Effect of external torques on the satellite angular motion

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Moscow, 2018

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- Introduction
- Gravity gradient torque
- Geomagnetic field
- Solar radiation pressure
- Shadow models
- Atmospheric drag

Introduction

External disturbances

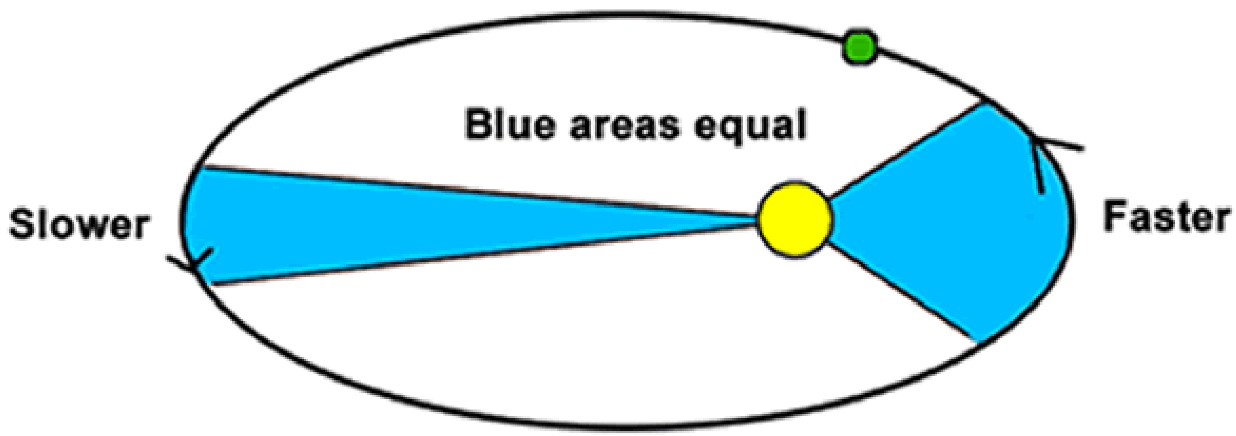
- depend on the satellite location
- depend on the velocity
- change with time

We want to predict satellite motion with high accuracy

Kepler's laws

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.

