IAC-18-B4.7.6

Deployment and Maintenance of Nanosatellite Tetrahedral Formation Flying Using Aerodynamic Forces

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Abstract

The paper is devoted to the study of decentralized control using differential lift and drag for the construction and maintenance of the tetrahedral configuration. The most popular in the class of nanosatellites 3U CubeSats are considered. They have a suitable form-factor which let the cross-sectional area of satellites relative to the incoming airflow vary by a factor 3 depending on the attitude. The satellites are assumed to be subsequently deployed from the launcher. Each satellite is equipped with a reaction wheel-based attitude control system which allows to provide the required angular motion. Each satellite has also information about the relative state vector of all neighboring satellites provided by the inter-satellite communication or sensors of relative motion determination system. In this paper a decentralized control algorithm is developed which ensures the tracking of the relative reference trajectory. Due to this reference trajectory the satellites moves at the vertices of the tetrahedron. The possibility of constructing a tetrahedral configuration after deployment of the satellites depending on the initial conditions is studied.

1. Introduction

For an experimental study of the spatial distribution of the Earth magnetosphere parameters it is necessary to conduct simultaneous measurements at several points in a given region of near-Earth space, which can be achieved using satellite formation flying. At least four satellites are required to carry out spatial measurements. In the ideal case the satellites should fly so that they are always at the vertices of the regular tetrahedron. To construct and maintain such a configuration the relative motion control must be applied. For the Low-Earth Orbits the control can be performed using aerodynamic forces which act on the satellites in the upper atmosphere.

The main difficulty in the satellite group flight is the navigation and control of the relative motion of the individual satellite. Currently, there are two main approaches to the autonomous control of a group of satellites: centralized and decentralized. Centralized control implies the presence of a head (or "mother") satellite in the formation, its motion is monitored by the remaining "daughter" satellites, which are controlled to achieve the required relative trajectory. An example of the mission with such a control scheme is the mission CanX-4&5, launched in June 2014 [1]. This mission consists of two nanosatellites and is aimed at autonomous control implementation and fuel consumption minimization during maneuvering. With decentralized control each satellite makes the decision to control individually based on the motion of the nearest neighbors. This approach to control is more suitable for a swarm of satellites taking into account natural restrictions on the number of connections with other satellites.

In the literature about swarm motion, the so-called "agents" are considered – independent and autonomous controllable units, in our case they are satellites. In most papers about multi-agent systems, a control model is introduced which consists of four rules: attraction, alignment, collision avoidance and achievement of the goal. In the paper [2] control based on a linear-quadratic regulator using these rules to control the swarm of satellites is considered and a comparison of the characteristic velocities of centralized and decentralized strategies is performed for various parameters and tasks of the mission. For a large number of satellites, a reduction in computational complexity was shown when a two-stage control (a combination of centralized and decentralized strategies) is used. The paper [3] focuses on the study of a decentralized approach using an artificial potential function for control based on the same rules as in the previously mentioned work.

Investigation of the controlled motion for achieving various goals were presented, however, the dynamics of the relative motion of satellites formation flying is not considered in this paper.

A propulsion can be used for the swarm deployment, but the differential drag-based control is more suitable for nanosatellites on the LEO. It does not require a fuel, however the active attitude control system is needed. The control approach based on the differential drag force was firstly proposed in 1980s by Leonard [4] under the assumption of a discrete change in the effective cross section of satellites flying in the group. He developed a control algorithm based on the proportional differential controller. A large amount of papers applied a big variety of the different control algorithms using differential drag: PID regulator [5], linear-quadratic regulator [6], Lyapunov-based control [7,8], sliding mode control [9], optimal control [10] etc. However almost all the papers consider only two satellites in formation flying with application of the centralized control approach. A few papers are devoted to differential drag control of the multiple satellites. The cyclic and optimal control strategies for a cluster flight with more than two satellites are proposed in the paper [11]. Stability and performance of cluster keeping while avoiding collisions is studied in the [12]. The papers mentioned above do not address communicational restrictions and decentralized control features.

These papers considered only the aerodynamic differential drag application. Some recent papers take into account the differential lift also. The application of the differential lift along with the drag for the small satellites rendezvous problem was first proposed by Horsley et. al. in [13]. Authors used the aerodynamic drag and lift forces model based on Sentman's treatment in the free molecular flow conditions [14]. Control strategy developed in [13] is based on the bang-bang approach when only the maximum values of the lift and drag are used. Paper [15] investigates some practical aspects of the trajectories proposed by Horsey et.al. and the collision risks during the rendezvous. The same model of the differential lift and drag is used in [16,17] where the neural-network-based sliding-mode adaptive controllers are proposed. Paper [18] addresses the problem of the satellite formation keeping by the differential lift and drag under the J_2 perturbation. In the papers cited above the lift force is perpendicular to the satellite velocity vector. The drag and lift coefficients depend on a set of atmosphere parameters and satellite attitude according to the used model. Here the simplified model of the aerodynamic force is used. The specific attitude is calculated with the goal to provide the required force in the end.

The purpose of the current paper is to develop and study decentralized algorithms for the group of satellites control after the launch to construct the tetrahedral configuration. The most popular in the class of nanosatellites 3U CubeSats are considered. The satellites are assumed to be subsequently deployed from the launcher. Each satellite is equipped with a reaction wheel-based attitude control system which allows to provide the required angular motion. In this paper a decentralized control algorithm is developed which ensures the tracking of the relative reference trajectory. Due to this reference trajectory the satellites moves at the vertices of the tetrahedron. The possibility of constructing and maintaining a tetrahedral configuration after deployment of the satellites depending on the initial conditions is studied.

2. The problem statement

The problem of the satellite tetrahedral formation deployment after their separation from the launcher is considered, i.e. the achievement of defined relative trajectories is required when each satellite is moving in the vertices of the tetrahedron of required size. It is assumed that each satellite knows relative motion of all other members of the group (see Fig. 1). This information can be obtained either via an inter-satellite link or using autonomous relative motion determination system (range finders, optical sensors, etc.).

