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Analytical approach to construction a reference motion for tetrahedral satellite formation

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Abstract

The paper considers the problem of maintaining a specific satellite structure mainly in connection with the problem of exploring the Earth's magnetic field. To provide accurate results in this case it is necessary to perform measurements not just in several different points of space: the satellites should form a 3-dimensional formation in space. Therefore, the minimal amount of satellites needed to perform such a measurement is four. Moreover, they should form the particular tetrahedral structure, which should not be degenerate and must remain as constant as possible. The goal of the present paper is to find reference motion of four satellites that is suitable for this description. We use assumptions that the satellites move along near circular orbits and major semiaxes of each satellite orbit are equal to prevent relative drift. Two satellites of the formation move on the same orbit. In the paper we define analytically such initial conditions and therefore such reference motion of satellites that the tetrahedron they form preserves its volume and shape in a linear model of motion. We also define an index describing the distortion of the tetrahedron, and show that the optimal solution approximately keeps the geometry fixed. General expressions for the initial parameters are obtained. Moreover, a simple atmospheric control algorithm, based on Lyapunov direct method that extends time of non-degenerate formation motion in the presence of J_2 disturbance is provided.

Keywords: tetrahedral formation, drag control, Lyapunov based control

Acronyms/Abbreviations

Inertial Reference Frame (IRF).
Orbital Reference Frame (ORF).
Clohessy-Wiltshire (CW)
Low Earth Orbit (LEO)

1. Introduction

Formation flying is known as a particularly effective concept of nanosatellite mission arrangement. Among others, this type of arrangement provides different points of view for the observations, which is very useful in certain situations. Particular attention is attracted to the constellation concepts that imply the satellites to fly following a given geometry. Among the successful constellations flying in formation of this type there are PRISMA, GRACE, and other missions [1–5].

The present paper is especially focused on the Magnetospheric Multiscale Mission (MMS). For this mission four satellites need to fly in a tetrahedron formation in a highly elliptical orbit. The goal is to keep the geometry of the tetrahedron as constant as possiblex. However, we will consider the case when satellites fly in circular Low Earth Orbits. We define an index describing the distortion of the tetrahedron and show that the optimal solution approximately keeps the geometry fixed.

Under the influence of external disturbances, e.g. J_2 perturbation, the satellites do not keep the necessary tetrahedron configuration. Moreover, the distance between them might increase over time. Therefore, the active control is necessary.

The paper organised as follows. In Section 2 we give the statement of the problem. In Section 3 we introduce a distortion parameter and derive necessary and sufficient conditions for its preservation. In Section 4 we obtain particular initial conditions for the problem. In addition, using numerical simulation we show that in real model of motion that includes J_2 perturbation the tetrahedron degrades over time. In Section 5 we present a simple control law that allow us to extend lifetime of a formation. In Section 6 we provide numerical simulation of the suggested control.

2. Problem statement and motion model

Let us consider the following problem

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- Four satellites orbit passively on near circular orbits, major semiaxes of their orbits are equal.
- Two of them move along the same circular orbit with some initial shift.
- Four satellites together form a tetrahedron for which we want to define some mathematical equivalent of size and shape.
- We need to find the initial parameters for the satellites motion so that the tetrahedron does not change its size and shape over time at least approximately
- Also, the tetrahedron should never reach zero volume.
- Moreover, we want to construct a simple control algorithm to maintain shape of the tetrahedron in a presence of disturbances.

We use the following right-handed Cartesian reference frames:

IRF. Its center $O_\oplus$ is at the Earth center of mass, the axis $O_\oplus Z$ is directed along the Earth axis of rotation, the axis $O_\oplus X$ is directed to the vernal equinox corresponding to the epoch J2000.

ORF. Its center O is at the one of satellites, the axis Ox is directed along the radius vector of the point O away from the Earth, the axis Oz is normal to the orbital plane and is directed along the orbital momentum.