2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

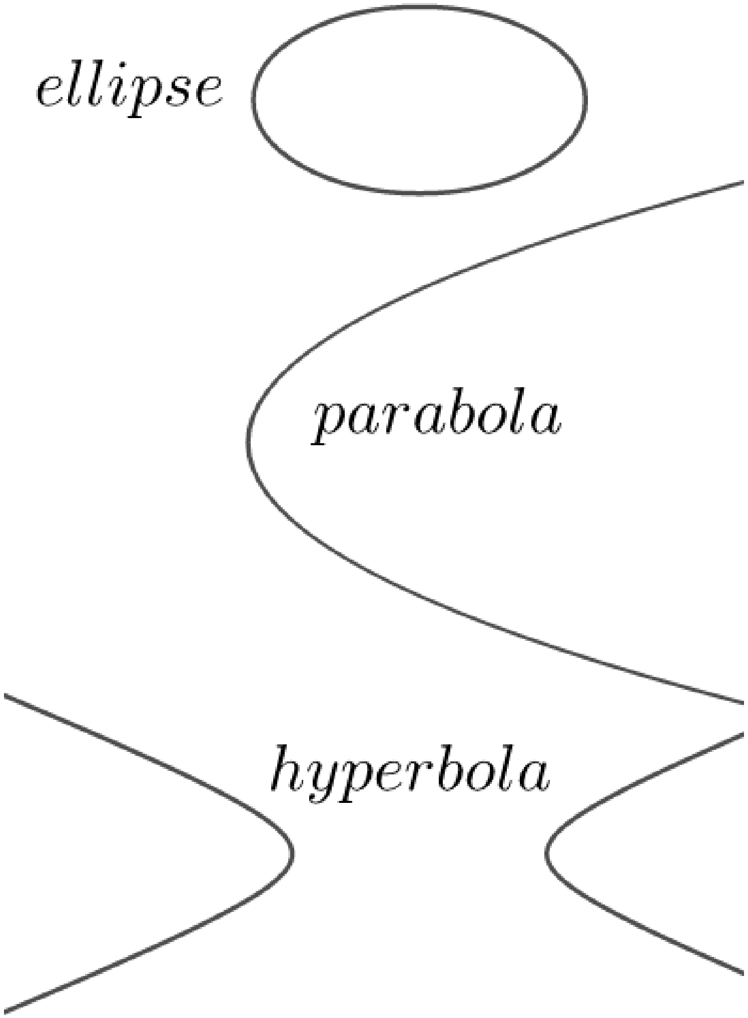


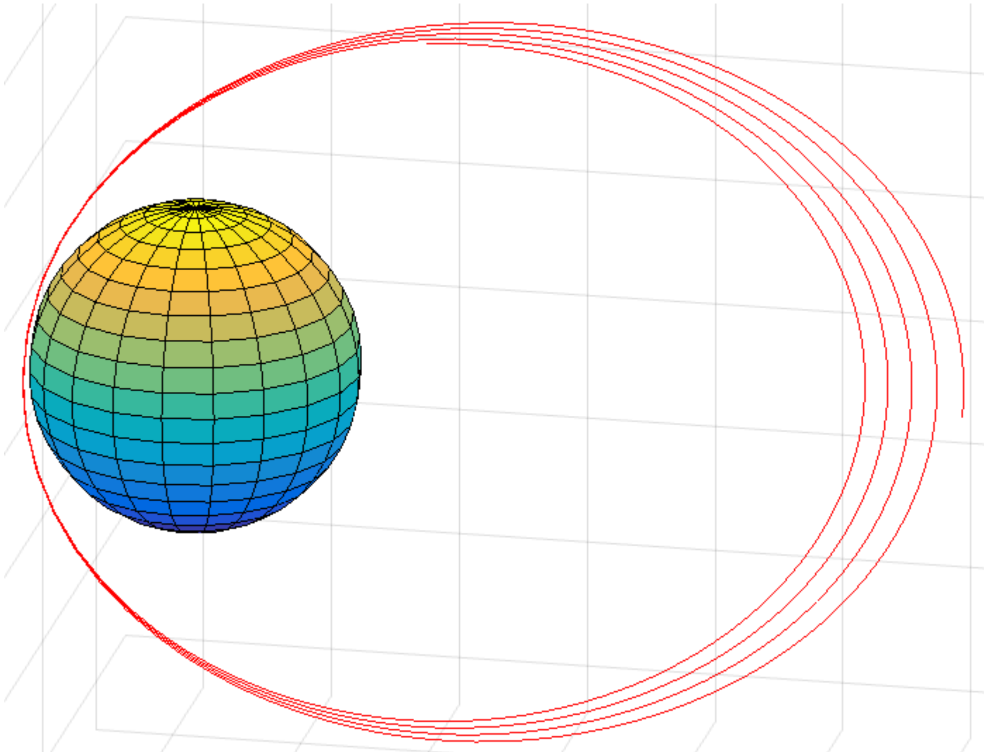
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

$$\frac{T_1^{\ 2}}{T_2^{\ 2}} = \frac{a_1^{\ 3}}{a_2^{\ 3}}$$

Keplerian orbits

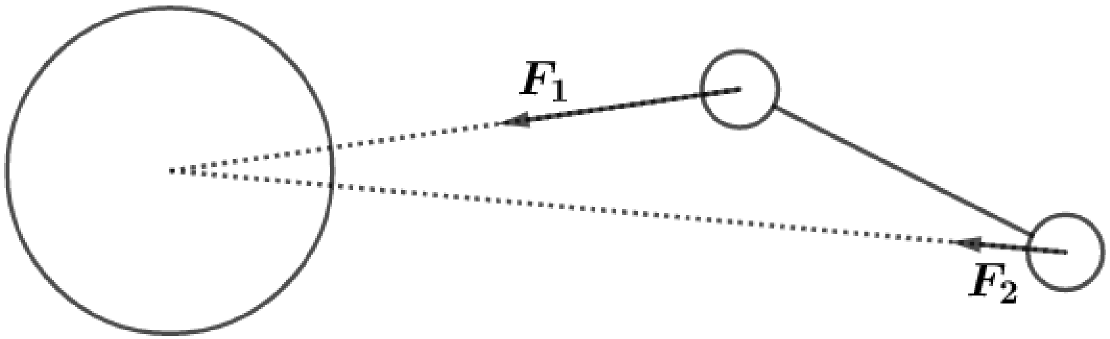
$F = G\frac{m_1 m_2}{r^2}$ – Newton's law of universal gravitation



