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Orbital Dynamics of Formation Flying Under Mass-Exchange Novel Control

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Fuelless FF Control Concepts

- Tethered satellite formations
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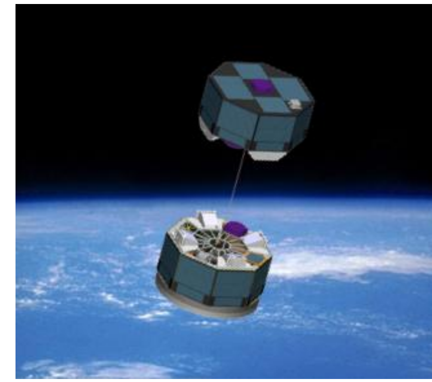
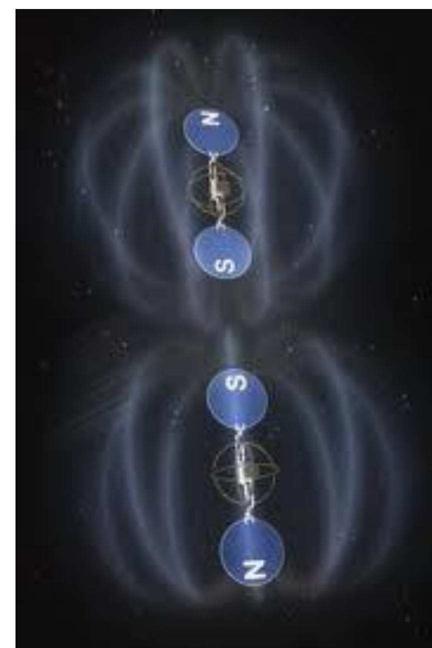




Momentum exchange FF Control Concepts

- One satellite creates momentum by ejecting an additional separable mass
- The other one then captures and redirects it back
- The repulsive force is obtained

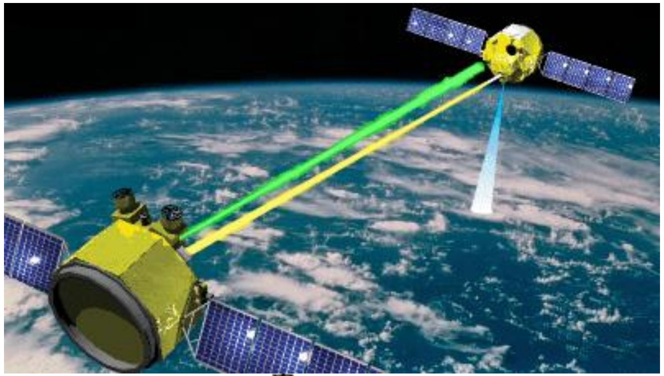
The former research

- The momentum from lasers for repulsive force

  Y. K. Bae. A contamination-free ultrahigh precision formation flying method for micro-, nano-, and pico-satellites with nanometer accuracy. In Space Technology and Applications International Forum- Staif 2006, volume 813, pages 1213–1223, 2006.

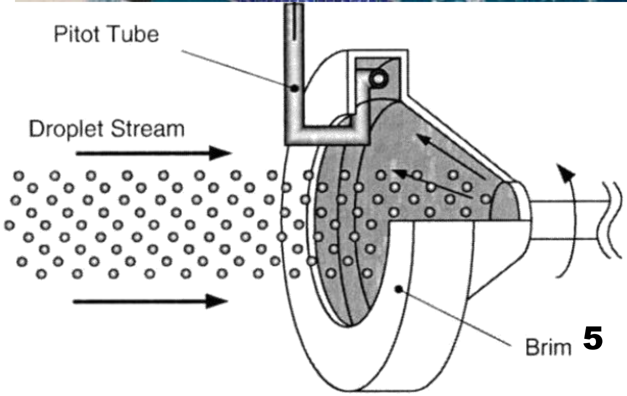
- Continuous stream of mass travelling between the satellites

  S. G. Tragesser. Static formations using momentum exchange between satellites. Journal of guidance, control, and dynamics, 32(4):1277 –1286, 2009.



