Angular motion synthesis for remote sensing satellite

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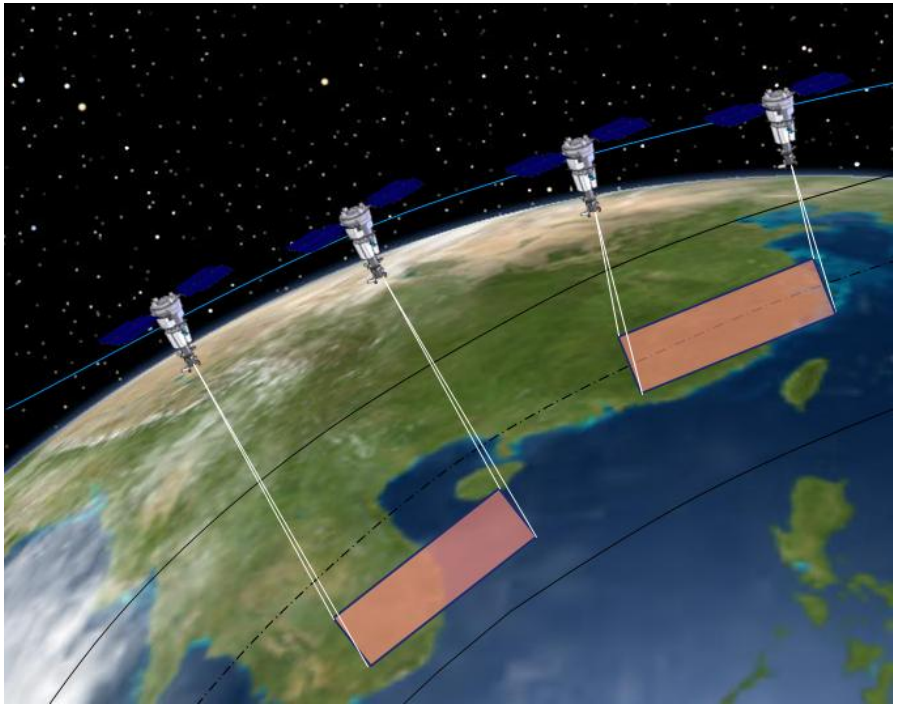
Introduction

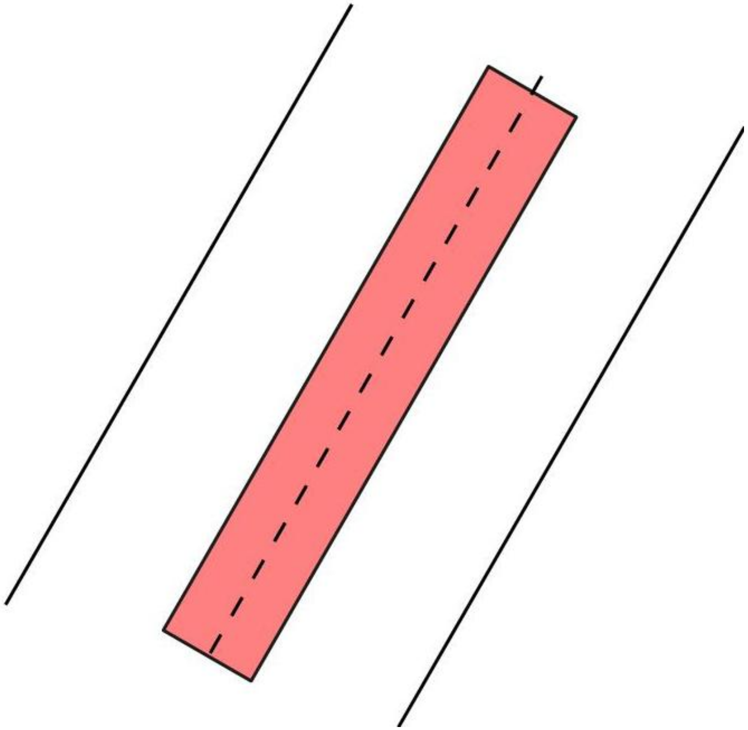
SRS applications:

- weather forecasting
- environmental monitoring
- water resource management

Resurs-P, modern Russian SRS satellite

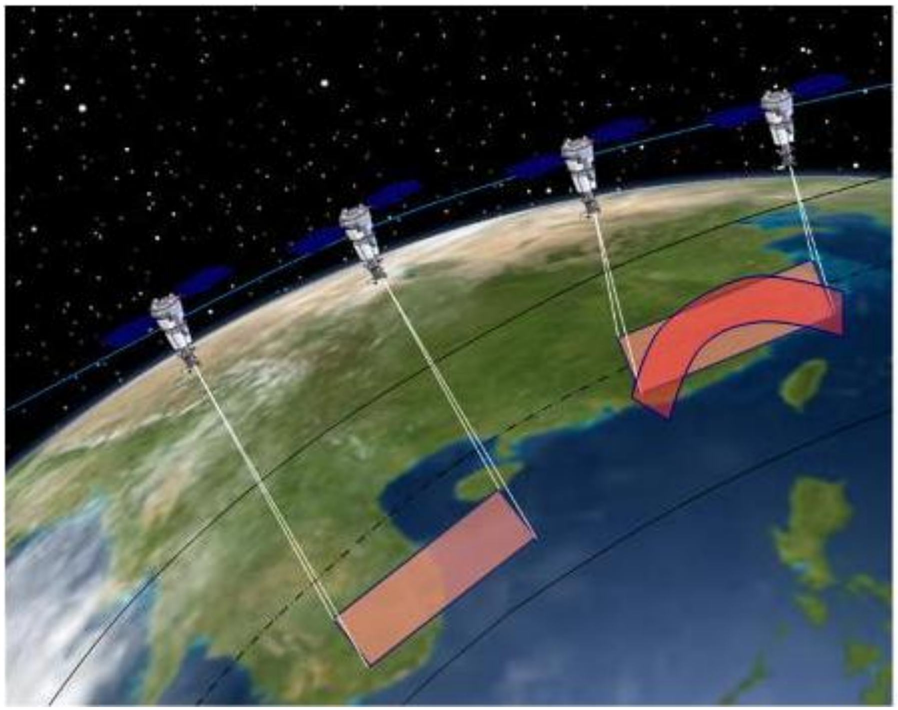
Introduction

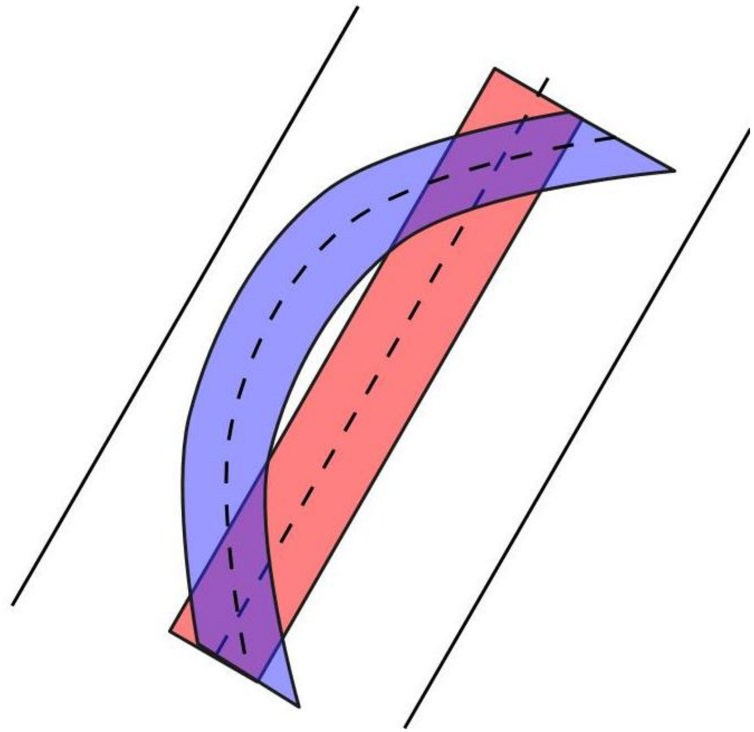




Different modes of surveying

Introduction





Different modes of surveying

Problem statement

What do we know:

- Satellite position and velocity for the whole time of observation
- Route on Earth surface that is described as a smooth curve of the parameter p
- Satellite parameters (e.g. inertia tensor, sensor properties etc.)

What do we want to know:

- Required angular motion of the satellite
- Control torques for this motion realization
- Quality of image acquisition

Coordinate systems

- Inertial System

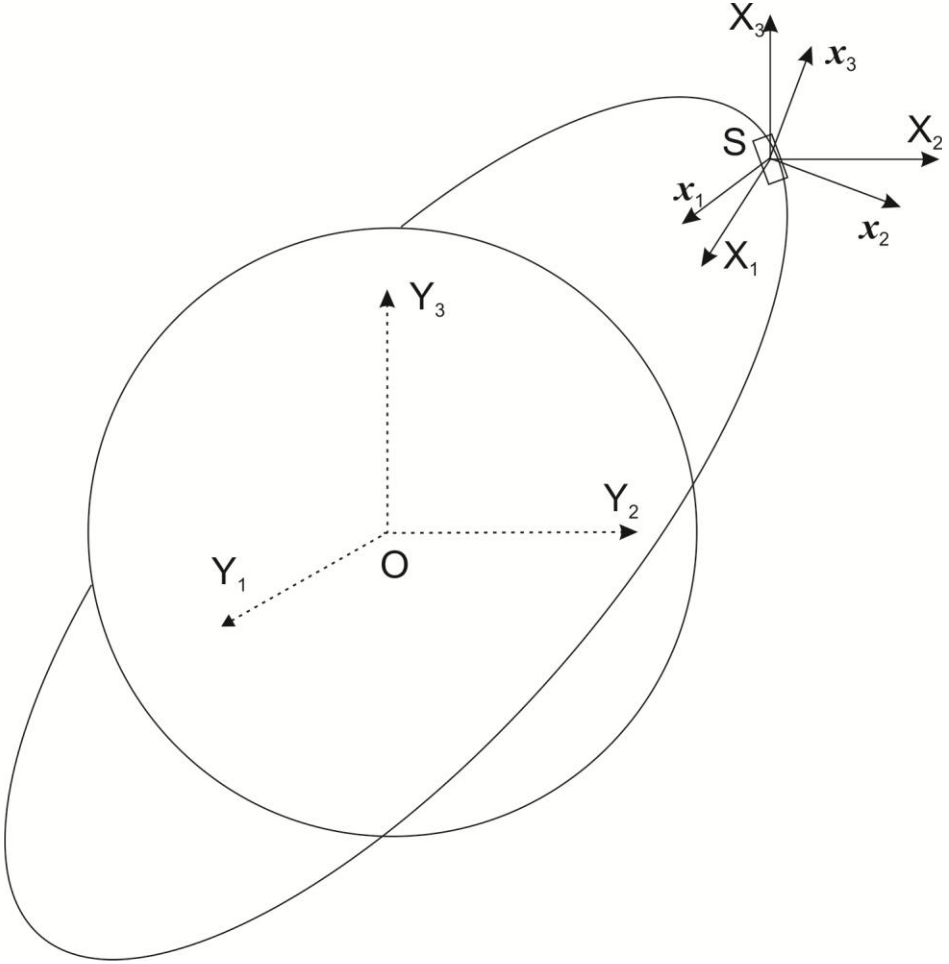
  $OY_1Y_2Y_3$

- Reference System

  $SX_1X_2X_3$

- Body System

  $Sx_1x_2x_3$



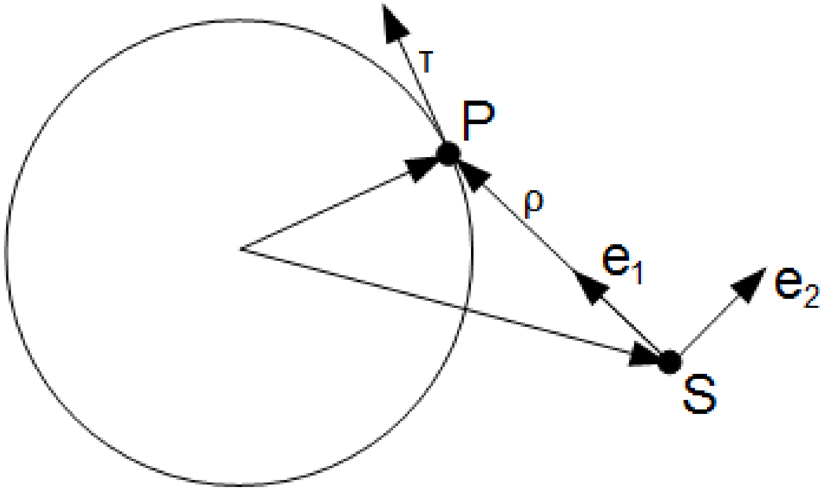
Coordinate systems

Basis vectors of Reference System

$$\mathbf{e}_1 = \frac{\mathbf{r}_\mathbf{p} - \mathbf{r}_\mathbf{s}}{\| \mathbf{r}_\mathbf{p} - \mathbf{r}_\mathbf{s} \|} = \frac{\boldsymbol{\rho}}{\rho}, \mathbf{r}_\mathbf{p} = \mathbf{r}_\mathbf{p}(\mathrm{t}, p(\mathrm{t})),$$

$$\mathbf{e}_2 = \frac{\boldsymbol{\tau} - \mathbf{e}_1(\mathbf{e}_1, \boldsymbol{\tau})}{\left\| \boldsymbol{\tau} - \mathbf{e}_1(\mathbf{e}_1, \boldsymbol{\tau}) \right\|}, \boldsymbol{\tau} = \frac{\partial \mathbf{r}_\mathbf{p}}{\partial p},$$

