Attitude and relative motion control of satellites in formation flying via solar sail with variable reflectivity properties

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Abstract

In this paper an algorithm of simultaneous relative motion and attitude control via solar radiation pressure is suggested. This allows us to deploy and maintain the given formation of two satellites. The principle idea is to use special materials for solar sail that are able to change its optical properties. It is considered that solar sail is divided into a number of cells. Each of them can be absolutely black, i.e. it absorbs completely the solar radiation, or absolutely specular (white), i.e. it reflects all solar radiation. The necessary control force is developed by varying the average reflectivity of solar sail, and the control torque is achieved by the appropriate pattern of black and white cells.

Keywords: formation flying, attitude control, relative motion control, solar sail, Lyapunov control function

Acronyms/Abbreviations

SRP – solar radiation pressure
IF – inertial frame
OF – orbital frame
BF – body-fixed frame
SF – solar frame
LCF – Lyapunov control function
SSN – solar sail normal

1. Introduction

Utilization of a group of satellites, for example formation flight, brings new possibilities in space missions. In addition, group of satellites is more reliable because even if one satellite fails, others can continue their operation.

The main problem of formation flying utilization is the deployment and maintenance of the particular group configuration. The simplest solution for this problem is to use thrusters that are installed onboard all or several satellites. On the other hand, thrusters require propellant, which can greatly affect the satellite lifetime or the payload mass. To overcome this problem environmental forces for formation flying motion control can be used [1]. This approach can be applied relatively easily by installing a special high area-to-mass ratio device such as a flat sail. There are two forces that can be used: aerodynamic drag [2–6] and solar radiation pressure (SRP) [7–11]. The principal idea here is to use a difference in environmental forces acting on each satellite in formation. This difference usually appears when a sail rotates but the effective size variation is also considered in literature [12].

In paper the case when both attitude and relative motion are controlled via solar sail with variable optical properties. It is considered that sail is divided into cells which can either absorb all solar radiation or fully reflect it.

2. Problem statement and reference frames

Deployment and maintenance of required relative orbit of two satellites is considered. It is assumed that each satellite has solar sail. The initial orbit of one satellite (leader) is circular. Second satellite (follower) is moving along the orbit which is close to the first one. Satellites move under the solar radiation pressure and J_2 perturbations.

In paper the following reference frames are used:

– O_1XYZ is the Inertial Frame (IF) with the origin in the Earth centre of mass, O_1Z is orthogonal to the equatorial plane, O_1X is directed to the vernal equinox;
– $Oxyz$ is the orbital frame (OF), its origin is located in the leader satellite centre of mass, Oz directed along its radius vector, Oy is orthogonal to the orbit plane;
– $O\xi\eta\zeta$ is the body-fixed frame (BF), its axes are the principal axes of inertia (it is also assumed that $O\zeta$ is orthogonal to the sail plane);
– $Ox_s y_s z_s$ is the solar frame (SF), Oz_s is directed to the Sun, Oy_s is orthogonal to the ecliptic plane.

Transition between IF and OF is performed by the following matrix

$$\mathbf{S} = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix},\ \mathbf{e}_3 = \frac{\mathbf{r}}{r},\ \mathbf{e}_2 = \frac{\mathbf{r}\times\mathbf{v}}{|\mathbf{r}\times\mathbf{v}|},\ \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_3,$$

where $\mathbf{r}$ is the radius vector and $\mathbf{v}$ is the velocity of the leader satellite. Transition between IF and SF is determined by the

$$\mathbf{S}_{\text{sun}} = \begin{pmatrix}\mathbf{l}_1 & \mathbf{l}_2 & \mathbf{l}_3\end{pmatrix},\ \mathbf{l}_3 = \begin{pmatrix}\cos\lambda \\ \sin\lambda\cos\varepsilon \\ \sin\lambda\sin\varepsilon\end{pmatrix},\ \mathbf{l}_2 = \begin{pmatrix}0 \\ -\sin\varepsilon \\ \cos\varepsilon\end{pmatrix},$$

$\mathbf{l}_1 = \mathbf{l}_2 \times \mathbf{l}_3$.

λ is the ecliptic longitude, ε is the obliquity of the ecliptic.

2. Motion equations

There are three types of motion equations that are used in this paper.

