Keldysh Institute of Applied Mathematics
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Fuelless Means of Reaction Wheels Desaturation

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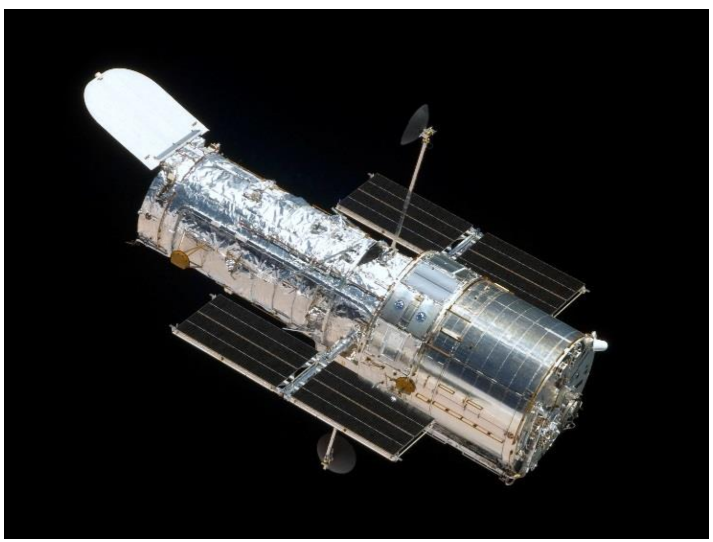
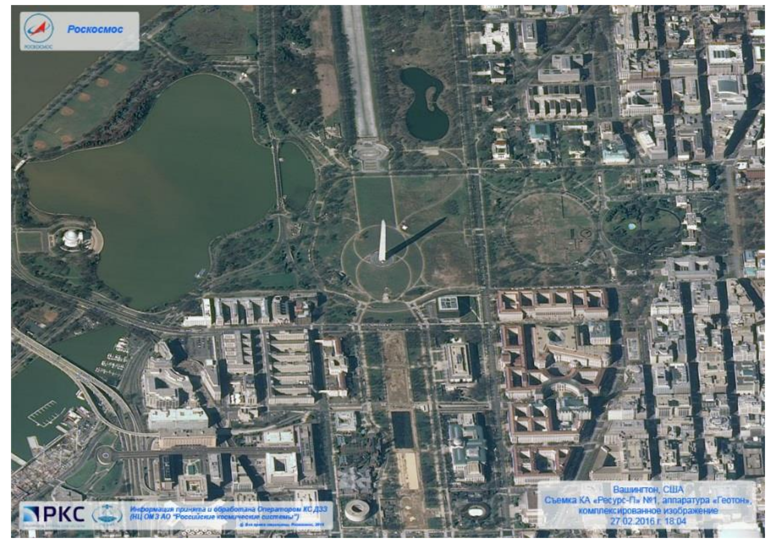
Introduction

Why do we need to control attitude?

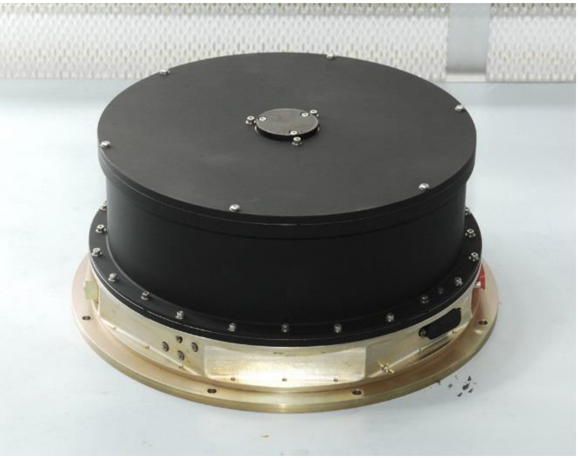
- Remote sensing
- Space telescopes
- Communication satellites
- Solar stabilization

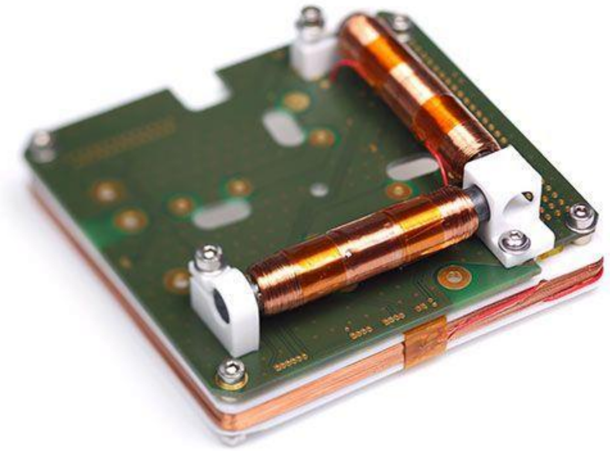




How we can do that?

