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Lyapunov control for attitude maneuvers with restricted areas

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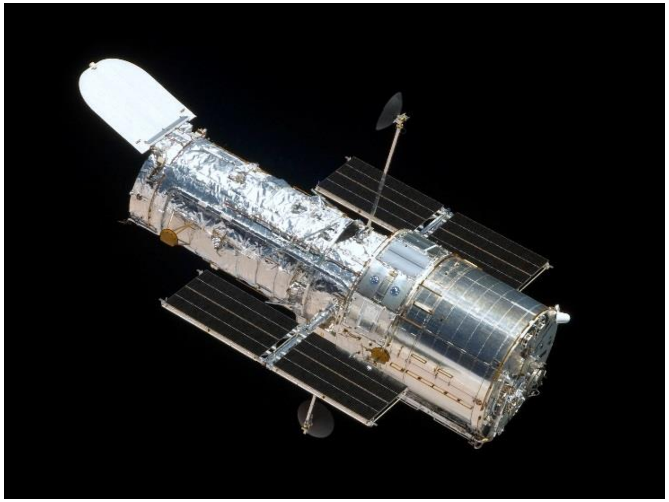
Introduction

We want to change satellite attitude

Restrictions:

- Camera cannot be aimed at bright objects
- Solar panels are directed to the Sun



Possible approaches:

- Optimal control
  - ✓ Fast
  - × Complicated for on-board computation
- Direct Lyapunov method
  - ✓ Rather simple control expressions
  - ✓ Asymptotic stability (robust)
  - × Not time optimal

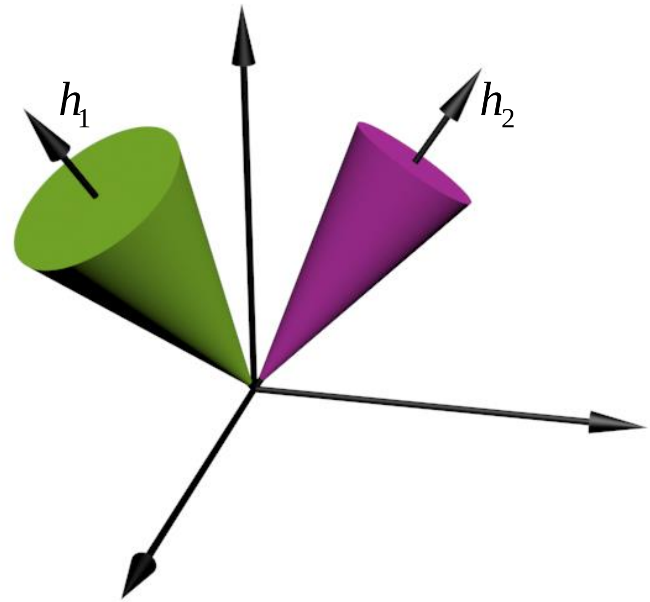
Problem statement

What do we know?

- Satellite parameters
- Restricted areas (fixed in Inertial Frame cones)
- Camera axis
- Initial and desired attitude
- Initial and desired angular velocity equal to zero

What do we want?

- Perform slew maneuver
- Avoid restricted cones



Lyapunov-based control

- Simple control expressions
- Ensures asymptotic stability of the reference motion
- Robust

Main idea:

- Positive definite function (Lyapunov function)
- Control ensures Barbashin-Krasovsky-LaSalle principle satisfaction

Examples

Single-axis attitude control

Function:

$$V_0 = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_a\left(1 - (\mathbf{D}\mathbf{n}_{ref}, \mathbf{n})\right)$$

$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs} - \mathbf{D}\left(\mathbf{n}_{ref} \times \dot{\mathbf{n}}_{ref} + \Omega\mathbf{n}_{ref}\right) = \boldsymbol{\omega}_{abs} - \mathbf{D}\boldsymbol{\omega}_{ref}$$

Control:

$$\mathbf{M}_{ctrl} = -k_\omega\boldsymbol{\omega}_{rel} - k_a\mathbf{D}\mathbf{n}_{ref} \times \mathbf{n} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - \\ - \mathbf{J}\boldsymbol{\omega}_{abs} \times \mathbf{D}\boldsymbol{\omega}_{ref} + \mathbf{J}\mathbf{D}\dot{\boldsymbol{\omega}}_{ref} - \mathbf{M}_{ext}$$

Three-axis attitude control

Function:

$$V_0 = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_q\left(1 - q_0\right)$$

$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs} - \mathbf{A}\boldsymbol{\omega}_{ref}$$

Control:

$$\mathbf{M}_{ctrl} = -\mathbf{M}_{ext} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} + \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_{ref} - \\ - \mathbf{J}\left[\boldsymbol{\omega}_{rel}\right]_\times \mathbf{A}\boldsymbol{\omega}_{ref} - k_q\mathbf{q} - k_\omega\boldsymbol{\omega}_{rel}$$