The center of ORF is located in one of the satellites that is moving along the circular orbit. Without loss of generality we refer to this satellite as "the fourth". Its motion in ORF is described by

$$\mathbf{r}_4(t) = \langle x_4(t), y_4(t), z_4(t) \rangle = \langle 0,0,0 \rangle.$$

Our assumptions allow us to describe the relative motion of other satellites using the linearized CW equations:

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - 3n^2 x = 0, \\ &\ddot{y} + 2n\dot{x} = 0, \\ &\ddot{z} + n^2 z = 0, \end{aligned}$$

where $n = \sqrt{\mu / \rho^3}$ is the mean motion, μ is the Earth gravitational parameter, ρ is the radius of circular orbit. Orbits major semiaxes being equal guarantees periodic motion of each satellite in ORF, so the tetrahedron size is bounded over time. This motion is described then by equations

$$\begin{aligned} &x_i(t) = A_i \sin\nu + B_i \cos\nu, \\ &y_i(t) = 2A_i \cos\nu - 2B_i \sin\nu + C_i, \\ &z_i(t) = D_i \sin\nu + E_i \cos\nu, \end{aligned} \qquad (1)$$

where $\nu = nt$. Here A_i, B_i, C_i, D_i, E_i are constants depending on the initial values of motion, index i attains values 1, 2, 3. The motion of the fourth satellite is described by the same set of equations with all the constants being equal to zero.

3. Size and shape of tetrahedron

We now derive the conditions for the tetrahedron to preserve size and shape. In this section we do not use the fact that two satellites move along the same orbit, rather we define size and shape in general case of tetrahedron.

The natural measure for the size of the tetrahedron is volume $\mathbb{V}$. In ORF the volume has the form

$$\mathbb{V} = \frac{1}{6}\det\|\mathbf{r}_1 - \mathbf{r}_4, \mathbf{r}_2 - \mathbf{r}_4, \mathbf{r}_3 - \mathbf{r}_4\| = \frac{1}{6}\det\|\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\|.$$

Substituting $\mathbf{r}_i$ with values from (1) we obtain volume as trigonometric polynomial of ν

$$\begin{aligned} &6\mathbb{V} = P\sin^3\nu + Q\cos^3\nu + R\sin^2\nu\cos\nu + T\sin\nu\cos^2\nu \\ &+U\sin^2\nu + V\cos^2\nu + W\sin\nu\cos\nu. \end{aligned}$$

The coefficients in the polynomial depend on initial conditions, i.e. on A_i, B_i, C_i, D_i, E_i . For the volume $\mathbb{V}$ of the tetrahedron to remain constant it is necessary that

$$\begin{aligned} &\mathrm{P = Q = T = W = R = 0}, \\ &\mathrm{U = V}. \end{aligned}$$

Under these conditions, the volume is equal to $\mathbb{V} = U/6$, hence they are also sufficient.

We want the tetrahedron to be non-degenerate. To simplify notation we combine constants A_i, B_i, C_i, D_i, E_i in (1) in vectors. Let $\mathbf{A} = \langle A_1, A_2, A_3 \rangle$, vectors $\mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}$ are defined analogously.

With this notation and after appropriate simplifications conditions have the form

$$\begin{aligned} &U = V \to (\mathbf{C}, \mathbf{D}, \mathbf{A}) = (\mathbf{C}, \mathbf{E}, \mathbf{B}), \\ &P = 0 \to (\mathbf{B}, \mathbf{A}, \mathbf{D}) = 0, \\ &Q = 0 \to (\mathbf{A}, \mathbf{E}, \mathbf{B}) = 0, \\ &W = 0 \to (\mathbf{C}, \mathbf{D}, \mathbf{B}) = (\mathbf{C}, \mathbf{A}, \mathbf{E}), \\ &R = 0 \to (\mathbf{B}, \mathbf{A}, \mathbf{E}) = 0, \\ &T = 0 \to (\mathbf{A}, \mathbf{D}, \mathbf{B}) = 0, \end{aligned} \qquad (2)$$

where $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ is mixed product of three vectors $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$.