$$\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2.$$



Angular motion synthesis

Constraints for image velocity

$$(\mathbf{V}_{rel},\mathbf{e_3})\frac{f}{\rho}=0,\ (\mathbf{V}_{rel},\mathbf{e_2})\frac{f}{\rho}=-V$$

$$\mathbf{V}_{rel}=\mathbf{\Omega_e}\times\mathbf{r_p}-\mathbf{V_s}-\boldsymbol{\omega}\times(\mathbf{r_p}-\mathbf{r_s})$$

Obtained equations for the satellite angular velocity

$$\omega_2=-\frac{(\mathbf{\Omega_e}\times\mathbf{r_p}-\mathbf{V_s},\mathbf{e_3})}{\rho},$$

$$\omega_3=\frac{(\mathbf{\Omega_e}\times\mathbf{r_p}-\mathbf{V_s},\mathbf{e_2})}{\rho}+\frac{V}{f},$$

$$\dot{\mathbf{B}}=\mathbf{WB},\mathbf{W}=\begin{pmatrix}0 & \omega_3 & -\omega_2\\ -\omega_3 & 0 & \omega_1\\ \omega_2 & -\omega_1 & 0\end{pmatrix},$$

$$\mathbf{B}=\begin{pmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3\end{pmatrix}^T,\dot{\mathbf{B}}=\begin{pmatrix}\dot{\mathbf{e}}_1 & \dot{\mathbf{e}}_2 & \dot{\mathbf{e}}_3\end{pmatrix}^T$$