2.1. Orbital dynamics

Orbital dynamics is described by the following vector equation

$$\ddot{\mathbf{r}} = -\mu_E \frac{\mathbf{r}}{r^3} + \mathbf{g},$$

where μ_E is the Earth gravity constant and $\mathbf{g}$ is the result vector of the external disturbances. As it was sad before the effects of J_2 and SRP force are taken into account only. The first has a form of

$$\mathbf{f}_{J_2} = \frac{3J_2\mu_E R_\oplus^2}{2r^4}\begin{pmatrix}3\sin^2 i\sin^2 u - 1 \\ -\sin^2 i \sin 2u \\ -\sin 2u \sin u\end{pmatrix}.$$

Here $J_2 = 1.082\times10^{-3}$, $R_\oplus$ is the mean Earth radius, i is the orbit inclination and u is the argument of latitude. SRP force on the elemental area can be written as follows

$$d\mathbf{F}_s = -\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\times$$
$$\times\left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s,\mathbf{n})\mathbf{n} + \alpha(1-\mu)\left(\mathbf{r}_s + \frac{2}{3}\mathbf{n}\right)\right)dS,$$

where $\Phi_0 = 1357\ W/m^2$ is the solar flux constant, $\mathbf{r}_s$ is the unit vector from the Sun to the satellite, $\mathbf{n}$ is the solar sail normal (SSN), α is the reflection coefficient, μ is the specularity coefficient. Further the case of $\mu = 1$ is considered, so

$$d\mathbf{F}_s = -\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\left((1-\alpha)\mathbf{r}_s + 2\alpha(\mathbf{r}_s,\mathbf{n})\mathbf{n}\right)dS.$$

Due to the variation of α from point to point the total SRP force becomes

$$\mathbf{F}_s = -\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\left(\left(S - \int\alpha dS\right)\mathbf{r}_s + 2(\mathbf{r}_s,\mathbf{n})\mathbf{n}\int\alpha dS\right).$$

If denote $f = \dfrac{\int\alpha dS}{S}$ $(0 \le f \le 1)$ and $A = -\dfrac{\Phi_0 S}{c}$, then

$$\mathbf{F}_s = A(\mathbf{r}_s,\mathbf{n})\left((1-f)\mathbf{r}_s + 2f(\mathbf{r}_s,\mathbf{n})\mathbf{n}\right).$$

These equations are written for both satellites and are used in numerical simulation.

2.2. Angular dynamics

Angular dynamics is described in the BF by the Euler equations

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega} = \mathbf{M}_{\text{control}} + \mathbf{M}_g, \quad (1)$$

where $\mathbf{J}$ is the satellite inertia tensor, $\boldsymbol{\omega}$ is the angular velocity, $\mathbf{M}_{\text{control}}$ is the control torque and $\mathbf{M}_{\text{g}} = 3\dfrac{\mu_E}{r^5}\mathbf{r}\times\mathbf{J}\mathbf{r}$ is the gravity torque.

Attitude kinematics is defined by the quaternion $\Lambda = (\lambda_0, \boldsymbol{\lambda})$, $\lambda_0^2 + \boldsymbol{\lambda}^2 = 1$. And corresponding equations are the following

$$\dot{\lambda}_0 = -0.5(\boldsymbol{\lambda}, \boldsymbol{\omega}),$$
$$\dot{\boldsymbol{\lambda}} = 0.5(\lambda_0\boldsymbol{\omega} + \boldsymbol{\lambda}\times\boldsymbol{\omega}).$$

These equations are used for the numerical simulation and control torque synthesis.

2.3. Relative motion dynamics

The control synthesis is based on the Hill-Clohessy-Wiltshire equations. It is assumed that the leader satellite moves along circular orbit while the relative orbit is small with respect to the size of the orbit. So this motion equations in the OF can be written as follows

$$\begin{aligned}&\ddot{x} + 2\omega\dot{z} = 0,\\ &\ddot{y} + \omega^2 y = 0,\\ &\ddot{z} - 2\omega\dot{z} - 3\omega^2 y = 0,\end{aligned} \quad (2)$$

where ω is the orbital angular velocity of the leader satellite, $\boldsymbol{\rho} = \begin{pmatrix}x & y & z\end{pmatrix}^T$ is the relative position, $\boldsymbol{\rho} = \mathbf{r}_2 - \mathbf{r}_1$. Index "1" corresponds to the leader satellite and "2" to the follower.

If control and disturbances are taken into account Eq.(2) can be rewritten in the following form

$$\ddot{x} + 2\omega\dot{z} = u_x + g_x,$$
$$\ddot{y} + \omega^2 y = u_y + g_y,$$
$$\ddot{z} - 2\omega\dot{z} - 3\omega^2 y = u_z + g_z.$$

Here u_x, u_y, u_z and g_x, g_y, g_z are the components of control vector $\mathbf{u} = \dfrac{\mathbf{F}_{s,2} - \mathbf{F}_{s,1}}{m}$ and disturbances vector $\mathbf{g}$, respectively.

