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Analytical approach to construction a reference motion for tetrahedral satellite formation

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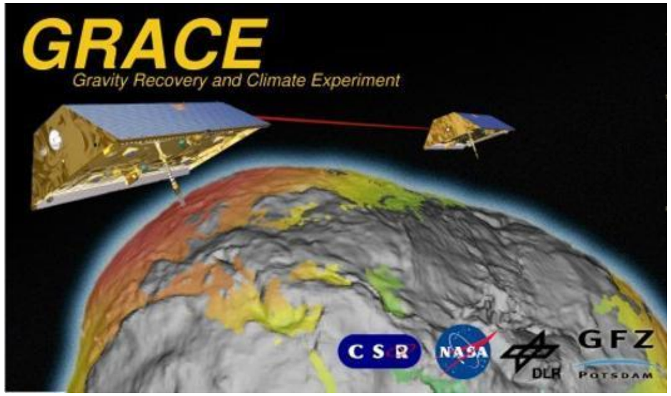
Introduction

Formation flight:

- Redundant
- Cheap
- Allow to perform simultaneous measurements
- Examples: Magnetospheric MultiScale, GRACE

How to maintain necessary geometry?



Problem statement

- Four satellites move on close LEOs
- Two satellites move along the same orbit
- Need to obtain a reference orbit in order that the volume and shape of the corresponding tetrahedron maintain over time
- Size and shape must be formalized
- Also provide a simple control algorithm for several satellites to neglect perturbations

Tetrahedral configuration

- Quality of the tetrahedron

$$\mathbb{Q} = 12\frac{(3|\mathbb{V}|)^{2/3}}{r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2} = 12\frac{(3|\mathbb{V}|)^{2/3}}{\mathbb{L}}$$

- Volume conservation is the conservation of the size
- Quality conservation is the conservation of the shape

- Quality is chosen to be meaningful in geometric sense and analytically analyzable

Reference orbit for circular motion

- Linearized HCW model, closed orbits described by

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0, \qquad x_i(t) = A_i \sin\nu + B_i \cos\nu,$$
$$\ddot{y} + 2n\dot{x} = 0, \qquad y_i(t) = 2A_i \cos\nu - 2B_i \sin\nu + C_i,$$
$$\ddot{z} + n^2 z = 0 \qquad z_i(t) = D_i \sin\nu + E_i \cos\nu,$$

- The main goal is to find such reference orbit that in passive motion in linearized model volume and quality of the tetrahedron remain constant

Reference orbit for circular motion

- Volume conserving conditions: define $\mathbf{A} = \langle A_1, A_2, A_3 \rangle$