If $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ or $\mathbf{E}$ is equal to zero, then $\mathbb{V} = 0$ that should be avoided. If none of these vectors are equal to zero, then from (2) we can derive that all four of them should be coplanar. If $\mathbf{A}$ and $\mathbf{B}$ are collinear, again $\mathbb{V} = 0$. If $\mathbf{A}$ and $\mathbf{B}$ are not collinear then they form a

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basis in a plane and coplanar $\mathbf{D}$, $\mathbf{E}$ are expressed as linear combinations

$$\mathbf{D} = a\mathbf{A} + b\mathbf{B},$$
$$\mathbf{E} = c\mathbf{A} + d\mathbf{B},$$

so that

$$b(\mathbf{C},\mathbf{B},\mathbf{A}) = c(\mathbf{C},\mathbf{A},\mathbf{B}),$$
$$a(\mathbf{C},\mathbf{A},\mathbf{B}) = d(\mathbf{C},\mathbf{A},\mathbf{B}),$$

which eventually leads to $b = -c$, $a = d$.

With such a conditions the volume $\mathbb{V}$ could be calculated from the formula

$$\mathbb{V} = \frac{b}{6}(\mathbf{A},\mathbf{C},\mathbf{B}).$$

The coefficient b should not be equal to zero in all subsequent calculations.

Unlike the volume, the shape of the tetrahedron does not have simple geometric or algebraic interpretation, partially due to the fact that the tetrahedron is not fully described by lengths of its edges. We do not demand the similarity of the tetrahedron in each moment of time, instead we use one parameter that depicts the shape of the tetrahedron in average. We also assume that conservation of this parameter implies conservation of the shape at least approximately. This parameter, which here and below is called the tetrahedron quality, is described as

$$\mathbb{Q} = 12\frac{(3\mathbb{V})^{2/3}}{\mathbb{L}}$$

Here $\mathbb{V}$ is the volume, $\mathbb{L}$ is the sum of squares of the tetrahedron edge lengths. For regular tetrahedron $\mathbb{Q} = 1$ and for degenerate one (when four satellites lie in the same plane) $\mathbb{Q} = 0$.

Similar to the volume derivation we derive the expression for $\mathbb{L}$:

$$\mathbb{L} = (\mathbf{r}_1 - \mathbf{r}_2)^2 + (\mathbf{r}_1 - \mathbf{r}_3)^2 + (\mathbf{r}_2 - \mathbf{r}_3)^2 + \mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2.$$

After substitutions, reductions of terms and all simplifications the derivation for $\mathbb{L}$ is a trigonometric polynomial. In general the polynomial has the form

$$\mathbb{L} = P\cos^2\nu + Q\cos\nu\sin\nu + R\sin^2\nu$$
$$+T\cos\nu + U\sin\nu + W$$

The necessary and sufficient conditions for conservation of $\mathbb{L}$ have the form

$$Q = T = U = 0,$$
$$P = R.$$

Using volume conservation expressions we can simplify the equations. Finally, for the non-degenerate tetrahedron preserving its volume and quality (size and shape) vectors $\mathbf{A}$ and $\mathbf{B}$ must be non-collinear and the following expressions must be true

$$\begin{aligned}
&\mathbf{D} = a\mathbf{A} + b\mathbf{B},\\
&\mathbf{E} = -b\mathbf{A} + a\mathbf{B},\\
&\\
&3A_1B_1 + 3A_2B_2 + 3A_3B_3\\
&-A_1B_2 - A_1B_3 - A_2B_1\\
&-A_2B_3 - A_3B_1 - A_3B_2 = 0,\\
&3\left(B_1^2 + B_2^2 + B_3^2 - A_1^2 - A_2^2 - A_3^2\right)\\
&+2\left(A_1A_2 + A_1A_3 + A_2A_3\right)\\
&-2\left(B_1B_2 - B_1B_3 - B_2B_3\right) = 0,\\
&\\
&C_1(3A_1 - A_2 - A_3)\\
&+C_2(3A_2 - A_1 - A_3)\\
&+C_3(3A_3 - A_1 - A_2) = 0,\\
&C_1(3B_1 - B_2 - B_3)\\
&+C_2(3B_2 - B_1 - B_3)\\
&+C_3(3B_3 - B_1 - B_2) = 0.
\end{aligned} \tag{3}$$