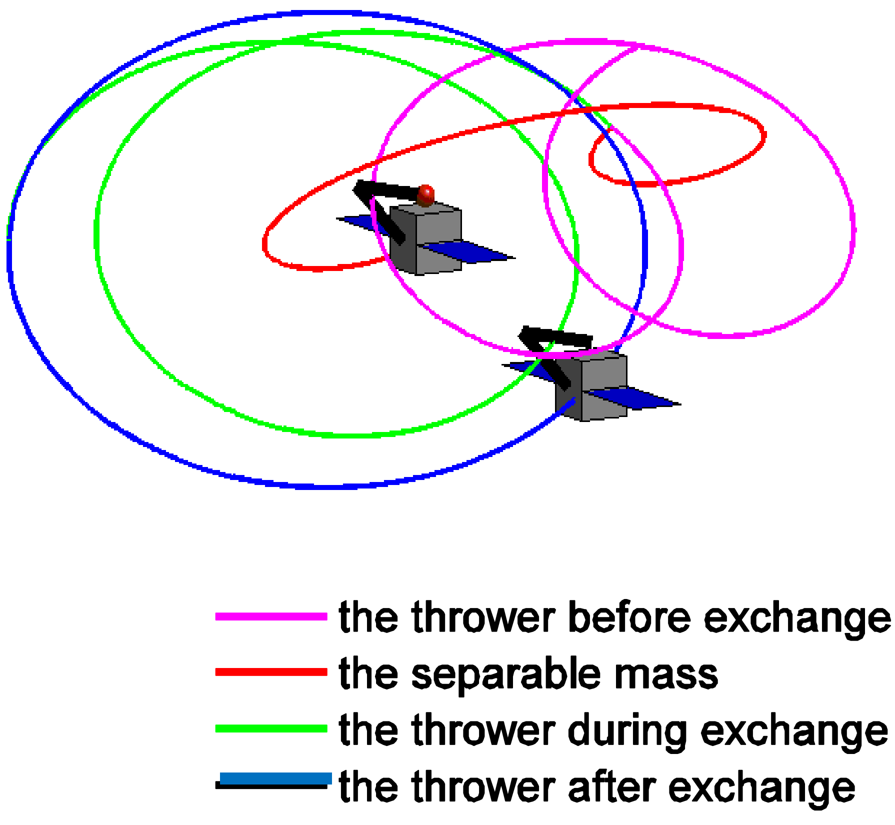
- Liquid droplet streams exchange

  T. Joslyn and A. Ketsdever. Constant momentum exchange between microspacecraft using liquid droplet thrusters. In 46th joint Propulsion Conference, volume 6966, pages 25–28, 2010.



Single mass exchange control concept

- By command the single mass separates from the satellite
- The separated mass moves to the other satellite and impacts it absolutely inelastically
- After the whole mass transfer the resulting relative trajectory changes in adjustable way



The Problem Formulation

Boundary problem:

What is the initial relative velocity of the mass required to hit the thrower?

Initial conditions:

$$x_0 = x(t_0),\ y_0 = y(t_0),\ z_0 = z(t_0),$$

The final position:

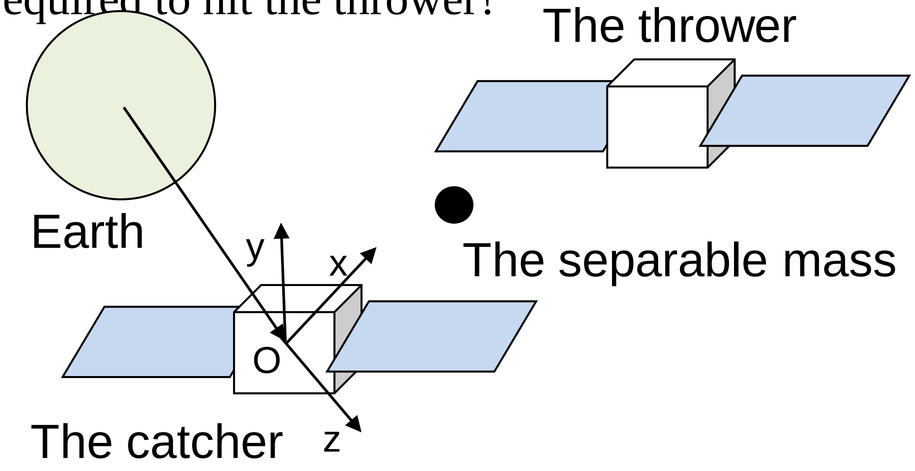
$$x_1 = x\left(t_1\right) = 0,\ y_1 = y\left(t_1\right) = 0,\ z_1 = z\left(t_1\right) = 0.$$

