2+\tilde{\mathbf{p}}_2\right)+2m_p\boldsymbol{\omega}_2\times\dot{\tilde{\mathbf{p}}}_2\right)+\mathbf{f}_a+\mathbf{f}_p-\mathbf{T}_s=0,$$

$$\mathbf{S}_{\omega p}^T\dot{\boldsymbol{\omega}}+\mathbf{S}_{\varphi p}^T\mathbf{e}\ddot{\varphi}+\mathbf{S}_{ap}^T\ddot{\mathbf{q}}_a+\mathbf{M}_p\ddot{\mathbf{q}}_p+\sum_i m_{pi}\mathbf{A}_{pi}^T\boldsymbol{\omega}\times\boldsymbol{\omega}\times\left(\mathbf{r}_{pi}+\mathbf{A}_{pi}\mathbf{q}_p+\mathbf{p}_1-\frac{1}{m}\left(m_a\mathbf{a}+\sum_i m_{ai}\mathbf{A}_{ai}\mathbf{q}_a+m_p\mathbf{p}+\sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right)\right)+$$

$$+\sum_i m_{pi}\mathbf{A}_{pi}^T\mathbf{e}\dot{\varphi}\times\mathbf{e}\dot{\varphi}\times\left(\left(\mathbf{r}_{pi}+\mathbf{A}_{pi}\mathbf{q}_p\right)-\frac{1}{m}\left(m_p\mathbf{p}_2+\sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right)\right)-\frac{1}{m}\mathbf{e}\dot{\varphi}\times\left(m_p\mathbf{p}_2+\sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right)+$$

$$+2\sum_i m_{pi}\mathbf{A}_{pi}^T\boldsymbol{\omega}\times\left(\mathbf{A}_{pi}\dot{\mathbf{q}}_p+\mathbf{e}\dot{\varphi}\times\left(\mathbf{r}_{pi}+\mathbf{A}_{pi}\mathbf{q}_p\right)-\frac{1}{m}\left(\sum_i m_{ai}\mathbf{A}_{ai}\dot{\mathbf{q}}_a+\sum_i m_{pi}\mathbf{A}_{pi}\dot{\mathbf{q}}_p\right)\right)-2\frac{1}{m}\sum_i m_{pi}\mathbf{A}_{pi}^T\mathbf{e}\dot{\varphi}\times\sum_i m_{pi}\mathbf{A}_{pi}\dot{\mathbf{q}}_p+$$

$$+\mathbf{f}_{qp}+\sum_i m_{pi}\mathbf{A}_{pi}^T\mathbf{A}_{pi}\left(\mathbf{D}\dot{\mathbf{q}}_p+\boldsymbol{\Omega}\mathbf{q}_p\right)=0.$$

$$\mathbf{S}\begin{pmatrix}\dot{\boldsymbol{\omega}}\\ \dot{\psi}\\ \ddot{\mathbf{q}}_a\\ \ddot{\mathbf{q}}_p\end{pmatrix}=\mathbf{N}\left(\omega,\psi,\dot{\mathbf{q}}_a,\dot{\mathbf{q}}_p,\mathbf{A},\varphi,\mathbf{q}_a,\mathbf{q}_p\right),\quad \mathbf{S}=\begin{pmatrix}\mathbf{J} & \mathbf{S}_{\omega\varphi}\mathbf{e} & \mathbf{S}_{\omega a} & \mathbf{S}_{\omega p}\\ \mathbf{e}^T\mathbf{S}_{\omega\varphi}^T & \mathbf{e}^T\mathbf{J}\mathbf{e} & \mathbf{e}^T\mathbf{S}_{\varphi a} & \mathbf{e}^T\mathbf{S}_{\varphi p}\\ \mathbf{S}_{\omega a}^T & \mathbf{S}_{\varphi a}^T\mathbf{e} & \mathbf{M}_a & \mathbf{S}_{ap}\\ \mathbf{S}_{\omega p}^T & \mathbf{S}_{\varphi p}^T\mathbf{e} & \mathbf{S}_{ap}^T & \mathbf{M}_p\end{pmatrix}$$