To fully describe all possible configurations preserving volume and quality one should solve this system for unknown vectors $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$ and $\mathbf{E}$. This is the system of 10 equations with 15 variables so the solutions are 5-parametric families. Two of the parameters are the constants a and b. One parameter should be proportional to the volume of the tetrahedron because enlarging or shrinking of the tetrahedron does not affect the quality. The fourth parameter is the initial phase of the motion, because $\nu = nt$ changes from 0 to 2π over time, so adding arbitrary number to the phase of all satellites in a group does not change the motion. The fifth parameter could be found using the following observation: vector $\mathbf{C}$ has three components, but only two last equations depend on them, so $\mathbf{C}$ could be found only up to a factor. This arbitrary factor is the fifth parameter – in our case it is the shift between two satellites orbiting on the same orbit. Note that the satellite renumbering does not affect the dynamics, so we refer two different solutions obtained from each other by renumbering the satellites to a single family of solutions.

4. Particular solutions analysis

In a search for particular solutions we do the following variables change

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$$A_1 = \alpha\cos\varphi, \quad B_1 = \alpha\sin\varphi,$$
$$A_2 = \beta\cos\psi, \quad B_2 = \beta\sin\psi,$$
$$A_3 = \gamma\cos\theta, \quad B_3 = \gamma\sin\theta,$$

here α, β, γ -- amplitudes of oscillations of first, second and third satellites in ORF respectively, φ, ψ, θ are the initial phases.

Now we use our main simplifying assumption: two satellites, the third and the forth are on the same orbit, so the third satellite rests in ORF. That means $\gamma = 0$.

$$2\left(\alpha\beta\cos(\varphi+\psi)\right) = 3\left(\alpha^2\cos 2\varphi + \beta^2\cos 2\psi\right),$$
$$2\left(\alpha\beta\sin(\varphi+\psi)\right) = 3\left(\alpha^2\sin 2\varphi + \beta^2\sin 2\psi\right).$$

Moreover, $\nu = nt$ changes from 0 to 2π upon the motion, so we choose initial moment of time so that $\psi = 0$. The only non-degenerate solution is

$$\alpha = \beta = K,$$
$$\cos\varphi = \frac{1}{3}.$$

Here K represents the linear size of tetrahedron (average length of edge) and φ is the phase shift between the first and the second satellites. When ψ is again nonzero arbitrary phase angle we obtain initial conditions

$$A_1 = K\left(\sqrt{6}/3\cos\psi + \sqrt{3}/3\sin\psi\right),$$
$$A_2 = K\left(\sqrt{6}/3\cos\psi - \sqrt{3}/3\sin\psi\right),$$
$$A_3 = 0,$$
$$B_1 = K\left(-\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right),$$
$$B_2 = K\left(\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right),$$
$$B_3 = 0.$$

Substituting obtained relations in (3) we obtain

$$C_1 = C_2 = c, \quad C_3 = 2c$$

with arbitrary c. Combining it with

$$\mathbf{D} = a\mathbf{A} + b\mathbf{B},$$
$$\mathbf{E} = -b\mathbf{A} + a\mathbf{B},$$

we obtain full solutions to the problem of initial values with K, ψ, a, b, c being parameters.

In this case

$$\mathbb{V} = \frac{b}{6}(\mathbf{A}, \mathbf{C}, \mathbf{B}) = cK^2 b\frac{2\sqrt{2}}{9},$$
$$\mathbb{L} = \frac{8}{3}K^2(a^2 + b^2 + 5) + 8c^2.$$

As being expected, neither volume, nor quality depends on ψ in linear case.

The maximum of quality is achieved when $a = 0, b = \pm\sqrt{5}, c = \pm K\sqrt{\frac{5}{3}}$ and is equal to $\mathbb{Q}_{max} = \frac{1}{\sqrt[3]{5}}$

Fig. 1 shows the evolution of the formation quality in the nonlinear motion model including J_2 geopotential harmonic, depending on the relative orbits of the satellites for an orbit 10 000 km radius. Different plots contain the information of tetrahedron degrading for different tetrahedron size; the graphics look alike, but the scale of y -axis is different, black line represents conserving quality in linear model.

