Keldysh Institute of Applied Mathematics
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Usage of solar and gravitational torques for reaction wheels desaturation

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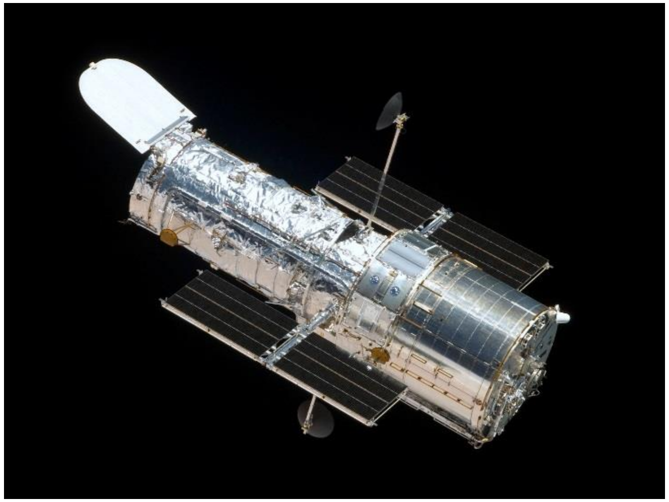
Introduction

Why do we need to control attitude?

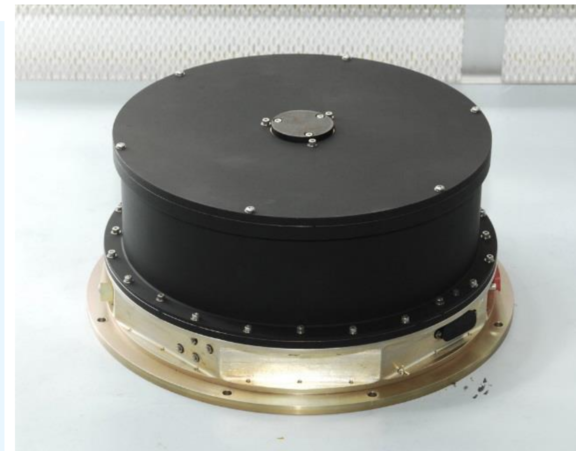
- Remote sensing
- Space telescopes
- Communication satellites
- Solar stabilization

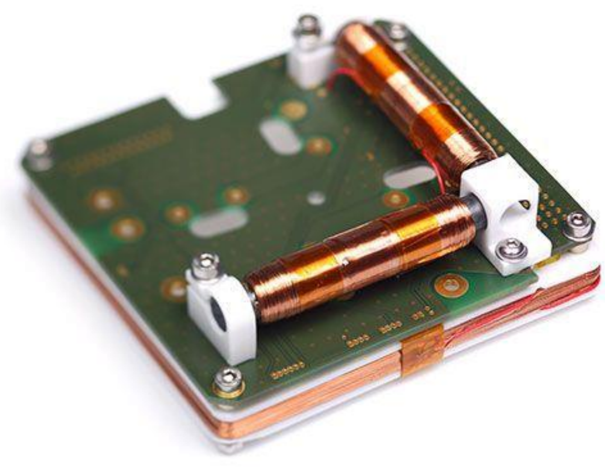


Introduction

How?

- Reaction wheels
- CMGs
- Thrusters
- Magnetorquers





Problems:

- Saturation of RW
- Singularity of CMG
- Propellant usage of thrusters
- Necessity of magnetic field presence for magnetorquers

Why is there a saturation?

Main reason is the external torques

- Gravitational torque

$$\mathbf{M}_G = 3\frac{\mu_E}{r^5}\mathbf{r}_{sat} \times \mathbf{J}\mathbf{r}_{sat}$$

- Solar radiation pressure torque

$$\mathbf{M}_{SRP} = \mathbf{R} \times \left[-S\frac{\Phi_0}{c}\left(\mathbf{r}_s, \mathbf{n}\right)\left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s, \mathbf{n}) + \alpha(1-\mu)\left[\mathbf{r}_s + \frac{2}{3}\mathbf{n} \right] \right) \right]$$

Problem statement

What do we know?