$(q_0, \mathbf{q})$, $\mathbf{A}$ – quaternion and DCM from Reference to Body Frame

$\mathbf{D}$ – DCM from Inertial Frame to Body Frame

$\boldsymbol{\omega}_{abs}, \boldsymbol{\omega}_{rel}, \boldsymbol{\omega}_{ref}$ – absolute, relative and reference angular velocity

Principal Idea

- We will use single axis attitude control
- It does not include restriction avoidance
- Modify Lyapunov function, so in restricted area it is very large
- Control ensures that Lyapunov function always decreases, hence we will never enter restricted areas
- Similar to the potential barrier introduction

Modification of Lyapunov Function

We will use this modification

$$V = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_a\left(1 - (\mathbf{D}\mathbf{n}_{ref}, \mathbf{n})\right) f$$

$$f = 1 + \sum_{k=1}^{n} f_i, \quad f_i = \begin{cases} H_i & \lambda_i < 0 \\ H_i\left(-3\lambda_i^2 + 2\lambda_i^3 + 1\right) & 0 \le \lambda_i \le 1, \\ 0 & 1 < \lambda_i \end{cases} \quad \lambda_i = \frac{\operatorname{acos}\left(\mathbf{n}, \mathbf{h}_i\right) - a_i}{b_i - a_i}$$

- Additional control will affect the motion only near restricted areas
- f is cubic Hermit spline, continuously differentiable
- Depend only on angle λ_i between cone axis $\mathbf{h}_i$ and camera axis $\mathbf{n}$
- a_i, b_i, H_i correspond to the size and “height” of the cone

Attitude control

- Reference motion is inertial stabilization, $\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs}$
- Take time derivative of V and ensure satisfaction of B.-K.-L. theorem
- Expression for the control torque

$$\mathbf{M}_{ctrl} = -M_{ext} - k_{\omega}\boldsymbol{\omega}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - k_a\left[\mathbf{n}_{ref} \times \mathbf{n}\right] +$$