Fig. 2 depicts quality changing for 40 000 km orbit, as can be seen due to the absence of J_2 disturbance, the quality exhibits much more regular behaviour. $K = 2000$ m is presented on both figures for comparison.

Fig. 3 shows the visualization of the resulting tetrahedron.

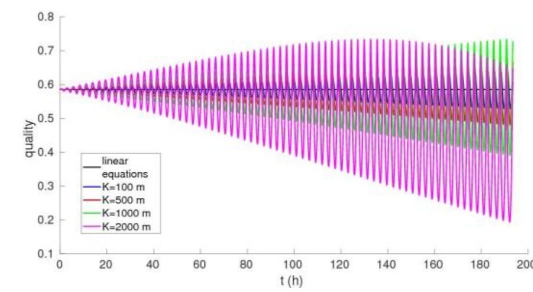

Fig. 1. Quality evolution over time for 10 000 km orbit

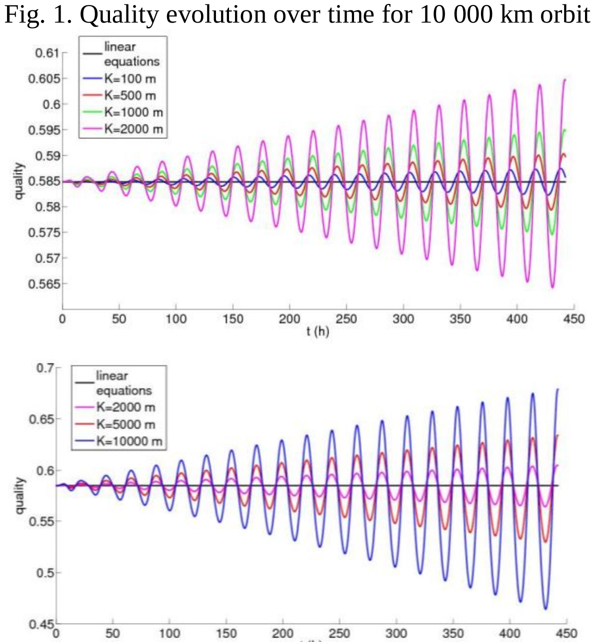

Fig. 2. Quality evolution over time for 40 000 km orbit

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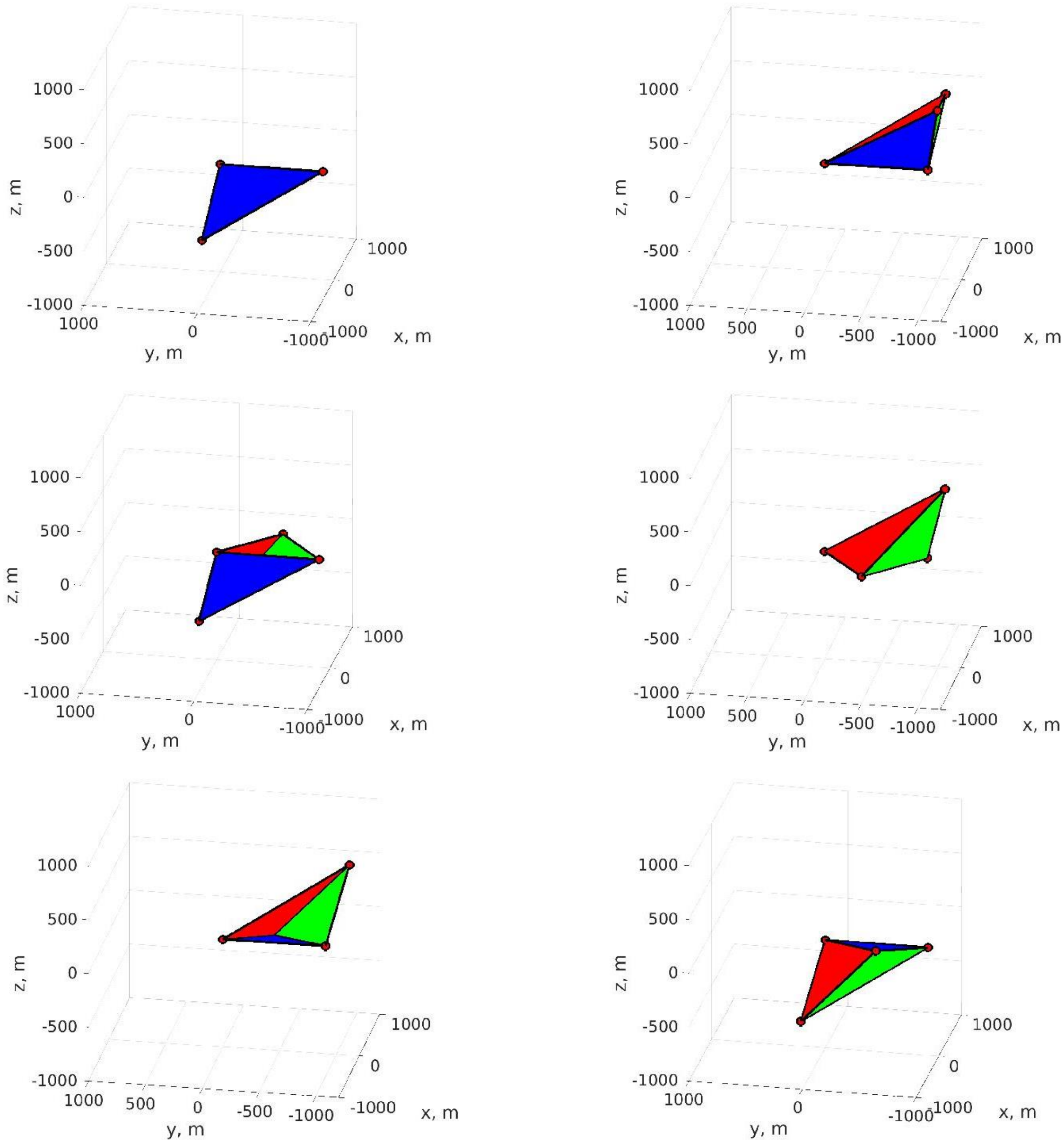