- Reaction wheels
- CMGs
- Thrusters
- Magnetorquers





Problems

- Saturation of RW
- Singularity of CMG
- Propellant usage of thrusters
- Necessity of magnetic field presence for magnetorquers

Why is there a saturation?

External torques compensation:

- Gravitational torque

$$\mathbf{M}_G = 3\frac{\mu_E}{r^5}\mathbf{r}\times\mathbf{J}\mathbf{r}$$

μ_E is gravitational parameter, $\mathbf{r}$ is satellite position, $\mathbf{J}$ is tensor of inertia

- Solar radiation pressure torque

$$\mathbf{M}_S = \mathbf{R}\times\left[-S\frac{\Phi_0}{c}\left(\mathbf{r}_s,\mathbf{n}\right)\left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s,\mathbf{n}) + \alpha(1-\mu)\left[\mathbf{r}_s + \frac{2}{3}\mathbf{n}\right]\right)\right]$$

$\mathbf{R}$ is solar panel position, S is its area, $\mathbf{n}$ is normal to SP, α, μ are reflectivity and specularity coefficients.

Problem statement

What do we know?

- Satellite moves along highly elliptical keplerian orbit
- Two identical rigidly fixed solar panels
- Reaction wheel attitude control

What do we want?

- Desaturate RW
- Recharge batteries

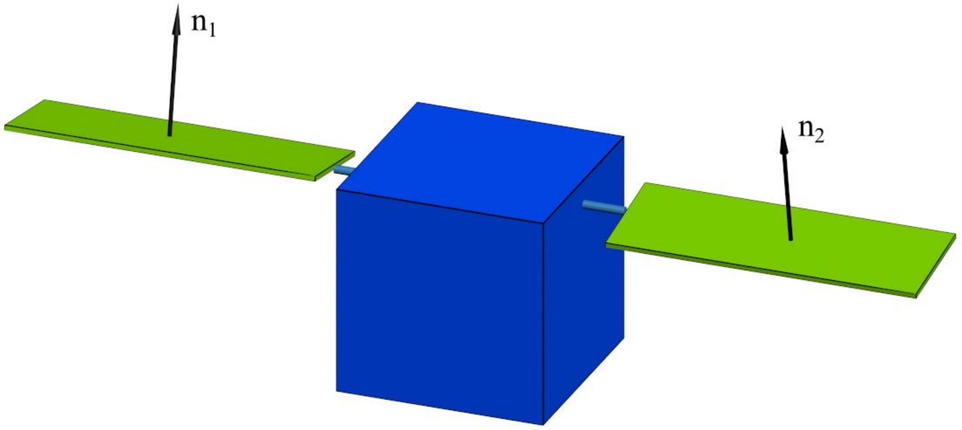
Solar radiation pressure torque

Two solar panels, hence more complex torque equation:

$$\mathbf{M}_i = \mathbf{R}_i \times \left[-S\frac{\Phi_0}{c}(\mathbf{r}_s, \mathbf{n}_i)\left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s, \mathbf{n}_i)\mathbf{n}_i + \alpha(1-\mu)\left[\mathbf{r}_s + \frac{2}{3}\mathbf{n}_i \right] \right) \right],\ i = 1, 2,$$

$$\mathbf{M}_S = \mathbf{M}_1 + \mathbf{M}_2$$

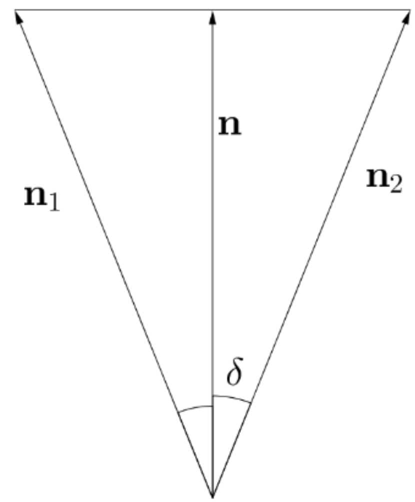
$\mathbf{n}_1 \neq \mathbf{n}_2$ in general case



