IAC-18-C1.4.7

Lyapunov control for attitude maneuvers with restricted areas

Yaroslav Mashtakov[a]*, Stepan Tkachev[b], Sergey Shestakov[c]

[a] *Attitude Control and Guidance Division, Keldysh Institute of Applied Mathematics of RAS, Miusskaya Sq.4, Moscow, Russia, 125047* , YarMashtakov@gmail.com
[b] *Attitude Control and Guidance Division, Keldysh Institute of Applied Mathematics of RAS, Miusskaya Sq.4, Moscow, Russia, 125047* , stevens_L@mail.ru
[c] *Attitude Control and Guidance Division, Keldysh Institute of Applied Mathematics of RAS, Miusskaya Sq.4, Moscow, Russia, 125047* , shestakov.sa@gmail.com

* Corresponding Author

Abstract

The problem of large slew maneuvers performing in the presence of prohibited areas is considered. It is shown that there is a modification of standard Lyapunov function that allows synthesising attitude control law that simultaneously ensures asymptotic stability of the required motion and avoids restricted areas. There are several issues that appear using this algorithm, such as new equilibriums that might be even asymptotically stable. All these problems are addressed in the paper.

Keywords: Lyapunov-based control, restriction avoidance

1. Introduction

Lyapunov-based attitude control algorithms offer good accuracy and robustness of the attitude control [1,2]. In addition, they are rather simple and can be easily implemented on-board the satellite. On the other hand, these algorithms have some peculiarities: they usually do not take into account the limitations that are imposed on the attitude. An example of such limitations is the presence of keep-out zones. They might appear because of sensitive equipment installed onboard the satellite, e.g. camera axis should not be aimed at bright objects. In present paper we discuss an approach that allows us to solve this problem.

It must be noted that optimal control algorithms can be used to calculate slew maneuvers in the case of restricted areas presence. Unfortunately, they usually require much computational time. Though there are some techniques that allow reducing of this time, e.g. the particle swarm optimization [3], it still can be hardly applied for a real time attitude control. On the other hand, modification of Lyapunov-based control law, which is similar to the introduction of potential barriers, is able to ensure the maneuver performing and restricted area avoidance in real time.

2. Problem statement

The following right-handed Cartesian coordinate systems are used:

$O_aY_1Y_2Y_3$ – Inertial Frame: its origin O_a is located in the Earth center of mass, O_aY_1 is directed to the Vernal equinox of the J2000 epoch, O_aY_3 is orthogonal to the ecliptic plane.

$Ox_1x_2x_3$ – Body-Fixed Frame: its origin O is located in the spacecraft center of mass, and the axes are its principal axes of inertia

Keep-out zones are the cones with known axis $\mathbf{h}_i$ and semiapex angle α_i . During the rotation they are considered fixed in Inertial Frame. It is necessary to align given axis $\mathbf{e}$ (fixed in Body Frame, e.g. camera axis) with the given axis $\mathbf{e}_{ref}$ (fixed in Inertial Frame). In addition, $\mathbf{e}$ should avoid keep-out zones. Obviously, the initial position of $\mathbf{e}$, as well as required one $\mathbf{e}_{ref}$, are outside restricted regions.

We consider the problem of performing rest-to-rest slew maneuvre in the presence of restricted attitudes. Usually such rotations do not require three axes attitude stabilization. In addition, single-axis stabilization control is much simpler for analysis. Hence, we will consider only the problem of single axis stabilization.

3. Single axis stabilization

In order to synthesize control algorithm for single axis stabilization we use Lyapunov direct method. The main idea of this method is to choose positive definite function V (also called Lyapunov function) and, using control, ensure that it satisfies Barbashin-Krasovsky-LaSalle principle.