At the initial time the satellites move in accordance with the specified initial conditions after the launch from the launcher. The launch of satellites is carried out using a certain launch system (usually with the help of special springs), and the system has execution errors. In the absence of control it leads to a gradual increasing distances between the satellites. Consider the formation launched in LEO. It is assumed that each satellite is equipped with the attitude control system, for example, reaction wheels-based system. So, the satellites are able to be controlled by the aerodynamic force, which depends on the attitude of satellite relative to the incoming airflow. In the paper the 3U CubeSats are considered. They are the most popular nanosatellites nowadays and they have a convenient for aerodynamic control form-factor because the ratio of the maximum to the minimum cross-sectional area is 3.

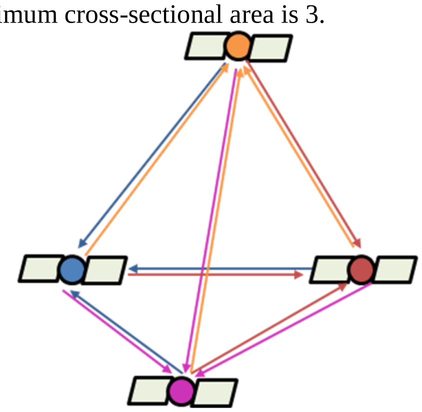

Fig.1 Tetrahedral formation of satellites with communication links

The main goal of the work is development of such a decentralized control of satellites, which leads to the tetrahedral formation flying construction. The possibility of constructing a tetrahedral formation using linear quadratic regulator (LQR) control depending on initial conditions is investigated. The effect of these parameters on the maintenance of formation flying during the implementation of the aerodynamic force control is considered.

2.1 Controlled Motion Equations

Consider a simplified formulation of a formation flying consisting of two satellites in near circular orbits. The general form of the equations of the relative motion of two satellites is too complex for analytical consideration, so Hill-Clohessy-Wiltshire equations [19,20] are used. This model describes the relative motion of two satellites flying in the near circular orbits in the central gravitational field. The orbital reference frame is used, its origin (reference point) moves along the circular orbit of radius r_0 with the orbital angular velocity $\omega=\sqrt{\mu/r_0^3}$, where μ is the Earth gravitational parameter. Axis Oz is aligned along the vector from the center of the Earth to the reference point, axis Oy is directed along the normal to the orbital plane, axis Ox complements the reference frame to the right-handed one (Fig.2).

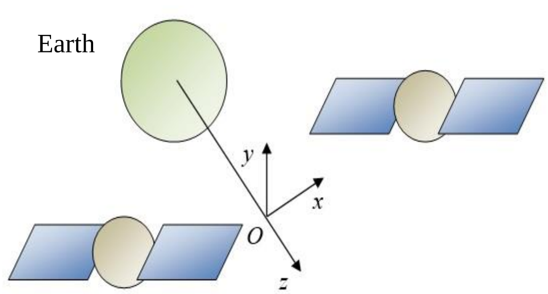

Fig.2. The reference frame associated with the point O moving along the circular orbit

Let $\mathbf{r}_i=(x_i,y_i,z_i)$ and $\mathbf{r}_j=(x_j,y_j,z_j)$ be the vectors of the conditional *i*-th and *j*-th satellites in the orbital reference frame, $i\neq j$, $i=1,...,N$, $j=1,...,N$, where N is the number of the satellites in the formation. Then the components of the relative position vector $\mathbf{r}_{ij}=\mathbf{r}_j-\mathbf{r}_i=(x_{ij},y_{ij},z_{ij})$ are governed by the following equations

$$\begin{cases}\ddot{x}_{ij}+2\omega\dot{z}_{ij}=u_x^{ij},\\ \ddot{y}_{ij}+\omega^2 y_{ij}=u_y^{ij},\\ \ddot{z}_{ij}-2\omega\dot{x}_{ij}-3\omega^2 z_{ij}=u_z^{ij},\end{cases} \tag{1}$$

where $\mathbf{u}_{ij}=\mathbf{\Delta f}_{ij}/m$, $\mathbf{\Delta f}_{ij}=\mathbf{f}_j-\mathbf{f}_i$ is the difference between aerodynamic drag forces acting on the *i*-th and *j*-th satellites, *m* is the mass of the satellite that is the same for all the satellites in group. In the case of free motion, i.e. if $\mathbf{\Delta f}_{ij}=0$, the exact solution of (1) is

$$\begin{cases}x_{ij}(t)=-3C_1^{ij}\omega t+2C_2^{ij}\cos(\omega t)-2C_3^{ij}\sin(\omega t)+C_4^{ij},\\ y_{ij}(t)=C_5^{ij}\sin(\omega t)+C_6^{ij}\cos(\omega t),\\ z_{ij}(t)=2C_1^{ij}+C_2^{ij}\sin(\omega t)+C_3^{ij}\cos(\omega t),\end{cases}$$

where $C_1^{ij}-C_6^{ij}$ are constants that depend on the initial conditions at $t=0$:

$$C_1^{ij}=\frac{\dot{x}_{ij}(0)}{\omega}+2z_{ij}(0),C_2^{ij}=\frac{\dot{z}_{ij}(0)}{\omega},C_3^{ij}=-3z_{ij}(0)-\frac{2\dot{x}_{ij}(0)}{\omega},$$
$$C_4^{ij}=x_{ij}(0)-\frac{2\dot{z}_{ij}(0)}{\omega},C_5^{ij}=\frac{\dot{y}_{ij}(0)}{\omega},C_6^{ij}=y_{ij}(0).$$

The term responsible for the relative drift is $-3C_1^{ij}\omega t$. Thus, the relative trajectory of two satellites is closed if and only if $C_1^{ij}=0$. However, in practice such an ideal initial conditions for free motion cannot be specified, and in the case of perturbations and nonlinear effects there is always a relative drift between the satellites.