Solution of (2) is

$$x = -3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t + C_4;$$
$$y = C_5\cos\omega t + C_6\sin\omega t;$$
$$z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t.$$

One can introduce new variables based on this solution.

$$
\begin{aligned}
&x = 2B_2\cos\psi_1 + B_3,\\
&z = B_2\sin\psi_1 + 2B_1,\\
&\dot{x} = -2B_2\omega\sin\psi_1 - 3B_1\omega,\\
&\dot{z} = B_2\omega\cos\psi_1,\\
&y = B_4\sin\psi_2,\\
&\dot{y} = B_4\omega\cos\psi_2.
\end{aligned}
$$

It should be noted that B_1 corresponds to the drift velocity of the follower satellite along axis *Ox* of the OF. The equations that correspond to these variables have form

$$
\begin{aligned}
&\dot{B}_1 = \frac{1}{\omega}\left(u_x + g_x\right),\\
&\dot{B}_3 = -3B_1\omega - \frac{2}{\omega}\left(u_z + g_z\right),\\
&\dot{B}_2 = \frac{1}{\omega}\left(\left(u_z + g_z\right)\cos\psi_1 - 2\left(u_x + g_x\right)\sin\psi_1\right),\\
&\dot{\psi}_1 = \omega - \frac{1}{B_2\omega}\left(\left(u_z + g_z\right)\sin\psi_1 + 2\left(u_x + g_x\right)\cos\psi_1\right),\\
&\dot{B}_4 = \frac{1}{\omega}\left(u_y + g_y\right)\cos\psi_2,\\
&\dot{\psi}_2 = \omega - \frac{1}{\omega B_4}\left(u_y + g_y\right)\sin\psi_2.
\end{aligned} \tag{3}
$$

These equations will be used for the relative motion control synthesis.

3. Control synthesis

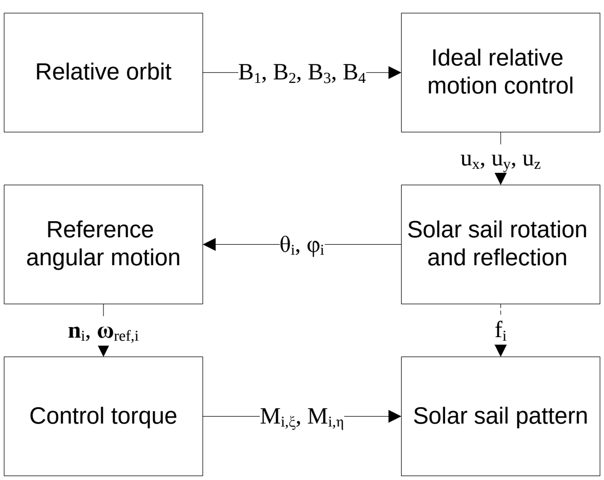

Figure 1. Control synthesis scheme

The control synthesis scheme is presented in Fig. 1. First of all, the ideal control that provides required relative motion is found. Then corresponding integral reflection coefficient f_i and angles of SSN θ_i, φ_i are determined. Normal directions define the reference angular motion of each satellite. After that the control torque is calculated $\mathbf{M}_{i,\,control}$ (in $O\xi\eta$ plane). Finally, $\mathbf{M}_{i,\,control}$ and f_i determine the solar sail reflection pattern. Further in this section each step is discussed.

3.1. Relative motion control

The purpose of the control is to deploy and maintain the required relative orbit. This orbit is defined by the B_i ($i = 1,2,3,4$). In paper the following relative orbit is considered

$B_1 = 0$, $B_2 = B_0$, $B_3 = 0$, $B_4 = 0$.

This means that the centre of the orbit is the origin of the OF and its shape is the ellipse with major and minor semi-axes $2B_0$ and B_0 respectively. The relative orbit stabilization is performed by two stages: firstly $B_1 = 0$ and $B_3 = 0$ are provided then $B_2 = B_0$ is achieved. The out-of-plane motion control is separated, so $B_4 = 0$ can be guaranteed independently.

On the first stage the following Lyapunov control function (LCF) is used

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^{\,2}.$$

Its time derivative is (the disturbances are omitted)

$$\dot{V} = B_1\dot{B}_1 + B_3\dot{B}_3 = \frac{1}{\omega}B_1u_x + B_3\left(-3B_1\omega - \frac{2}{\omega}u_z\right).$$

So the control that ensures global asymptotic stability of $B_1 = 0$ and $B_3 = 0$ is the following (Barbashin-Krassovskii theorem)

$$
\begin{aligned}
&u_x = -k_1B_1,\ k_1 > 0,\\
&u_z = \frac{1}{2}\left(-3B_1\omega^2 + k_2\omega B_3\right),\ k_2 > 0.
\end{aligned} \tag{4}
$$

Once $B_1 = 0$ and $B_3 = 0$ are achieved or at least B_1 and B_3 are small the second stage of control begins. The LCF here is

