Study of the accuracy of active magnetic damping algorithm

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Main assumptions

- Rigid body
- Circular orbit
- Geomagnetic field is represented with direct and simplified dipoles
- Euler angles (rotation sequence 3-1-2) describe satellite attitude
- Satellite moves in inertial space
- External disturbances are not considered

Motion nature

- Satellite motion is governed by -Bdot damping algorithm $\mathbf{m} = -k\, d\mathbf{B}/dt$
- Field variation is largely determined by satellite rotation in transient motion
- Satellite rotation velocity is close to the double orbital one after transient motion. Inherent field rotation in absolute space should be taken into account
- Resulting rotation velocity, axis of rotation and its attitude in inertial space present some interest

Rotation stability

- Simplified dipole model is used (uniform field vector rotation along the circular cone)
- Rotation along third satellite axis with double orbital velocity exists
- Linearized near this rotation equations of motion

$$\dot{\omega}_1 = 2\lambda_A \omega_2 - \theta_A \varepsilon \left(\omega_1 + 2\gamma \right),$$

$$\dot{\omega}_2 = 2\lambda_B \omega_1 + \theta_B \varepsilon \left(-\omega_2 \cos^2 \Theta + \omega_3 \sin \Theta \cos \Theta + 2 \cos^2 \Theta \beta \right),$$

$$\dot{\omega}_3 = \varepsilon \left(\omega_2 \cos \Theta \sin \Theta - \omega_3 \sin^2 \Theta - 2 \sin \Theta \cos \Theta \beta \right),$$

$$\dot{\alpha} = \omega_3, \qquad \dot{\beta} = \omega_1 + 2\gamma, \qquad \dot{\gamma} = \omega_2 - 2\beta.$$

Characteristic exponents

- Equations of motion and secular equation are

$$\dot{\mathbf{x}} = \mathbf{A}_0\mathbf{x} + \varepsilon\mathbf{A}_1\mathbf{x}, \quad P(\mu + \varepsilon\eta) = \sum_{j=0}^{5}\left(A_j + \varepsilon B_j\right)(\mu + \varepsilon\eta)^j = 0$$

- Zero-order approximation

$$\mu_{1,2} = \pm 2\sqrt{-\lambda_A\lambda_B}\,i, \quad \mu_3 = 0, \quad \mu_{4,5} = \pm 2i.$$

- First order approximation

$$\eta_{1,2} = -\theta_A\lambda_B(1+\lambda_A) + \theta_B\cos^2\Theta\lambda_A(\lambda_B - 1)/2(\lambda_A\lambda_B + 1), \; \eta_3 = -\sin^2\Theta,$$

$$\eta_{4,5} = -\theta_A(1-\lambda_B) + \theta_B\cos^2\Theta(\lambda_A + 1)/2(\lambda_A\lambda_B + 1)$$

- Rotation along maximum moment of inertia is stable

Pointing accuracy

- Simplified dipole provides precise rotation axis alignment along geomagnetic field cone axis
- Direct dipole model is used to refine rotation velocity, direction and pointing accuracy
- Satellite motion is characterized with periodical solutions
- Inclination deviation from 0 and 90° is used as small parameter

Subequatorial orbit

- Equations of motion have the form

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}\right) + i\mathbf{g}_1\left(\mathbf{x}\right) + i^2\mathbf{g}_2\left(\mathbf{x}\right)$$

- Asymptotically stable generating solution

$$\mathbf{x}_0 = \left(\Omega u,\ 0,\ 0,\ 0,\ 0,\ \Omega\right)$$

- Rotation velocity is 9/5 of the orbital one. This is determined from the existence of planar periodical solutions of the second order equations

$$\dot{\omega}_3^{(2)} = 3\varepsilon\left(\Omega - 1\right)\cos 2u - 5\Omega + 9,\ \dot{\alpha}^{(2)} = \omega_3^{(2)}$$

First approximation

- Spatial motion first order approximation equations

$$\dot{\mathbf{z}}_1 = \mathbf{F}\mathbf{z}_1 + \varepsilon \begin{pmatrix} \theta_A \left[3/2 (\Omega - 2) \sin(\Omega - 2) u - 1/2 \sin \Omega u \right] \\ \theta_B \left[3/2 (\Omega - 2) \cos(\Omega - 2) u - 1/2 \cos \Omega u \right] \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} -\varepsilon\theta_A & \Omega\lambda_A & 0 & -\varepsilon\Omega\theta_A \\ \Omega\lambda_B & -\varepsilon\theta_B & \varepsilon\Omega\theta_B & 0 \\ 1 & 0 & 0 & \Omega \\ 0 & 1 & -\Omega & 0 \end{pmatrix}$$

- Their solution is

$$\beta^{(1)} \approx 1/2 \sin \Omega u, \;\; \gamma^{(1)} \approx -1/2 \cos \Omega u$$

- Rotation axis deviates form the orbital normal by up to approximately half the inclination angle