- Satellite moves along highly elliptical Keplerian orbit
- Normal to solar panels is aligned with principal axis of Inertia
- Reaction wheel attitude control

What do we want?

- Desaturate RW
- Recharge batteries, i.e. angle between normal to solar panels and sun direction has to be rather small

How will we desaturate?

Our goal is to desaturate reaction wheels. $\mathbf{K} = \mathbf{J}\boldsymbol{\omega} + \mathbf{H}$, during solar stabilization $\boldsymbol{\omega} \approx \mathbf{0}$ and

$$\frac{d}{dt}\left|\mathbf{K}\right|^2 = 2\left(\mathbf{M}, \mathbf{K}\right)$$

Hence, we have to ensure

$$\left(\mathbf{M}, \mathbf{K}\right) < 0$$

and for fastest desaturation

$$\left(\mathbf{M}, \mathbf{K}\right) \to \min$$
$$\left(\mathbf{n}, \mathbf{r}_s\right) \geq \cos\theta_{max}$$

Solar radiation pressure torque

Solar radiation pressure torque (absorption, specular and Lambertian reflection)

$$\mathbf{M}_{SRP} = \mathbf{R} \times \left[-S \frac{\Phi_0}{c} \left(\mathbf{r}_s, \mathbf{n} \right) \left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s, \mathbf{n})\mathbf{n} + \alpha(1-\mu)\left[\mathbf{r}_s + \frac{2}{3}\mathbf{n} \right] \right) \right]$$

Using some notation and considering small angle between normal to solar panel and sun direction we obtain

$$\mathbf{M}_{SRP} = \mathbf{p} \times \mathbf{r}_s + \mathbf{q}$$

where

$$\mathbf{p} = -S \frac{\Phi_0}{c} (1-\alpha\mu)\mathbf{R}, \quad \mathbf{q} = -S\alpha \frac{2\Phi_0}{3c} \left(1 + \frac{1}{3}\mu \right) \mathbf{R} \times \mathbf{n},$$

Solar radiation pressure torque

In this case minimization problem

$$(\mathbf{M},\mathbf{K}) \to \min$$
$$(\mathbf{n},\mathbf{r}_s) \ge \cos\theta_{max}$$

transforms to

$$f\sin(\varphi+\psi)+\theta\left(g\sin(\varphi)-h\sin(\psi)\right) \to \min_{\psi,\theta,\varphi}$$
$$-\theta_{max} \le \theta \le \theta_{max}.$$

where f,g,h are depend on current angular momentum and satellite parameters, φ,ψ,θ are the Euler angles that describe the attitude relative to the Frame determined by sun direction and angular momentum

Solar radiation pressure torque

How to find the solution?

Lagrange multipliers method:

$$\frac{\partial L}{\partial \theta} = g\sin(\varphi) - h\sin(\psi) + \lambda_2 - \lambda_1 = 0,$$
$$\frac{\partial L}{\partial \varphi} = f\cos(\varphi+\psi) + \theta g\cos(\varphi) = 0,$$
$$\frac{\partial L}{\partial \psi} = f\cos(\varphi+\psi) - \theta h\cos(\psi) = 0,$$
$$\lambda_1(\theta_{max} - \theta) = 0,$$
$$\lambda_2(\theta_{max} + \theta) = 0.$$

This system can be reduced to the cubic equation

Solar radiation pressure torque

Approximate solution:

$$\psi = -\frac{\pi}{2} - \gamma_0, \quad \theta = \theta_{max}, \quad \varphi = \left(1 - \operatorname{sign}(f)\right)\frac{\pi}{2} + \gamma_0$$

$$\sin\gamma_0 = \frac{-g\operatorname{sign}(f)}{\sqrt{g^2 + h^2}}, \quad \cos\gamma_0 = \frac{-h}{\sqrt{g^2 + h^2}}$$