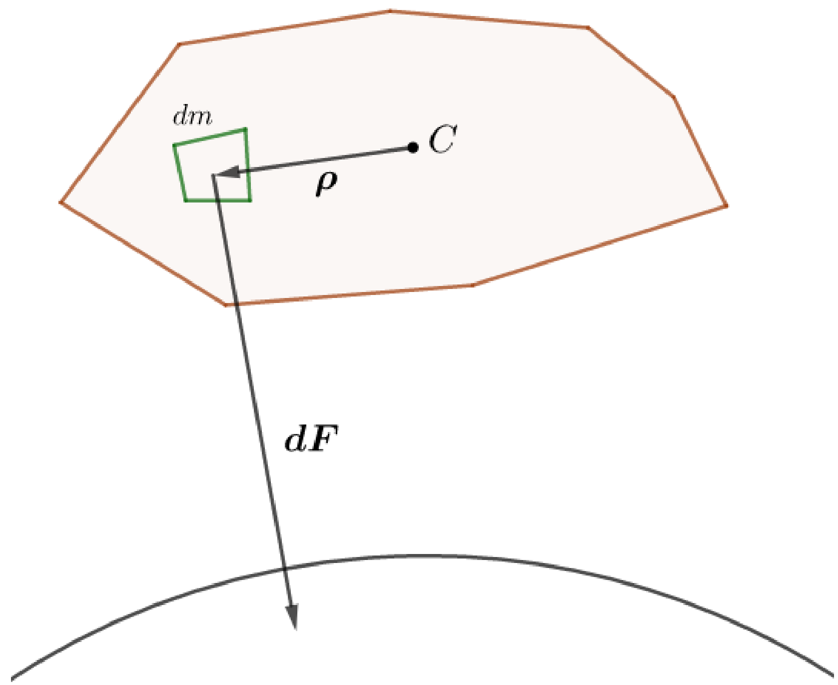


real satellite orbit

Gravity gradient torque



$F_1 > F_2$ — torque appears, satellite rotates



$$d\mathbf{M}_{grav} = \boldsymbol{\rho} \times d\mathbf{F} = \boldsymbol{\rho} \times \frac{\mu \mathbf{r}}{r^3} dm$$

Gravity gradient torque

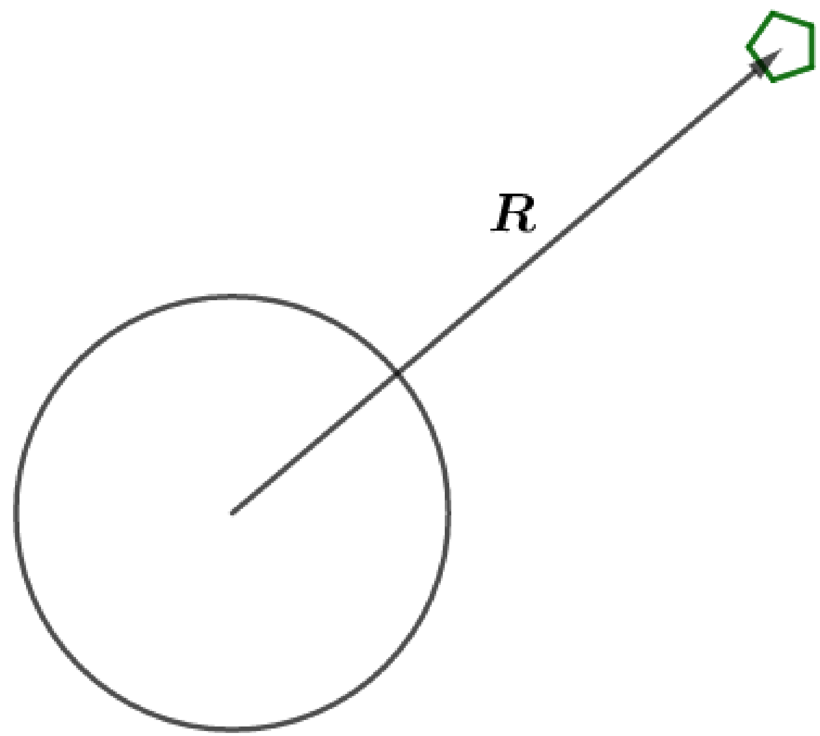
$$\mathbf{M}_{grav} = \int_V \boldsymbol{\rho} \times \frac{\mu \mathbf{r}}{r^3} dm$$