Consider the most general case when it is necessary to align constant in Body Frame axis $\mathbf{e}$ with given in Inertial Frame time-dependent $\mathbf{e}_{ref}$. In addition, it is necessary to provide a specific angular velocity

$$\boldsymbol{\omega}_{ref} = \mathbf{e}_{ref} \times \dot{\mathbf{e}}_{ref} + \Omega \mathbf{e}_{ref}.$$

Ω here is time-dependant scalar function. As we can see, this angular velocity ensures necessary motion of reference axis $\mathbf{e}_{ref}$, i.e. $\dot{\mathbf{e}}_{ref} = \boldsymbol{\omega}_{ref} \times \mathbf{e}_{ref}$. Let $\mathbf{D}$ is direction cosine matrix that describes transfer from Inertial to Body Frame. Consider Lyapunov function

$$V = \left(1 - \left(\mathbf{D}\mathbf{e}_{ref}, \mathbf{e}\right)\right) + \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right),$$
$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs} - \mathbf{D}\boldsymbol{\omega}_{ref},$$

where $\mathbf{J}$ is satellite tensor of inertia, $\boldsymbol{\omega}_{abs}$ is its angular velocity. Equations of motion are

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} = \mathbf{M}_{ctrl} + \mathbf{M}_{ext},$$
$$\dot{\mathbf{D}} = -[\boldsymbol{\omega}_{abs}]_{\times}\mathbf{D}.$$

Here $\mathbf{M}_{ext}, \mathbf{M}_{ctrl}$ are external and control torques respectively, and

$$[\boldsymbol{\omega}_{abs}]_{\times} := \begin{pmatrix} 0 & -\omega_{abs,3} & \omega_{abs,2} \\ \omega_{abs,3} & 0 & -\omega_{abs,1} \\ -\omega_{abs,2} & \omega_{abs,1} & 0 \end{pmatrix}.$$

Hence, time derivative of Lyapunov-function is

$$\dot{V} = -k_a\left(\dot{\mathbf{D}}\mathbf{e}_{ref} + \mathbf{D}\dot{\mathbf{e}}_{ref}, \mathbf{e}\right) + \left(\boldsymbol{\omega}_{rel}, \mathbf{J}\dot{\boldsymbol{\omega}}_{rel}\right).$$

Expression $\dot{\mathbf{e}}_{ref} = \boldsymbol{\omega}_{ref} \times \mathbf{e}_{ref}$ and equations of motion allow us to rewrite it:

$$\dot{V} = \left(\boldsymbol{\omega}_{rel}, \mathbf{J}\dot{\boldsymbol{\omega}}_{rel} + k_a \mathbf{D}\mathbf{e}_{ref} \times \mathbf{e}\right). \tag{1}$$

Take into account time derivative of relative angular velocity:

$$\begin{aligned}\dot{\boldsymbol{\omega}}_{rel} &= \dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{D}\left(\mathbf{e}_{ref} \times \dot{\mathbf{e}}_{ref} + \Omega \mathbf{e}_{ref}\right) \\ &\quad - \mathbf{D}\left(\mathbf{e}_{ref} \times \ddot{\mathbf{e}}_{ref} + \Omega \dot{\mathbf{e}}_{ref} + \dot{\Omega}\mathbf{e}_{ref}\right) \\ &= \dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{D}\boldsymbol{\omega}_{rel} - \mathbf{D}\dot{\boldsymbol{\omega}}_{ref}\end{aligned}$$

After substitution it into (1) and ensuring that Lyapunov function derivative is nonpositive, we obtain the following control law:

$$\begin{aligned}\mathbf{M}_{ctrl} &= -k_\omega \boldsymbol{\omega}_{rel} - k_a \mathbf{D}\mathbf{e}_{ref} \times \mathbf{e} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} \\ &\quad - \mathbf{J}\boldsymbol{\omega}_{abs} \times \mathbf{D}\boldsymbol{\omega}_{ref} + \mathbf{J}\mathbf{D}\dot{\boldsymbol{\omega}}_{ref} - \mathbf{M}_{ext}\end{aligned}$$

where k_a, k_ω are positive constants.