Fig. 3. Tetrahedron evolution

5. Atmospheric control algorithm

Simulations show that for high orbits degradation of the tetrahedron quality is rather small, so active control of relative orbits is not necessary. However, at LEO tetrahedron degradation rate leaves much to be desired so some control algorithm, for example using thrusters [6], is necessary.

The motion model including non-linearity and control is of the form

$$\ddot{x} - 2n\dot{y} - 3n^2 x = g_x + u_x,$$
$$\ddot{y} + 2n\dot{x} = g_y + u_y,$$
$$\ddot{z} + n^2 z = g_z + u_z,$$

Introduce new variables $A, B, C, D, \lambda, \zeta$:

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$$
\begin{aligned}
&x = A\sin\zeta + 2C,\\
&\dot{x} = An\cos\zeta,\\
&y = 2A\cos\zeta + D,\\
&\dot{y} = -2An\sin\zeta - 3Cn,\\
&z = B\sin\lambda,\\
&\dot{z} = Bn\cos\lambda.
\end{aligned}
$$

Their derivative in accordance with equations of motion can be written as follows:

$$
\begin{aligned}
&\dot{A} = \frac{1}{n}\left(\left(u_x + g_x\right)\cos\zeta - 2\left(u_y + g_y\right)\sin\zeta\right),\\
&\dot{B} = \frac{1}{n}\left(u_z + g_z\right)\cos\lambda,\\
&\dot{C} = \frac{1}{n}\left(u_y + g_y\right),\\
&\dot{D} = -3Cn - \frac{2}{n}\left(u_x + g_x\right),\\
&\dot{\lambda} = n - \frac{1}{nB}\left(u_z + g_z\right)\sin\lambda.\\
&\dot{\zeta} = n - \frac{1}{An}\left(\left(u_x + g_x\right)\sin\zeta + 2\left(u_y + g_y\right)\cos\zeta\right).
\end{aligned}
$$

The physical meaning of new variables is closely related to the meaning of CW constants: the relative trajectory is close to ellipse with semiaxes A and B, this ellipse drifts from the origin of ORF with rate C and initial shift D. Also, λ and ζ correspond to the position of the satellite on the relative orbit. The motion orthogonal to the orbit plane is quasiperiodic and therefore is not much of an interest.