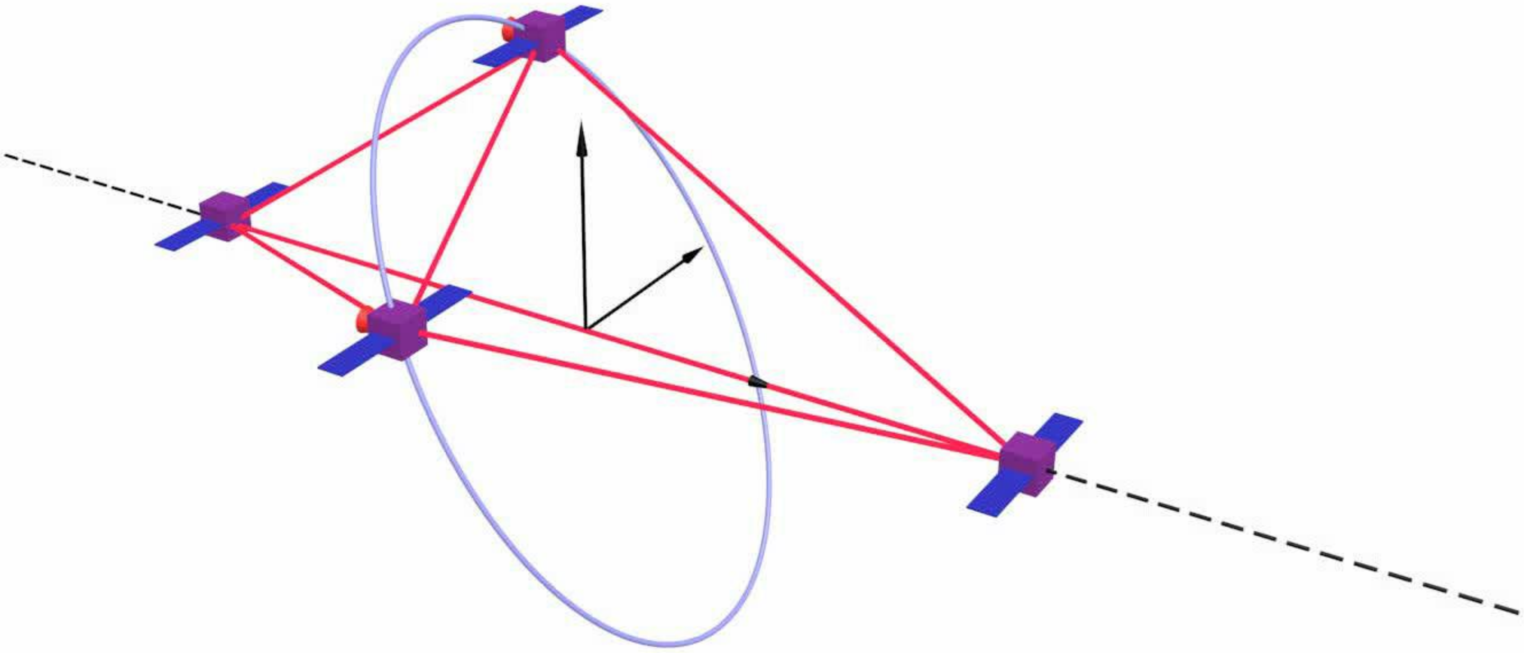
and the other vectors analogously

- The reference orbit so that volume and quality of the tetrahedron conserve

$$
\begin{aligned}
x_i(t) &= A_i \sin\nu + B_i \cos\nu, \\
y_i(t) &= 2A_i \cos\nu - 2B_i \sin\nu + C_i, \\
z_i(t) &= D_i \sin\nu + E_i \cos\nu,
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
A_1 &= K\left(\sqrt{6}/3\cos\psi + \sqrt{3}/3\sin\psi\right), \\
A_2 &= K\left(\sqrt{6}/3\cos\psi - \sqrt{3}/3\sin\psi\right), \\
A_3 &= 0, \\
B_1 &= K\left(-\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right), \\
B_2 &= K\left(\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right), \\
B_3 &= 0.
\end{aligned}
$$

$$C_1 = C_2 = c, \quad C_3 = 2c$$

$$\mathbf{D} = a\mathbf{A} + b\mathbf{B}, \quad \mathbf{E} = -b\mathbf{A} + a\mathbf{B},$$



