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Microsatellite Attitude Motion Determination Using Measurements of Electromotive Force Inducted in Magnetic Torquers

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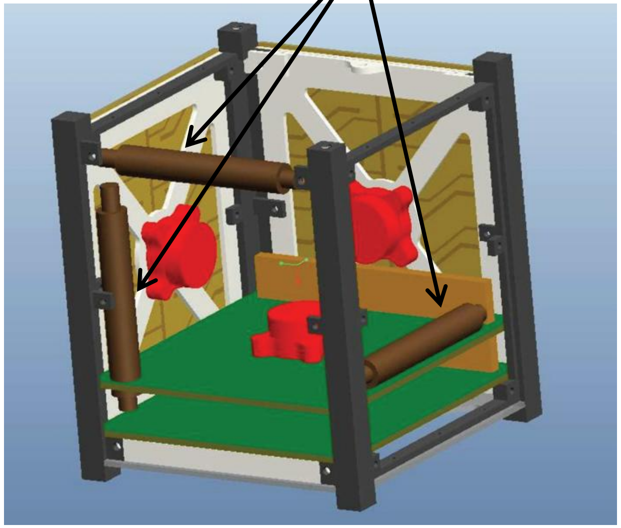
Active Magnetic ACS

- **The most common for micro- and nanosatellites**
- **Used for:**
  - **angular velocity damping**
  - **orientation along geomagnetic field**
  - **three-axis stabilization**
- **Consists of magnetic torquers and sensors for attitude determination**

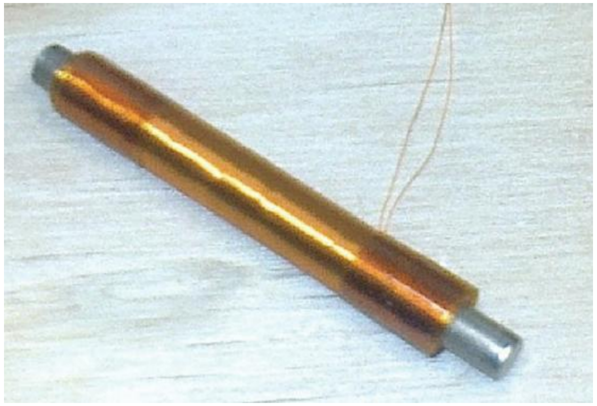
Magnetic torquers



CubeSat magnetic torquers

Magnetic Torquers Configurations

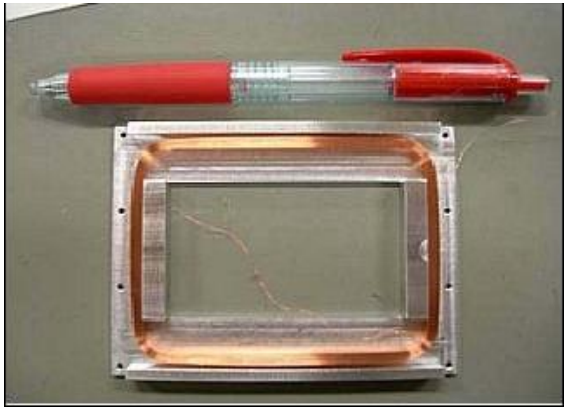
- **Rod-type**



Developed in York University

•**Compactness**
•**High magnetic moment**
•**Ferromagnetic core**
•**Small cross area**

- **Coil-type**



CUTE-1.7 magnetic torquers

•**Convenience in its placement**
•**Less amount of turns**
•**High cross area**

- **Combined**

Magnetorquer board
“SatBus MTQ”

•**Easy to install inside the CubeSat**
•**Consists of two magnetorquer rods and one magnetorquer coil**

Three-axis Attitude Determination Using Minimum Set of Sensors

- **Magnetometer only measurements**

  Abdelrahman, M., and Park, S.-Y., “Integrated attitude determination and control system via magnetic measurements and actuation,” *Acta Astronautica*, vol. 69, Aug. 2011, pp. 168–185.