Obtained control law is suitable for any single-axis attitude tracking. However, in the case of inertial stabilization it takes much simpler form

$$\mathbf{M}_{ctrl} = -k_\omega \boldsymbol{\omega}_{rel} - k_a \mathbf{D}\mathbf{e}_{ref} \times \mathbf{e} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - \mathbf{M}_{ext}.$$

Suggested control ensures asymptotic stability of required motion, hence it provides decent accuracy even in the presence of unaccounted disturbances.

4. Restriction avoidance

There are several techniques that allow us to perform slew maneuvre and avoid restricted areas. Most of them utilize either optimal control or technique similar to energy barrier construction [4–6]. The technique we suggest here can be related to the latter ones. As we have seen in previous section, Lyapunov-based attitude control ensures that Lyapunov function never increases. Hence, if in restricted areas it is sufficiently large, the satellite would never enter them.

Consider almost the same Lyapunov function as in the previous section:

$$V = \left(1 - \left(\mathbf{D}\mathbf{e}_{ref}, \mathbf{e}\right)\right)\left(1 + F\right) + \frac{1}{2}\left(\boldsymbol{\omega}, \mathbf{J}\boldsymbol{\omega}\right).$$

Here F is a scalar function that depends on $\mathbf{e}$ and ensures restricted area avoidance, $\boldsymbol{\omega}$ is satellite angular velocity ($\boldsymbol{\omega}_{ref} = 0$ and index *abs* is omitted). Function F can be chosen in different ways. Let us choose it as follows:

$$F = \sum_{i=1}^{n} f_i, \quad f_i = \begin{cases} H_i & \lambda_i < 0 \\ H_i\left(-3\lambda_i^2 + 2\lambda_i^3 + 1\right) & 0 \le \lambda_i \le 1, \\ 0 & 1 < \lambda_i \end{cases}$$
$$\lambda_i = \frac{\operatorname{acos}\left(\mathbf{e}, \mathbf{D}\mathbf{h}_i\right) - a_i}{b_i - a_i}$$

Here $\mathbf{h}_i$ is axis of i-th keep-out cone, a_i corresponds to its semiapex angle, $H_i, b_i > a_i$ are positive constants. Constant b_i can be described as a boundary where the presence of keep-out cone starts affecting the satellite motion. This function equals zero when the satellite is far from keep-out cones, equals H_i when it is inside the i-th cone and between this positions it is smoothly connected.

In order to obtain control law we have to take time-derivative of Lyapunov function. Technique is the same as one described in Sec. 3. After the mathematics:

$$
\begin{aligned}
\mathbf{M}_{ctrl} = & -M_{ext} - k_{\omega}\boldsymbol{\omega}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - k_a\left[\mathbf{De}_{ref} \times \mathbf{e}\right] + \\
& + k_a\left(1 - \left(\mathbf{De}_{ref}, \mathbf{e}\right)\right)\sum_{i=1}^{n} f_i' \frac{\mathbf{e} \times \mathbf{Dh}_i}{\sqrt{1 - \left(\mathbf{e}, \mathbf{Dh}_i\right)^2}} \\
& - k_a\left[\mathbf{De}_{ref} \times \mathbf{e}\right] F
\end{aligned}
$$

$$
f_i' = \begin{cases} \dfrac{H_i\left(6\lambda_i^2 - 6\lambda_i\right)}{b_i - a_i}, & 0 \le \lambda_i \le 1 \\ 0, & \text{otherwise} \end{cases}
$$

The first raw of this expression corresponds to the conventional single-axis stabilization. Additional terms appear only when satellite enters the vicinity of restricted region.

4.1 Saddle points

Consider the following simulation:

- There is no external torques affecting the satellite
- Only one keep-out cone
- Tensor of inertia $\mathbf{J} = \mathrm{diag}(2,3,4)\ \mathrm{kg \cdot m^2}$
- It is necessary to align third axis of Body Frame with third axis of Inertial Frame.