Reference orbit for circular motion

- Volume and quality have the form

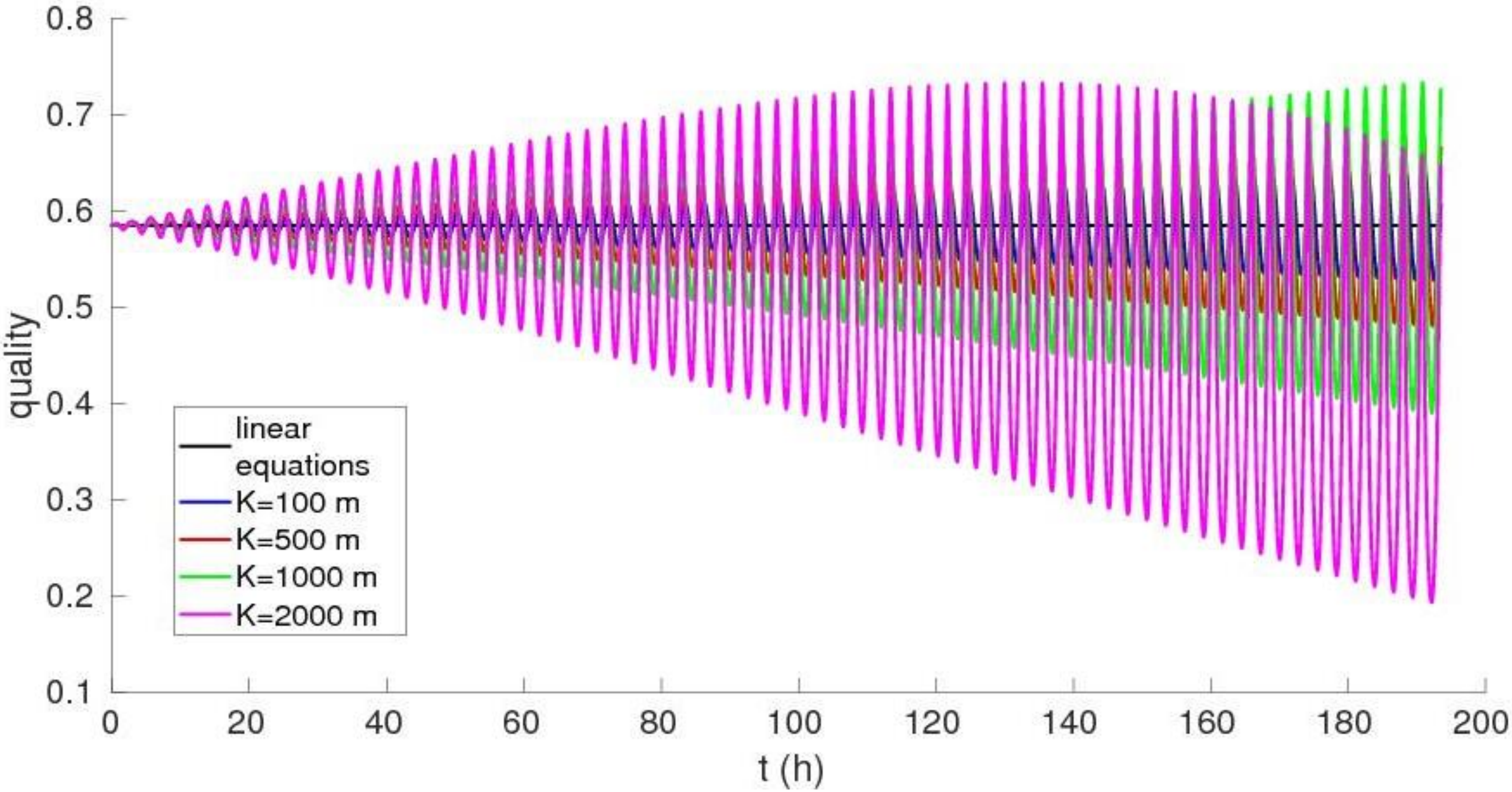
$$\mathbb{V} = \frac{b}{6}(\mathbf{A},\mathbf{C},\mathbf{B}) = cK^2 b\frac{2\sqrt{2}}{9},$$

$$\mathbb{L} = \frac{8}{3}K^2(a^2 + b^2 + 5) + 8c^2.$$

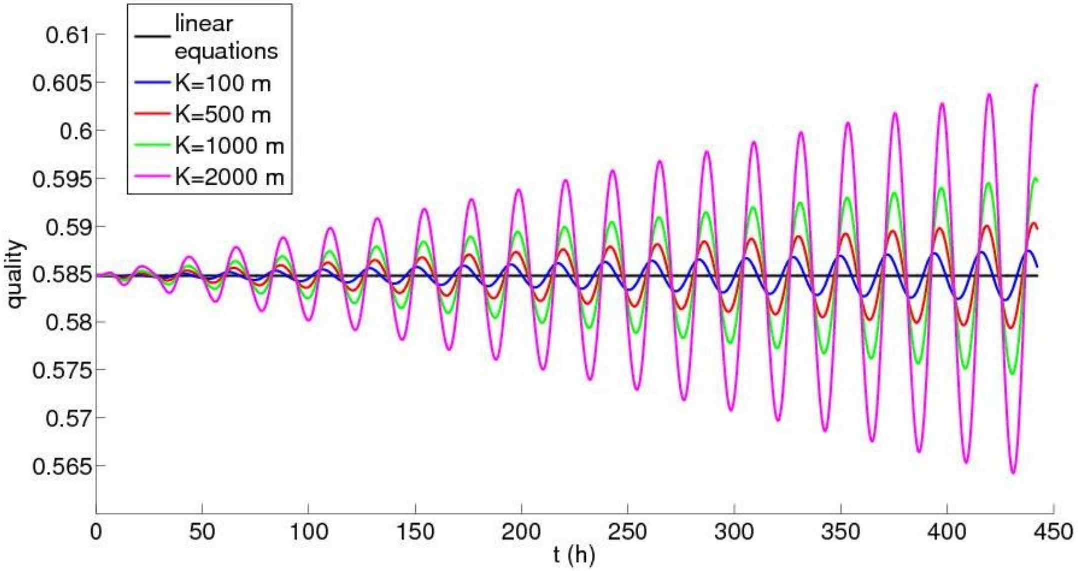
- This shape quality parameter achieves its maximal