Hill - Clohessy - Wiltshire equations:

$$\ddot{x} + 2\omega\dot{z} = 0,$$

$$\ddot{y} + \omega^2 y = 0,$$

$$\ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0$$



The exact solution:

$$x = C_4 - 3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t,$$

$$y = C_5\sin\omega t + C_6\cos\omega t,$$

$$z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t$$

$$C_1 = 2z(t_0) + \frac{\dot{x}(t_0)}{\omega}, C_2 = \frac{\dot{z}(t_0)}{\omega},$$

$$C_3 = -3z(t_0) - \frac{2\dot{x}(t_0)}{\omega}, C_4 = x(t_0) - \frac{2\dot{z}(t_0)}{\omega},$$

$$C_5 = \frac{\dot{y}(t_0)}{\omega}, C_6 = y(t_0).$$

The Analytical Problem Solution

Throwing mass relative velocity:

$$\delta\dot{x} = -\dot{x}_0 - 2z_0\omega + \frac{1}{\Delta}[x_0\omega\sin u + 2z_0\omega(\cos u - 1)],$$

$$\delta\dot{y} = -\dot{y}_0 - y_0\omega\frac{\cos u}{\sin u},$$

$$\delta\dot{z} = -\dot{z}_0 - \frac{1}{\Delta}[2x_0\omega(1-\cos u) + z_0\omega(3u\cos u - 4\sin u)],$$

where $u = \omega(t_m - t_e)$, $\Delta = 3u\sin u - 8(1-\cos u)$.

The resulting thrower satellite velocity after mass throwing

$$\mathbf{v}_t = \mathbf{v}_{t,0} - \frac{m}{M}\delta\mathbf{v}.$$

The resulting *catcher* **satellite velocity after mass catching**

$$\mathbf{v}_c(t_m) = \frac{m}{M+m}\mathbf{v}_s(t_m).$$

For instance, the final relative trajectory $\tilde{C}_1$ constant:

$$\tilde{C}_1 = \left(2z_0 + \frac{\dot{x}_0}{\omega}\right) + \frac{k(k+2)}{(k+1)^2}\cdot\frac{x_0\cos s - 2z_0\sin s}{8\sin s - 6s\cos s}.$$

An Example: Obtaining a closed relative trajectory

Let $C_{1,c} = C_{1,t}$ for the closed relative trajectory,

then obtain equation

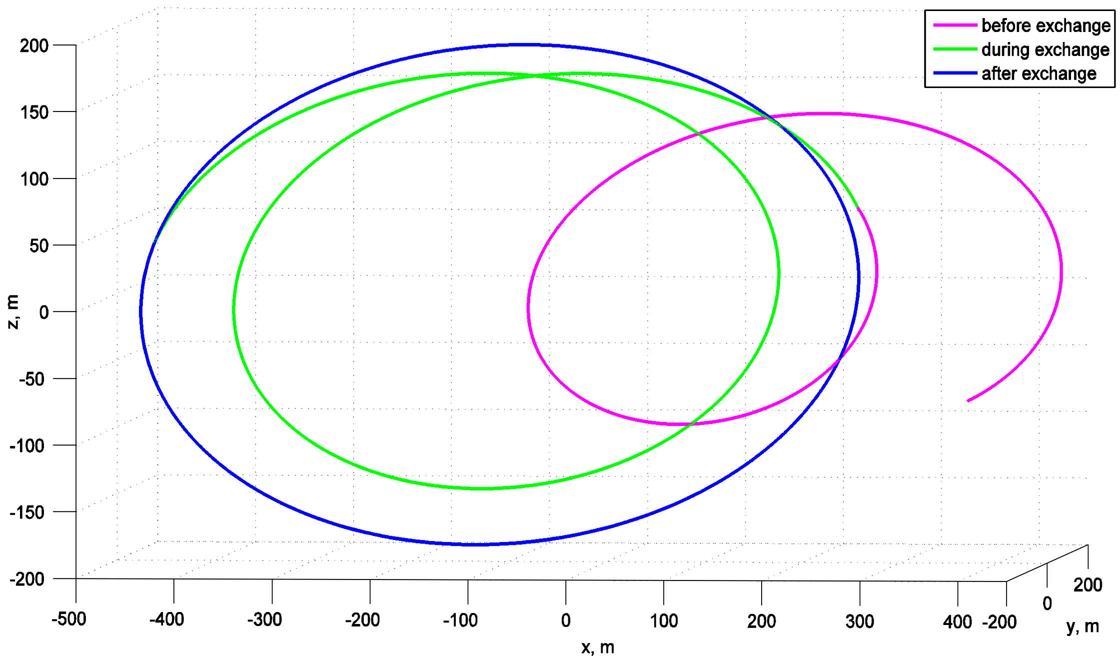
$$\left(2z_0 + \frac{\dot{x}_0}{\omega}\right) + \frac{k(k+2)}{(k+1)^2} \cdot \frac{x_0 \cos s - 2z_0 \sin s}{8\sin s - 6s\cos s} = 0.$$

Solution of equation with respect to s=$\omega(t_m - t_e)/2$

is a time required for mass exchange.