2.2 Aerodynamic Force Model

Assume that the *i*-th and *j*-th satellites are the identical 3U CubeSats. The satellites are equipped with solar panels that covers the body of the satellites. Consider for simplicity that the satellites rotates in the way that only one of the greater side of the satellite (30x10cm^2) is affected by the incoming airflow. The other two greater sides are always perpendicular to the velocity vector and the last one is in the shadow relative to the incident flow. And only one of two smaller sides (10x10 cm^2) can be directed toward the velocity vector of the satellites.

The physical processes of the interaction of the atmospheric particles with the satellite surface are complex. However, one may construct rather simple model of these interactions using a limited number of empirical coefficients. Assume that the interaction proceeds mechanically through two schemes – a mirror one, when the reflection of the molecule from the surface is absolutely elastic, and diffuse one in the case of an absolutely inelastic collision. Define the actual reflection as a linear interpolation of these two interaction schemes, assuming that a certain part of the molecules ε are reflected in the mirror way, and the rest $(1-\varepsilon)$ part is reflected inelastically with the

Maxwell distribution corresponding to the temperature T_r. In this case [21] the expression for the aerodynamic force acting on the panel is

$$\mathbf{f}_i = -\frac{1}{m}\rho V^2 S\left\{(1-\varepsilon)(\mathbf{e}_V,\mathbf{n}_i)\mathbf{e}_V + 2\varepsilon(\mathbf{e}_V,\mathbf{n}_i)^2\mathbf{n}_i + (1-\varepsilon)\frac{\upsilon}{V}(\mathbf{e}_V,\mathbf{n}_i)\mathbf{n}_i\right\}. \quad (2)$$

Here ρ is the atmosphere density, m is the satellite mass, V is the airflow velocity, S is the solar panel area, $\mathbf{n}_i$ is the unit vector of the normal to the panel, $\mathbf{e}_V$ is a unit vector directed along the velocity of the incoming airflow, $\upsilon = \sqrt{\pi R T_r / 2}$ is a parameter proportional to the most probable thermal velocity of the diffusely reflected molecules, R is the gas constant, $i = 1,...,N$. The first term in (2) determines the aerodynamic drag force directed against the velocity of the air flow. The second and third terms are the force components directed against the normal to the plate, which define the lift force. In the aerodynamic force model (2) there are two interaction parameters, ε and $\eta = \upsilon / V$. Generally, they are not constant and depend on the angle of attack, the velocity of the incident particles and other characteristics of the gas and the surface. However, in this paper we consider some average values of the parameters ε and η and it is assumed that they are constant and do not depend on attitude. In [21], the inverse problem was solved to determine these parameters using the flight data of the motion of the Proton satellites. According to these estimates the interaction schemes of the gas with the satellite surface in LEO are such that $\varepsilon \approx 0.1$ and $\eta \approx 0.1$. This force model is firstly used for formation flying control in [22].

Let us rewrite the expression for the force components (2) using angles θ and φ representing the normal vector to the panel surfaces (see Fig. 2)

$$\mathbf{n}_i = \begin{bmatrix} \sin\theta_i \\ \cos\theta_i \cos\varphi_i \\ \cos\theta_i \sin\varphi_i \end{bmatrix}.$$

The angle θ is chosen such that the aerodynamic force does not act on the satellite when $\theta = 0$. $\theta \in [0;\pi/2]$, if $\theta < 0$ then the other side of the panel may be considered. Angle $\varphi \in [0, 2\pi)$.

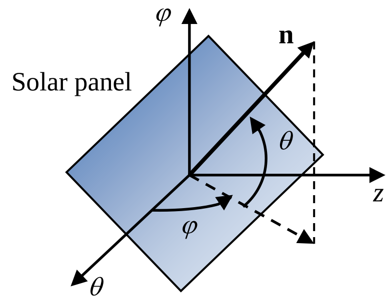

Fig.3. Angles defining the panel normal vector

The vector of incoming airflow is $\mathbf{e}_V = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T$ in orbital reference frame. Substitute the values of $\mathbf{n}_i$ and $\mathbf{e}_V$ in expression (2) of aerodynamic force,

$$\mathbf{f}_i = k\begin{bmatrix} -2\varepsilon(\sin\theta_i)^3 + \eta(\varepsilon-1)(\sin\theta_i)^2 + (\varepsilon-1)\sin\theta_i \\ -\cos\theta_i \sin\theta_i(\eta - \varepsilon\eta + 2\varepsilon\sin\theta_i)\cos\varphi_i \\ -\cos\theta_i \sin\theta_i(\eta - \varepsilon\eta + 2\varepsilon\sin\theta_i)\sin\varphi_i \end{bmatrix}$$

where $k = \frac{1}{m}\rho V^2 S$. Let us specify the following functions

$$p(\theta_i) = -2\varepsilon(\sin\theta_i)^3 + \eta(\varepsilon-1)(\sin\theta_i)^2 + (\varepsilon-1)\sin\theta_i,$$

$$g(\theta_i) = -\cos\theta_i \sin\theta_i(\eta - \varepsilon\eta + 2\varepsilon\sin\theta_i).$$

Then the expression for the aerodynamic force is simplified as

$$\mathbf{f}_i = k\begin{bmatrix} p(\theta_i) \\ g(\theta_i)\cos\varphi_i \\ g(\theta_i)\sin\varphi_i \end{bmatrix}. \quad (3)$$

From (3) one can see that the Ox component of the aerodynamic force depends on the angle θ only. The projection of the force on the plane Oyz is defined by the function $g(\theta_i)$ and its attitude is determined by the angle φ. The functions $p(\theta_i)$ and $g(\theta_i)$ are presented in Fig. 3 for $\varepsilon \approx 0.1$ and $\eta \approx 0.1$. The maximum value of f_x component of the aerodynamic force is achieved at $\theta = 90$ deg (in the case $p \approx 1.2$), i.e. when the plate is perpendicular to the incoming airflow. The maximum projection of the force on the plane Oyz is smaller by the order of magnitude. It is about $g \approx 0.12$ at $\theta \approx 52$ deg. Besides at $\theta = 0$ and $\theta = 90$ deg the function g is equal to zero. It means that the application of the force in the plane Oyz is possible only in the case of nonzero and non-maximal force along the axis Ox.