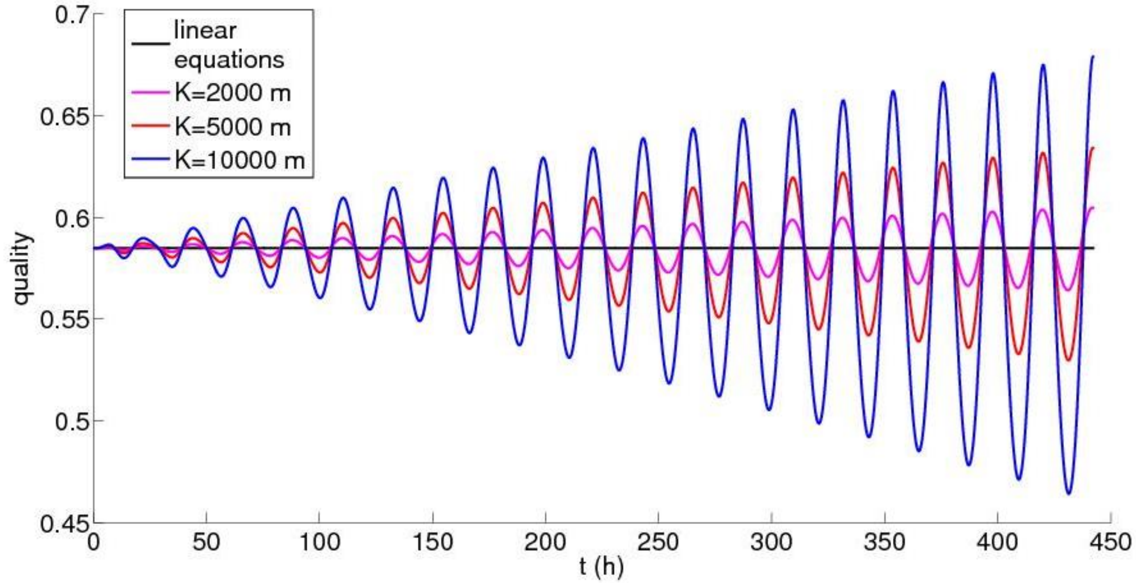
value $\mathbb{Q}_{max} = 1/\sqrt[3]{5}$ when $a = 0, b = \pm\sqrt{5}, c = \pm K\sqrt{\frac{5}{3}}$

Degradation of tetrahedron for 10 000 km orbits



Degradation of tetrahedron for 40 000 km orbits





Atmospheric control algorithm

$$x(t) = A\sin(nt+\psi)+2C,$$
$$y(t) = 2A\cos(nt+\psi)-3Cnt+D,$$
$$z(t) = B\sin(nt+\lambda),$$

Undisturbed CW linear solutions

$$\ddot{x}-2n\dot{y}-3n^2x = g_x+u_x,$$
$$\ddot{y}+2n\dot{x} = g_y+u_y,$$
$$\ddot{z}+n^2z = g_z+u_z,$$

But we have disturbances and control

$$x(t) = A(t)\sin\psi(t)+2C(t),$$
$$\dot{x}(t) = A(t)n\cos\psi(t),$$
$$y(t) = 2A(t)\cos\psi(t)+D(t),$$
$$\dot{y}(t) = -2A(t)n\sin\psi(t)-3C(t)n,$$
$$z(t) = B(t)\sin\lambda(t),$$
$$\dot{z}(t) = B(t)n\cos\lambda(t).$$

Introducing new variables (the idea is similar to osculating orbital elements)

Atmospheric control algorithm

$$\dot{A}(t) = \frac{1}{n}\left(\left(u_x + g_x\right)\cos\psi - 2\left(u_y + g_y\right)\sin\psi\right),$$

$$\dot{B}(t) = \frac{1}{n}\left(u_z + g_z\right)\cos\lambda,$$

$$\dot{C}(t) = \frac{1}{n}\left(u_y + g_y\right),$$

$$\dot{D}(t) = -3Cn - \frac{2}{n}\left(u_x + g_x\right),$$

$$\dot{\lambda}(t) = n - \frac{1}{nB}\left(u_z + g_z\right)\sin\lambda.$$

$$\dot{\psi}(t) = n - \frac{1}{nA}\left(\left(u_x + g_x\right)\sin\psi + 2\left(u_y + g_y\right)\cos\psi\right).$$