Consider an example with $C_1 = 10$m,

then obtain $t_m = 8070s, |\delta \mathbf{v}| = 0.85 m/s.$



Relative trajectory

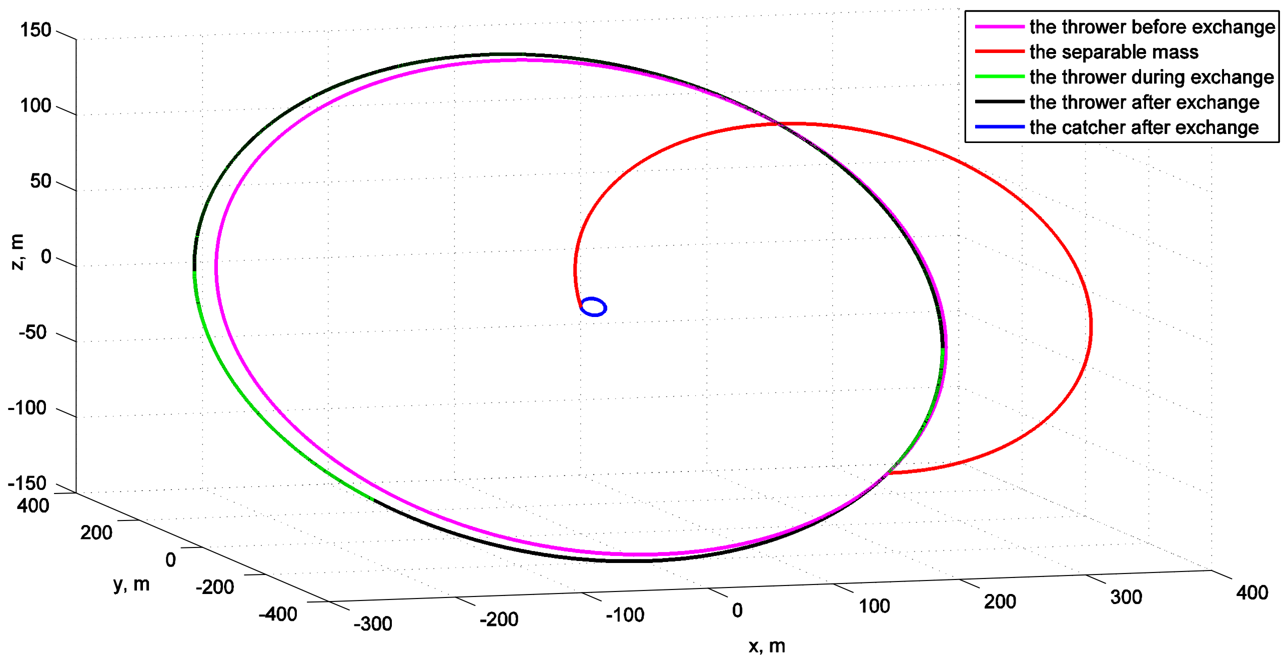
Reconfiguration

Let $C_{1,c} = C_{1,t} = 0$ for the final relative trajectory, then obtain equation

$$x_0 \cos\frac{u}{2} = 2z_0 \sin\frac{u}{2}, \frac{3u}{2}\cos\frac{u}{2} - 4\sin\frac{u}{2} \neq 0.$$

Solution of equation is a time required for mass exchange. Consider an example - circular relative trajectory

$$x^2 + y^2 + z^2 = 8a^2$$



Relative trajectory

Optimization problems

Trajectory parameters $C_1 \ldots C_6(t_e, t_m)$ are functions of the time of throwing t_e and time of catching t_m

Consider following minimization tasks:

The trajectory shape is closest to the initial one

$$\Phi_1(t_e, t_m) = (\bar{A} - A)^2 + (\bar{B} - B)^2,$$

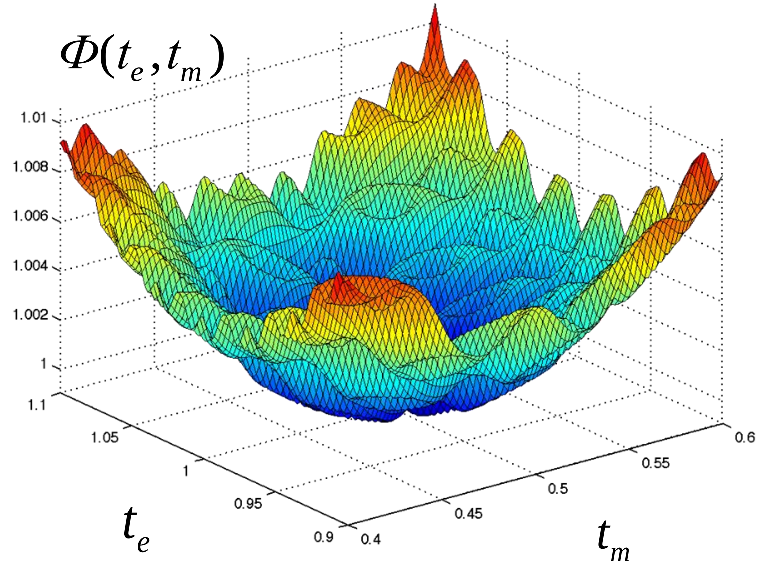
where $A = \sqrt{C_2^2 + C_3^2}$, $B = \sqrt{C_5^2 + C_6^2}$

The energy - optimum exchange

$$\Phi_2(t_e, t_m) = \|\delta \mathbf{v}\|^2 = \delta \dot{x}^2 + \delta \dot{y}^2 + \delta \dot{z}^2$$

The time - optimum exchange

$$\Phi_3(t_e, t_m) = t_m - t_e$$



Demand to be the final trajectory closed $\tilde{C}_1 = 0$.

The problems are solved numerically.

Optimization problems: Examples

Consider an example

$x_0 = 242\text{m}, y_0 = 67\text{m}, z_0 = 140\text{m}, \dot{x}_0 = -0.2244\text{m/s},$

$\dot{y}_0 = 0.11\text{m/s}, \dot{z}_0 = 0.11\text{m/s}, \omega = 0.0011\text{s}^{-1}, k = 1/20,$

then

$A = 100.7\text{m}, B = 120.4\text{m}, C_1 = 76\text{m}, |\mathbf{v}| = 1.30\text{m/s}.$

The trajectory shape is closest to the initial one

$t_e = 4380\text{s}, t_m = 5286\text{s}, \Delta t = 906\text{s}$

$|\mathbf{v}| = 1.3 \text{ m/s}, \ \Delta A = 0.5\text{m}, \Delta B = 4.6\text{m}$

The energy - optimum exchange

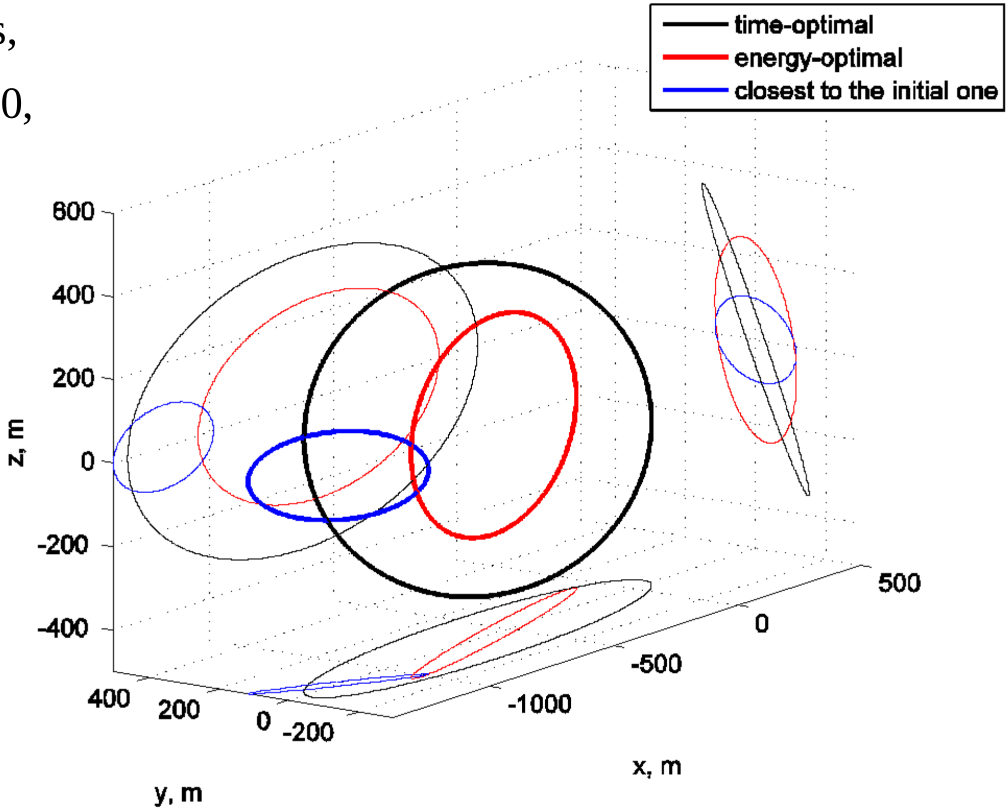
$t_e = 1806\text{s}, t_m = 2424\text{s}, \Delta t = 618\text{s}$