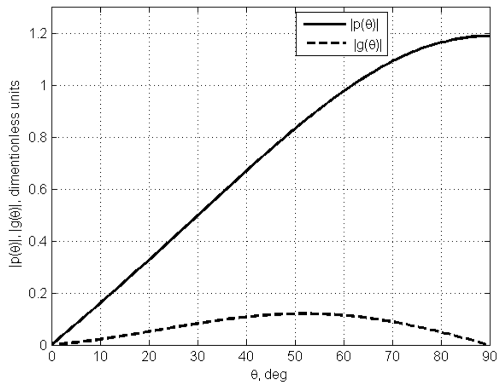

Fig.4. Components of the aerodynamic force with respect to angle θ

The paper considers the satellite relative motion control, so it is necessary to use the difference in the aerodynamic forces acting on two satellites,

$$\Delta\mathbf{f} = \mathbf{f}_1 - \mathbf{f}_2 = k\begin{bmatrix} p(\theta_1) - p(\theta_2) \\ g(\theta_1)\cos\varphi_1 - g(\theta_2)\cos\varphi_2 \\ g(\theta_1)\sin\varphi_1 - g(\theta_2)\sin\varphi_2 \end{bmatrix}. \quad (4)$$

Thus, the differential aerodynamic force is defined by four angles $\theta_1, \theta_2, \varphi_1, \varphi_2$. The value of the force projection on the plane *Oyz* is small as one can see from Fig. 3. So it better be maximized by setting $\varphi_2 = \varphi_1 + \pi$, i.e. the satellites rotate in opposite directions of *Ox* axis. In this case the value of Δf_{yz} component is defined by function $g(\theta_1) + g(\theta_2)$,

$$\Delta\mathbf{f} = k\begin{bmatrix} p(\theta_1) - p(\theta_2) \\ \left(g(\theta_1) + g(\theta_2)\right)\cos\varphi_1 \\ \left(g(\theta_1) + g(\theta_2)\right)\sin\varphi_1 \end{bmatrix}. \quad (5)$$

The maximum value of Δf_{yz} is two times more than the value of the projection of a single force in Fig.3. This follows from the dependence of the differential aerodynamic force $\Delta\mathbf{f}$ components on the angles θ_1 and θ_2 in Fig. 4. The acceptable control region in the dimensionless components of $\Delta\mathbf{f}$ is shown in Fig. 5. Since the attitude of the vector component Δf_{yz} in the plane *Oyz* is defined by the angle $\varphi_1 \in [0, 2\pi)$ it is necessary to rotate the area shown in Fig. 5 around the *Oy* axis to obtain the acceptable control region in three dimensions.

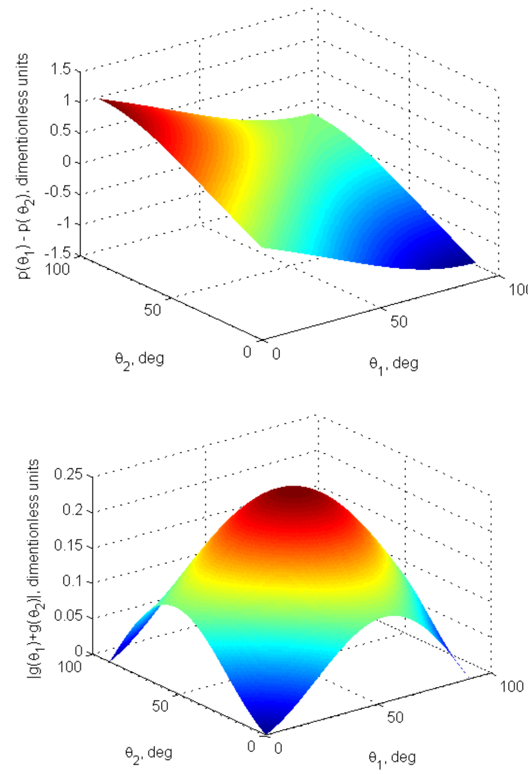

Fig.5. Functions $p(\theta_1) - p(\theta_2)$ and $|g(\theta_1) + g(\theta_2)|$

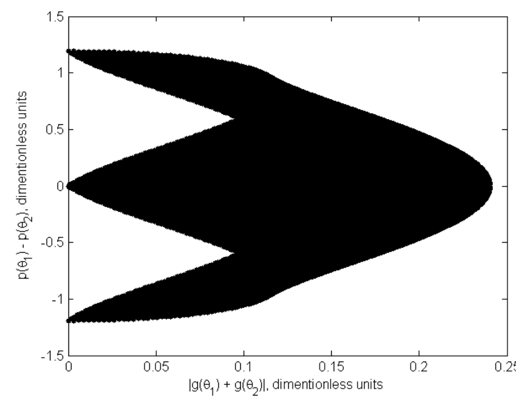

Fig.6. Acceptable control region

3. Control algorithm

Consider an application of the LQR regulator to track the predefined relative trajectory. This control algorithm is quite simple to implement in the case of two satellites in the group, but in the case of 4 satellites an additional decentralize control rule is needed. In this section an LQR is constructed and the eecentralized approach is proposed.