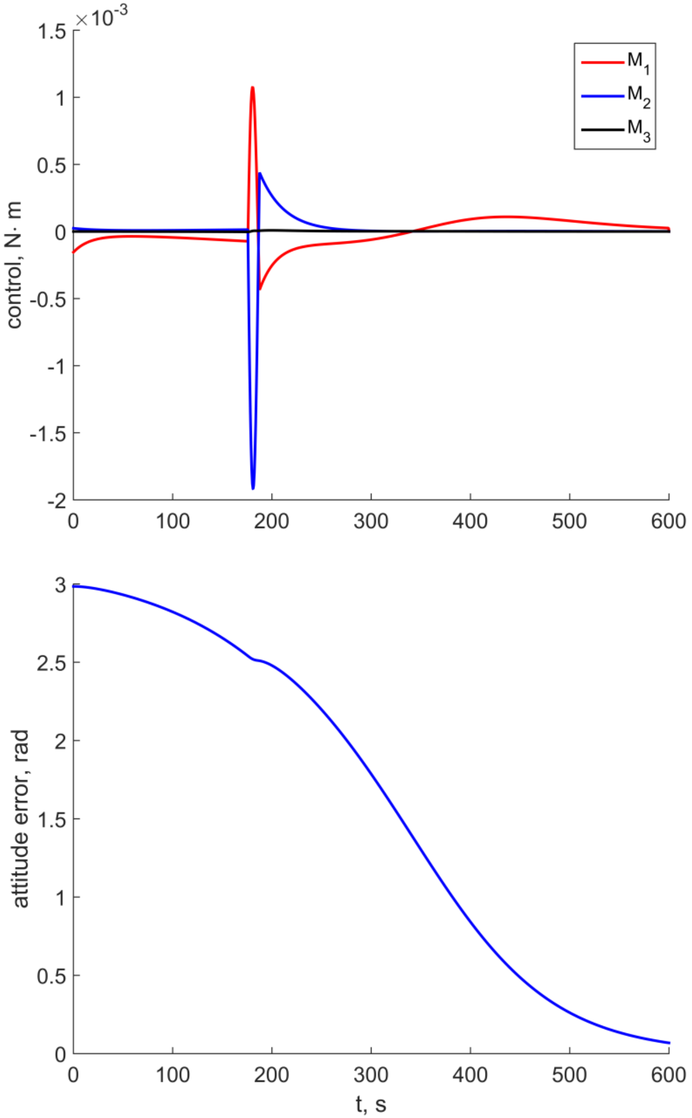
Standard control

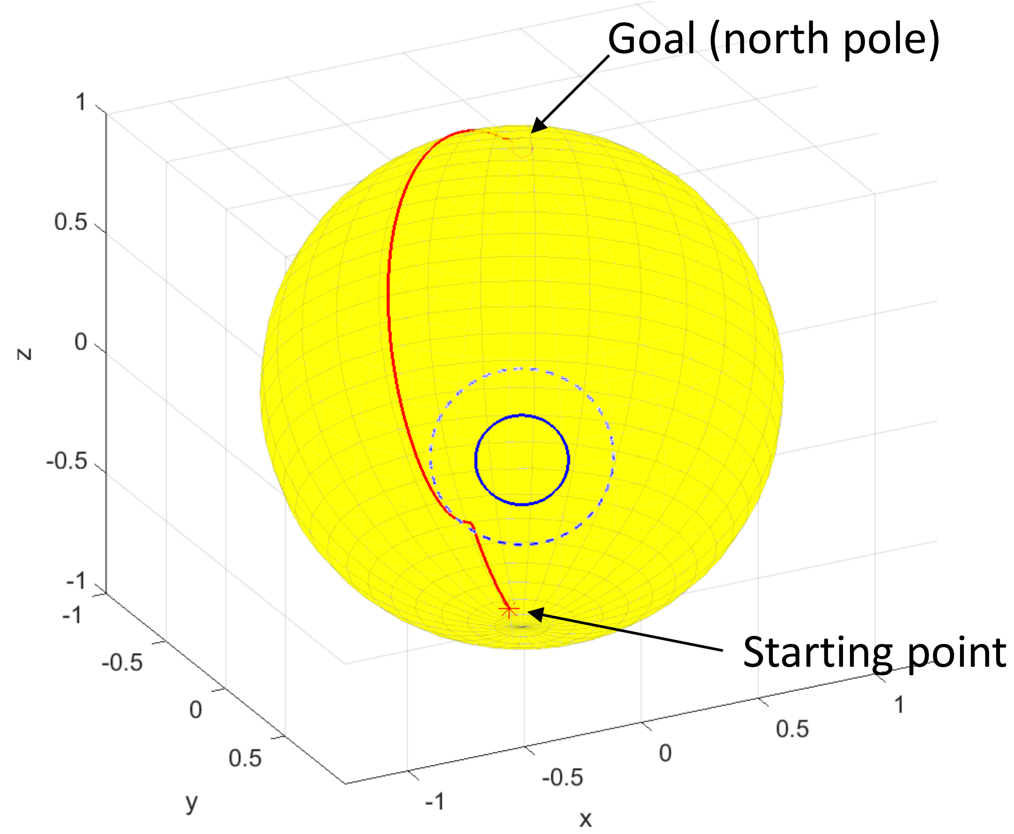
$$+k_a\left(1-\left(\mathbf{n}_{ref},\mathbf{n}\right)\right)\sum_{i=1}^{n}\frac{H_i\left(6\lambda_i^2 - 6\lambda_i\right)}{b_i - a_i}\frac{\mathbf{n}\times\mathbf{h}_i}{\sqrt{1-\left(\mathbf{n},\mathbf{h}_i\right)^2}}$$