Circumpolar orbit

- Rotation velocity is governed by the equation

$$\dot{\omega}_3 + \varepsilon\left(1+3\sin^2 u\right)\omega_3 = 3\varepsilon\left(1+\sin^2 u\right)$$

- Approximate solution is

$$\omega_3 \approx e^{-\varepsilon\left(\frac{5}{2}u-\frac{3}{4}\sin 2u\right)}\left(\omega_3(0)+6\varepsilon\left[\frac{e^{5/2\varepsilon u}\sin u}{25\varepsilon^2+16}(5\varepsilon\sin u-4\cos u)\right.\right.$$

$$\left.\left.+\frac{1}{\varepsilon}\left(\frac{8}{125\varepsilon^2+80}+\frac{1}{5}\right)\left(e^{5/2\varepsilon u}-1\right)\right]\right)$$

- On a long time interval

$$\omega_3 = 9/5 + 3/5\,\varepsilon\sin 2u + O\left(\varepsilon^2\right)$$

First approximation

- First order equations are further analyzed with Poincare method. Torque value is used as a small parameter.
- First order solution is a generating one for that purpose. It is found from the existence of periodical solutions of equations of the next order,

$$\omega_1 = \omega_2 = 0,$$

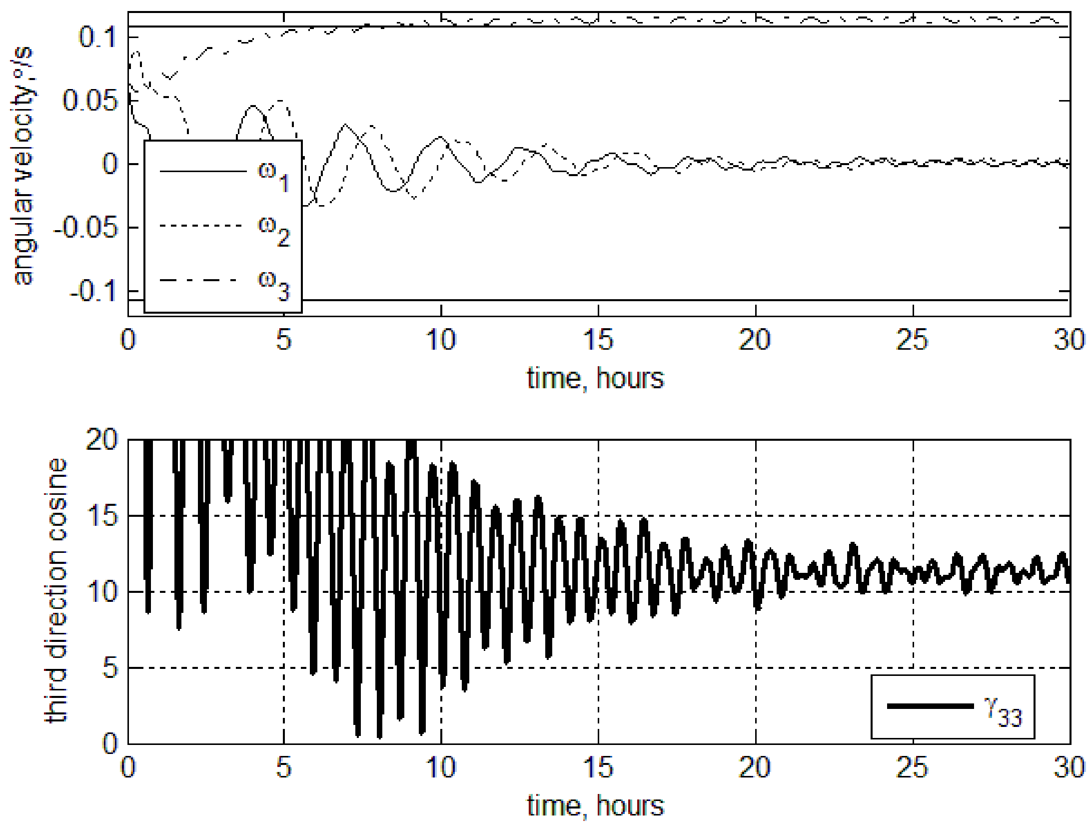
$$\beta = 4/9\left(\pi/2 - i\right)\cos 1.8u,\ \gamma = -4/9\left(\pi/2 - i\right)\sin 1.8u$$

- Rotation axis deviates form the orbital normal by approximately $4/9\left(\pi/2 - i\right)$

Numerical example

Inertia tensor(1.4, 1.6, 2.0) kg·m^2, altitude 750 km, inclination 75°, $\varepsilon \approx 0.11$

Numerical rotation axis deviation 5.5-7°, approximate 6.5°



Results

- Rotation along the maximum moment of inertia is shown, rotation velocity is found
- Stability is proven for the motion in simplified geomagnetic field
- Rotation velocity, direction and stabilization accuracy are found for subequatorial and circumpolar orbits in direct dipole geomagnetic field