- **Measurements of solar panels currents**

  Ruiter, A., Tran, L., Kumar, B. S., and Muntyanov, A., “Sun Vector–Based Attitude Determination of Passively Magnetically Stabilized Spacecraft,” *Journal of Guidance, Control, and Dynamics*, vol. 39, Jul. 2016, pp. 1551–1562.

- **Temperature measurements**

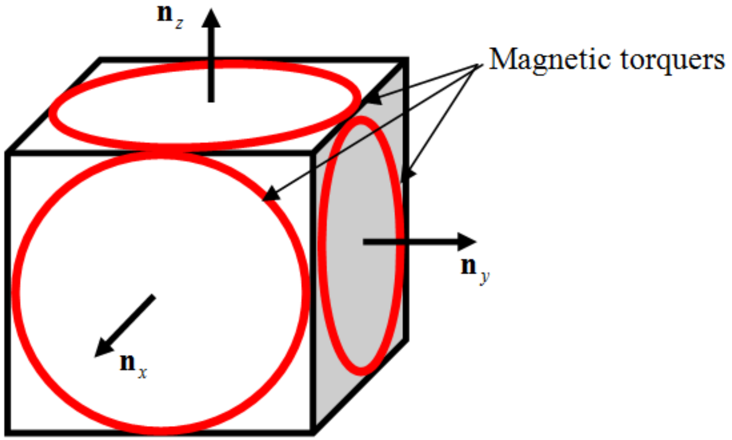
  Labibian, A., Pourtakdoust, S. H., Kiani, M., Sheikhi, A. A., and Alikhani, A., “Experimental validation of a novel radiation based model for spacecraft attitude estimation,” *Sensors and Actuators A: Physical*, vol. 250, 2016, pp. 114–122.

- **Measurements of electromotive force induced in magnetic torquers**

  Considered in present work

Problem Statement

- Consider a free motion of 1U CubeSat with a three orthogonal magnetic torquers
- The satellite rotates in the Earth geomagnetic field



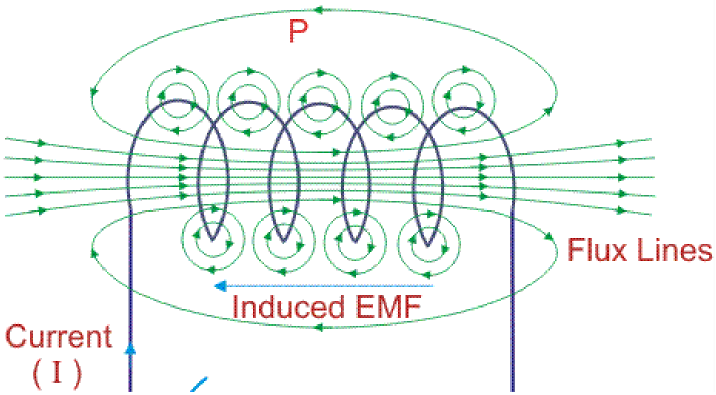
- It is necessary to determine attitude motion using measurements of electromotive force (EMF) induced in the magnetic torquers

Magnetic Torquers Measurement Model

- Magnetic torquers during free attitude motion act like three-axis induction coil sensor
- The EMF induced in torquers:

$$\mathbf{V} = -N\frac{d\Phi}{dt} + \delta\mathbf{V} = -NS\frac{d\mathbf{B}}{dt} + \delta\mathbf{V},$$

$\delta\mathbf{V}$ is measurement noise



Scheme of measurements

- In the case of rod-type magnetorquers:

$$\mathbf{V} = -NS\mu\frac{d\mathbf{H}}{dt} + \delta\mathbf{V}$$

Extended Kalman Filter

The satellite motion model

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},t) + \mathbf{q}_k,$$

$$\mathrm{M}(\mathbf{q}_k) = 0,\ \mathrm{M}\left(\mathbf{q}_k\mathbf{q}_k^T\right) = Q_k,$$