$$-k_a\left[\mathbf{n}_{ref} \times \mathbf{n}\right]\left(f-1\right)$$

Restriction areas avoidance

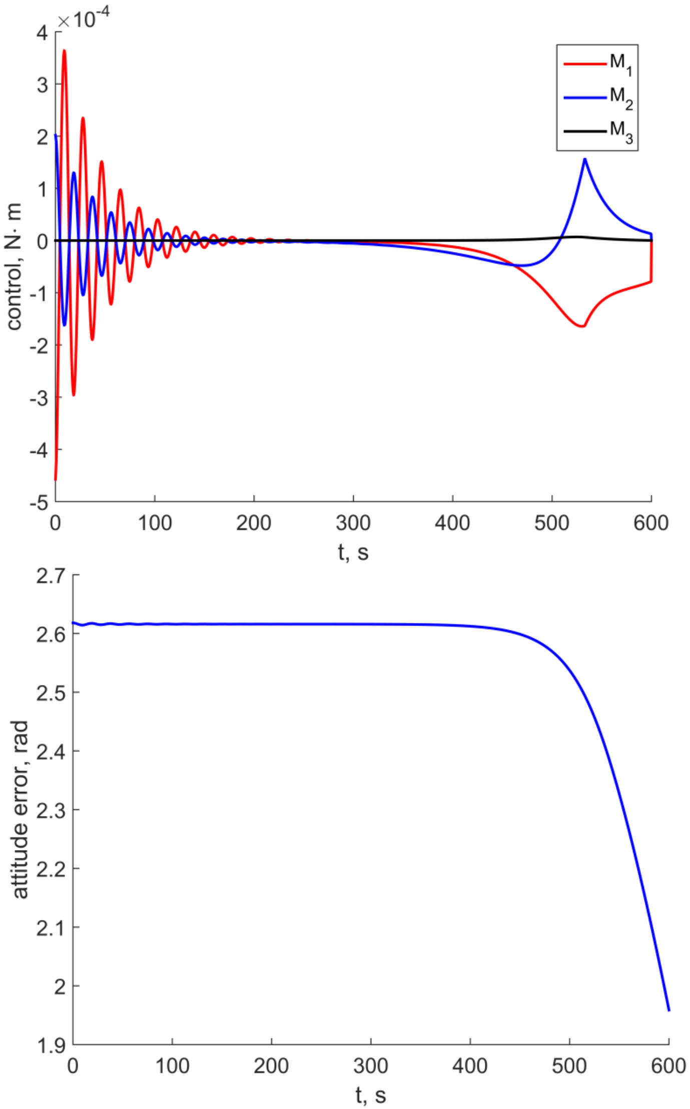
Simulation results

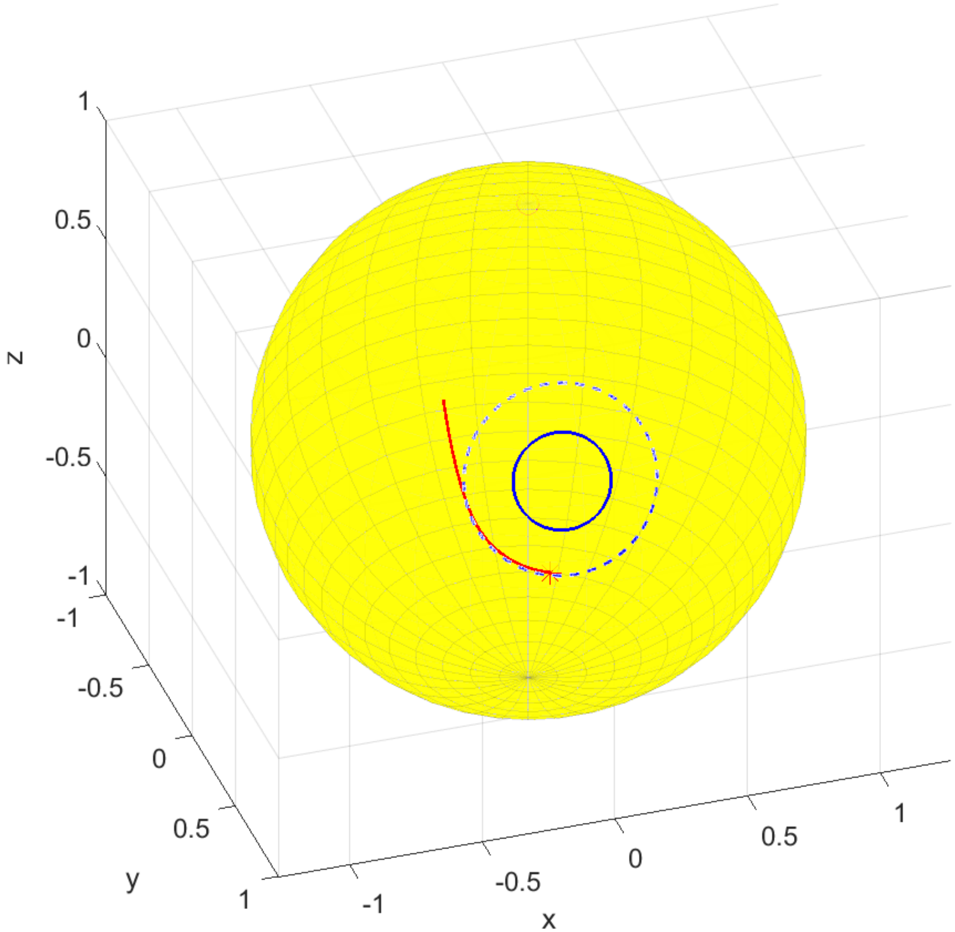




$\mathbf{J} = \mathrm{diag}(2,\ 3,\ 4)$

Pecularities





Saddle points appear with asymptotically stable manifold along meridian

$\mathbf{J} = \mathrm{diag}(2,\ 3,\ 4)$