3.1 LQR Construction

Rewrite (1) in the matrix-vector form

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \quad (6)$$

where $\mathbf{x} = [\mathbf{r}^T\ \mathbf{v}^T]^T$ is the state vector, A is the dynamic matrix

$A = \begin{bmatrix} 0_{3x3} & E \\ C & D \end{bmatrix}$, E is the identity matrix with size 3x3,

$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega^2 & 0 \\ 0 & 0 & 3\omega^2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 & -2\omega \\ 0 & 0 & 0 \\ 2\omega & 0 & 0 \end{bmatrix}$,

B is the control matrix

$B = \begin{bmatrix} 0_{3x3} \\ E \end{bmatrix}$,

$\mathbf{u} = \mathbf{u}_i - \mathbf{u}_j$ is the control vector. For the formation flying controlled by the differential aerodynamic force the control vector $\mathbf{u} = \Delta\mathbf{f} / m$.

The desired relative motion corresponds to the free motion of the system described by the equation

$$\dot{\mathbf{x}}_d = A\mathbf{x}_d$$

where $\mathbf{x}_d$ is the desired state vector. Then one can obtain linear equation of the dynamics of the deviation from the desired trajectory

$$\dot{\mathbf{e}} = A\mathbf{e} + B\mathbf{u}, \quad (7)$$

where $\mathbf{e} = [\mathbf{x}^T - \mathbf{x}_d^T]^T$.

Linear quadratic regulator is the feedback control $\mathbf{u} = K\mathbf{e}$ which ensures the minimum of the functional

$$J = \int_0^\infty (\mathbf{e}^T Q\mathbf{e} + \mathbf{u}^T R\mathbf{u})dt \quad (8)$$

along the trajectory [23]. Here Q, R are the positive definite matrices that determine the weight of errors for the state vector and the weight of the control resource consumption respectively.

The feedback minimizing the functional is determined by the equation

$$\mathbf{u} = -R^{-1}B^T P\mathbf{e}, \quad (9)$$

where the matrix P is obtained as a solution of the Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \quad (10)$$

The Riccati equation (10) can be solved to obtain the matrix P if the weight matrices Q and R are known. Then the control vector $\mathbf{u}$ is calculated according to (9) using the current vector of the trajectory deviation $\mathbf{e}$. The matrices Q and R are the parameters of the algorithm. They characterize the transient processes. The problem is to choose such matrices Q and R that they would ensure the required performance of the algorithm under the given control constraints.

3.2 Average deviation from the desired trajectories

The main problem of the LQR application to the tetrahedral formation is that for each satellite there are three desires trajectories relative to each of the rest satellites. The desired relative trajectories are chosen in the way that all the four satellites are located in the vertices of the tetrahedron of specified size during the motion. So, each satellite need to apply the control (9) for each trajectory deviation. But the deviations $\mathbf{e}_{ij}$ could lead to the completely different control vectors $\mathbf{u}_{ij}$. That is why one need to propose the strategy for the constructing the tetrahedral formation flying.

We proposed the following scheme to solve this problem. For each satellite one can calculate the mean vector of the deviations $\bar{\mathbf{e}}_i$ as follows:

$$\bar{\mathbf{e}}_i = \sum_{j=1}^{3} \mathbf{e}_{ij} / 3,$$

Then using (9) the control vector is calculated:

$$\bar{\mathbf{u}}_i = -R^{-1}B^T P\bar{\mathbf{e}}_i. \quad (11)$$

Thus, the relative trajectory of the *i*-th satellite will converge to some average desired relative trajectory, but in the end all of the relative deviations will decrease and the required tetrahedron will be obtained.

3.3 Decentralized control approach constraints

The centralized control implies the presence of a head satellite in the formation, its motion is monitored by the remaining satellites, which are controlled to achieve the required relative trajectory, or the head satellite sends the control commands to the other satellites. In contrary the decentralized control approach means that each satellite is controlled individually and independently based on the relative motion information. It is assumed that the calculated control applied to the other satellites could be unknown.

Since in the decentralized scheme each satellite is controlled independently then the *i*-th satellite can just partly implement the calculated value. According to the aerodynamic force model $u_i^x \in [0; -u_{\max}^x]$, where $u_{\max}^i > 0$ is the absolute maximum value of $u_{ij}^x = u_j^x - u_i^x$ the acceleration.

Thus, it is assumed that in the case of the control saturation it is necessary to implement maximum possible component along the Ox axes, but according to the force model in this case the other components are zero: $\mathbf{u}_{\max}^x = \begin{bmatrix} u_{\max}^x & 0 & 0 \end{bmatrix}$. In the case the calculated control $\bar{\mathbf{u}}_i$ is in the acceptable control region, then it could be implemented. But in the case if the calculated average deviation control $\bar{u}_i^x$ is of negative value, then its vector $\mathbf{u}_i$ cannot be implemented and set to zero. In the case when the $0 < \bar{u}_i^x < u_{\max}^x$ in the acceptable control region, but the sum of the other two components are saturated, i.e. $\sqrt{\left(\bar{u}_i^y\right)^2 + \left(\bar{u}_i^z\right)^2} > u_{\max}^{yz}$, then it is reasonable to implement its maximum value $u_{\max}^{yz}$.

However in that case according to the Fig.4 the angle a $\theta \approx 52$ deg and the Ox component $\tilde{u}_i^x$ at this angle is $u_i^x / k \approx 0.8$. So, the control vector is to be applied in that case is $\mathbf{u}_{\max}^{yz} = \left[\tilde{u}_i^x \;\; \bar{u}_i^y / u_{\max}^{yz} \;\; \bar{u}_i^z / u_{\max}^{yz}\right]$, i.e. the calculated values for the Oy and Oz values are normalized to the maximum possible value $u_{\max}^{yz}$.