$|\mathbf{v}| = 1.25 \text{ m/s}$

The time - optimum exchange

$t_e = 925\text{s}, t_m = 1025\text{s}, \Delta t = 100\text{s}$

$|\mathbf{v}| = 2.86 \text{ m/s}.$

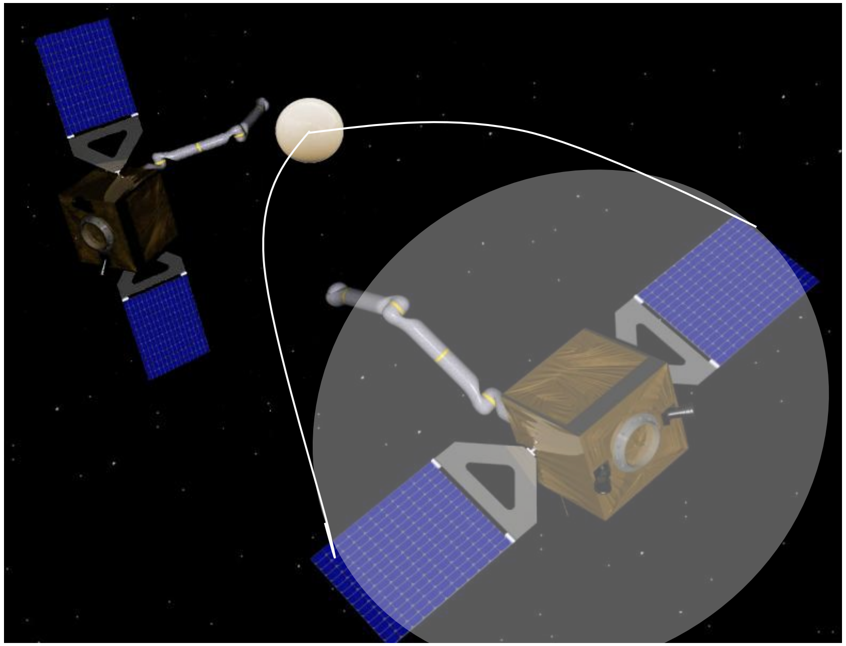


Relative trajectory

Errors analysis

Assume errors in mass ejecting mechanism.

Consider a manipulator capable to catch the mass in a sphere with radius *R*.



Errors in time

If the mass is ejected at time $t = t_e + \varepsilon_t$,

then for previous example $\varepsilon_t \le 1.13\text{s}, \text{ if } R = 1\text{m}.$

Errors in velocity

Let separation velocity be $\delta\mathbf{v} + \boldsymbol{\varepsilon},$

then the ellipsoid of errors is

$$R^2\omega^2 = A\varepsilon_x^2 + B\varepsilon_y^2 + C\varepsilon_z^2 + 2D\varepsilon_x\varepsilon_z$$

where

$$A = 8(1-\cos u) + 12\sin u(\sin u - 2u) + 9u^2,$$

$$B = \sin^2 u, \; C = 5 + 3\cos^2 u - 8\cos u,$$

$$D = 6(\sin u - u)(\cos u - 1).$$

For previous example $\varepsilon_t \le 0.11\text{cm/s}, \text{ if } R = 1\text{m}.$

Conclusions

- Formation flying control by mass exchange is proposed
- A set of optimization problems of concept is presented and solved
- The errors in ejecting/collecting mechanism could be estimated and accounted
- So, the mass exchange control concept could be applied successfully to solve a problem of relative motion maneuvering

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Thank you for your attention!