As we can see in Fig. 1, satellite successfully avoids restricted area. However, for almost the same initial conditions, convergence time greatly increases (see Fig. 2). It is due to the fact that suggested control creates two additional equilibriums along the "meridian" of the sphere: one of them is unstable, and the other one is saddle, so it has one asymptotically stable manifold and one unstable. If initial conditions are near this meridian, it will take a lot of time to get out of saddle point. Hence, it is necessary to somehow avoid them. Introduce the following disturbance that act in the vicinity of the saddle point:

$$
\mathbf{M}_{dist} = \mathrm{sgn}\left(\left(\boldsymbol{\omega}_{abs}, \mathbf{De}_{ref} \times \mathbf{e}\right)\right)\mathbf{J}\frac{\mathbf{De}_{ref} - \mathbf{e}\left(\mathbf{De}_{ref}, \mathbf{e}\right)}{\left|\mathbf{De}_{ref} - \mathbf{e}\left(\mathbf{Dn}_{ref}, \mathbf{e}\right)\right|} g
$$

$$
g = \begin{cases} G\left(2\mu^3 - 3\mu^2 + 1\right) & \mu \le 1 \\ 0 & \mu > 1 \end{cases}, \quad \mu = \frac{\mathrm{acos}\left(\mathbf{e}, \mathbf{s}\right)}{d}
$$

- Here $\mathbf{s}$ is saddle point radius-vector, d is positive constant that corresponds to the size of the area where disturbance applied, G is the maximum magnitude of the disturbance. This disturbance helps to evade saddle point much faster (see Fig. 3).