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^{\,2} + \frac{1}{2}\left(B_2 - B_{20}\right)^2$$

and its time derivative

$$
\begin{aligned}
\dot{V} = {} & \frac{1}{\omega}\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right)u_x + \\
& + \frac{1}{\omega}\left(-2B_3 + \left(B_2 - B_0\right)\cos\psi_1\right)u_z - 3B_1B_3\omega.
\end{aligned}
$$

As the last term is small then it is enough to make first and second term negative. Hence, the control is as follows

$$
\begin{aligned}
&u_x = -k_3\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right),\ k_3 > 0,\\
&u_z = -k_4\left(-2B_3 + \left(B_2 - B_0\right)\cos\psi_1\right),\ k_4 > 0.
\end{aligned} \tag{5}
$$

The stability condition for the control (5) ($\dot{V} < 0$) is

$$u_{\max} > \left|3B_1B_3\omega^2\right|.$$

Where $u_{\max}$ is the maximum possible control force. Since expressions (4) and (5) are different two states should be switched between each other. Control (4) is used when B_1B_3 is large, otherwise (5) is used.

Additionally, as B_1 determine the drift velocity it could be used to control the convergence speed of B_3 to zero. If B_3 is large, then instead the first expression of (4) the following control is used

$$u_x = -k_1\left(B_1 - B_{10}\right).$$

The out-of-plane motion control has a form

$$u_y = -k_y B_4 \cos\psi_2,\ k_y > 0.$$

Control u_x, u_y, u_z is an ideal one. It should be implemented through the solar sails rotation and integral reflectivity coefficients f_i.

3.2. Relative motion control implementation

Let θ be the angle between the SSN and Sun direction, φ is the angle between normal projection to the plane Ox_sy_s of the SF and axis Ox_s. Then in the SF the SSN is as follows

$$\mathbf{n} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}.$$

Relative motion control force $\mathbf{u}$ in the SF

$$\begin{aligned} u_{x_s} &= 2Af_2\cos^2\theta_2\sin\theta_2\cos\varphi_2 - \\ &\quad - 2Af_1\cos^2\theta_1\sin\theta_1\cos\varphi_1, \\ u_{y_s} &= 2Af_2\cos^2\theta_2\sin\theta_2\sin\varphi_2 - \\ &\quad - 2Af_1\cos^2\theta_1\sin\theta_1\sin\varphi_1, \\ u_{z_s} &= A\left(1-f_2\right)\cos\theta_2 - A\left(1-f_1\right)\cos\theta_1 + \\ &\quad + 2Af_2\cos^3\theta_2 - 2Af_1\cos^3\theta_1. \end{aligned} \tag{6}$$

As the SRP force is decreasing when θ_i tends to 90 degrees, it is reasonable to suppose that θ_i are small, so (6) transforms to

$$\begin{aligned} u_{x_s} &= 2Af_2\theta_2\cos\varphi_2 - 2Af_1\theta_1\cos\varphi_1, \\ u_{y_s} &= 2Af_2\theta_2\sin\varphi_2 - 2Af_1\theta_1\sin\varphi_1, \\ u_{z_s} &= Af_2 - Af_1. \end{aligned} \tag{7}$$

System (7) has six unknown variables and only three equations. From the last equation one can see that f_i determine u_{z_s}. It should be noted that the maximum torque will be when $f = 0.5$ while for $f = 0$ and $f = 1$ the torque is zero. So f_i can be found from the optimization of

$$\left(f_1 - 0.5\right)^2 + \left(f_2 - 0.5\right)^2 \to \min$$

with the constraint

$$f_2 - f_1 = \frac{u_{z_s}}{A},$$

$$0 < f_{\min} \le f_i \le f_{\max} < 1,\ i = 1, 2.$$

The solution in the inner domain is as follows

$$f_1 = 0.5 - \frac{u_{z_s}}{2A}, \qquad f_2 = 0.5 + \frac{u_{z_s}}{2A}. \tag{8}$$

It exists when

$$2f_{\min} - 1 \le \frac{u_{z_s}}{A} \le 2f_{\max} - 1. \tag{9}$$

In the further discussion it is supposed that $f_{\min} = 0.2$ and $f_{\max} = 0.8$. So (9) can be rewritten as

$$-0.6 \le \frac{u_{z_s}}{A} \le 0.6.$$

When f_i is known one can find φ_i from the maximization (with θ_i fixed) of

$$\begin{aligned} L &= \left(f_2\theta_2\cos\varphi_2 - f_1\theta_1\cos\varphi_1\right)^2 + \\ &\quad + \left(f_2\theta_2\sin\varphi_2 - f_1\theta_1\sin\varphi_1\right)^2. \end{aligned}$$