Thus, summarily, for the $\mathbf{u}_i$ one can propose the following decentralized control law:

$$\mathbf{u}_i = \begin{cases} -\mathbf{u}_{\max}^x, \text{if } \bar{u}_i^x > u_{\max}^x, \\ -\mathbf{u}_{\max}^{yz}, \text{if } 0 < \bar{u}_i^x < u_{\max}^x, \\ \qquad \text{and } \sqrt{\left(\bar{u}_i^y\right)^2 + \left(\bar{u}_i^z\right)^2} > u_{\max}^{yz}, \\ -\bar{\mathbf{u}}_i, \quad \text{if } 0 < \bar{u}_i^x < u_{\max}^x, \\ 0, \quad \text{if } \bar{u}_i^x < 0. \end{cases} \tag{12}$$

The proposed decentralized control strategy is derived imperially based in the practical logics, its values are just partly based on the LQR because it is takes into account the aerodynamic force value constrains. So, the algorithm performance is needed to be demonstrated. Due to actual aerodynamic force restrictions the convergence of the relative deviations of the trajectories cannot be proved analytically. That is why only numerical simulations is used for the controlled motion study.

4. Numerical study

Consider the application of the proposed control rule for the problem of the nanosatellites tetrahedral formation flying construction after the launch. The scheme of the launch of the satellites is the same that used by PlanetLabs company in 2017 for the launch of 88th 3U CubeSats [24], it is presented in Fig. 7. It is assumed that the satellites separate from the launcher in the *Ox* axis direction one after another with the time interval Δt between the launches. The velocity of the ejection V_e is assumed to be the same for all the CubeSats, however due to launch system inaccuracy the ejection velocity V_e is subjected to errors. So, the initial velocity vector $\mathbf{V}_0$ in orbital reference frame is modelled as follows:

$$\mathbf{V}_0 = \begin{bmatrix} V_e + \delta V \\ \delta V \\ \delta V \end{bmatrix}, \tag{13}$$

where δV is ejection error considered as normally distributed random value with zero mean and covariance $\sigma_{\delta V}^2$.

a) b)

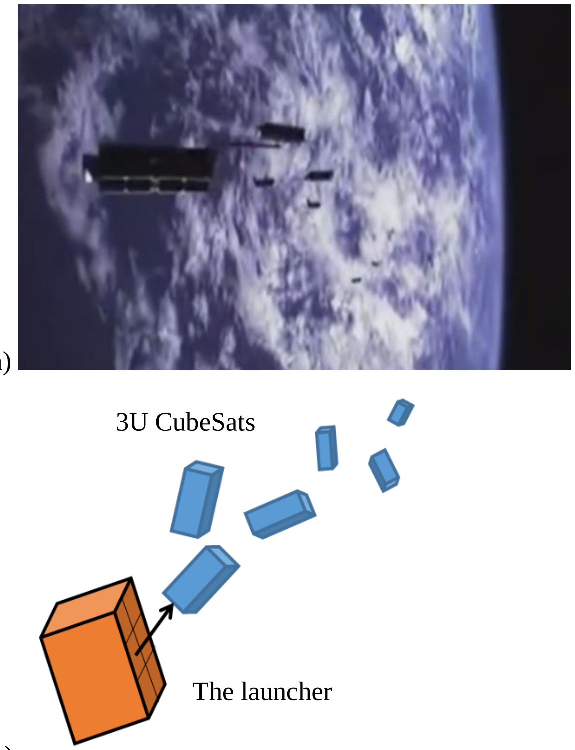

Fig. 7. The screenshot of the video of the launch of PlanetLabs 3U CubeSats [24] (a) and scheme of the launch (b)

After separation the implementation of control which is aimed at achieving the tetrahedral formation begins. A tetrahedron with the best quality is accomplished when the satellites move along the following reference orbits [25]:

$$\begin{array}{ll} x_1 = 2A\cos(\omega t - \arccos(1/3)), & x_3 = D, \\ y_1 = A\sqrt{3}\sin(\omega t), & y_3 = 0, \\ z_1 = A\sin(\omega t - \arccos(1/3)), & z_3 = 0, \\ & \\ x_2 = 2A\cos(\omega t), & x_4 = -D, \\ y_2 = A\sqrt{3}\sin(\omega t + \arccos(1/3)), & y_4 = 0, \\ z_2 = A\sin(\omega t), & z_4 = 0, \end{array} \tag{14}$$

where A, D are constants. According to that equations two of the satellites are moving along the same the circular with a constant separation equal to 2D. The other two satellites are moving along the circular relative trajectories as shown in Fig. 8.

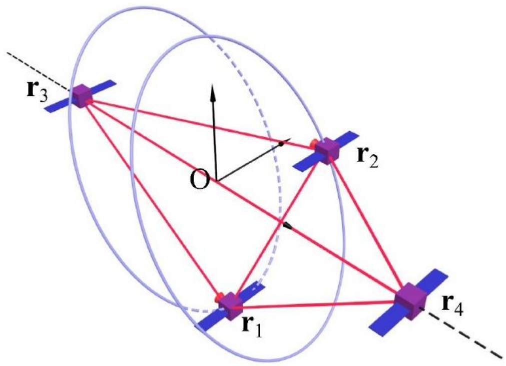

Fig.8. Scheme of the tetrahedral formation in LEO [25]

All the parameters used in the simulation of the controlled motion of the CubeSats formation flying are presented in Table 1. In this section the constant atmosphere density model is used. The value of the density is chosen as an average atmosphere density along the orbit with 340 km altitude according to the Russian GOST model of upper atmosphere [26].