Solar radiation pressure torque

Let $\delta << 1$, so is the angle between solar vector and $\mathbf{n}$:

$$\mathbf{M}_s \approx \mathbf{R} \times 2a\mathbf{r}_s + \left[\mathbf{R} \times 2\mathbf{n} + \boldsymbol{\rho} \times 2\boldsymbol{\nu}\right]\left(b + d\right).$$



where

$$a = -S\frac{\Phi_0}{c}(1-\alpha\mu), \quad b = -S\frac{2\Phi_0}{3c}\alpha(1-\mu), \quad d = -2S\frac{\Phi_0}{c}\alpha\mu.$$

$$\mathbf{R} = \frac{1}{2}\left(\mathbf{R}_1 + \mathbf{R}_2\right), \quad \boldsymbol{\rho} = \frac{1}{2}\left(\mathbf{R}_1 - \mathbf{R}_2\right),$$

$$\mathbf{n} = \frac{1}{2}\left(\mathbf{n}_1 + \mathbf{n}_2\right), \quad \boldsymbol{\nu} = \frac{1}{2}\left(\mathbf{n}_1 - \mathbf{n}_2\right),$$

How will we desaturate?

Our goal is to achieve specific nominal angular momentum $\mathbf{K}_n$

Consider:

$$\frac{d}{dt}\left|\mathbf{K}-\mathbf{K}_n\right|^2=\frac{d}{dt}\left(K^2+K_n^2-2\left(\mathbf{K}_n,\mathbf{K}\right)\right)=\frac{d}{dt}\left(K^2\right)-2\left(\mathbf{K}_n,\frac{d}{dt}\mathbf{K}\right)$$

where $\mathbf{K}$ is total angular momentum. Using

$$\frac{d\mathbf{K}}{dt}=\mathbf{M}_s,$$

we get

$$\frac{d}{dt}\left|\mathbf{K}-\mathbf{K}_n\right|^2=2\left(\mathbf{M}_s,\mathbf{K}-\mathbf{K}_n\right)$$

How will we desaturate?

Therefore, we have to ensure

$$\left(\mathbf{M}_s, \mathbf{K} - \mathbf{K}_n\right) < 0$$

for desaturation. For the fastest desaturation

$$\left(\mathbf{M}_s, \mathbf{K} - \mathbf{K}_n\right) \to \min$$

This problem will allow us to find the necessary attitude. Since solar torque is small, we will provide inertial stabilization and update it once in a while

Minimization problem solution

After mathematics

$$q_3 K_3 + K_2\left(q_2 \cos(\varphi+\psi) + \sin(\varphi+\psi)(q_1 + p_2)\right) +$$

$$+\theta\left(K_3\left(q_1 \sin(\varphi) + q_2 \cos(\varphi)\right) - K_2\left(p_3 \sin(\psi) + q_3 \cos(\psi)\right)\right) \to \min_{\psi,\theta,\varphi},$$

$$-\theta_{max} \le \theta \le \theta_{max}$$

$$\mathbf{p} = 2a\mathbf{R}, \quad \mathbf{q} = \left[\mathbf{R} \times 2\mathbf{n} + \boldsymbol{\rho} \times 2\boldsymbol{\nu}\right](b+d)$$

where K_i elements of $\mathbf{K} - \mathbf{K}_n$. Using Lagrange multiplier method and Karush-Kuhn-Tucker conditions, we will obtain system of equations for minimum point. But it is hard to find its solution

Minimization problem solution

Consider

$$\mathbf{q} = \left[\mathbf{R} \times 2\mathbf{n} + \boldsymbol{\rho} \times 2\mathbf{v}\right](b+d).$$

Since $\mathbf{v} << 1$ and b, d are proportional to reflectivity, which is small for solar panels, we can rewrite minimization problem

$$f \sin(\varphi+\psi) + \theta\left(g \sin(\varphi) - h \sin(\psi)\right) \to \min_{\psi,\theta,\varphi}$$