It means that the result values of φ_i should correspond to the maximum values of the control force. This problem has two groups of solutions

$$\varphi_1 = \varphi_2,\ \theta_1\theta_2 < 0,$$

$$\varphi_1 = \varphi_2 + \pi,\ \theta_1\theta_2 > 0.$$

It should be noted that relative attitude of two satellites is the same for both solutions. So further the case $\varphi_1 = \varphi_2 = \varphi$ is considered. The first and second equations of (7) one can rewrite as follows

$$\left(f_2\theta_2 - f_1\theta_1\right)\cos\varphi = \frac{u_{x_s}}{2A},$$

$$\left(f_2\theta_2 - f_1\theta_1\right)\sin\varphi = \frac{u_{y_s}}{2A}.$$

Then

$$tg\varphi = \frac{u_{y_s}}{u_{x_s}},$$

$$\left|f_2\theta_2 - f_1\theta_1\right| = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}.$$

If $u_{x_s}\cos\varphi > 0$

$$f_2\theta_2 - f_1\theta_1 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}. \tag{10}$$

To find θ_i one can solve the following optimization problem

$$L = \theta_1^2 + \theta_2^2$$

with constraints (10) and $-\theta_{\max} \le \theta_i \le \theta_{\max}$.

The solution in the inner domain is as follows

$$\theta_1 = -\frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_1}{f_1^2 + f_2^2},$$

$$\theta_2 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_2}{f_1^2 + f_2^2}.$$

Thus, once $\mathbf{u}$ is known the attitude of SSN can be found.

3.3. Attitude control

The next step is to provide attitude control that guarantees the required SSN motion. The LCF in this case is as follows

$$V = \frac{1}{2}\left(J_\xi \omega_{\mathrm{rel},1}^2 + J_\eta \omega_{\mathrm{rel},2}^2\right) + \\ + k_{\mathrm{a}}\left(1 - \left(\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}, \mathbf{Bn}\right)\right). \quad (11)$$

Here J_ξ, J_η are the in-plane moments of inertia, $\omega_{rel,1}$, $\omega_{rel,1}$ are the corresponding relative angular velocity components ($\boldsymbol{\omega}_{rel} = \boldsymbol{\omega} - \boldsymbol{\omega}_{ref}$), $\boldsymbol{\omega}_{ref} = \mathbf{n} \times \dot{\mathbf{n}}$, $\mathbf{n}$ is the required SSN attitude in the IF, $k_a > 0$ and $\mathbf{B}$ is the transition matrix between the IF and the BF. The goal of the control is to guarantee asymptotic stability of the motion when the axes $O\zeta$ of the BF and $\mathbf{n}$ coincide.

Derivative of (11) is

$$\dot{V} = J_\xi \omega_{\mathrm{rel},1}\dot{\omega}_{\mathrm{rel},1} + J_\eta \omega_{\mathrm{rel},2}\dot{\omega}_{\mathrm{rel},2} - \\ - k_{\mathrm{a}}\left(\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}, \frac{d}{dt}(\mathbf{Bn})\right). \quad (12)$$

As there is no need to control the third component of the angular velocity it is reasonable to take relative angular velocity vector as $\boldsymbol{\omega}_{rel} = \begin{pmatrix}\omega_{rel,1} & \omega_{rel,2} & 0\end{pmatrix}^T$. In this case

$$\frac{d}{dt}(\mathbf{Bn}) = -\boldsymbol{\omega}_{rel} \times \mathbf{Bn}.$$

And (12) one can rewrite as follows

$$\dot{V} = \boldsymbol{\omega}_{rel}^T\left(\mathbf{J}\dot{\boldsymbol{\omega}}_{\mathrm{rel}} + k_{\mathrm{a}}\mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}\right).$$

To guarantee $\dot{V} < 0$ it is sufficient if

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{\mathrm{rel}} + k_{\mathrm{a}}\mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}} = -k_\omega \boldsymbol{\omega}_{\mathrm{rel}}$$

and the control torque

$$\mathbf{M}_{\mathrm{control}} = -k_\omega \boldsymbol{\omega}_{\mathrm{rel}} - \mathbf{M}_{\mathrm{ext}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \\ - \mathbf{J}\boldsymbol{\omega} \times \mathbf{B}\boldsymbol{\omega}_{\mathrm{ref}} + \mathbf{J}\mathbf{B}\dot{\boldsymbol{\omega}}_{\mathrm{ref}} - k_{\mathrm{a}}\mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}. \quad (13)$$

It should be noted that only two first components of $\mathbf{M}_{\mathrm{control}}$ are taken. The last component will be determined when the cell pattern is defined.