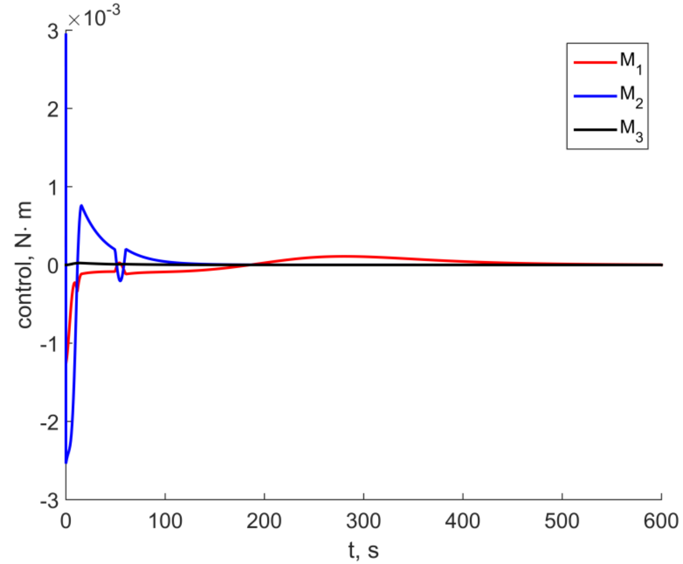
Saddle point avoidance

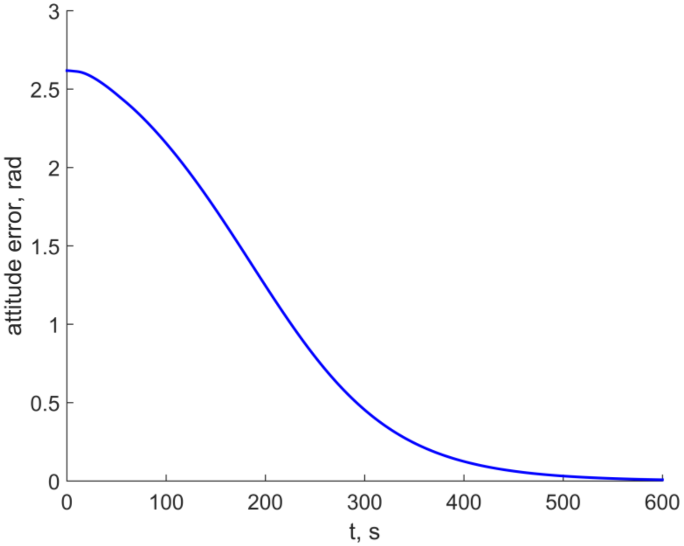
- Saddle point might greatly affect convergence time
- We will add disturbance in the vicinity of saddle point
- Principal idea is to increase angular velocity along the lattitude
- Disturbance torque ($\mathbf{s}$ is unit vector to a saddle point on a sphere):

