Attitude and relative motion control of satellites in formation flying via solar sail with variable reflectivity properties

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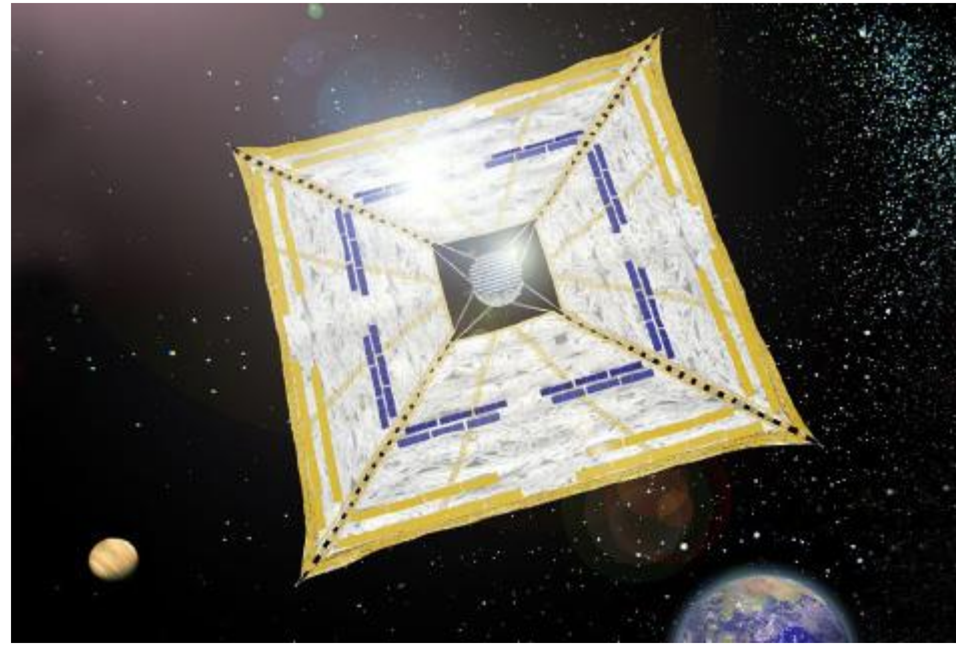
Introduction

IKAROS - Reflection properties variation to control attitude



IKAROS mission (global.jaxa.jp)

General idea
Solve attitude and relative two satellite motion control problem by means of solar sail

Goal
Get the elliptic orbit

Assumptions
Satellites identical
Specular reflection only
Center of mass motion: J_2 + solar radiation pressure (SRP)
Angular motion: Gravity gradient torque + SRP

Motion equations

Orbital dynamics

$$\ddot{\mathbf{r}} = -\mu_E \frac{\mathbf{r}}{r^3} + \mathbf{g},\ \ \mathbf{g} = \mathbf{g}_{J_2} + \mathbf{g}_s,\ \mathbf{g}_s = A(\mathbf{r}_s, \mathbf{n})\left((1-f)\mathbf{r}_s + 2f(\mathbf{r}_s, \mathbf{n})\mathbf{n}\right),\ f = \frac{\int \alpha\, dS}{S}$$

Angular dynamics

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M}_s + \mathbf{M}_g,\ \mathbf{M}_{\text{grav}} = 3\omega_0^2 \mathbf{e}_3 \times \mathbf{J}\mathbf{e}_3,\ \mathbf{M}_s = \int \mathbf{r} \times d\mathbf{F}_s$$

Relative motion dynamics (control synthesis only)

$$\begin{aligned} &\ddot{x} + 2\omega\dot{z} = u_x, \\ &\ddot{y} + \omega^2 y = u_y, \\ &\ddot{z} - 2\omega\dot{z} - 3\omega^2 y = u_z. \end{aligned} \qquad \mathbf{u} = 0 \ \Rightarrow \qquad \begin{aligned} &x = -3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t + C_4; \\ &y = C_5\cos\omega t + C_6\sin\omega t; \\ &z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t. \end{aligned}$$

$$\mathbf{u} = \frac{\mathbf{F}_{s,2} - \mathbf{F}_{s,1}}{m}$$

Motion equations (relative motion parameterization)

$$
\begin{aligned}
x &= 2B_2 \cos\psi_1 + B_3, \\
z &= B_2 \sin\psi_1 + 2B_1, \\
\dot{x} &= -2B_2\omega \sin\psi_1 - 3B_1\omega, \\
\dot{z} &= B_2\omega \cos\psi_1, \\
y &= B_4 \sin\psi_2, \\
\dot{y} &= B_4\omega \cos\psi_2.
\end{aligned}
$$