3.4. Solar sail pattern

The sail is divided into $n \times n$ cells. Each cell one can define by pair $(i,\ j)$, where i and j are the row and the column numbers. Let $\alpha_{i,j}$ is the reflection coefficient of the $(i,\ j)$ cell and has values either 0 or 1. So integral coefficient of reflection is

$$f = \left(\frac{1}{n}\right)^2 N.$$

Here N is the total number of cells which $\alpha_{i,j} = 1$.

The SRP control torque

$$\mathbf{M}_{\mathrm{control}} = \int \mathbf{r} \times d\mathbf{F}_{\mathbf{s}}.$$

The integral is taken over all surface of the sail, $\mathbf{r}$ is the radius vector of some point of the sail in the BF and $d\mathbf{F}_s$ is the elemental SRF force. For the discrete case the control torque becomes

$$\mathbf{M}_{\mathrm{control}} = \frac{1}{2}\left(\frac{a}{n}\right)^3 \cos^2\theta \begin{pmatrix} P \\ Q \\ -n\tan\theta\sin\alpha Q - \tan\theta\cos\alpha P\end{pmatrix},$$

$$P = 2I - (n+1)N,\ Q = 2J - (n+1)N$$

$$I = \sum_{(i,j):\alpha_{i,j}=1} i,\ J = \sum_{(i,j):\alpha_{i,j}=1} j.$$

From the control torque expression one can see that there are only two independent components. The third component is defined when the first and the second are known. Thus, once f is defined from (8) one can determine N. After that I and J can be calculated using the control torque components from (13).

3.5. Solar sail cell pattern

Solar sail pattern is described by the string of n^2 length, where *l*-th element define $(i,\ j)$ cell:

$$j = \begin{cases} n, \text{if } \mathrm{mod}(l,n) = 0, \\ \quad \mathrm{mod}(l,n); \end{cases},\ i = \frac{l-j}{n} + 1.$$

One in this string means that $\alpha_{i,j} = 1$, zero otherwise. For allowed values of $N \in [N_{\min}, N_{\max}]$, where $N_{\min} = n^2 f_{\min}$, $N_{\max} = n^2 f_{\max}$ the matrix G_N is built. Element (I, J) of this matrix is the set of all cell patterns for each the following expressions are valid

$$\sum_l i = I,\ \sum_l j = J.$$

All this data is then saved and is used when the certain cell pattern have to be chosen. During control algorithm operation once N is known the corresponding matrix

G_N is taken. In this matrix the element (I, J) is being looking for. If this element is not empty then there are two options: if the cell pattern is unique then this is the pattern one is looking for otherwise the pattern that is close to the previous one is selected. In case when the element is empty the closest one is taken.

4. Numerical example

The proposed scheme was built for the case when there are no disturbances and the relative motion model is linear. To show the control performance in case of non-linear model the numerical example is provided (Fig.2-5). The following parameters and initial conditions are taken

Orbit radius: $R_{orb} = 9000\ km$,

Initial relative orbit: $\mathbf{r}_{rel} = (10 \quad 10 \quad 5)\ m$, $\mathbf{V}_{rel} = (0.05 \quad 0.1 \quad 0.1)\ m/s$,

Satellite mass: $m = 10\,kg$,

Sail size: square with $5\ m$ side,

Inertia tensors: $\mathbf{J} = diag(2.1 \quad 2.1 \quad 3.8)\ kg \cdot m^2$,

Initial angular velocity:

$\boldsymbol{\omega}_1 = (0.002 \quad 0.003 \quad 0.001)\ rad/s$,

$\boldsymbol{\omega}_2 = (0.001 \quad 0.003 \quad 0.002)\ rad/s$,

Control parameters: $k_1 = k_3 = k_4 = 20$, $k_2 = 10^{-6}\ s^{-1}$,

$k_\omega = 0.02\ N \cdot m \cdot s$, $k_a = 10^{-4}\ N \cdot m$,

Maximum control force: $u_{\max} = 10^{-6}\ N$,

Maximum control torque: $M_{trq} = 3 \times 10^{-5}\ N \cdot m$

Switch condition: $B_1 B_3 < 1\ m^2$.