The measurement model

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, t) + \mathbf{r}_k.$$

$$\mathrm{M}(\mathbf{r}_k) = 0,\ \mathrm{M}\left(\mathbf{r}_k\mathbf{r}_k^T\right) = R_k.$$

Linearization

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}(\mathbf{x},t)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^-}, \mathbf{H}_k = \left.\frac{\partial \mathbf{h}(\mathbf{x},t)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^-}$$

$$\mathbf{\Phi}_k = \mathbf{E} + \mathbf{F}_k(t_k - t_{k-1}).$$

Prediction stage

1. Project the state ahead

$$\hat{\mathbf{x}}_k^- = \hat{\mathbf{x}}_{k-1}^+ + \int_{t_{k-1}}^{t_k} \mathbf{f}(\mathbf{x},t)dt,$$

2. Project the error covariance ahead

$$\mathbf{P}_k^- = \mathbf{\Phi}_k \mathbf{P}_{k-1}^+ \mathbf{\Phi}_k^{\mathrm{T}} + \mathbf{Q}_k,$$

Correction stage

1. Compute the Kalman Gain

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^{\mathrm{T}} (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1},$$

2. Update the estimate via $\mathbf{z}_k$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_k^-, t)],$$

3. Update the error covariance

$$\mathbf{P}_k^+ = [\mathbf{E} - \mathbf{K}_k \mathbf{H}_k]\mathbf{P}_k^-.$$

Initial state $\mathbf{x}_0$,

Initial error covariance $\mathbf{P}_0$, $\mathbf{Q}$, $\mathbf{R}$

Kalman Filter
Based on EMF Measurements

State vector: $\mathbf{x}(t) = [\mathbf{q}(t)\ \boldsymbol{\omega}(t)]^T$

Attitude motion equations: $\mathbf{J}\dot{\boldsymbol{\omega}}_{si} = \mathbf{N}_{gg} + \mathbf{N}_{dist} - \boldsymbol{\omega}_{si} \times \mathbf{J}\boldsymbol{\omega}_{si}$

$$\dot{\mathbf{q}}_{so} = \frac{1}{2}\mathbf{q}_{so} \circ \boldsymbol{\omega}_{so},$$

Linearized motion equations:

$$\delta\dot{\mathbf{x}} = \mathbf{F}\delta\mathbf{x},\ \mathbf{F}(t) = \begin{bmatrix} -\mathbf{W}_{\omega} & \frac{1}{2}\mathbf{E}_{3\times3} \\ \mathbf{J}^{-1}\left(k\mathbf{F}_{gr}\right)_{3\times3} & -\mathbf{J}^{-1}\mathbf{F}_{gir} \end{bmatrix}_{6\times6}$$

Measurement model:

$$\mathbf{z} = -NS\frac{d(A\mathbf{B}_{orb})}{dt} + \boldsymbol{\eta}_V = -NS\left[-\boldsymbol{\omega} \times \mathbf{B} + A\left(\boldsymbol{\omega}_0 \times \mathbf{B}_{orb}\right)\right] + \boldsymbol{\eta}_V$$

Linearized measurement model:

$$\delta\mathbf{z} = \mathbf{H}\delta\mathbf{x}, \quad -NS\left[-2\mathbf{W}_{\boldsymbol{\omega}}\mathbf{W}_{\hat{\mathbf{B}}} - 2\mathbf{W}_{A\dot{\mathbf{B}}_{orb}} \quad 2\mathbf{W}_{\hat{\mathbf{B}}}\right].$$

Numerical simulation

Inertia tensor:

$$\mathbf{J} = \mathrm{diag}\left(5\cdot10^{-3}, 6\cdot10^{-3}, 7\cdot10^{-3}\right)\ \mathrm{kg}\cdot\mathrm{m}^2$$

Orbit altitude: $h = 400$ km

Orbit inclination: $i = 51.7\,\mathrm{deg}$

Initial angular velocity:

$$\boldsymbol{\omega}(t=0) = \left(10\omega_0, 10\omega_0, 10\omega_0\right),\ \omega_0 = 0.06\ \mathrm{deg}/\,s.$$