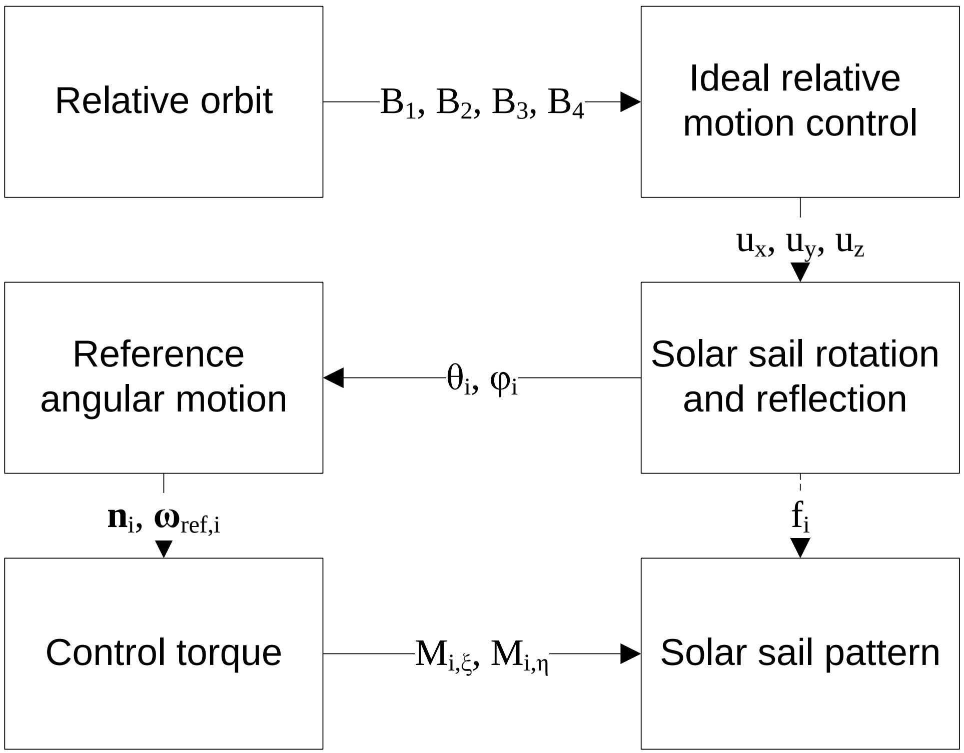
$$
\begin{aligned}
\dot{B}_1 &= \frac{1}{\omega} u_x, \\
\dot{B}_3 &= -3B_1\omega - \frac{2}{\omega} u_z, \\
\dot{B}_2 &= \frac{1}{\omega}\left(u_z \cos\psi_1 - 2u_x \sin\psi_1\right), \\
\dot{\psi}_1 &= \omega - \frac{1}{B_2\omega}\left(u_z \sin\psi_1 + 2u_x \cos\psi_1\right), \\
\dot{B}_4 &= \frac{1}{\omega} u_y \cos\psi_2, \\
\dot{\psi}_2 &= \omega - \frac{1}{\omega B_4} u_y \sin\psi_2.
\end{aligned}
$$

Goal

Ellipse with center at (0,0) and semi-axes $2B_{20}$ and B_{20}

$$B_1 = 0,\ B_2 = B_{20},\ B_3 = 0,\ B_4 = 0$$

Control synthesis scheme



Control force synthesis

Relative orbit center stabilization

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^{\ 2}$$

$$\dot{V} = B_1\dot{B}_1 + B_3\dot{B}_3 = \frac{1}{\omega}B_1 u_x + B_3\left(-3B_1\omega - \frac{2}{\omega}u_z\right)$$

$$u_x = -k_1 B_1,\ k_1 > 0,$$

$$u_z = \frac{1}{2}\left(-3B_1\omega^2 + k_2\omega B_3\right),\ k_2 > 0.$$

Relative orbit size stabilization (B_1 and B_3 is small)