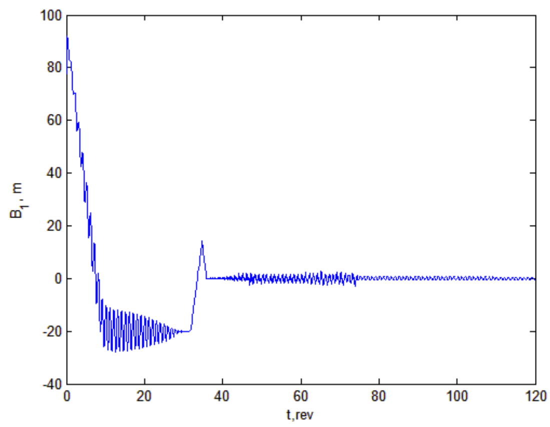

Figure 2. Parameter B_1

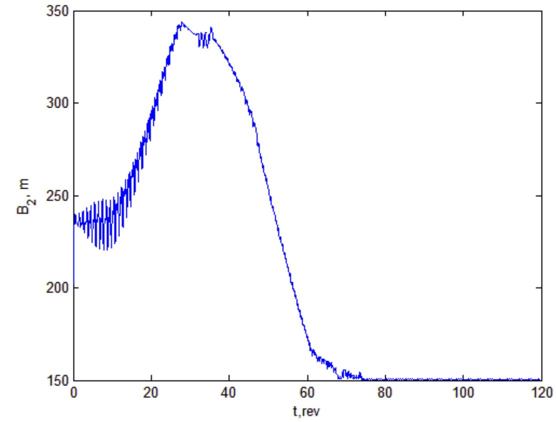

Figure 3. Parameter B_2

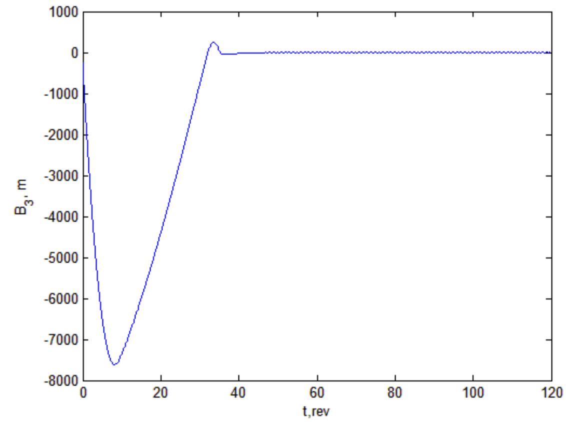

Figure 4. Parameter B_3

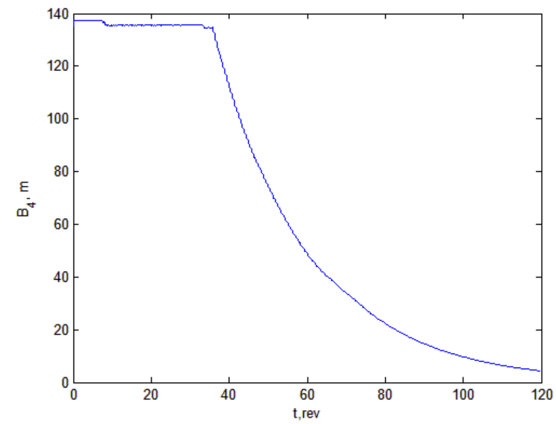

Figure 5. Parameter B_4

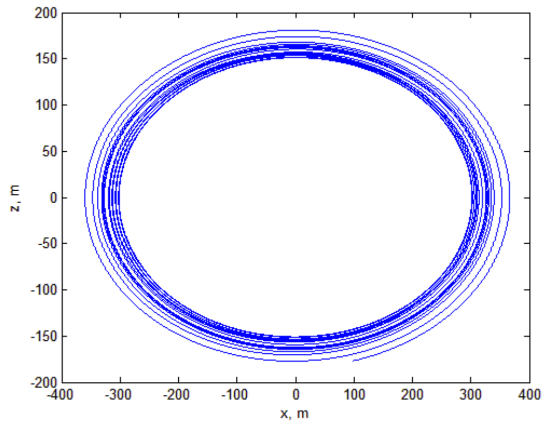

Figure 6. In-plane relative motion (result motion)

Numerical simulation results show that the control solve its task. In Fig.2 one can see that between 5th and 10th revolution the parameter $B_1 = -20\ m$. This allows to stabilize the B_3 much faster (see Fig.4). The model relative motion control is presented in Figures 7,8. Its implementation is presented in Figures 9,10.