$\mathbf{M}_{grav} = 3\frac{\mu}{R^5}\mathbf{R} \times \mathbf{J}\mathbf{R}$ — expression for the gravity gradient torque

$\mu = 3.968 \cdot 10^{14}\ m^3/s^2$ – Earth's gravity parameter

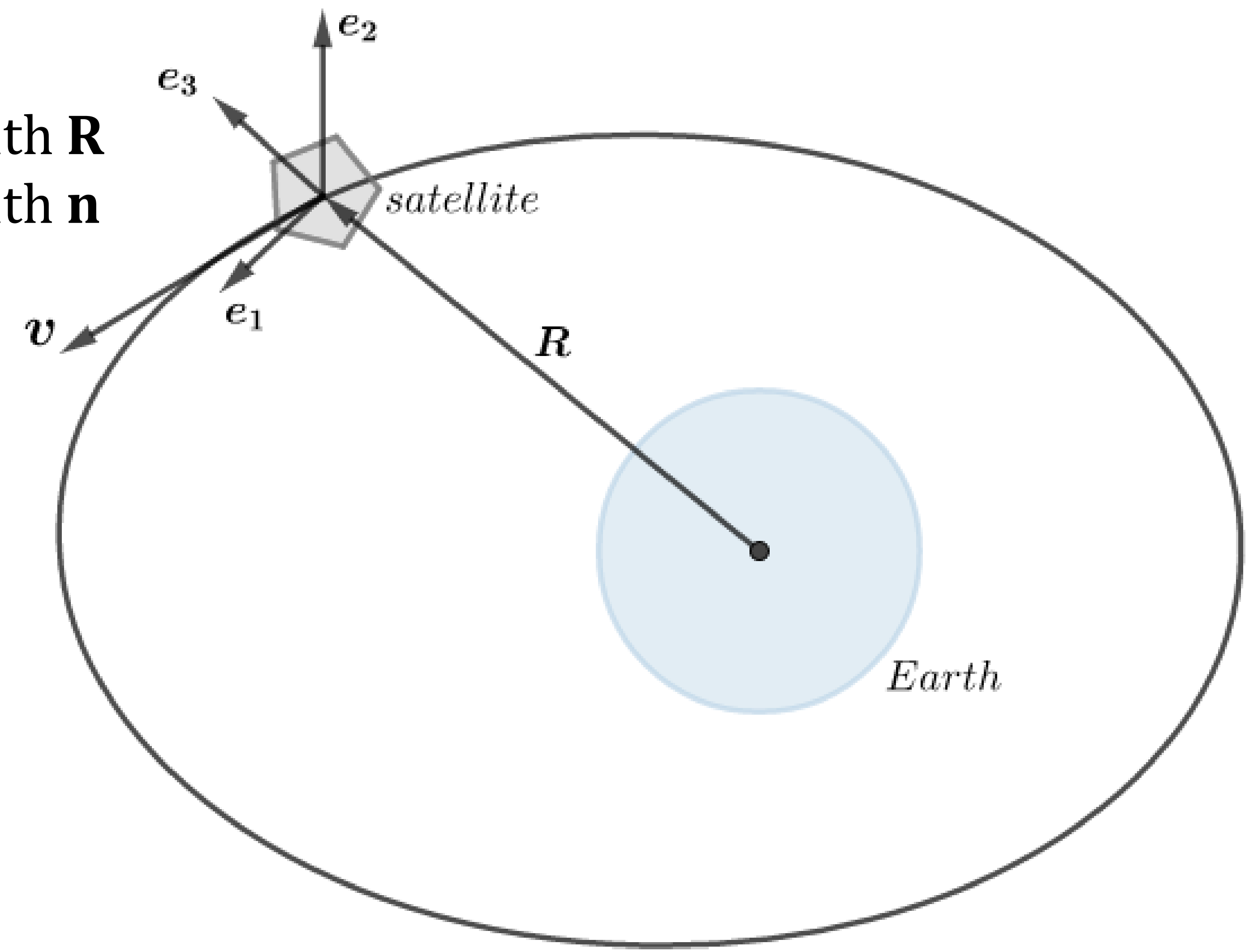
$\mathbf{J}$ — inertia tensor

Radius-vector $\mathbf{R}$ - vector from the center of Earth to the center of mass of satellite