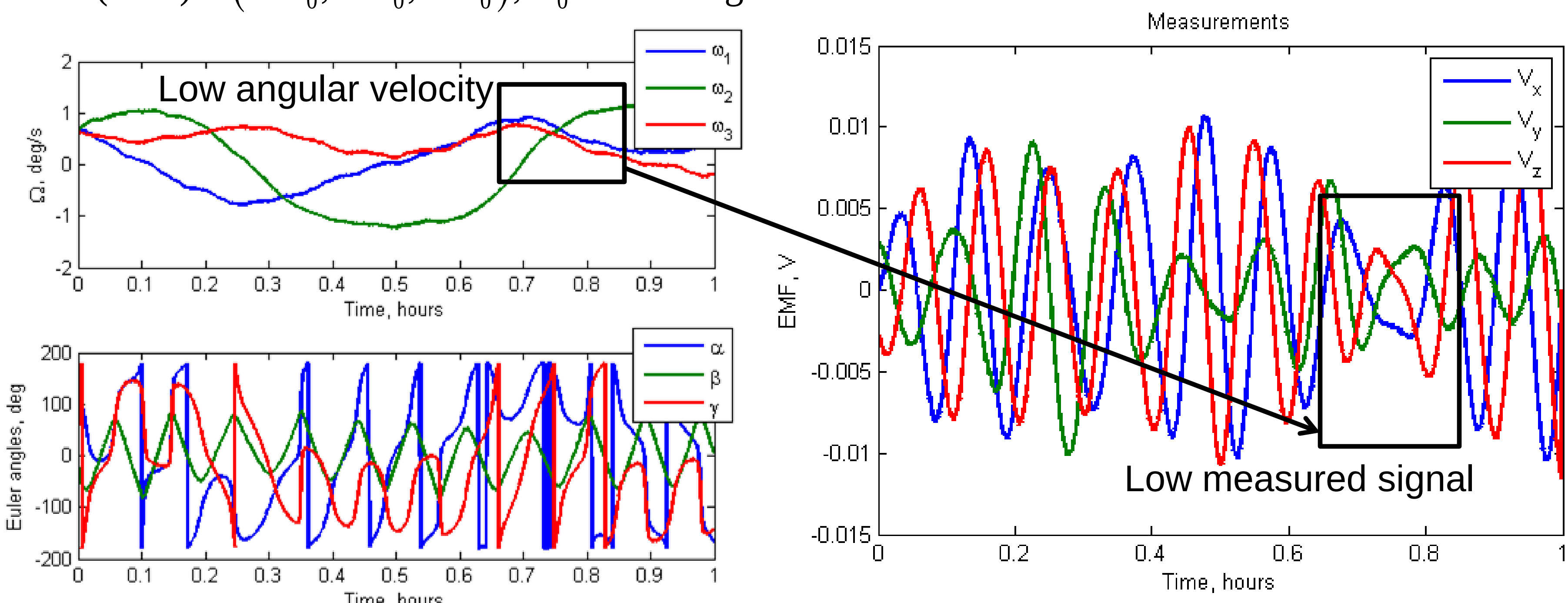
Three ortogonal rod-type magnetorquers

Rod diameter $d = 5.7$ mm

Relative permeability $\mu_r = 75000$

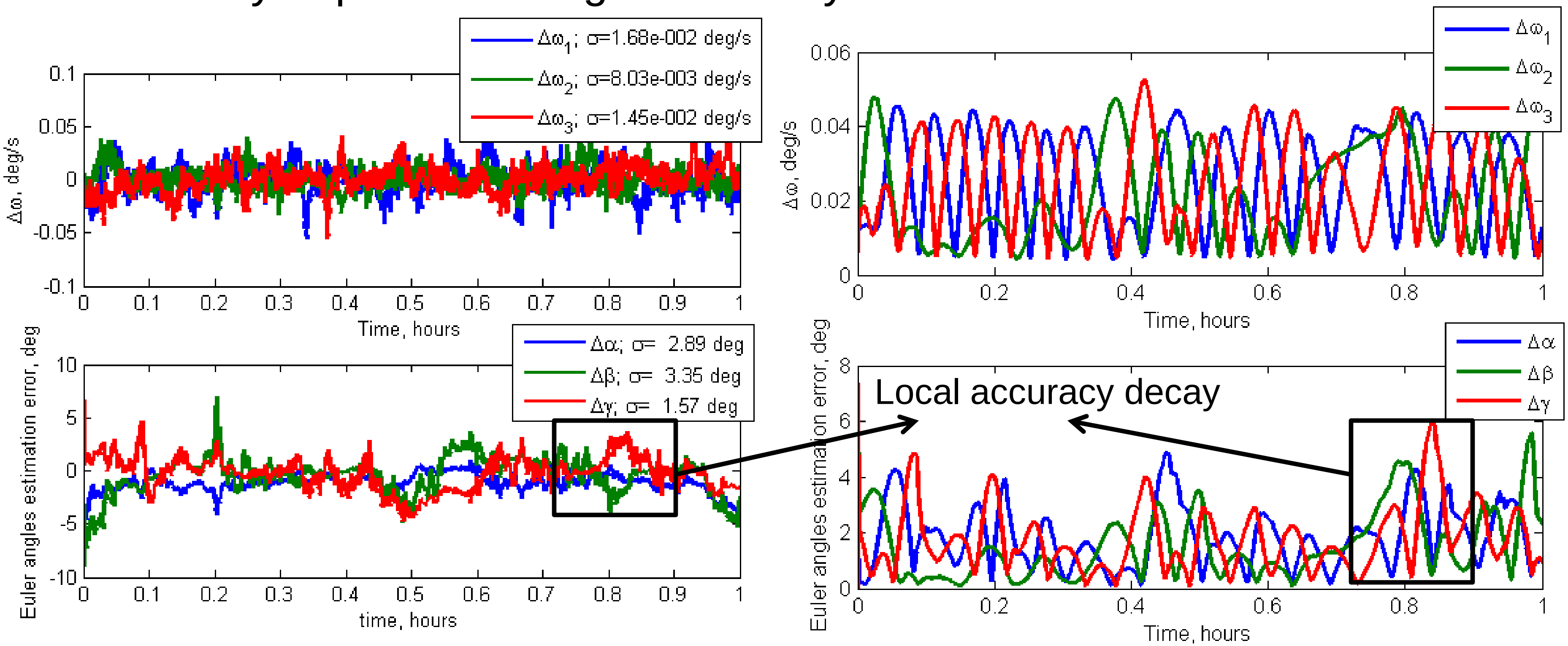
Number of turns $N = 6000$ each