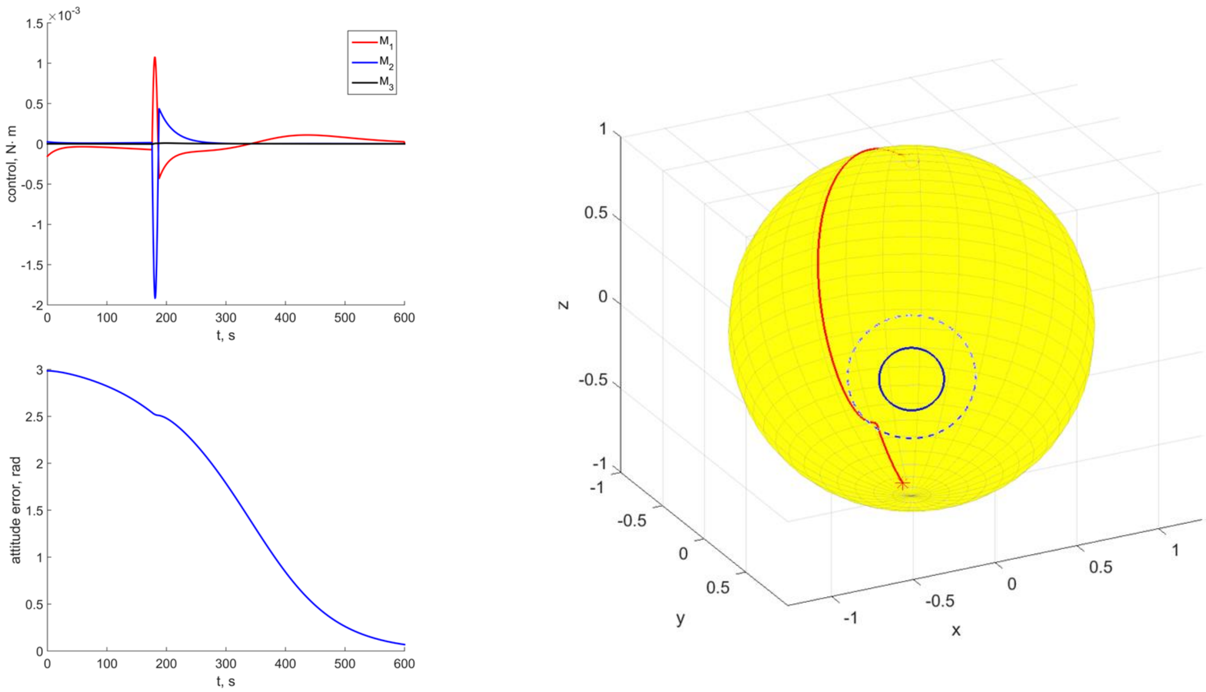

Fig. 1. Attitude control performance

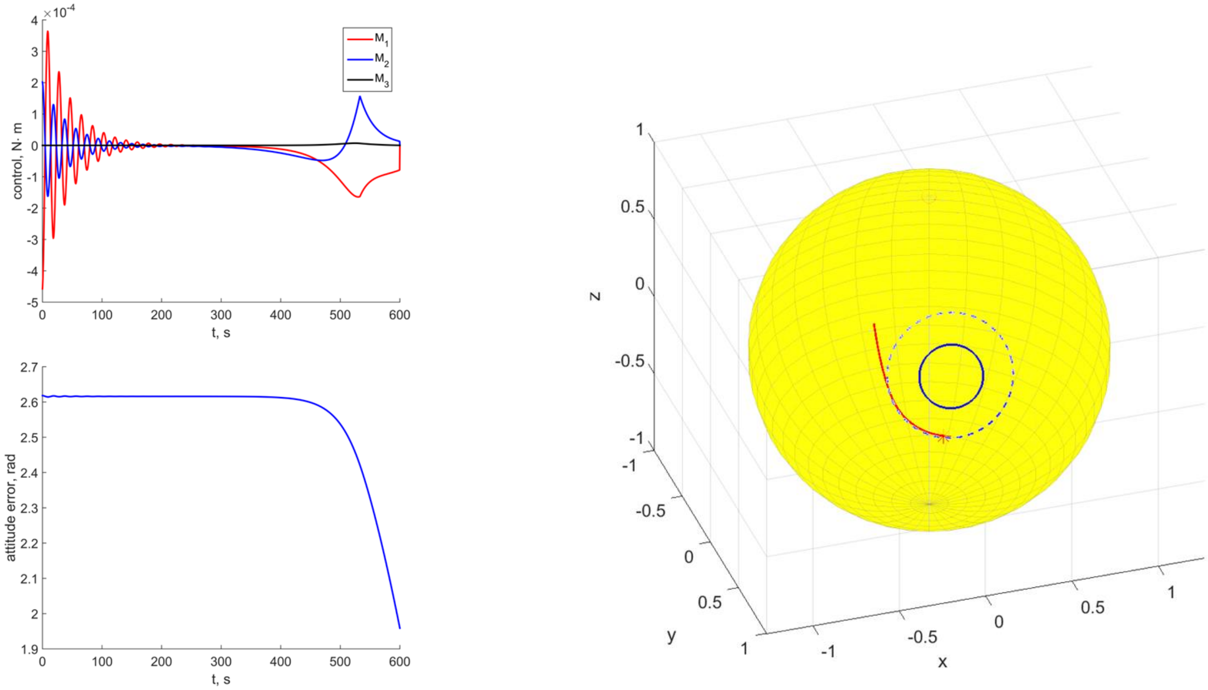

Fig. 2. Saddle point

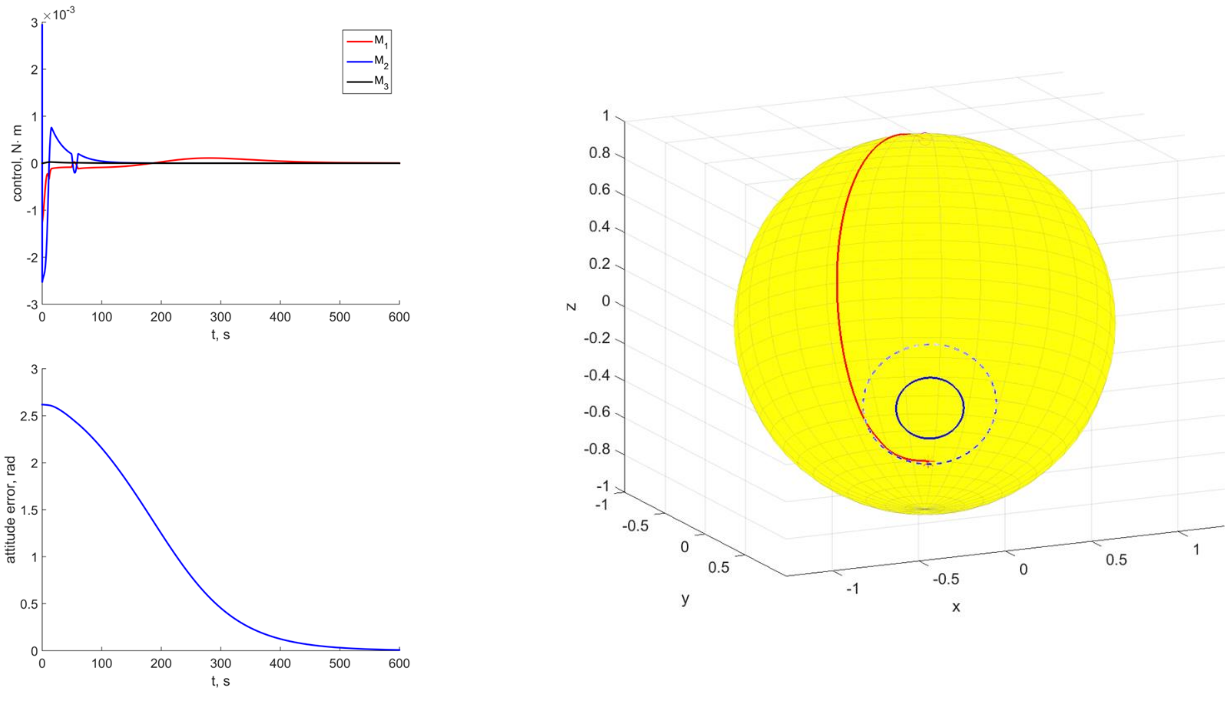

Fig. 3. Saddle point avoidance

4.2 Asymptotically stable equilibriums

Another peculiarity of suggested control law arises in the case when two restricted cones intersect. It might create additional asymptotically stable equilibrium, so satellite might get stuck in this point (see Fig.4). Solution of this problem might be the change of Lyapunov function: for example, two restricted cones might be included in one bigger cone. However, this might be inappropriate, because resulting cone is too large. Therefore, it is suitable to choose another figure on the sphere that includes both cones. We suggest to use "ellipses". It is the locus of the points which are on the same (spherical) distance from two "focuses". In other words, these points should satisfy two conditions. Firstly, they must be located on a sphere. Secondly

$$\operatorname{acos}(\mathbf{p},\mathbf{h}_1)+\operatorname{acos}(\mathbf{p},\mathbf{h}_2)=L$$

In previous sections functions f_i depend on dot product $(\mathbf{e},\mathbf{Dh}_i)$. For ellipses it is suitable to choose the same functions, but now it should depend on $\operatorname{acos}(\mathbf{p},\mathbf{h}_1)+\operatorname{acos}(\mathbf{p},\mathbf{h}_2)$. Therefore control torque is