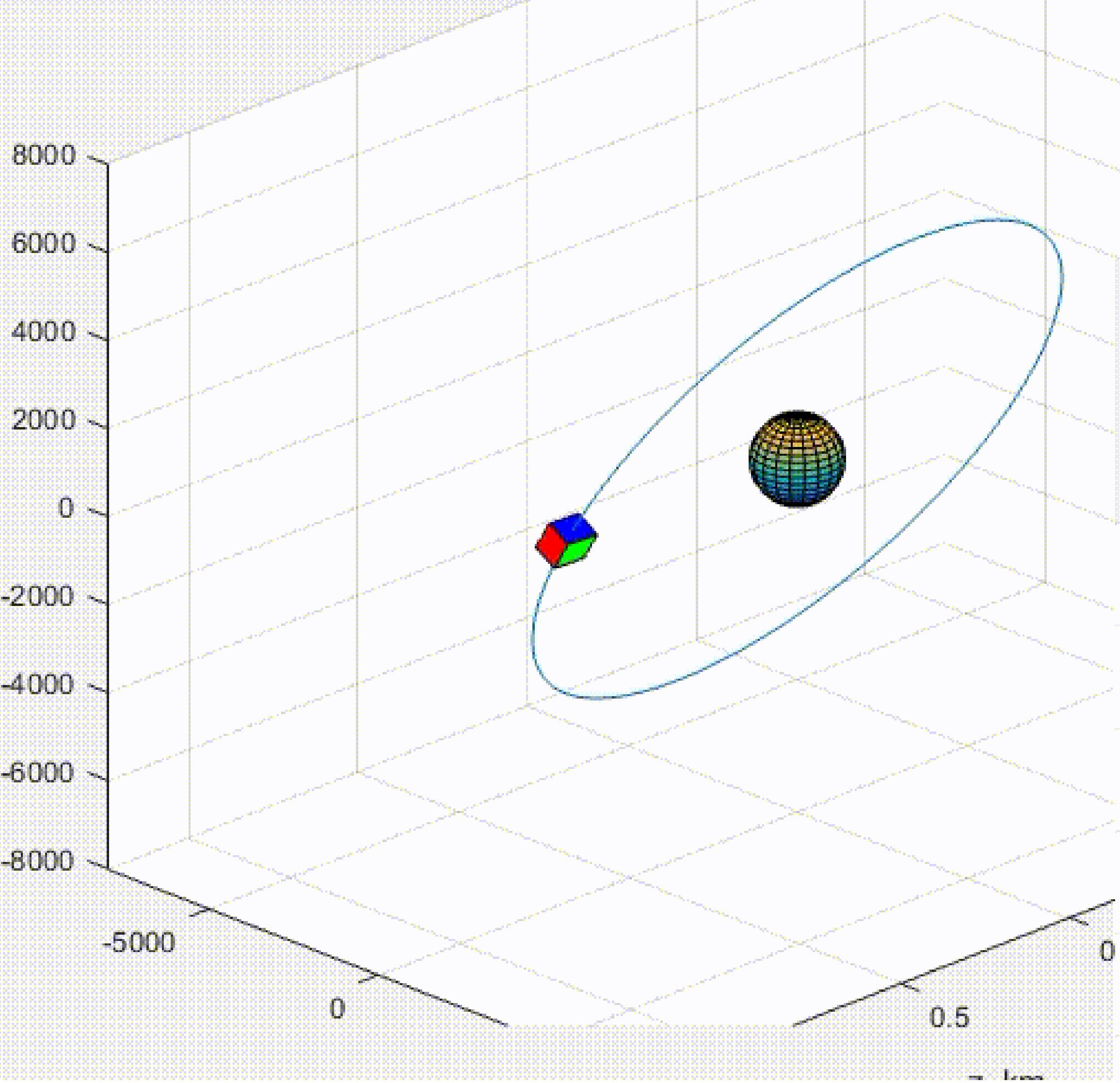


Orbital coordinate system

$\mathbf{e}_3$ aligned with $\mathbf{R}$
$\mathbf{e}_2$ aligned with $\mathbf{n}$