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^{\ 2} + \frac{1}{2}\left(B_2 - B_{20}\right)^2$$

$$\dot{V} = \frac{1}{\omega}\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right)u_x + $$

$$+\frac{1}{\omega}\left(-2B_3 + \left(B_2 - B_0\right)\cos\psi_1\right)u_z - 3B_1B_3\omega$$

$u_{\max} > \left|3B_1B_3\omega^2\right|$ stability condition

$$u_x = -k_3\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right),\ k_3 > 0$$

$$u_z = -k_4\left(-2B_3 + \left(B_2 - B_0\right)\cos\psi_1\right),\ k_4 > 0$$

Out-of-plane stabilization

$$u_y = -k_y B_4\cos\psi_2,\ k_y > 0$$

Sail attitude and reflection

Solar sail normal

$$\mathbf{n} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

Control for small θ_i

$$u_{x_s} = 2Af_2\theta_2\cos\varphi_2 - 2Af_1\theta_1\cos\varphi_1,$$
$$u_{y_s} = 2Af_2\theta_2\sin\varphi_2 - 2Af_1\theta_1\sin\varphi_1,$$
$$u_{z_s} = Af_2 - Af_1.$$

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f=0 and f=1 zero torque
f=0.5 maximum torque

$$\left(f_1 - 0.5\right)^2 + \left(f_2 - 0.5\right)^2 \to \min$$

$$f_2 - f_1 = \frac{u_{z_s}}{A},$$

$$0 < f_{\min} \le f_i \le f_{\max} < 1,\ i = 1,2.$$

$$f_1 = 0.5 - \frac{u_{z_s}}{2A} \qquad f_{\min} = 0.2,\ f_{\max} = 0.8$$

$$f_2 = 0.5 + \frac{u_{z_s}}{2A} \qquad -0.6 \le \frac{u_{z_s}}{A} \le 0.6$$

Maximum u_x and u_y

$$L = \left(f_2\theta_2\cos\varphi_2 - f_1\theta_1\cos\varphi_1\right)^2 + \\ + \left(f_2\theta_2\sin\varphi_2 - f_1\theta_1\sin\varphi_1\right)^2.$$

$$\varphi_1 = \varphi_2,\ \theta_1\theta_2 < 0$$
$$\varphi_1 = \varphi_2 + \pi,\ \theta_1\theta_2 > 0$$

Minimum θ_i

$$L = \theta_1^2 + \theta_2^2,$$

$$f_2\theta_2 - f_1\theta_1 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}$$

$$\theta_1 = -\frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_1}{f_1^2 + f_2^2},$$

$$\theta_2 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_2}{f_1^2 + f_2^2}.$$

Attitude motion control

Lyapunov control function

$$V = \frac{1}{2}\left(J_\xi \omega_{\mathrm{rel},1}^2 + J_\eta \omega_{\mathrm{rel},2}^2\right) + k_{\mathrm{a}}\left(1 - \left(\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}, \mathbf{Bn}\right)\right)$$

$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega} - \boldsymbol{\omega}_{ref}\,,\ \boldsymbol{\omega}_{ref} = \mathbf{n} \times \dot{\mathbf{n}}$$

$$\dot{V} = \boldsymbol{\omega}_{rel}^T \left(\mathbf{J}\,\dot{\boldsymbol{\omega}}_{\mathrm{rel}} + k_{\mathrm{a}} \mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}\right)$$

$$\mathbf{M}_{\mathrm{control}} = \left(-k_\omega \boldsymbol{\omega}_{\mathrm{rel}} - \mathbf{M}_{\mathrm{ext}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{J}\boldsymbol{\omega} \times \mathbf{B}\boldsymbol{\omega}_{\mathrm{ref}} + \mathbf{JB}\dot{\boldsymbol{\omega}}_{\mathrm{ref}} - k_{\mathrm{a}} \mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}\right)_{\xi,\eta}.$$