$$\begin{aligned}\mathbf{M}_{ctrl} &= -M_{ext} - k_\omega \boldsymbol{\omega}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} \\ &- k_a\left[\mathbf{De}_{ref} \times \mathbf{e}\right] + \\ &+ k_a\left(1-\left(\mathbf{De}_{ref},\mathbf{e}\right)\right) \times \\ &\sum_{i=1}^{n} f_i' \left(\frac{\mathbf{e}\times\mathbf{Dh}_1^i}{\sqrt{1-\left(\mathbf{n},\mathbf{h}_1^i\right)^2}} + \frac{\mathbf{e}\times\mathbf{h}_2^i}{\sqrt{1-\left(\mathbf{n},\mathbf{h}_2^i\right)^2}} \right) \\ &- k_a\left[\mathbf{De}_{ref} \times \mathbf{e}\right] F\end{aligned}$$

$$f_i' = \begin{cases} \dfrac{H_i\left(6\nu_i^2 - 6\nu_i\right)}{b_i - a_i}, & 0 \le \nu_i \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Here

$$\nu_i = \frac{\operatorname{acos}(\mathbf{e},\mathbf{Dh}_1)+\operatorname{acos}(\mathbf{e},\mathbf{Dh}_2)-a_i}{b_i - a_i}.$$

It should be mentioned that for this control there is again the problem of saddle points. It can be solved in the same way as for the cones. In Fig. 5 simulation for the general case (several cones, one ellipse) is provided. As we can see, all restricted areas are successfully avoided.