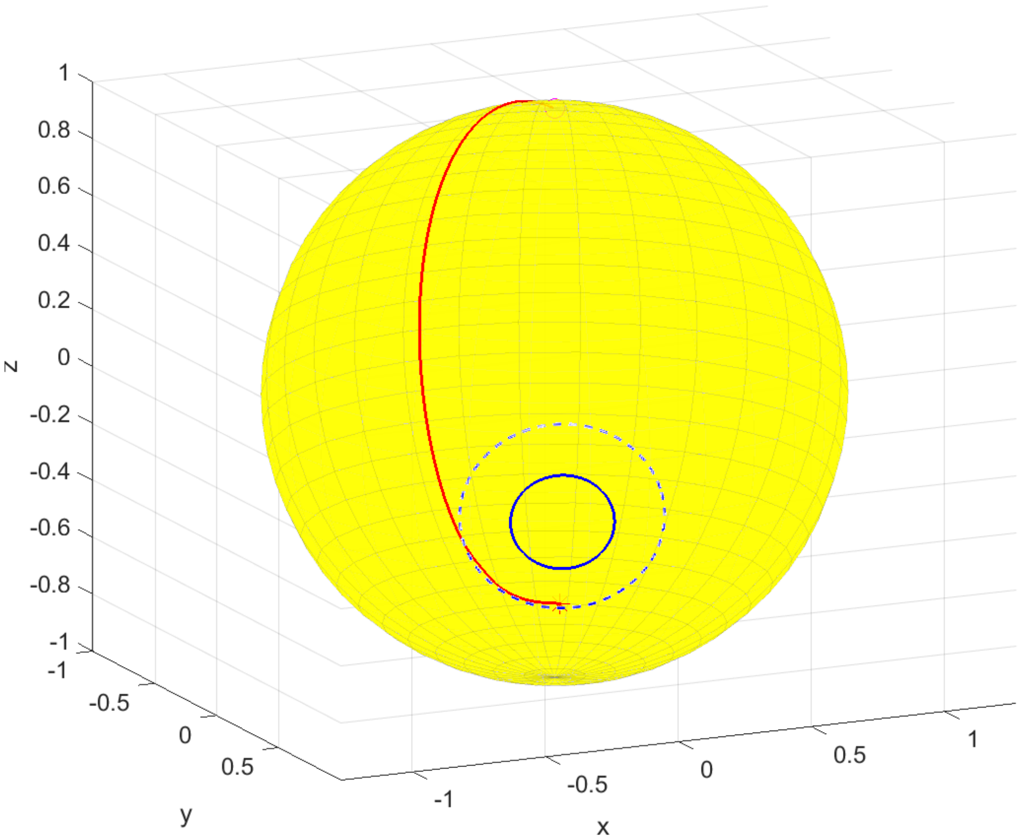
$$\mathbf{M}_{dist} = \operatorname{sign}\left(\left(\boldsymbol{\omega}_{abs}, \mathbf{n}_{ref} \times \mathbf{n}\right)\right)\mathbf{J}\,\frac{\mathbf{n}_{ref} - \mathbf{n}\left(\mathbf{n}_{ref}, \mathbf{n}\right)}{\left|\mathbf{n}_{ref} - \mathbf{n}\left(\mathbf{n}_{ref}, \mathbf{n}\right)\right|}\,g,$$