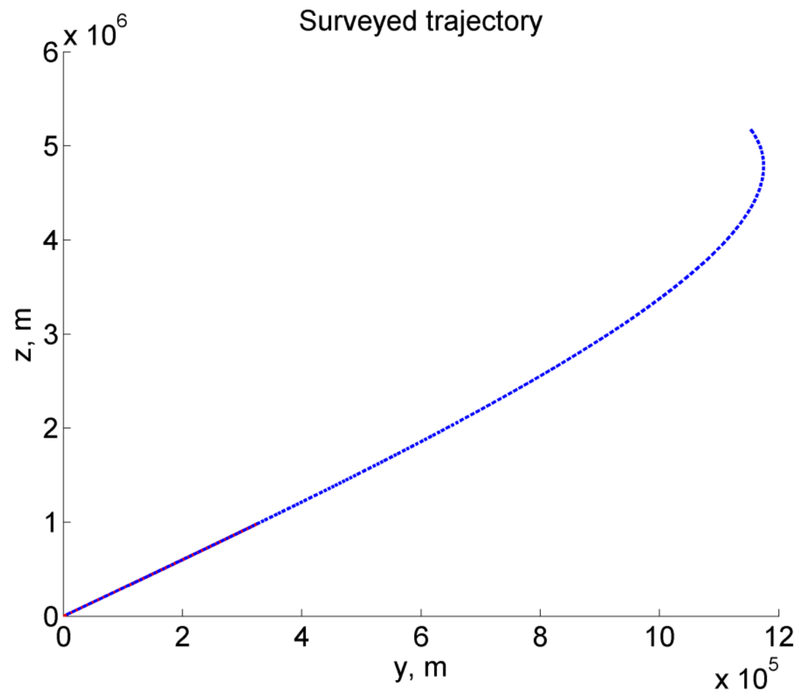
Angular motion synthesis

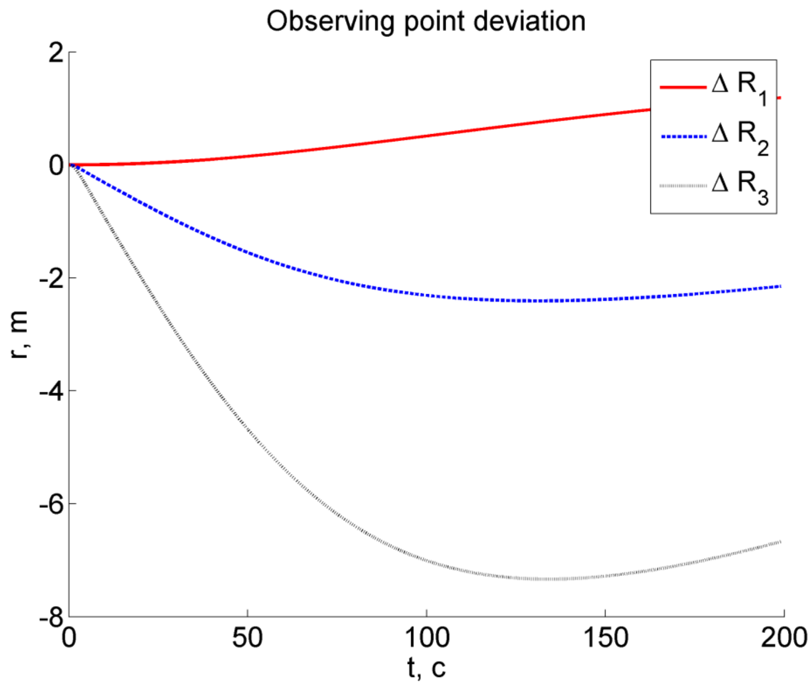
Differential equation for route parameter

$$\dot{p} = \frac{\rho(p,t)V}{f} \frac{1}{(\boldsymbol{\tau}(p,t), \mathbf{e}_2(p,t))}.$$