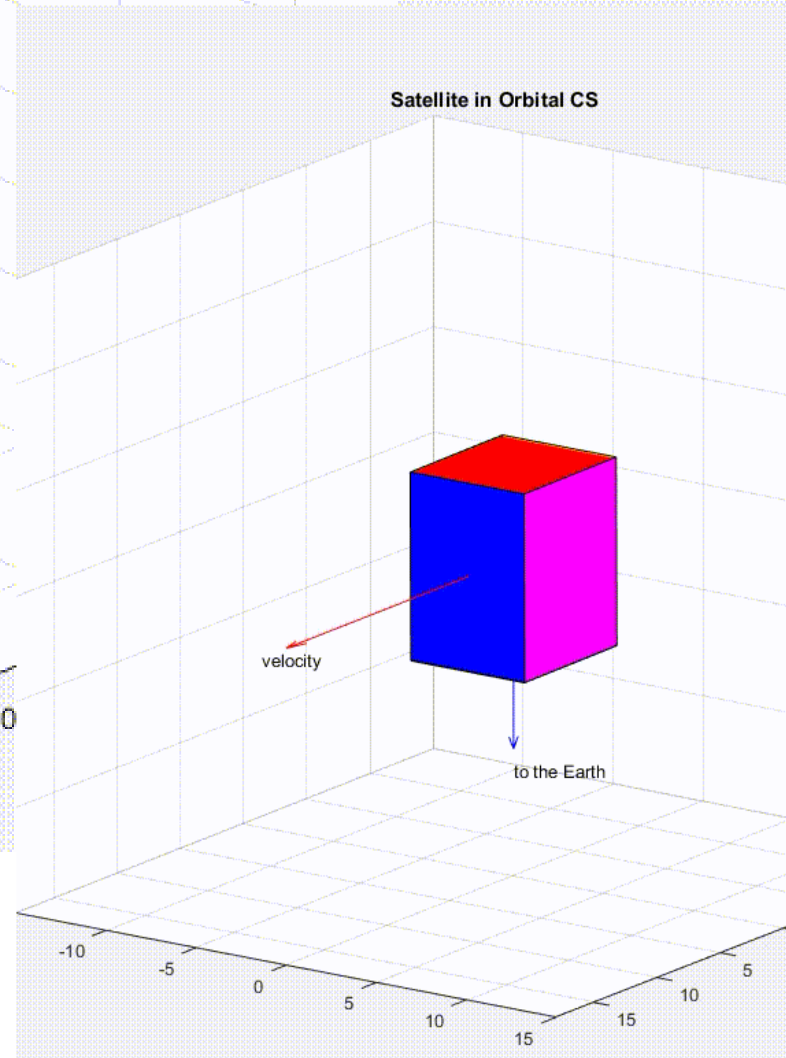