$$g = \begin{cases} G\left(2\mu^3 - 3\mu^2 + 1\right) & \mu \le 1 \\ 0 & \mu > 1 \end{cases}, \quad \mu = \frac{\operatorname{acos}\left(\mathbf{n}, \mathbf{s}\right)}{d}$$

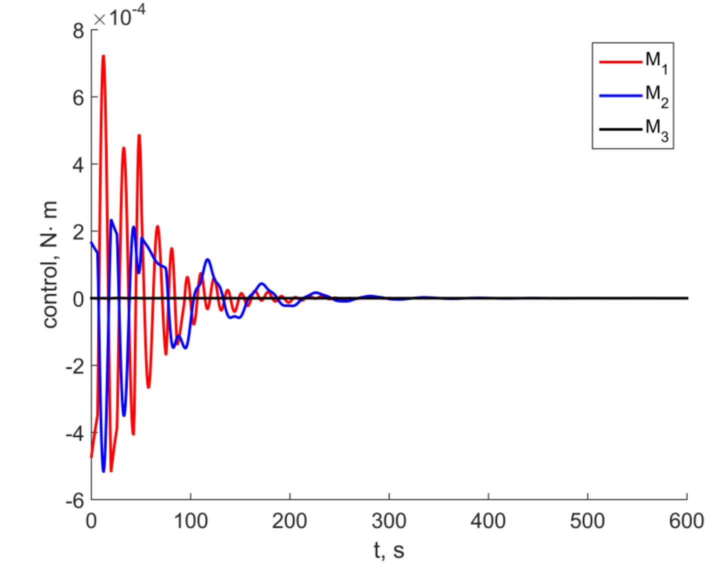
Saddle point avoidance

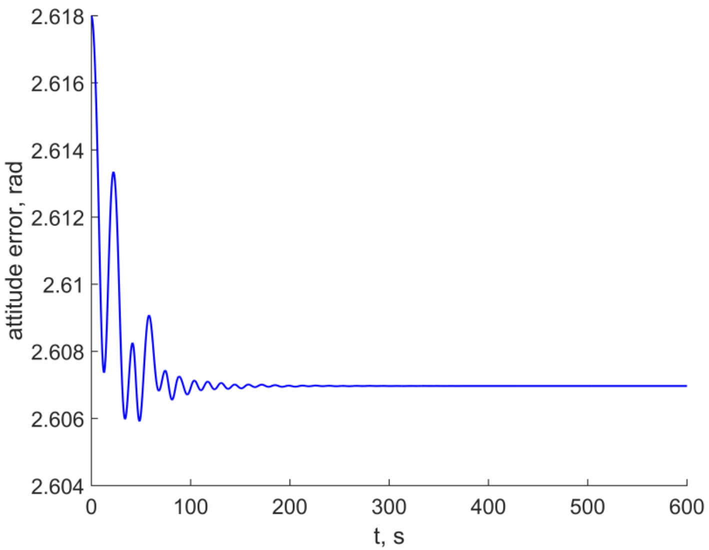


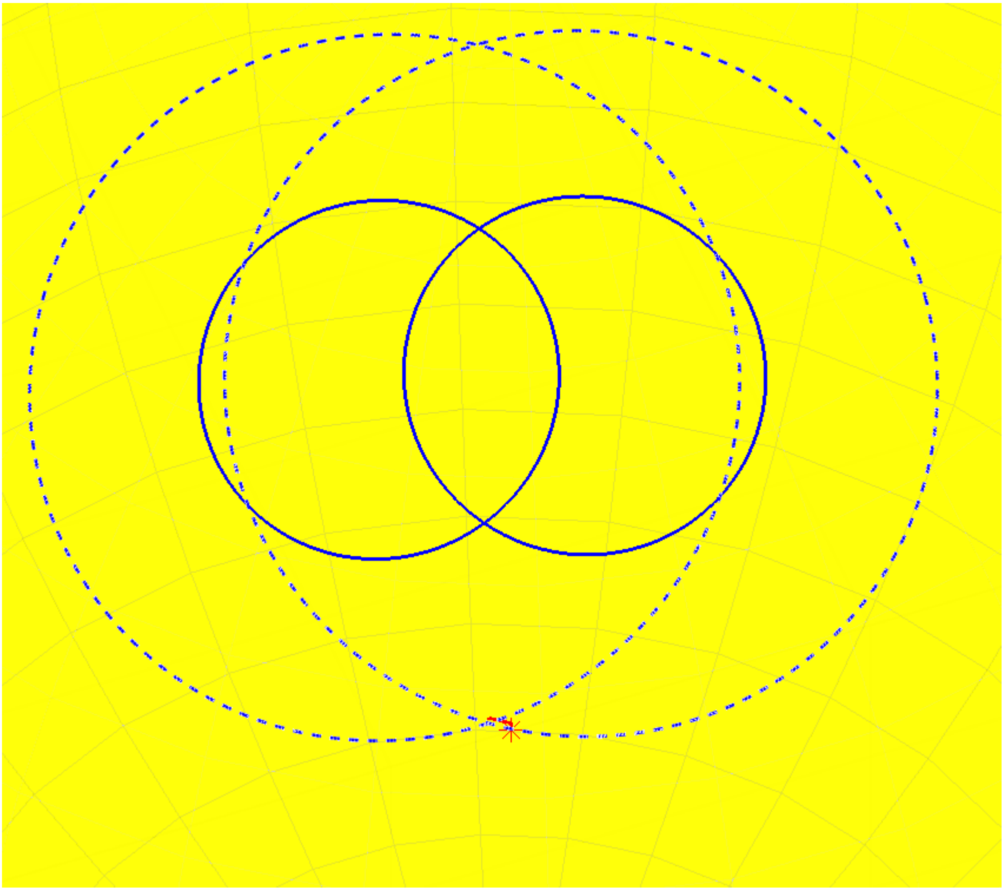




Several keep-out cones







When two cones intersect new asymptotically stable equilibrium might appear

Avoidance of as. stable equilibriums

- We cannot apply suggested control in case of two intersected cones
- It is necessary to “cover” intersected cones by something else
- We suggest to use ellipses: it is the locus of points for which the following is satisfied:

$$\left|\mathbf{P}\right| = 1, \quad \operatorname{acos}\left(\mathbf{P},\mathbf{F}_1\right) + \operatorname{acos}\left(\mathbf{P},\mathbf{F}_2\right) = L$$