Solar sail pattern

Integral reflection $f = \left(\frac{1}{n}\right)^2 N,$ N is the number of “white” cells

$$\mathbf{M}_{\mathrm{control}} = \frac{1}{2}\left(\frac{a}{n}\right)^3 \cos^2\theta \begin{pmatrix} P \\ Q \\ -n\tan\theta\sin\alpha Q - \tan\theta\cos\alpha P \end{pmatrix}, \quad \begin{array}{l} P = 2I - (n+1)N,\ I = \sum\limits_{(i,j):\alpha_{i,j}=1} i \\ Q = 2J - (n+1)N,\ J = \sum\limits_{(i,j):\alpha_{i,j}=1} j \end{array}$$

Numerical example

Orbit size $R_{orb} = 9000\ km$

Relative orbit $\mathbf{r}_{rel} = (10 \quad 10 \quad 5)\ m,$

$\mathbf{V}_{rel} = (0.05 \quad 0.1 \quad 0.1)\ m/s,$

Mass $m = 10\,kg$

Size $5\ m$

Inertia tensor $\mathbf{J} = diag(2.1 \quad 2.1 \quad 3.8)\ kg \cdot m^2$

Angular velocity $\boldsymbol{\omega}_1 = (0.002 \quad 0.003 \quad 0.001)\ rad/s$

$\boldsymbol{\omega}_2 = (0.001 \quad 0.003 \quad 0.002)\ rad/s$

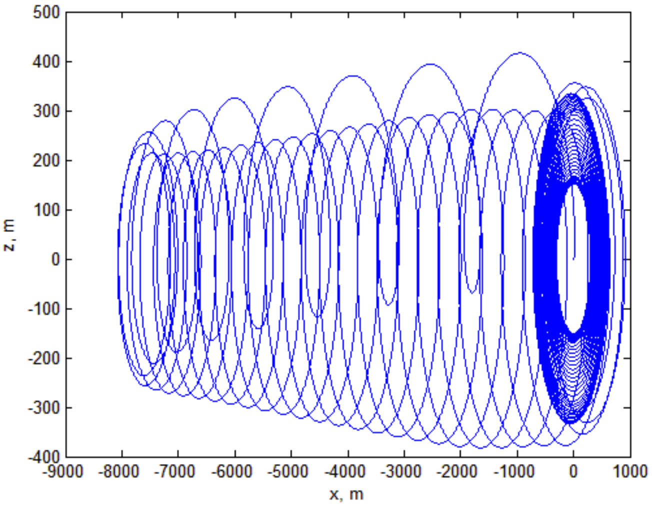
Control parameters $k_\omega = 0.02\ N \cdot m \cdot s,\ k_a = 10^{-4}\ N \cdot m$

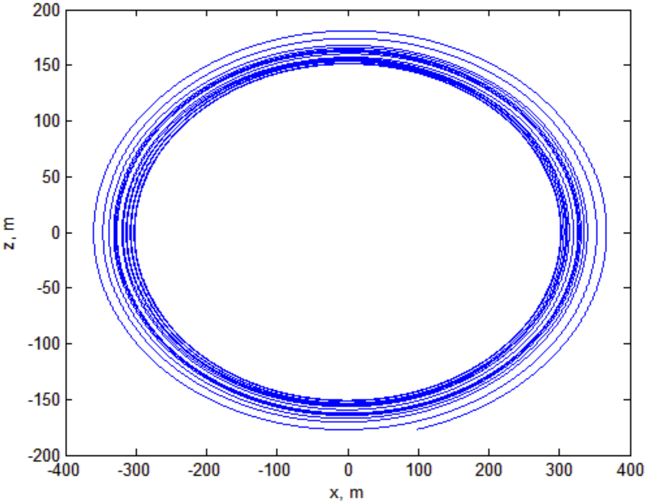
$k_1 = k_3 = k_4 = 20,\ k_2 = 10^{-6}\ s^{-1}$