5. Conclusions

In present paper direct Lyapunov method was used for attitude control synthesis in the presence of restrictions. In order to avoid keep-out zones a modification of the standard Lyapunov function was suggested. Two problems that might affect convergence rate of the control algorithm were discussed.

In order to avoid saddle points an additional disturbance torque was introduced. It allows us to leave vicinity of saddle point much faster.

In the case when two restricted cones intersect new asymptotically stable equilibrium appears. To solve this problem it was suggested to replace two cones by ellipse.

Suggested control law might be used either to control the satellite directly or to generate the reference attitude motion, which afterwards is implemented by conventional Lyapunov-based control.

Acknowledgements

This work is supported by Russian Science Foundation (grant no. 17-71-20117)

List of references

1. Mashtakov Y.V., Ovchinnikov M.Y., Tkachev S.S. Study of the disturbances effect on small satellite route tracking accuracy // Acta Astronaut. 2016. Vol. 129. P. 22–31.
2. Tsiotras P. New Control Laws for the Attitude Stabilization of Rigid Bodies // 13th IFAC Symposium on Automatic Control in Aerospace. 1994. P. 316–321.
3. Melton R.G. Hybrid methods for determining time-optimal, constrained spacecraft reorientation maneuvers // Acta Astronaut. Elsevier, 2014. Vol. 94, № 1. P. 294–301.
4. McInnes C.R. Large angle slew maneuvers with autonomous sun vector avoidance // J. Guid. Control. Dyn. 1994. Vol. 17, № 4. P. 875–877.
5. Salama O. Autonomous Spacecraft Attitude Constraints Avoidance // Proceedings of the 65th IAC. 2014. P. 9.
6. Shen Q., Goh C.H., Yue C. Constrained attitude control of agile spacecraft using CMGs // Region 10 Conference (TENCON). 2016. P. 3664–3669.