The relative orbit size changes slowly with respect to shift and drift that can change rapidly and with increasing rate. So the first step of a control algorithm should nullify drift and set shift to a desired value

$$C_{ref} = 0, \quad D_{ref} = D_0.$$

In order to increase lifetime of the satellite it would be useful to suggest fuelless control algorithm. For example, we can use atmospheric drag. It means, that control can be applied only along the velocity vector of the satellite, i.e. $u_x = u_z = 0$. Here we do not take into account the possible reflection of air molecules from the satellite body, i.e. aerodynamic lift – we will consider it as a disturbance and, as well as other disturbances, will not include it in control synthesis. Hence, simplified system dynamics is

$$
\begin{aligned}
&\dot{A} = \frac{-2u_y\sin\zeta}{n},\\
&\dot{C} = \frac{u_y}{n},\\
&\dot{D} = -3Cn.
\end{aligned}
$$

For control construction we use Lyapunov direct method. Consider the Lyapunov-candidate function

$$V = C^2 + k_D\left(D - D_{ref}\right)^2, \quad k_D > 0.$$

Its derivative is

$$\dot{V} = C\left(\frac{u_y}{n} - 3nk_D\left(D - D_{ref}\right)\right).$$

If $\dot{V} = -k_c C, \ k_c > 0$, the derivative is non-positive and equal to zero on set containing only one whole trajectory $C = 0, D = D_{ref}$. According to Barbashin-Krasovski-LaSalle theorem the control

$$u_y = 3n^2 k_D\left(D - D_{ref}\right) - k_c C$$

provides asymptotic stability of a desired motion. However, due to the presence of external disturbances this control will only ensure that drift and shift of the orbit are within the vicinity of the required ones.

In addition to shift elimination, we should also provide the correct size of relative orbit

$$A = A_{ref}.$$

We suggest simple idea: when shift and drift are within acceptable vicinity of required values, control should work only when it helps to achieve correct size of relative orbit i.e. when

$$\operatorname{sign}\left[\left(A - A_{ref}\right)u_y\sin\zeta\right] > 0$$

otherwise control equals to zero. Such an algorithm provides desired zero drift, shift and size of relative orbit.

It should be noted that drag gives us an opportunity to produce control only in the direction opposite to the current satellite velocity. However, since we got two satellites, one of which is located in the origin of ORF, total control can be described by

$$u_y = f_{sat} - f_{origin}$$

where f_{origin} affects the satellite located in the origin of ORF and f_{sat} affects the other one. Hence, it is possible to create control not only in the direction opposite to the velocity, but also in the same direction.

We now apply designed control algorithm to maintain the tetrahedron. Since we got four satellites (the fourth is located in the origin of ORF), the following control scheme is suggested. For three

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satellites the reference values of shift, drift and relative orbit is set. For each of them ideal necessary control is calculated separately, so we have three different (in general case) values of required control. Therefore,

$$f_{sat,1} - f_{origin} = u_{y,1},$$
$$f_{sat,2} - f_{origin} = u_{y,2},$$
$$f_{sat,3} - f_{origin} = u_{y,3},$$

must be satisfied. This system consists of three equations and three variables. Also, there are additional constraints:

$$0 \le f_{sat,i} \le f_{\max}, \quad 0 \le f_{origin} \le f_{\max}.$$

The ambiguity can be solved as follows.

The first step ensures that all forces applied are nonnegative:

$$\langle f_{origin}, f_{sat,1}, f_{sat,2}, f_{sat,3} \rangle := \langle 0, u_{y,1}, u_{y,2}, u_{y,3} \rangle - \min\left(0, u_{y,1}, u_{y,2}, u_{y,3}\right)$$

The second step takes into account the magnitude constraint

$$\langle f_{origin}, f_{sat,1}, f_{sat,2}, f_{sat,3} \rangle := \frac{f_{\max}}{\max\left(f_{origin}, f_{sat,1}, f_{sat,2}, f_{sat,3}\right)} \times \langle f_{origin}, f_{sat,1}, f_{sat,2}, f_{sat,3} \rangle$$

Suggested technique allows us to take into account all the constraints and get rid of the ambiguity.