Table 1. Parameters of simulation

Main parameters of the formation	
Number of satellites in the formation, N	4
Time interval between control calculation, ΔT	150 s
Parameter of tetrahedron A	100 m
Parameter of tetrahedron D	115 m
Launch parameters	
Time interval between the launches, Δt	10 s
Ejection velocity, V_e	0.5 m/s
Ejection error deviation, $\sigma_{\delta V}$	0.015 m/s
CubeSats parameters	
Mass of satellite, m	3 kg
Difference between maximum and minimum value of the cross-sectional area, ΔS	0.02 m^2
Aerodynamic drag coefficient, C_a	2
LQR parameters	
Matrix Q	E_{6x6}
Matrix R	diag ([1e-13; 1e-14; 1e-14])
Aerodynamic drag force parameters	
Constant atmosphere density, ρ	10^{-11} kg/m^3
Orbit altitude, h	340 km
Airflow velocity, $V=\sqrt{\mu/(R_E+h)}$	7.69 km/s
Parameters ε and η	0.1
Maximum of the control source, $u_{\max}$	$4.1\cdot10^{-6}$ m/s^2

Fig. 10 shows the relative trajectories of the three satellites relative to the forth satellites in the case of the uncontrolled motion. One can see that the relative trajectories are diverging along to the Ox axes due to the launch initial velocity errors. Fig. 10 demonstrates the relative motion trajectories under the proposed decentralized control (12). One can see that the trajectories relative to the fourth satellite are converges to the desired trajectory described by (14).

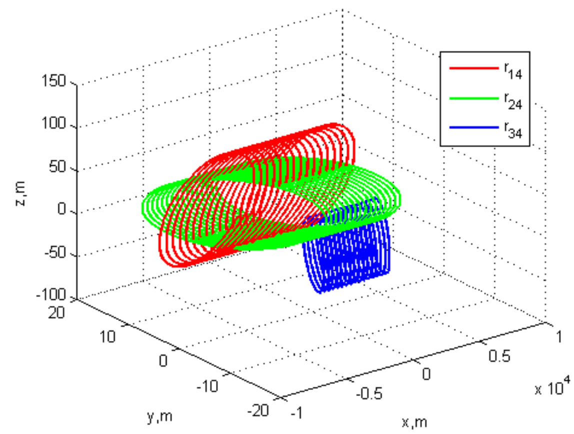

Fig.9. Uncontrolled motion relative to the fourth s atellite after the launch

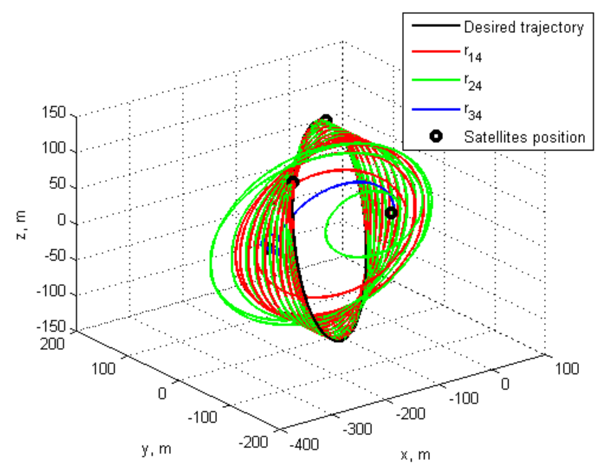

Fig. 10. Relative trajectories under the proposed control

Fig.11 presents the deviations vectors relative to the fourth satellite. One can see that all the deviations after approximately 50 hours finally converges to zero. The slowest convergence has the Oy component of the deviations vectors due to relatively small lift component of the aerodynamic force.

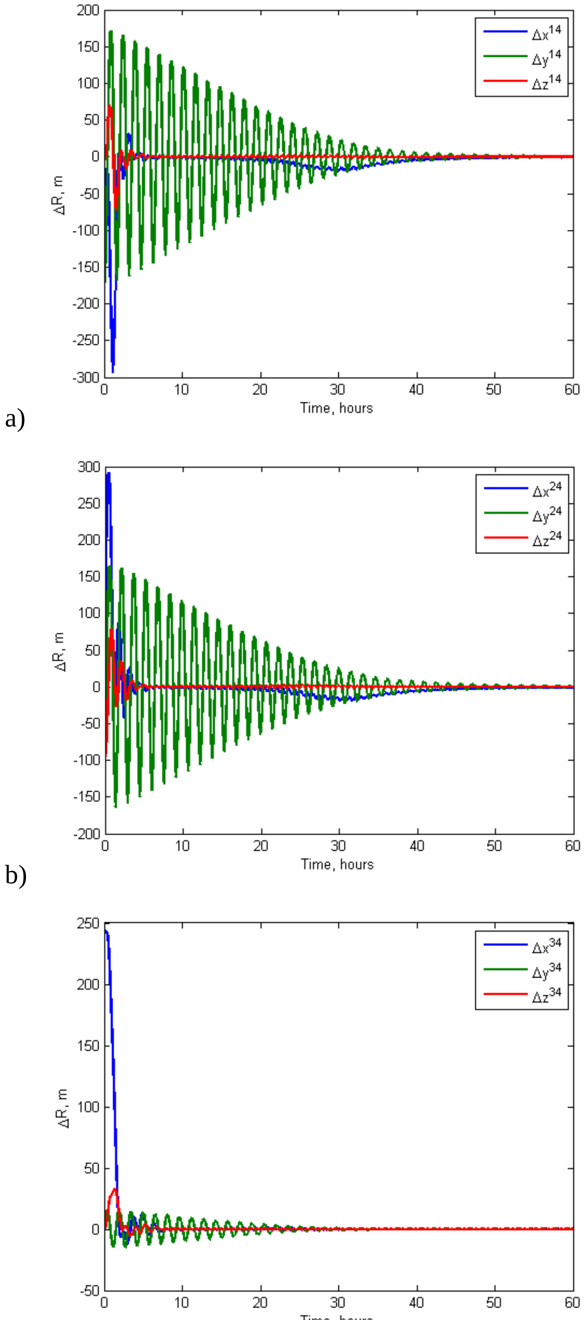

Fig. 11. Satellites position vector deviation relative to the forth satellite

The calculated control according to (11) for average relative vector of deviations for the first satellite is presented in Fig. 12 for example. And the corresponding implemented value according to the (12) is in Fig. 13. One can see that in the beginning the calculated value for the Ox component is positive. It cannot be realized by the aerodynamic force, so its value is set to zero. After approximately 3 hours the deviation along the Ox axis is considerably decreased, but all the positive calculated values are still not implemented. After the 3 hours the deviation along Oy axis still was large enough that lead to the saturation for the corresponding control vector component. During this case that lasted about next 20 hours the second control situation from (12) was implemented. It caused a temporary increased deviation along the Ox axis. But since 28 hours the deviations along Oy axis decreased and all of the components of the calculated control became in the acceptable control region. The similar plots can be shown for the other satellites, but not presented in the paper for the short.