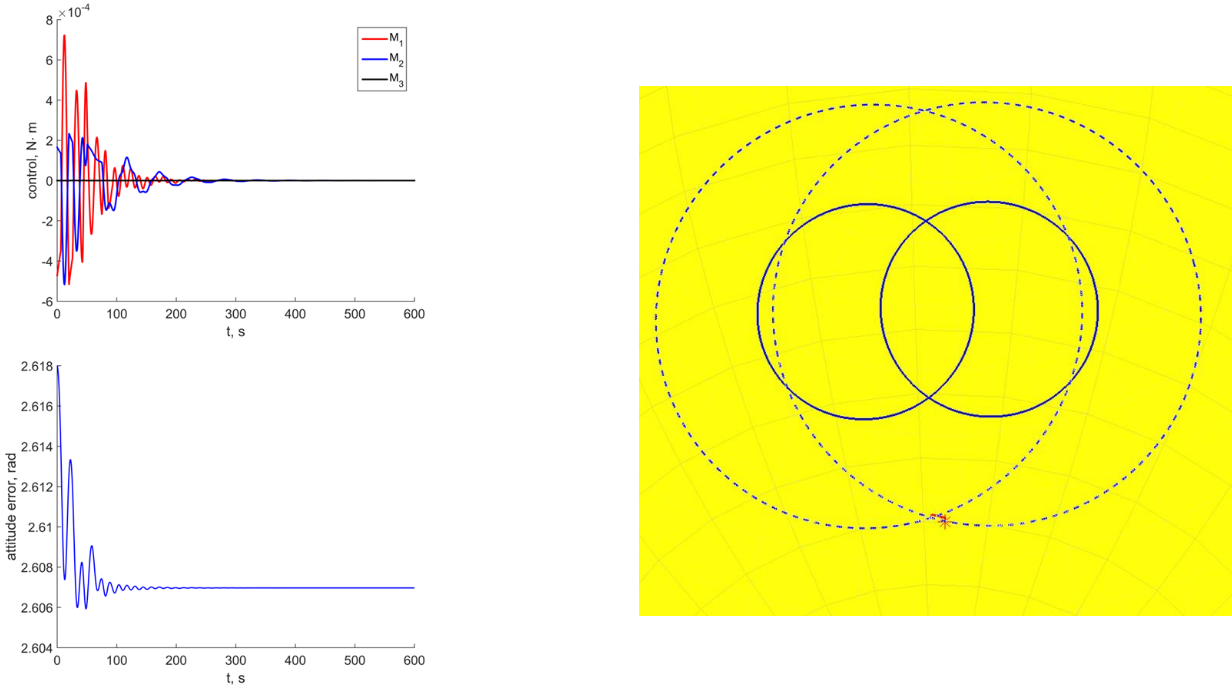

Fig. 4. Asymptotically stable equilibrium

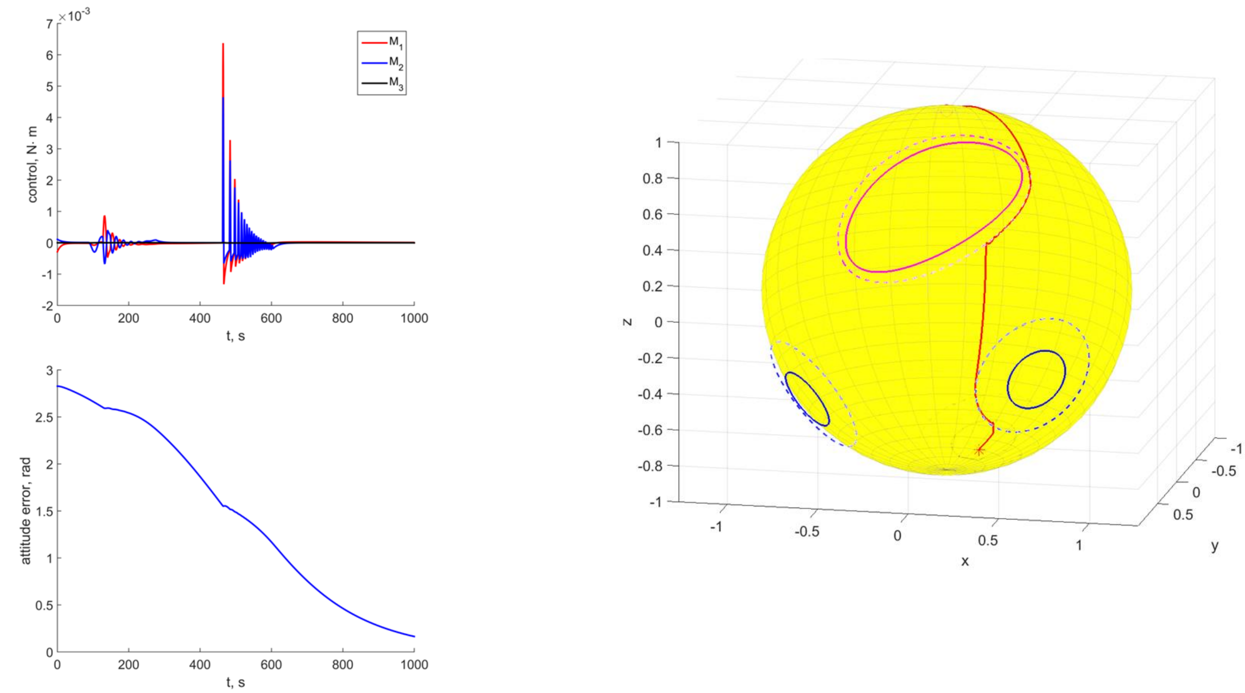

Fig. 5. Simulation in general case with enabled saddle point avoidance