6. Simulation results

In this section we show a simulation results of the control technique proposed in Section 5. J_2 effects, as well as lift force were included in simulation. Characteristics of relative orbits are chosen in accordance with results obtained in Section 4:

$$A_{1,ref} = A_{2,ref} = K, \quad A_{3,ref} = 0,$$
$$C_{1,ref} = C_{2,ref} = C_{3,ref} = 0,$$
$$D_{1,ref} = D_{2,ref} = \sqrt{\frac{5}{3}}K, \quad D_{3,ref} = 2\sqrt{\frac{5}{3}}K,$$

Here $K = 300\,\text{m}$. In Fig. 4 evolution of tetrahedron quality is presented with active atmospheric drag control is presented. Figs. 5 and 6 contain relative orbit parameters evolution.

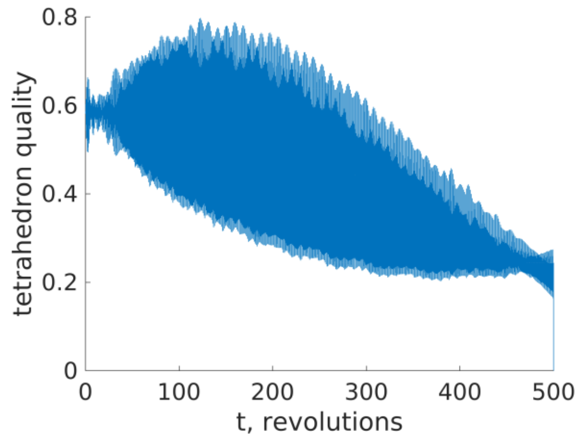

Fig. 4. Evolution of tetrahedron quality under control

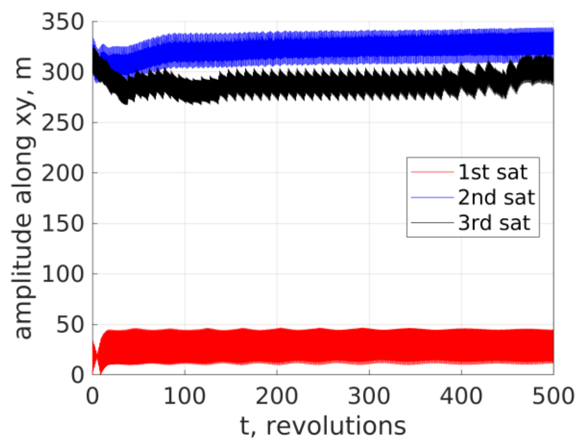

Fig. 5. Amplitude of relative orbit evolution

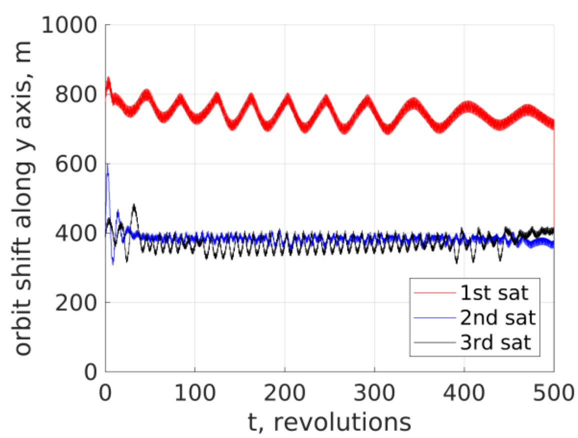

Fig. 6. Shift of relative orbit evolution

As we can see, suggested control technique allows us to maintain the necessary relative orbit parameters, at least in orbital plane.

7. Conclusion

In the paper we construct reference orbits for four satellites in linear model for the tetrahedron to preserve volume and quality according to introduced

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criterion. In Keplerian motion model including Earth's J_2 perturbation it is shown that in low orbits the tetrahedron shape remains approximately constant and degradation rate notably grows with tetrahedron size increase. It is shown that simple fuellsess control algorithm based on atmospheric drag utilization allows expanding mission lifetime to a month.

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