Linearized equations

Considering $\frac{m_a}{m} \ll 1 \quad \frac{m_p}{m} \ll 1$

$$\mathbf{S} = \begin{pmatrix} \mathbf{J} & \mathbf{S}_{\omega\varphi}\mathbf{e} & \mathbf{S}_{\omega a} & \mathbf{S}_{\omega p} \\ \mathbf{e}^T\mathbf{S}_{\omega\varphi}^T & \mathbf{e}^T\mathbf{J}\mathbf{e} & 0 & \mathbf{e}^T\mathbf{S}_{\varphi p} \\ \mathbf{S}_{\omega a}^T & 0 & \mathbf{M}_a & 0 \\ \mathbf{S}_{\omega p}^T & \mathbf{S}_{\varphi p}^T\mathbf{e} & 0 & \mathbf{M}_p \end{pmatrix}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{S}_{\omega\varphi}\mathbf{e}\ddot{\varphi} + \mathbf{S}_{\omega a}\ddot{\mathbf{q}}_a + \mathbf{S}_{\omega p}\ddot{\mathbf{q}}_p + \mathbf{f}_a + \mathbf{f}_p - \mathbf{M}_s = 0,$$

$$\mathbf{e}^T\mathbf{S}_{\omega\varphi}^T\dot{\boldsymbol{\omega}} + \mathbf{e}^T\mathbf{J}_e\mathbf{e}\ddot{\varphi} + \mathbf{e}^T\mathbf{S}_{\varphi p}\ddot{\mathbf{q}}_p + \mathbf{e}^T\mathbf{f}_p = M_e,$$

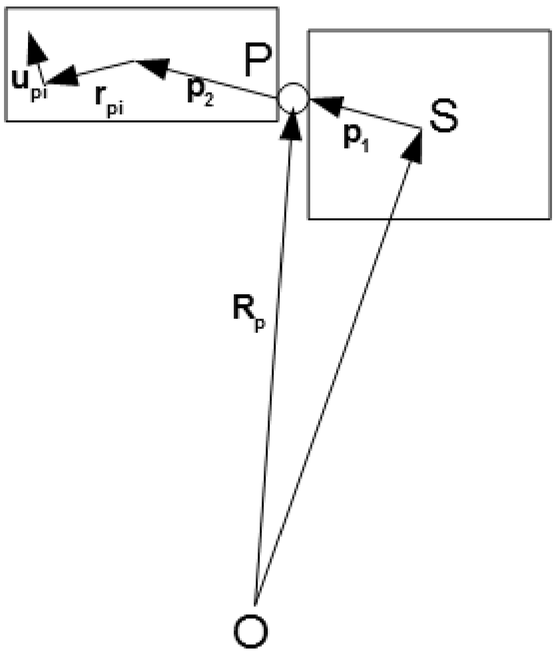
$$\mathbf{S}_{\omega a}^T\dot{\boldsymbol{\omega}} + \mathbf{M}_a\left(\ddot{\mathbf{q}}_a + \mathbf{D}\dot{\mathbf{q}}_a + \boldsymbol{\Omega}\mathbf{q}_a\right) + \mathbf{f}_{qa} = 0,$$

$$\mathbf{S}_{\omega p}^T\dot{\boldsymbol{\omega}} + \mathbf{S}_{\varphi p}^T\mathbf{e}\ddot{\varphi} + \mathbf{M}_p\left(\ddot{\mathbf{q}}_p + \mathbf{D}\dot{\mathbf{q}}_p + \boldsymbol{\Omega}\mathbf{q}_p\right) + \mathbf{f}_{qp} = 0.$$

$3 + 1 + N_p + N_a$ degrees of freedom

Satellite with hinged solar panel

$$\mathbf{S}\begin{pmatrix}\dot{\boldsymbol{\omega}}\\ \dot{\psi}\\ \ddot{\mathbf{q}}_p\end{pmatrix}=\mathbf{N}\left(\omega,\psi,\dot{\mathbf{q}}_p,\mathbf{q},\varphi,\mathbf{q}_p\right)\qquad \mathbf{S}=\begin{pmatrix}\mathbf{J} & \mathbf{S}_{\omega\varphi}\mathbf{e} & \mathbf{S}_{\omega p}\\ \mathbf{e}^T\mathbf{S}_{\omega\varphi}^T & \mathbf{e}^T\mathbf{J}\mathbf{e} & \mathbf{e}^T\mathbf{S}_{\varphi p}\\ \mathbf{S}_{\omega p}^T & \mathbf{S}_{\varphi p}^T\mathbf{e} & \mathbf{M}_p\end{pmatrix}$$