Standard measurement deviation $\sigma_{meas} = 50\mu V$



Estimation Accuracy

- Accuracy depends on angular velocity



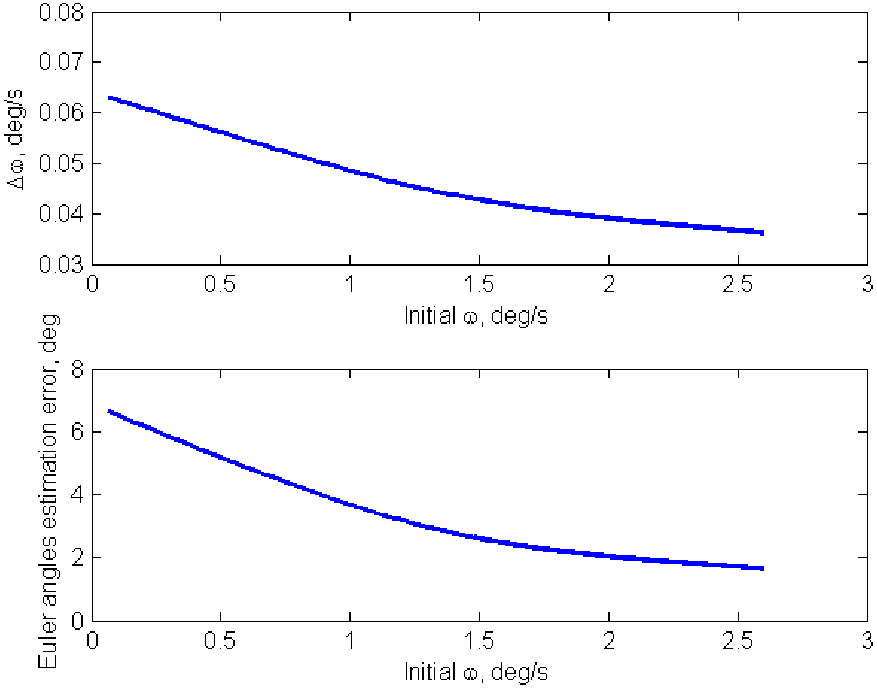
Attitude motion determination accuracy calculated as deviation of estimation from modeled state vector