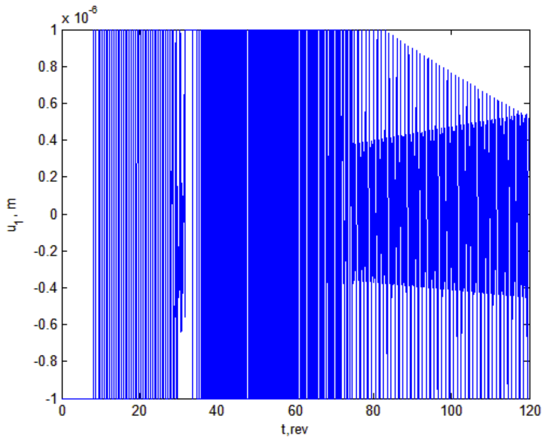

Figure 7. Control u_1

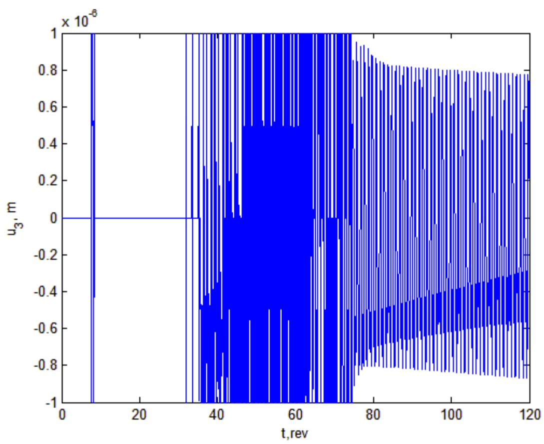

Figure 8. Control u_3

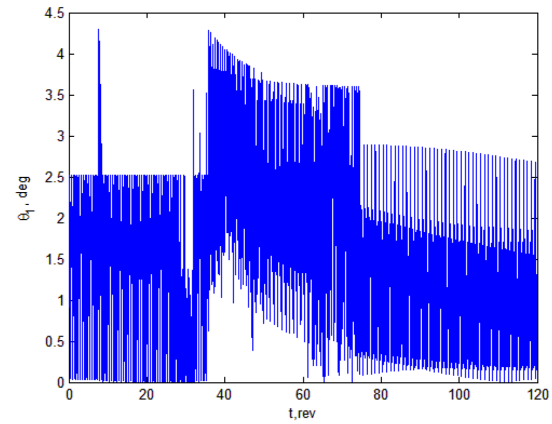

Figure 9. Angle θ_1

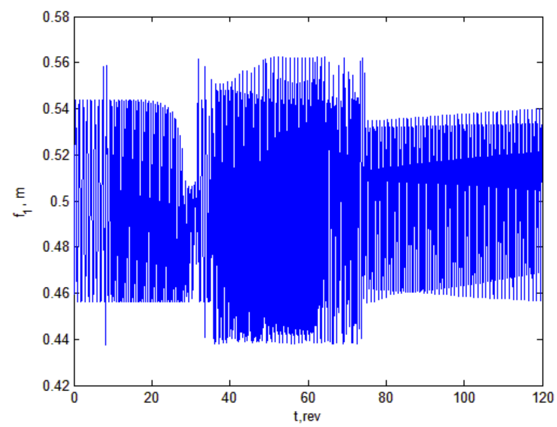

Figure 10. Integral reflectivity f_1

From Fig.7 and 8 one can see that control components don't exceed $u_{\max}$ and so f_1 and θ_1 (as well as f_2 and θ_2) are stayed within the desired ranges.

Finally, the model control torque one can find in Figures 11 and 12.

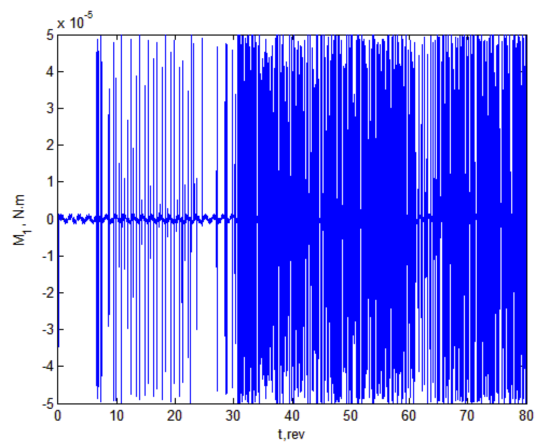

Figure 11. Control torque M_1

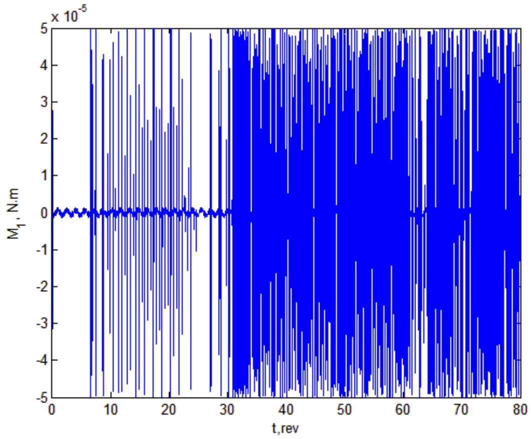

Figure 12. Control torque M_2

Figures 11 and 12 show that the control torque is also within the desired range.

5. Conclusions

In paper the scheme of the two satellites formation flying control using the solar sail is proposed. It was shown that it is possible to control relative motion and corresponding attitude control using solar sail only.

The provided numerical example shows the control scheme operation in case of J_2 disturbance and gravity gradient torque presence.

Acknowledgements

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