$$\dot{\psi}=0,\ \varphi=const$$

$$\mathbf{J}\dot{\boldsymbol{\omega}}+\mathbf{S}_{\omega p}\ddot{\mathbf{q}}_p=0,$$

$$\mathbf{S}_{\omega p}^T\dot{\boldsymbol{\omega}}+\mathbf{M}_p\ddot{\mathbf{q}}_p+\sum_i m_{pi}\mathbf{A}_{pi}^T\mathbf{A}_{pi}\boldsymbol{\Omega}\mathbf{q}_p=0,$$

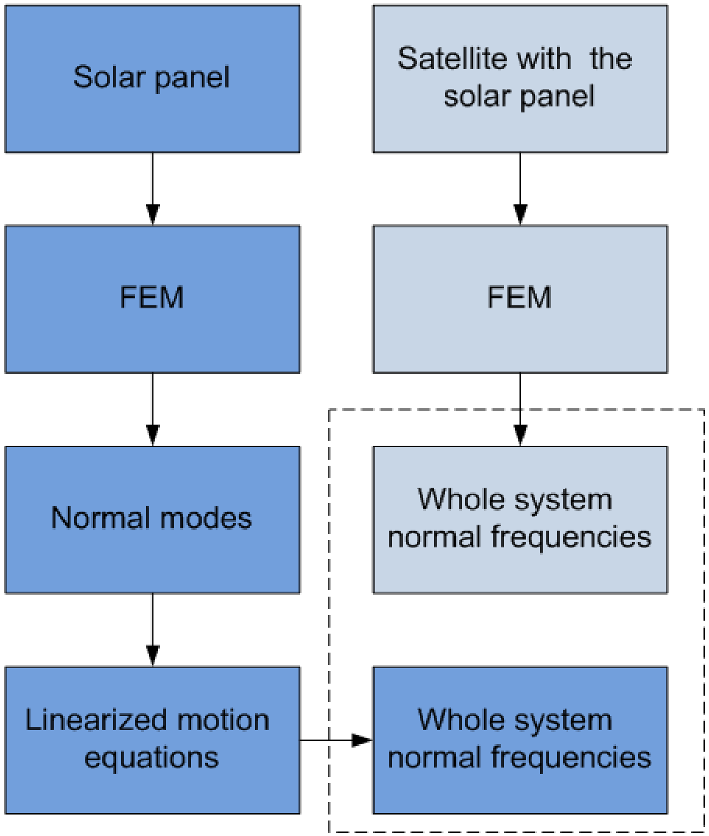
$$\mathbf{S}_{\omega p}=\sum_i\left(\mathbf{p}_1+\mathbf{r}_{pi}\right)\times m_{pi}\mathbf{A}_{pi}-\frac{1}{m}m_p\mathbf{p}\times\sum_i m_{pi}\mathbf{A}_{pi},$$