Then minimized function equals

$$\left(\mathbf{M}_{SRP}, \mathbf{K}\right) = -\left|f\right| - \theta_{max}\sqrt{g^2 + h^2} < 0$$

We can always choose an attitude that ensures desaturation. The ratio between the optimal and approximate solution is less than $\sqrt{2}$

Gravitational torque

- Projection of SRP torque onto the sun direction is second order infinitesimal

$$(\mathbf{M}_{SRP}, \mathbf{r}_s) = (\mathbf{q}, \mathbf{r}_s) = -S\alpha \frac{2\Phi_0}{3c}\left(1 + \frac{1}{3}\mu\right)(\mathbf{R} \times \mathbf{n}, \mathbf{r}_s)$$

- We will use gravitational torque for desaturation along sun direction only
- Angular momentum orthogonal to sun direction will be desaturated using SRP torque

The problem to be solved:

$$\left(\mathbf{r}_s(\mathbf{K}, \mathbf{r}_s), 3\frac{\mu_E}{r_{sat}^5}\mathbf{r}_{sat} \times \mathbf{J}\mathbf{r}_{sat}\right) \to \min,$$

$$(\mathbf{r}_s, \mathbf{n}) \geq \cos\theta_{max}$$

Gravitational torque

It can be simplified:

$$F\sin\left(\alpha+\beta\right)+\Theta\left(G\sin(\alpha)-H\sin(\beta)\right)\to\min_{\alpha,\beta,\Theta},$$

$$-\theta_{max}\le\Theta\le\theta_{max}.$$

It matches the one obtained for SRP desaturation.

New result for approximate solution: spacecraft attitude relative to the Frame determined by satellite position and sun direction is constant:

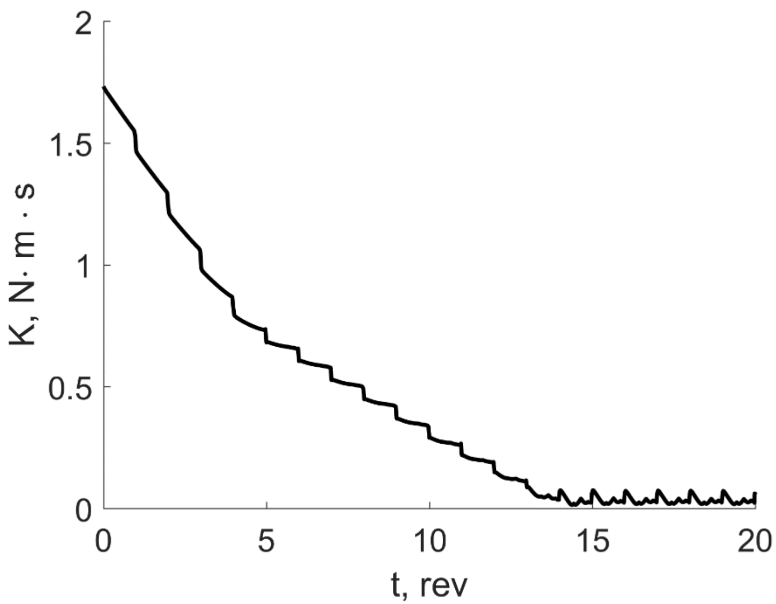
$$\Phi=\gamma_0+\frac{2-\mathrm{sign}\left(\left(\mathbf{K},\mathbf{r}_s\right)\left(B-A\right)\right)}{2},\ \ \Psi=-\gamma_0-\pi,\ \ \Theta=\theta_{max}$$