Switch condition $B_1 B_3 < 1\ m^2$

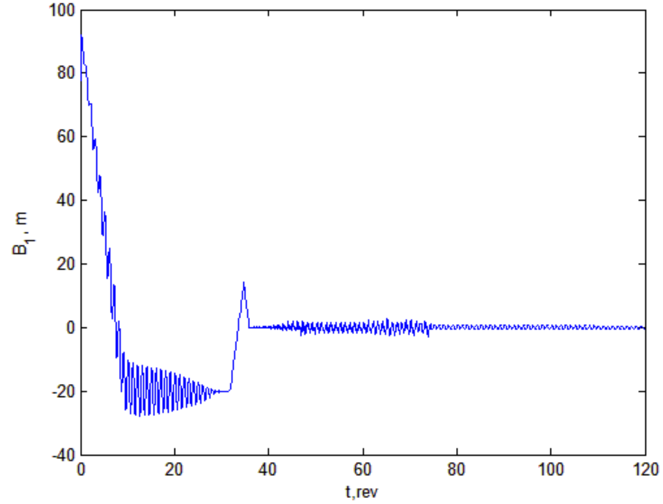
Maximum control $u_{\max} = 10^{-6}\ N$

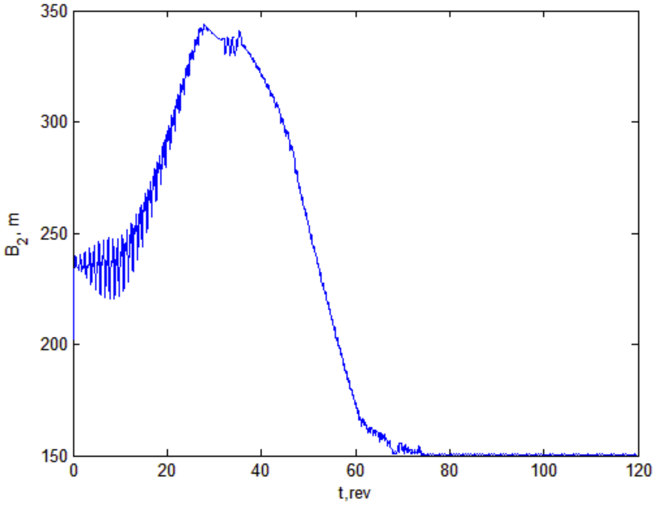
$M_{trq} = 3 \times 10^{-5}\ N \cdot m$

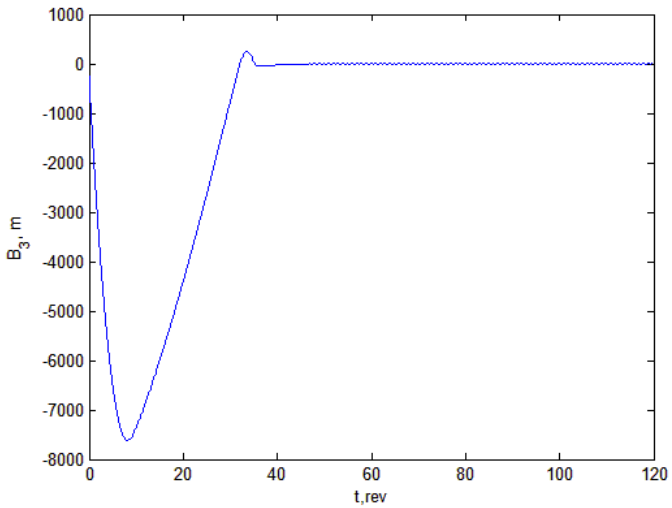


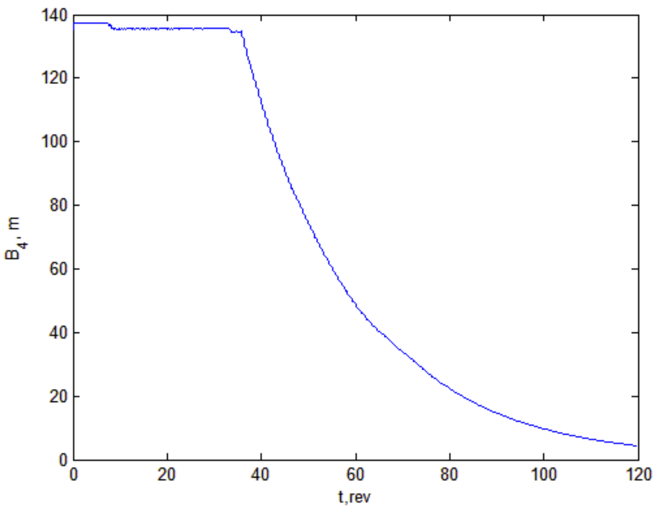


Numerical example (B-parameters)