We produce asymptotically stable controller, so we neglect additional accelerations

Also acceleration is only along velocity vector, so

$$\dot{A}(t) = \frac{-2u_y \sin\psi}{n},$$

$$\dot{C}(t) = \frac{u_y}{n},$$

$$\dot{D}(t) = -3Cn.$$

Atmospheric control algorithm

$$V = C^2 + k_D \left(D - D_{ref}\right)^2, \quad k_D > 0.$$

$$\dot{V} = C\left(\frac{u_y}{n} - 3nk_D \left(D - D_{ref}\right)\right).$$

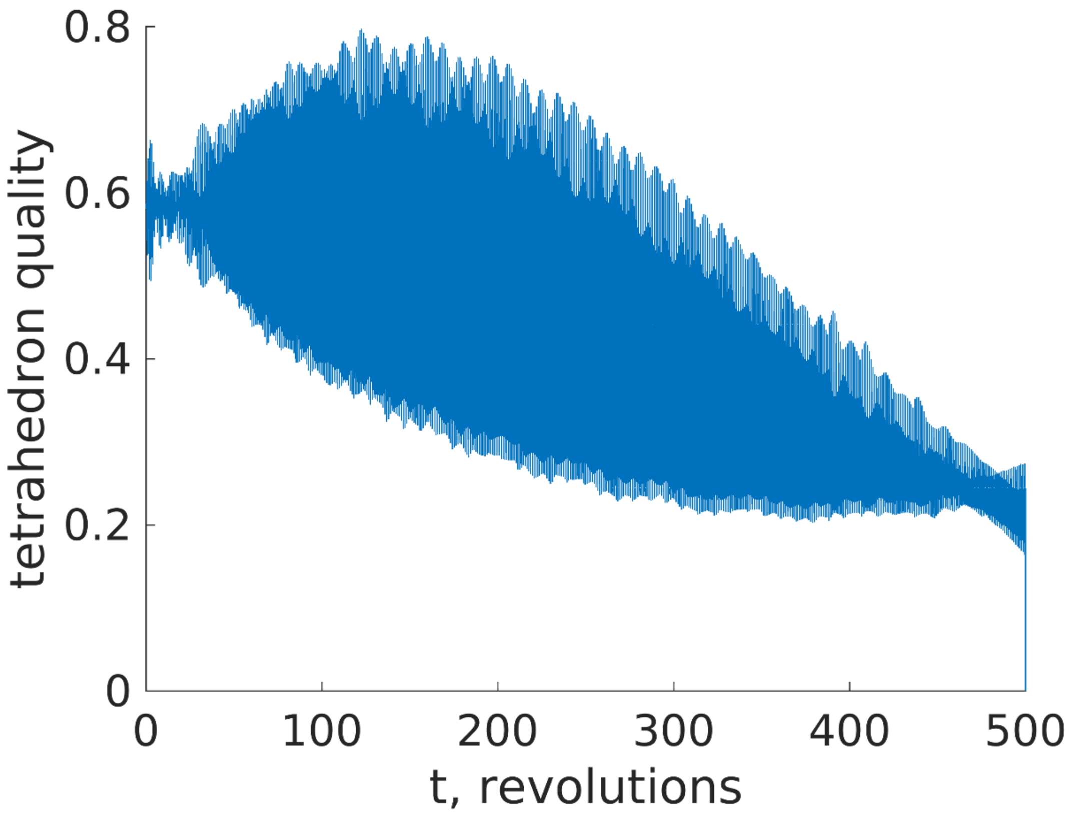
$$\dot{V} = -k_c C^2, \quad k_c > 0$$

$$u_y = 3n^2 k_D \left(D - D_{ref}\right) - k_c C$$

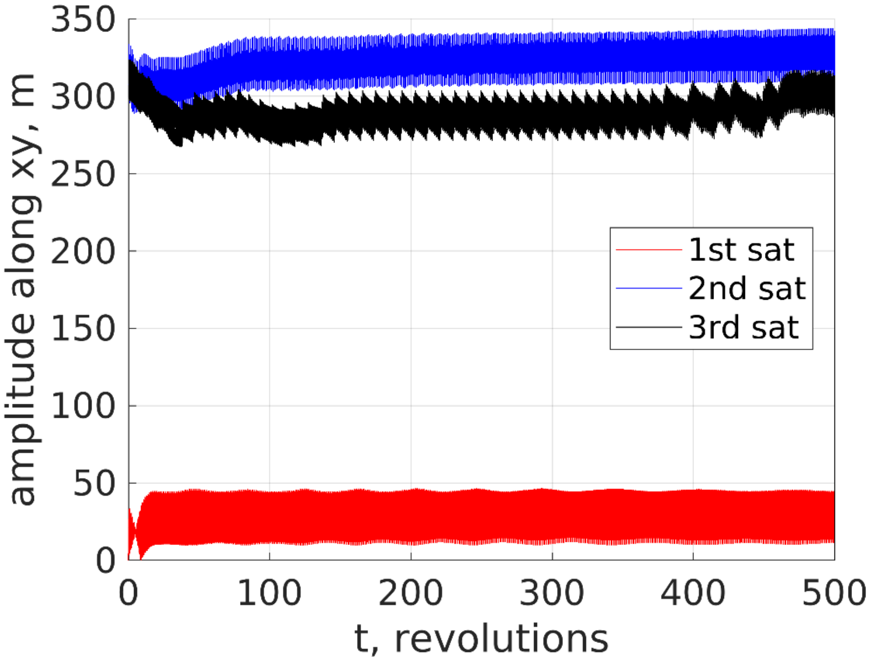
We use direct Lyapunov method to produce asymptotically stable controller