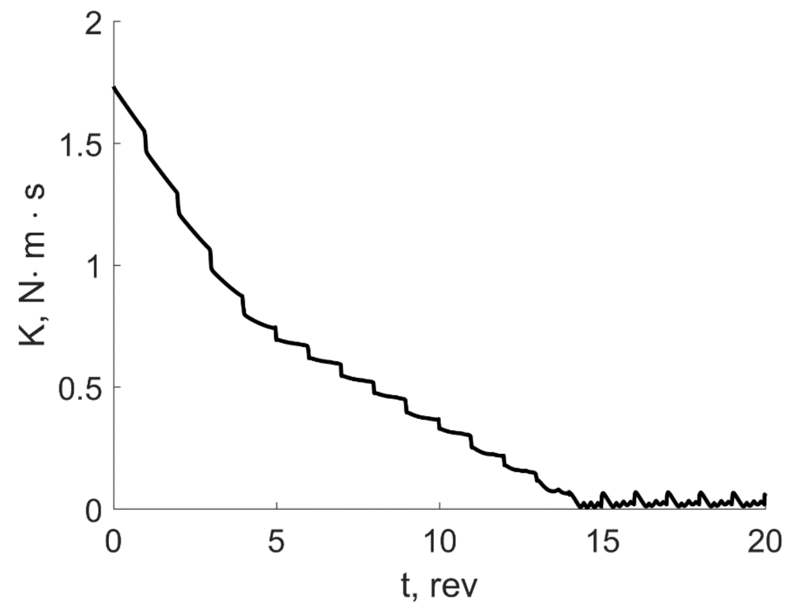
$$\sin\gamma_0=\frac{-\mathrm{sign}\left(r_3\right)\left|B-A\right|}{\sqrt{\left(A-B\right)^2+\left(A+B-2C\right)^2}},\quad \cos\gamma_0=\frac{-\mathrm{sign}\left(\left(\mathbf{K},\mathbf{r}_s\right)r_3\right)\left(A+B-2C\right)}{\sqrt{\left(A-B\right)^2+\left(A+B-2C\right)^2}}$$

Simulation results

- $r_\pi = 9\,000\ \text{km}, \quad r_a = 130\,000\ \text{km}, \quad i = 60^o$
- $\mathbf{J} = \text{diag}(160, 130, 220)\ \text{kg}\cdot\text{m}^2$
- Initial angular velocity is zero
- Attitude for SRP updates every 40 000 s, for gravitational torque it is continious
- Initial stored momentum is $\mathbf{H}_0 = \begin{pmatrix}-1 & -1 & -1\end{pmatrix}\ \text{N}\cdot\text{m}\cdot\text{s}.$
- Total solar panel area is $3\ \text{m}^2$
- Reflectivity is 0.1, specularity is 0.5
- Lyapunov based control algorithm is used to provide the necessary attitude