$\mathbf{F}_1, \mathbf{F}_2$ are the ellipse focuses

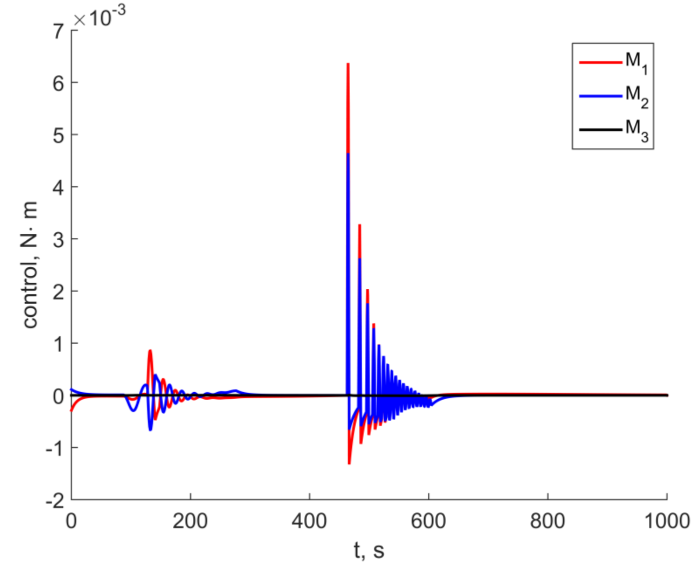
- Control torque

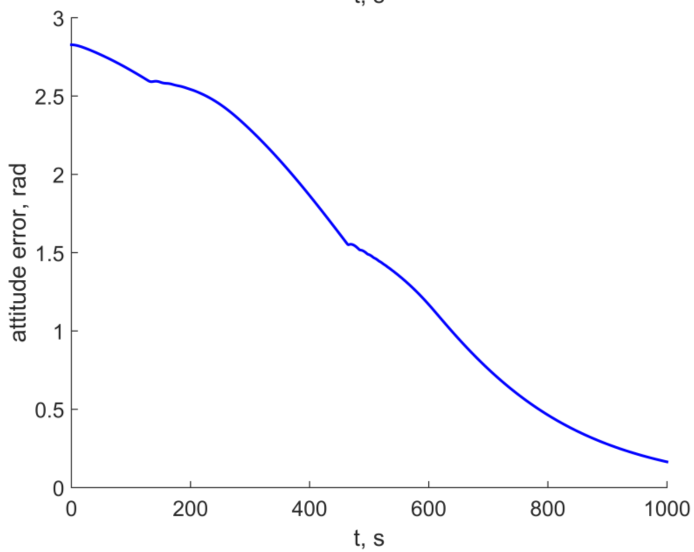
$$\mathbf{M}_{ctrl} = -M_{ext} - k_\omega \boldsymbol{\omega}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - k_a\left[\mathbf{n}_{ref} \times \mathbf{n}\right] +$$

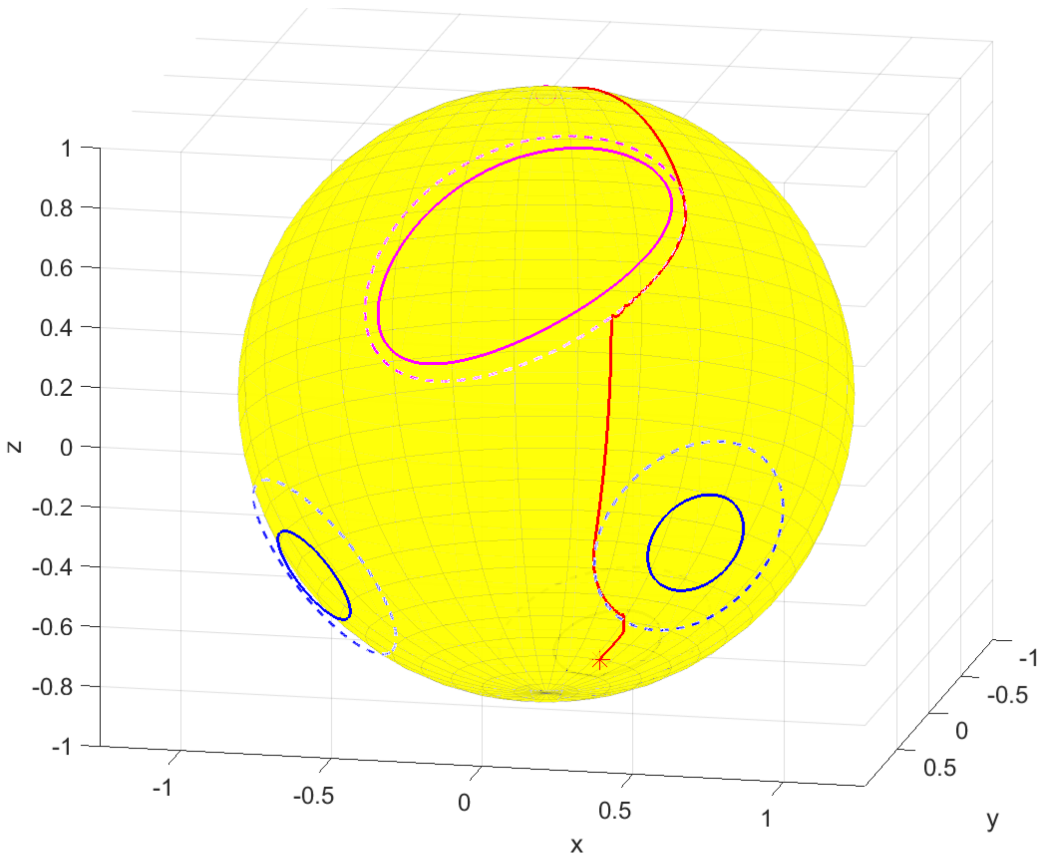
$$+ k_a\left(1 - \left(\mathbf{n}_{ref}, \mathbf{n}\right)\right)\sum_{i=1}^{n} \frac{H_i\left(6\nu_i^2 - 6\nu_i\right)}{B_i - A_i}\left(\frac{\mathbf{n} \times \mathbf{F}_1^i}{\sqrt{1 - \left(\mathbf{n},\mathbf{F}_1^i\right)^2}} + \frac{\mathbf{n} \times \mathbf{F}_2^i}{\sqrt{1 - \left(\mathbf{n},\mathbf{F}_2^i\right)^2}}\right)$$

$$- k_a\left[\mathbf{n}_{ref} \times \mathbf{n}\right]\left(f - 1\right)$$

Simulation in general case







Conclusion

- The attitude control algorithm that takes into account keep-out cones is suggested
- Several cases when this approach might fail or give unsatisfying results are considered, and solution for them is suggested
- Control torques are find analytically, they can be easily calculated on-board
- Control law can be used in two ways: “as is” during the mission, or as a tool of reference motion construction

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