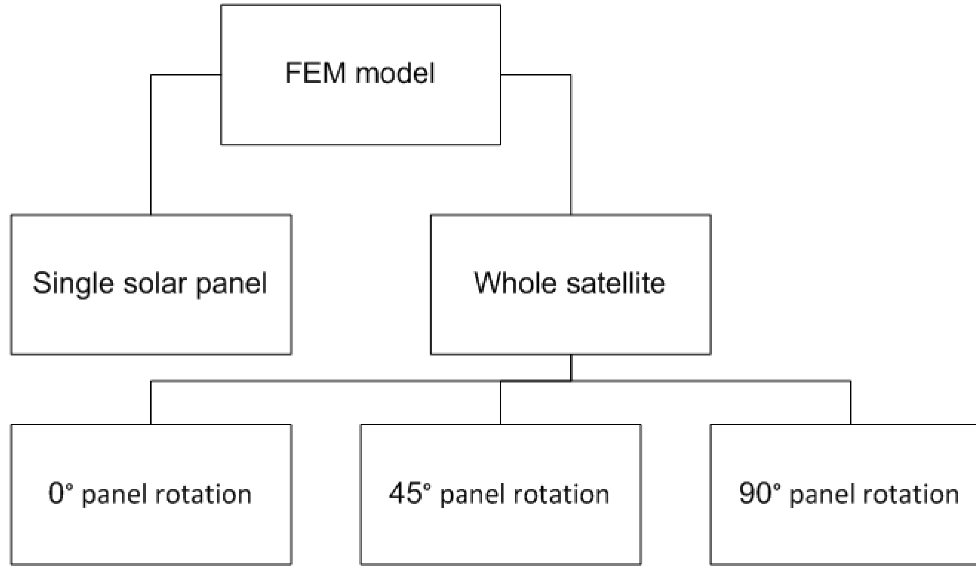
$$\mathbf{J}=\mathbf{J}_s+\tilde{\mathbf{J}}_p+\mathbf{K}\left(\mathbf{p}_1,m_p\mathbf{p}_1\right)+\mathbf{K}\left(\mathbf{p}_1,m_p\mathbf{p}_2\right)+\mathbf{K}\left(m_p\mathbf{p}_2,\mathbf{p}_1\right)-\frac{1}{m}\mathbf{K}\left(m_p\mathbf{p},m_p\mathbf{p}\right),$$

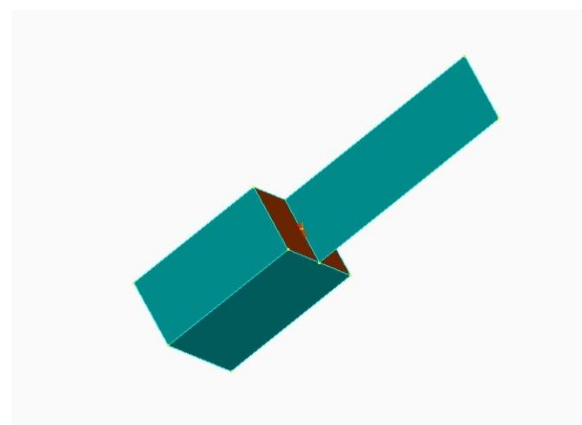
$$\mathbf{M}_p=\sum_i m_{pi}\mathbf{A}_{pi}^T\mathbf{A}_{pi}-\frac{1}{m}\sum_i m_{pi}\mathbf{A}_{pi}^T\sum_i m_{pi}\mathbf{A}_{pi}.$$

$$\mathbf{K}(\mathbf{a},\mathbf{b})=\begin{pmatrix}a_2b_2+a_3b_3 & -a_2b_1 & -a_3b_1\\ -a_1b_2 & a_1b_1+a_3b_3 & -a_3b_2\\ -a_1b_3 & -a_2b_3 & a_1b_1+a_2b_2\end{pmatrix}$$