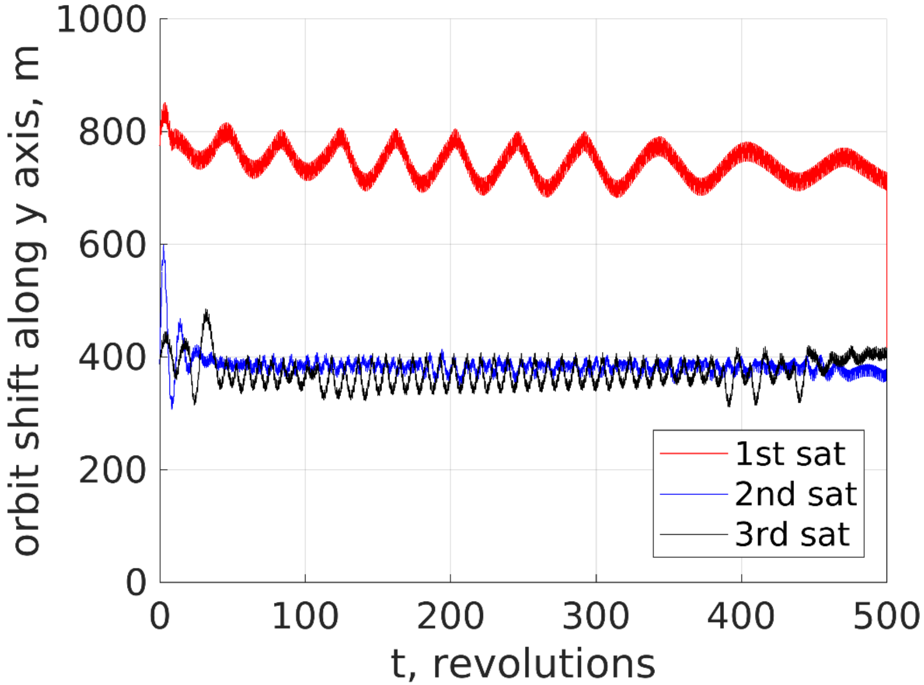
Atmospheric control algorithm

- Tetrahedron configuration conserving quality in linear model
- Tetrahedron size K = 300 m
- Orbit radius = 6700 km
- Masses = 5 kg
- Area = 0.1 m^2
- Inclination = 56 deg
- Disturbances – J2 and atmosphere



Atmospheric control algorithm





Conclusions

- The reference orbit for 4 satellites conserving volume and shape of the tetrahedron is found in linear model on circular orbit
- It is shown that simple fuelless control algorithm based on atmospheric drag utilization allows expanding mission lifetime to a month

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