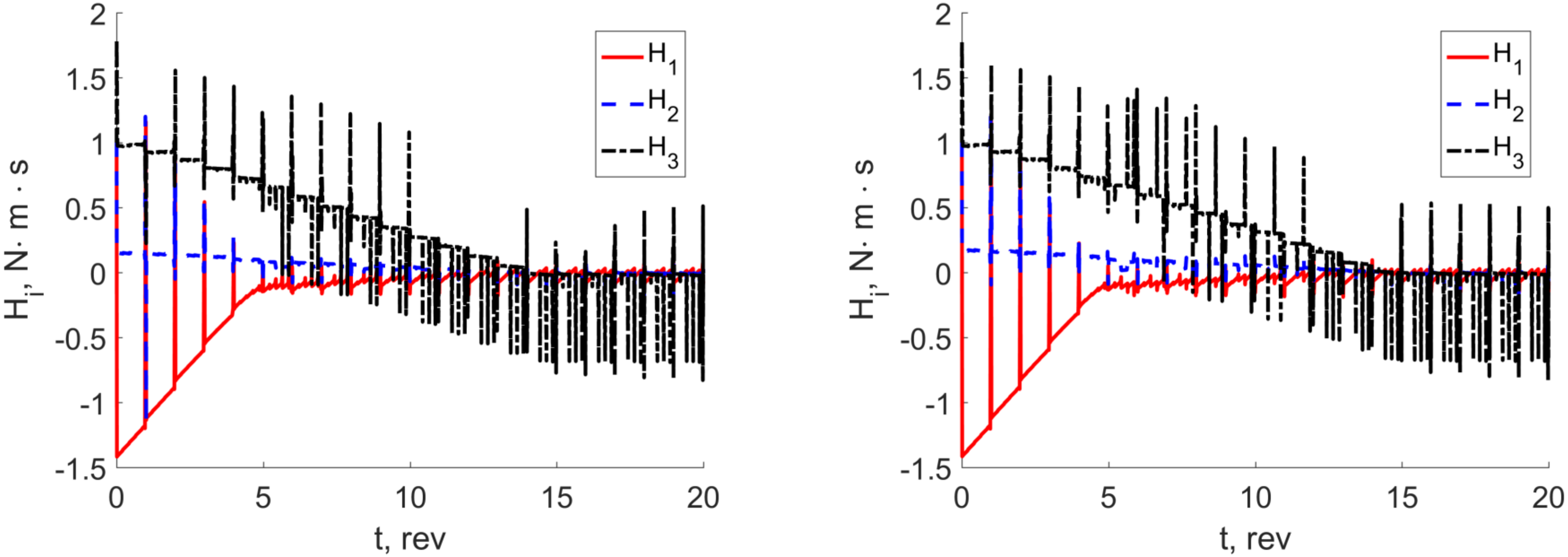
Simulation results





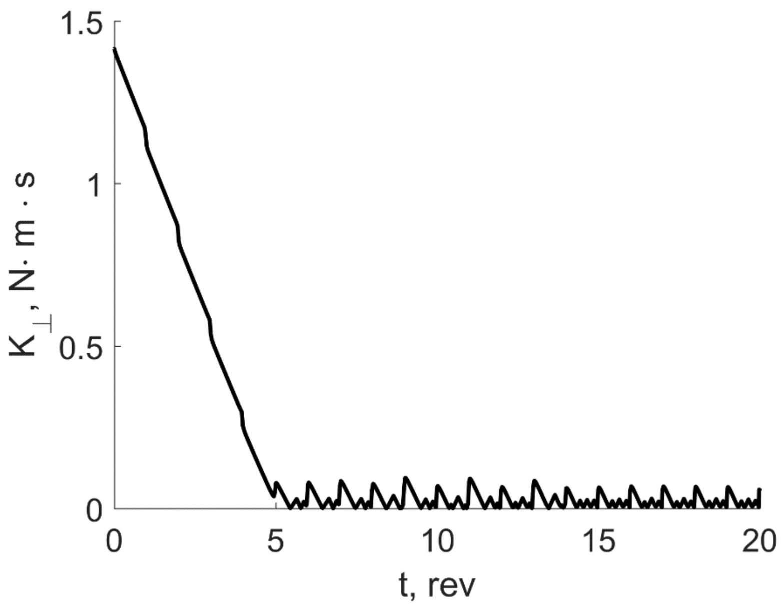
Total angular momentum. Left figure corresponds to the exact solution, and right figure corresponds to the approximate one

Simulation results



Stored angular momentum. Exact (left) and approximate (right) algorithms

Simulation results



Angular momentum orthogonal to sun direction (left) and aligned with sun direction (right)

Conclusion

- It was shown that there always exists an attitude that ensures simultaneous sun pointing and reaction wheels desaturation
- Exact and approximate solutions were obtained. The ratio between them is less than $\sqrt{2}$
- Numerical examples shown that the difference between the optimal attitude and near-optimal one is not critical, therefore, considering much simpler expressions for near-optimal solution, it is reasonable to use the latter one

Questions?

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