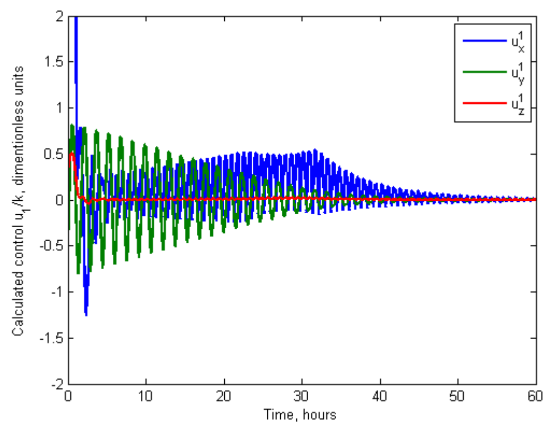

Fig. 12 Calculated control vector according to the (11) control algorithm

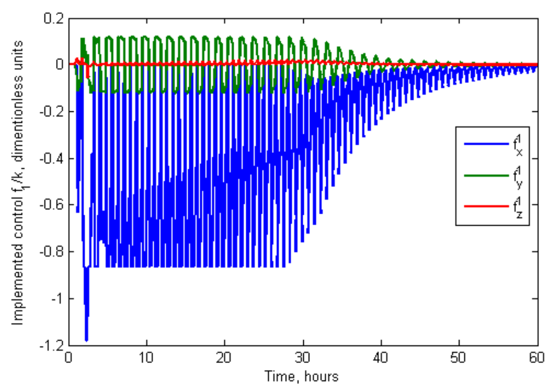

Fig. 13. The implemented control according to the (12) decentralized strategy

It can be interesting to investigate the time that is needed to construct the tetrahedral formation flying depending on the launch conditions. A set of the possible time intervals between the launches is considered, and the time of tetrahedral construction is numerically estimated. It is assumed that the configuration is formed if the deviations is less than 2m. Fig. 14 presents the dependence of the time of tetrahedron construction on the time between the launches. One can see that there are a minimum time of the construction that corresponds to the 23 s between the launches of the CubeSats.

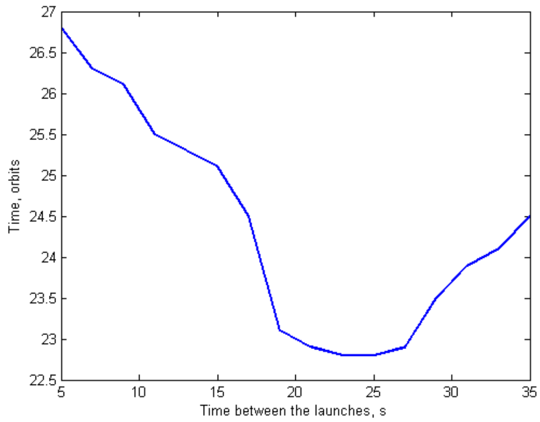

Fig. 14. The dependence of the time of tetrahedron construction on the time between the launches

For magnetosphere measurements it is important to scale the size of the tetrahedron to investigate the magnetic effects at different scales. For the reconfiguration to the same tetrahedron as described by (14) but of the different size the numerical experiments is performed. The time of the reconfiguration dependence on the similarity coefficient if presented in Fig. 15.

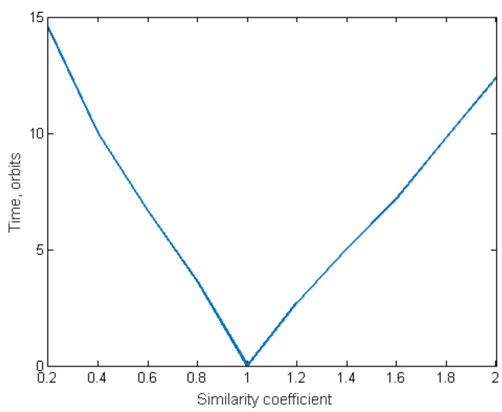

Fig. 15. Time for reconfiguration of the resizing of the tetrahedron

5. Discussions

The proposed control scheme of the tetrahedral formation flying construction requires further investigation. This paper can be considered as only the beginning of the work. A set of the real system features, parameters uncertainties and disturbances should be taken into account. Particularly, the motion equations with J_2 consideration should be applied to the system of satellites. The real atmospheric density uncertainty has a great effect on the controlled motion. Moreover, the atmospheric force parameters also could be hardly known for the real system. From the practical point of view it is interesting to investigate the proposed control performance with different errors of the launch system. All of the mentioned features are planned to be addressed in the authors future work.

6. Conclusions

The decentralized control scheme is proposed for the tetrahedral formation flying using the aerodynamic force with the lift component. It takes into account the constraints on the force maximum values. It was shown that for the considered tetrahedron the control successfully leads to the required configuration. The study of the time of formation flying construction after the launch showed that there are minimum depending on the time of the launches of the satellites. Despite the optimistic results of the numerical simulations the further investigation is needed to study the influence of the disturbances and uncertainties on the controlled motion.

Acknowledgements

The work supported by Russian Foundation for Basic Research, grant № 17-01-00449_a.

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