$$\delta \mathbf{x} = \mathbf{x} - \hat{\mathbf{x}}$$

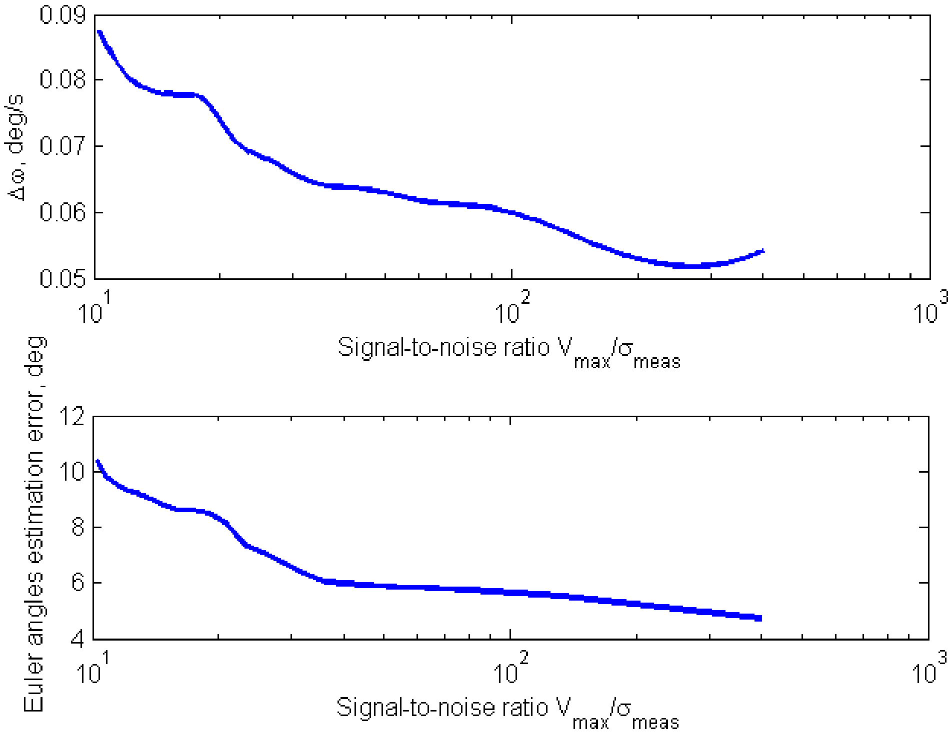
Attitude motion determination accuracy calculated from covariance matrix

$$\sigma_i = \sqrt{p_{ii}}, \; i = 1,\dots,6$$

Accuracy Study



The worst accuracy of attitude motion estimation dependence on initial angular velocity



The worst accuracy of attitude motion estimation dependence on signal-to-noise ratio

Conclusions

- The algorithm is able to determine the attitude motion using EMF measurements induced in magnetic torquers
- The estimated state vector accuracy is about several degrees in attitude and about ω_0 in angular velocity
- It is not very accurate for general application but usually CubeSat missions do not require precise orientation
- When attitude sensors are out of order due to a failure the algorithm can save the mission

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Thanks for your attention!