There is always an optimal solution if

$$\theta = \pm\theta_{max}$$

and simple (but not optimal) solution

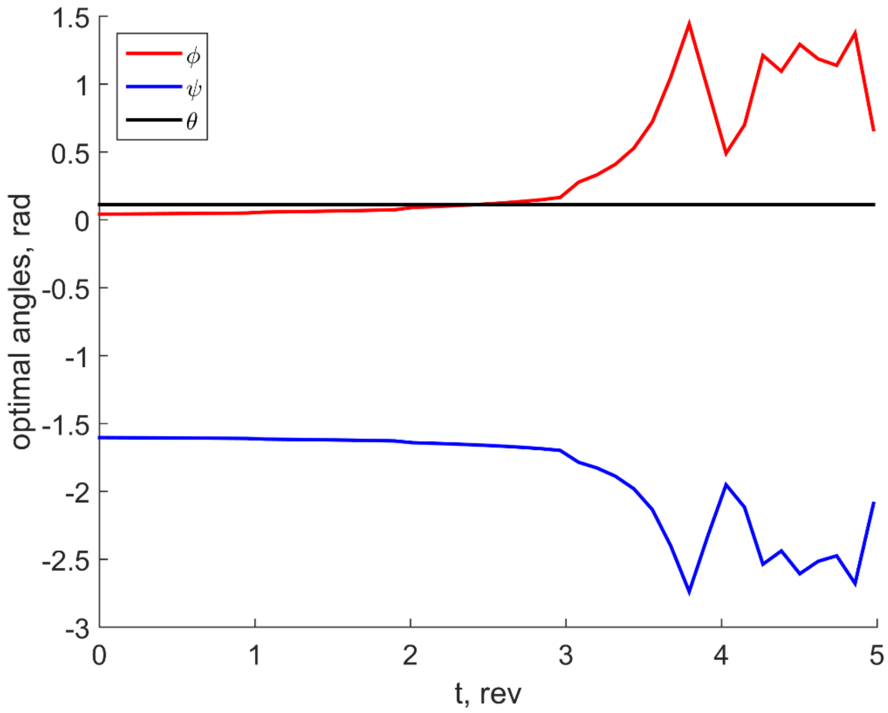
$$\psi = -\frac{\pi}{2} - \gamma_0, \quad \theta = \theta_{max}, \quad \varphi = \left(1 - \mathrm{sign}(f)\right)\frac{\pi}{2} + \gamma_0$$

$$\left(\mathbf{M}_s, \mathbf{K} - \mathbf{K}_n\right) = -\left|f\right| - \theta_{max}\sqrt{g^2 + h^2}$$

Simulation results

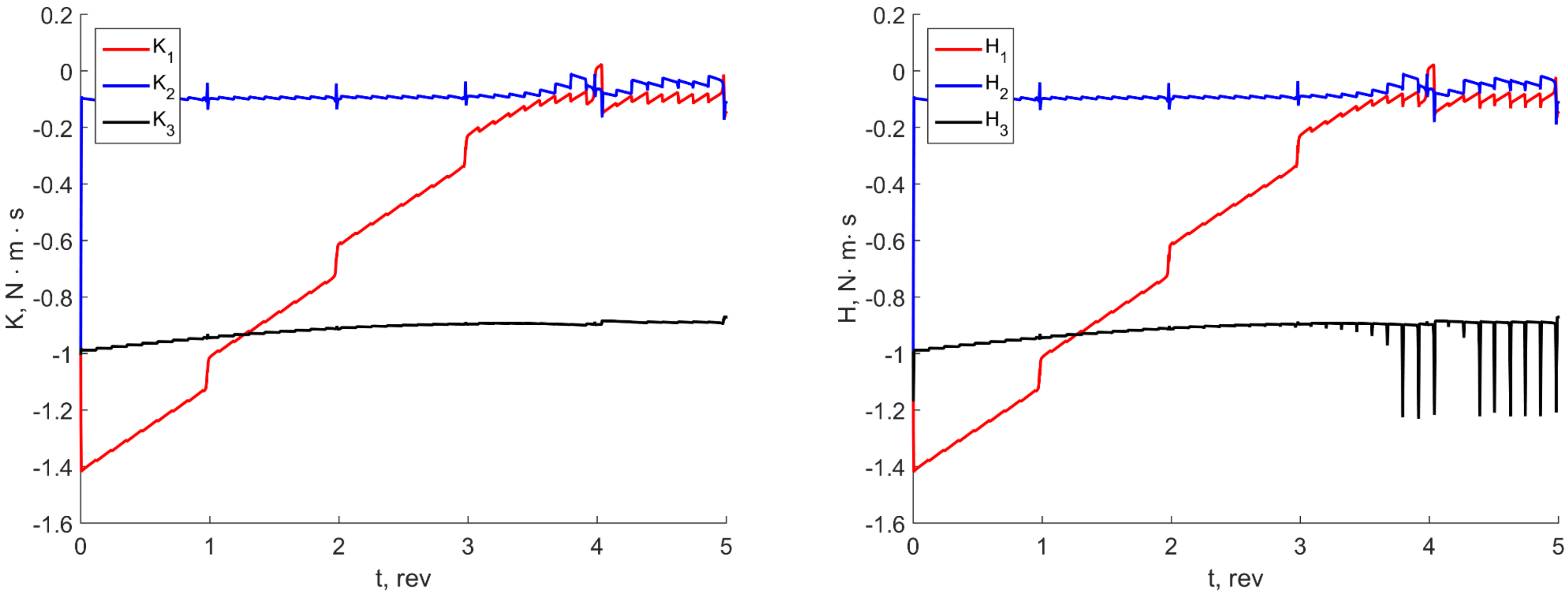
- $r_\pi = 8\,000\ \mathrm{km}, \quad r_a = 200\,000\ \mathrm{km}, \quad i = 60^o$
- $\mathbf{J} = \mathrm{diag}(150, 120, 200)\ \mathrm{kg}\cdot\mathrm{m}^2$
- Initial angular velocity is zero
- Attitude updates every 40 000 s
- Initial stored momentum is $\mathbf{H}_0 = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix}\ \mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}.$
- Nominal angular momentum is $\mathbf{K}_n = \mathbf{0}$
- Total solar panel area is $3\ \mathrm{m}^2$
- Reflectivity is 0.1, specularity is 0.5
- Lyapunov based control algorithm is used to provide the necessary attitude