Numerical modeling







panel grid: 200x600
satellite grid: 20x20x20

FEM modeling

Flexible solar panel

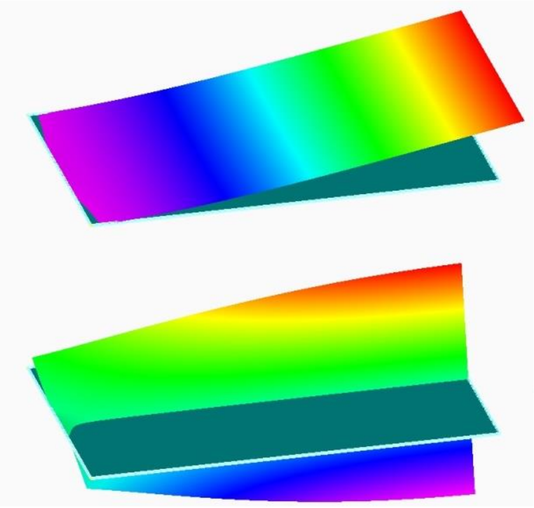
Purposes

Main

Determine quantities for motion equations

Auxiliary

Eigen modes forms and frequencies determination

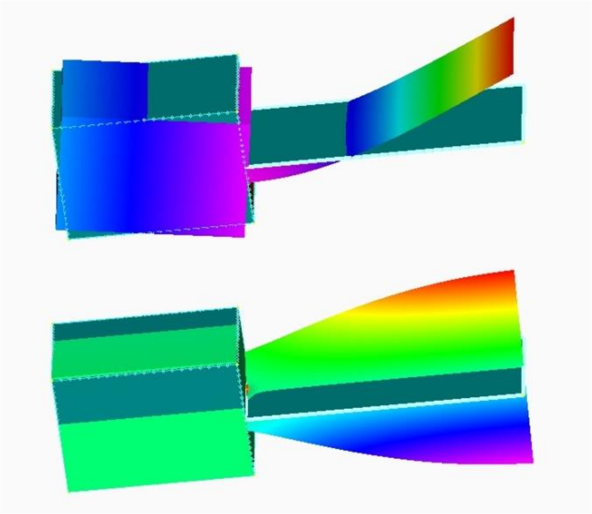


Satellite with flexible panel

Purpose

Auxiliary

Eigen modes forms and frequencies determination



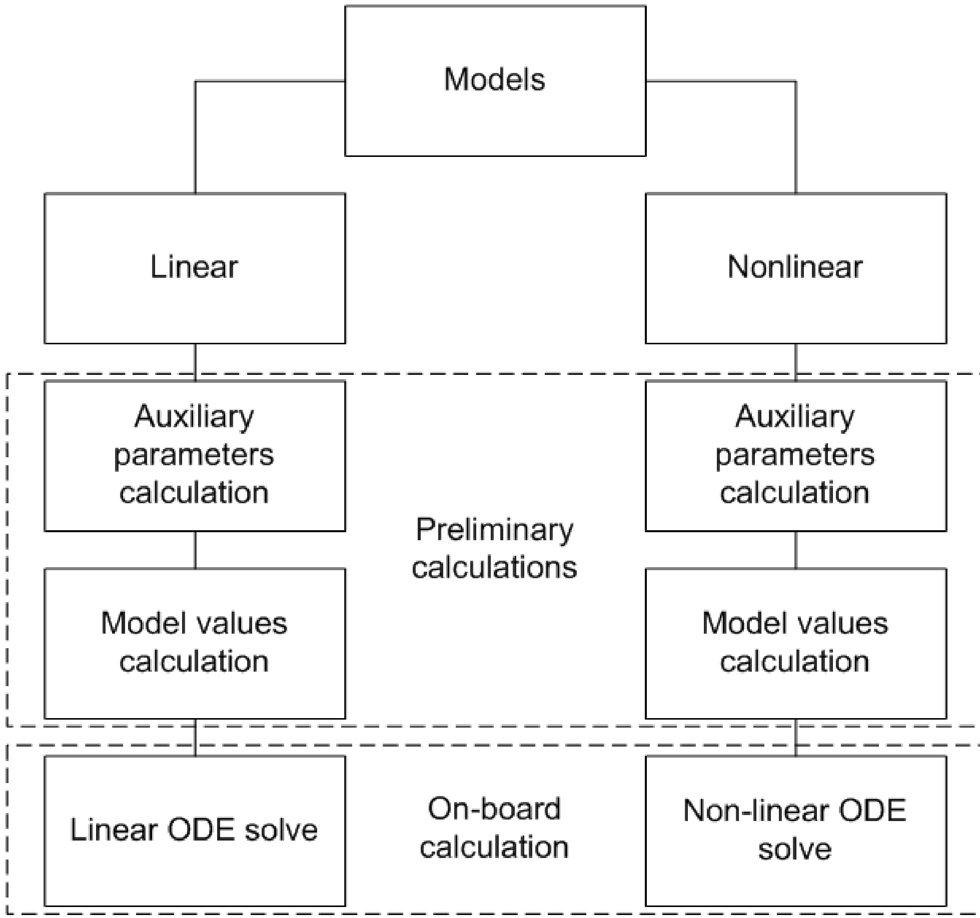
Eigen frequencies comparison

Mode No.	Single panel	Satellite with panel $\varphi = 0$		Satellite with panel $\varphi = \pi/4$		Satellite with panel $\varphi = \pi/2$	
		FEM	Model	FEM	Model	FEM	Model
1	5.2487	7.6310	7.6311	7.8223	7.8234	8.0301	8.0316
2	32.057	32.393	32.483	32.378	32.483	32.361	32.483
3	33.256	36.063	36.068	36.195	36.203	36.364	36.370
4	34.485	52.679	53.148	51.235	51.679	49.722	50.137
5	98.195	99.381	99.420	99.488	99.533	99.596	99.643
6	100.90	100.78	101.03	100.73	101.03	100.69	101.02