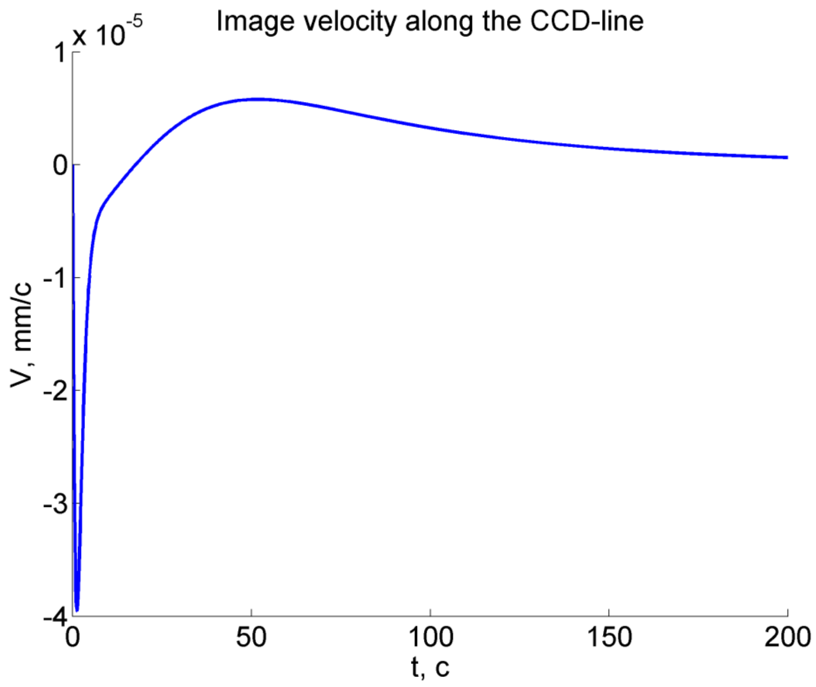
After solving it, we can define angular velocity and attitude of the satellite

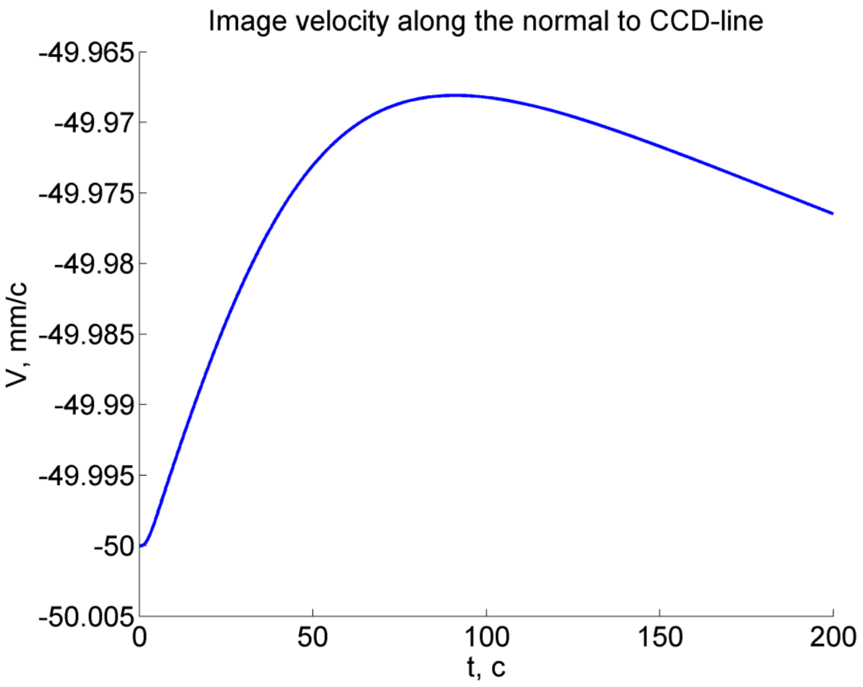
Computer simulation





Computer simulation





Errors

Aiming point displacement:

$$\delta\mathbf{r}_p = \left\{ \mathbf{e}_1 \frac{(\mathbf{r}_s,\mathbf{e}_1)(\mathbf{r}_s,\mathbf{e}_3)}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2-(\mathbf{r}_s^2-R^2)}} + \mathbf{e}_1(\mathbf{r}_s,\mathbf{e}_3) - \mathbf{e}_3\mathrm{t} \right\}\delta\alpha +$$

$$+\left\{ \mathbf{e}_2\mathrm{t} - \mathbf{e}_1 \frac{(\mathbf{r}_s,\mathbf{e}_1)(\mathbf{r}_s,\mathbf{e}_2)}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2-(\mathbf{r}_s^2-R^2)}} - \mathbf{e}_1(\mathbf{r}_s,\mathbf{e}_2) \right\}\delta\beta +$$

$$+\left\{ \mathbf{E}_{3\times 3} - \mathbf{e}_1\mathbf{e}_1^{\,T} + \frac{\mathbf{e}_1\mathbf{r}_s^{\,T}}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2-(\mathbf{r}_s^2-R^2)}} - (\mathbf{r}_s,\mathbf{e}_1)\frac{\mathbf{e}_1\mathbf{e}_1^T}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2-(\mathbf{r}_s^2-R^2)}} \right\}\delta\mathbf{r}_s$$

Errors

$$\delta V_{\parallel} = \left| \frac{f}{\rho} \mathbf{e}_3^T \left\{ ([\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,1-3} + [\boldsymbol{\rho}]_\times \mathbf{A}^T [\boldsymbol{\omega}]_\times \mathbf{N}) \delta\boldsymbol{\psi} + [\boldsymbol{\rho}]_\times \mathbf{A}^T \delta\boldsymbol{\omega} + \right.\right.$$

$$\left.\left. + ([\mathbf{A}^\mathbf{T} \boldsymbol{\omega}]_\times + [\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,7-9}) \delta\mathbf{r}_s - \delta\mathbf{V}_s \right\} + \frac{f}{\rho} \left\{ (\mathbf{V}_{rel}, \mathbf{e}_1) \delta\alpha - (\mathbf{V}_{rel}, \mathbf{e}_2) \delta\gamma \right\} \right|$$

$$\delta V_{\perp} = \left| \frac{f}{\rho} \mathbf{e}_2^T \left\{ ([\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,1-3} + [\boldsymbol{\rho}]_\times \mathbf{A}^T [\boldsymbol{\omega}]_\times \mathbf{N}) \delta\boldsymbol{\psi} + [\boldsymbol{\rho}]_\times \mathbf{A}^T \delta\boldsymbol{\omega} + \right.\right.$$

$$\left. + ([\mathbf{A}^\mathbf{T} \boldsymbol{\omega}]_\times + [\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,7-9}) \delta\mathbf{r}_s - \delta\mathbf{V}_s \right\} - \frac{f}{\rho} (\mathbf{V}_{rel}, \mathbf{e}_1) \delta\beta -$$

$$\left. + (\mathbf{V}_{rel}, \mathbf{e}_2) \frac{f}{\rho^3} \boldsymbol{\rho}^T \delta\mathbf{r}_s - (\mathbf{V}_{rel}, \mathbf{e}_2) \frac{f}{\rho^3} \boldsymbol{\rho}^T \mathbf{D} \delta\mathbf{x} \right|$$

Errors

Main parts of errors (for the 400 km height orbits)

$$|\delta \mathbf{r}_p| \approx |\delta \mathbf{r}_s| + 5 \cdot 10^5 |\delta \boldsymbol{\psi}|,$$

$$|\delta V| \approx f\,|\delta \boldsymbol{\omega}|$$

Conclusion

- Angular motion for SRS satellite while tracking routes on Earth surface is constructed
- Control algorithm is suggested
- Connection between attitude and angular motion errors and aiming point displacement is investigated

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Thank you for your attention