Simulation results



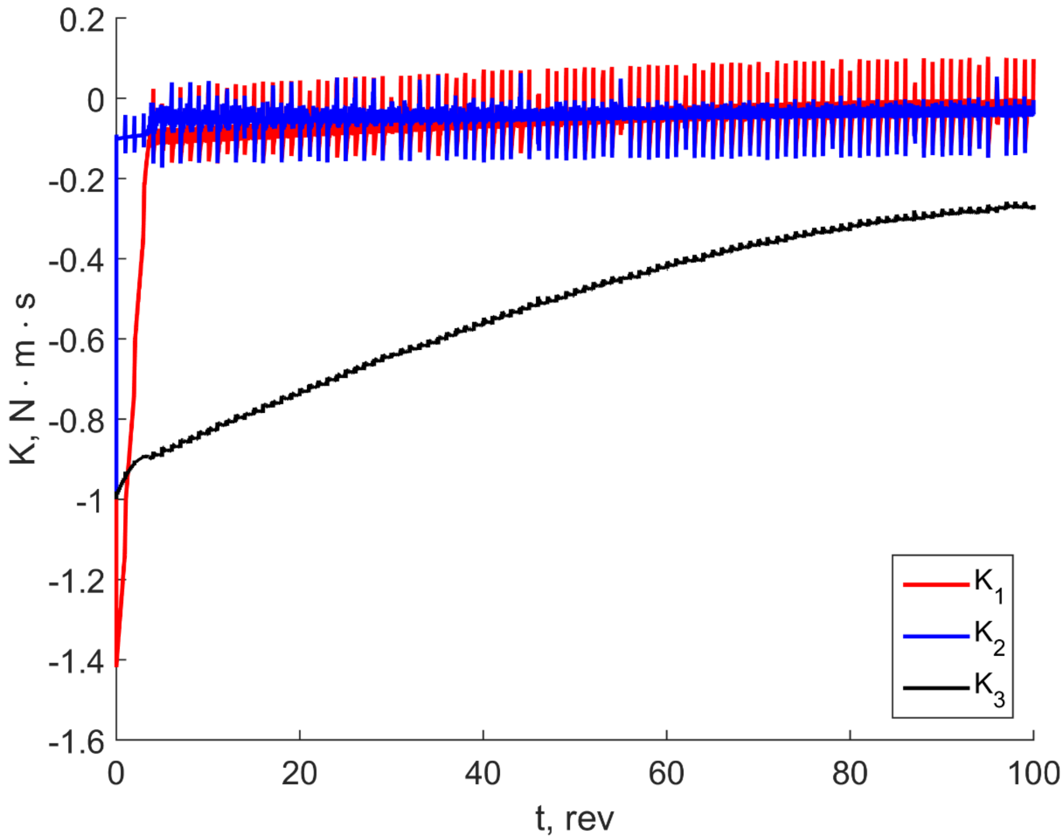
Solution of minimization problem. One revolution is about four days

Simulation results



Total angular momentum and angular momentum stored in reaction wheels.
Black line corresponds to angular momentum along solar vector

Simulation results



Total angular momentum for a one year mission

Slow rate of angular momentum decreasing along solar vector is because solar torque along this axis is small and there is constant torque along this axis

Conclusion

- Complex and simplified models of solar radiation pressure torque in case of two identical solar panels are obtained
- It was shown that there is always a solution for minimization problem for reaction wheels desaturation
- If normals to solar panels coincide, it is always possible to desaturate reaction wheels