7	181.44	181.11	181.51	181.04	181.51	180.97	181.51
8	190.81	191.90	192.06	191.98	192.17	192.07	192.28
9	277.50	276.98	277.54	276.89	277.54	276.79	277.54
10	294.77	295.27	295.89	295.26	295.98	295.25	296.08

Model calculation
FEM for panel: 30 minutes
Model auxiliary parameters: 5 minutes
One position mode calculation: less 1 second
Total time: about 35 minutes

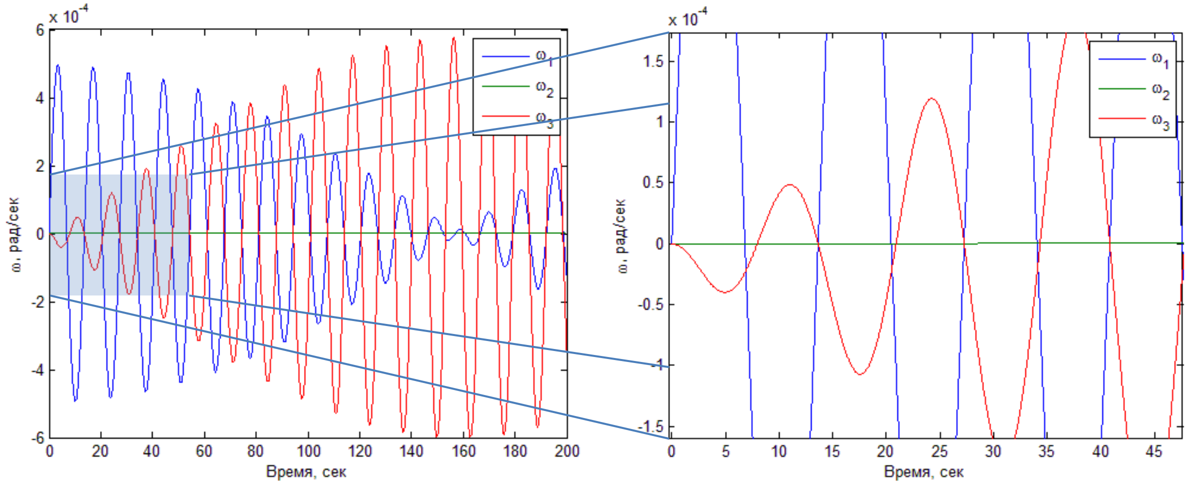
FEM calculation:
One position calculation: 40 minutes
Total time: about 120 minutes

Model on-board application

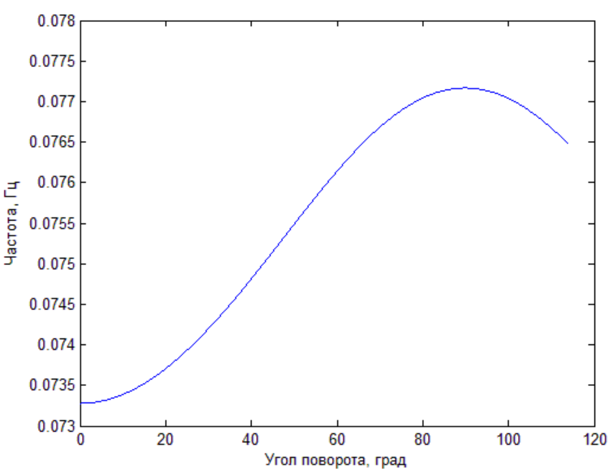


Motion numerical modeling

Angular velocity



1st mode frequency variation



Conclusions

To describe multi-element angular dynamics it is possible to use normal modes for each element instead of those for the whole system. The approach

decreases dramatically numerical modeling time,

allows to model dynamics with hinged flexible element.

The work was supported be the RFBR No. **16-01-00634 A**