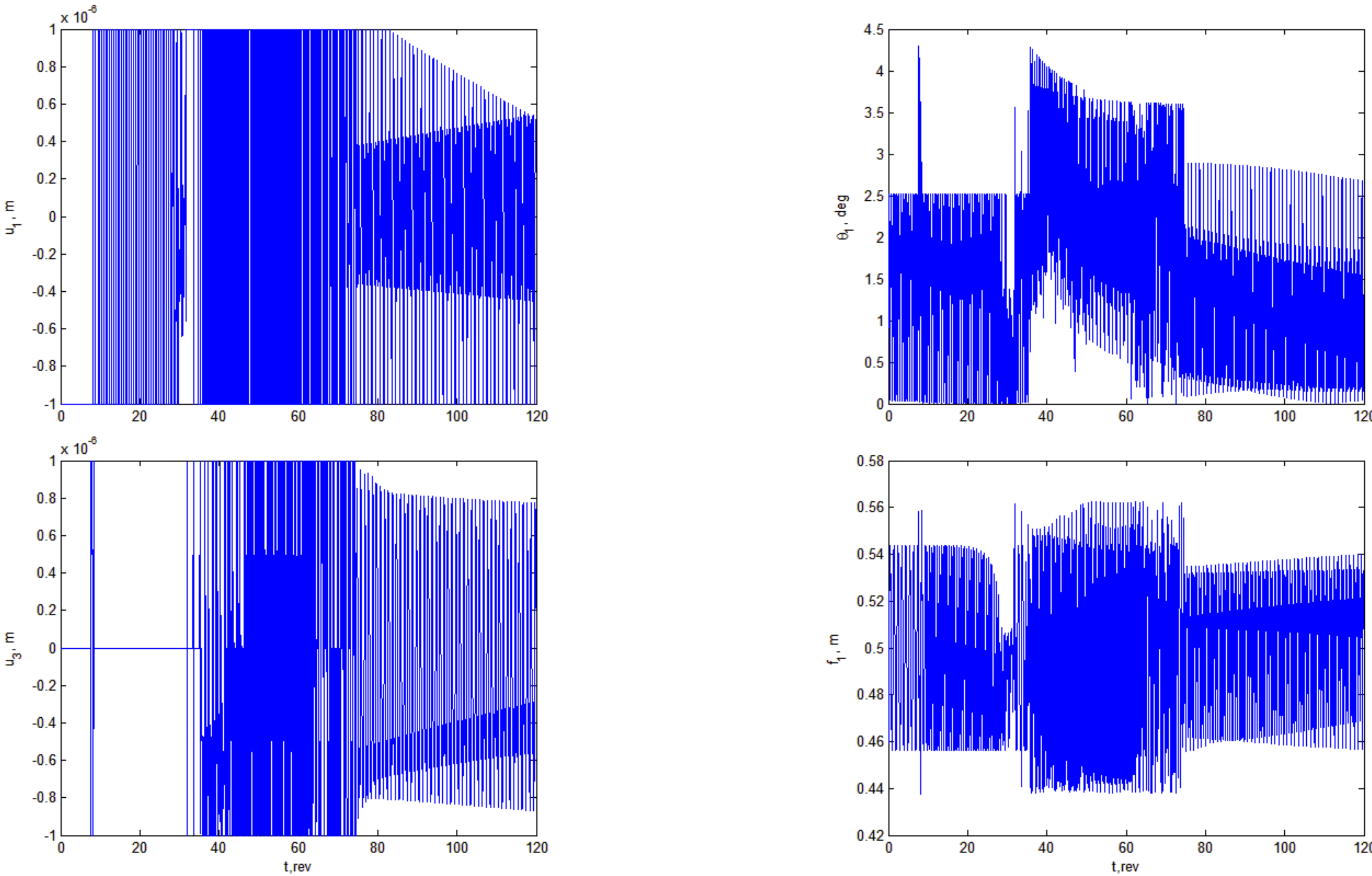








Numerical example (control)



Conclusion

It is shown that it is possible to stabilize elliptic relative orbit by means of the SRP only

The control synthesis scheme include the control of the angular and relative motion simultaneously

The relative motion is provided by the SRP force and control source is the solar sail attitude and variable reflectivity

The desired solar sail attitude is also proved by the SRP (torque)

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