Equilibrium

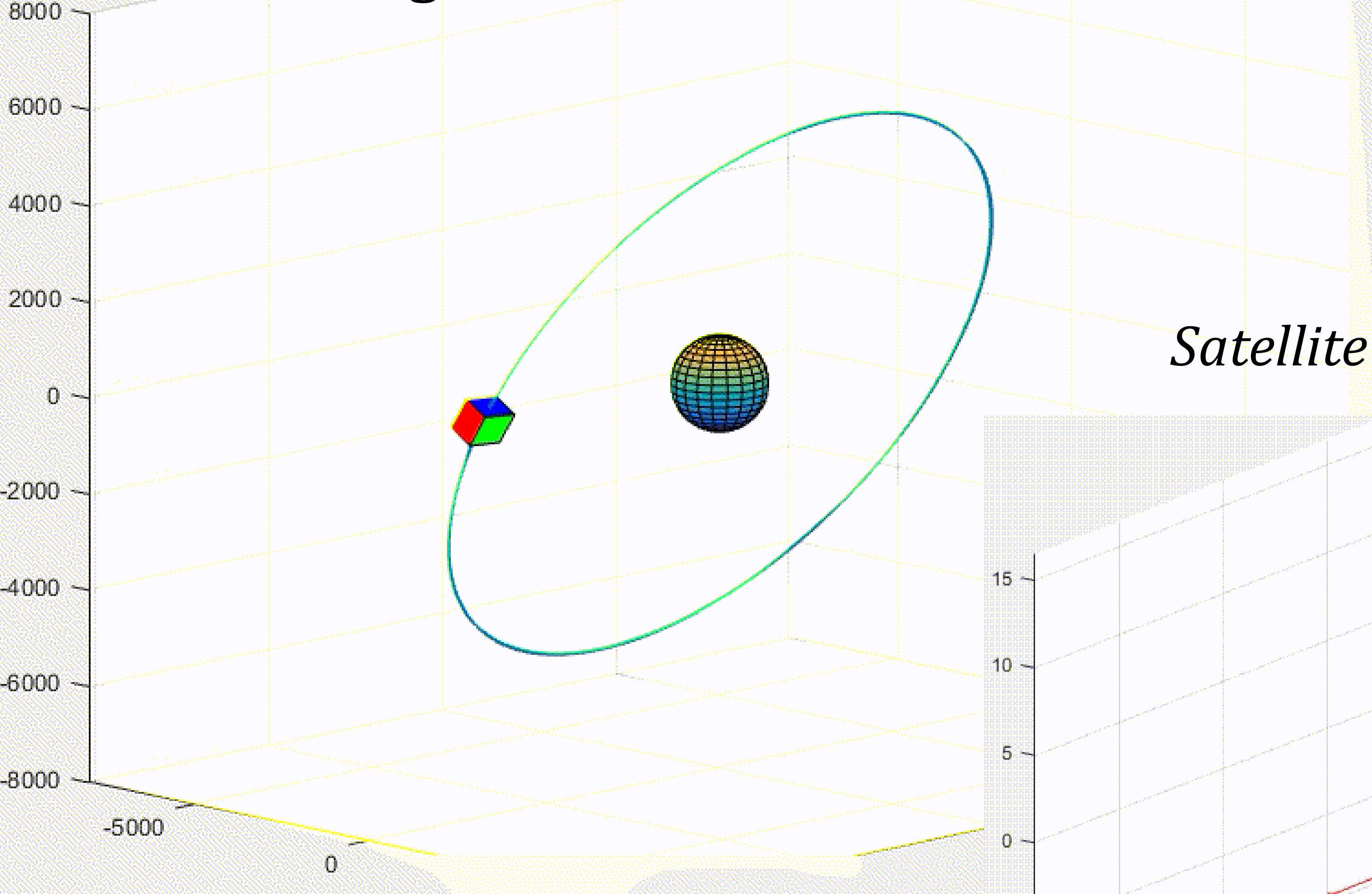


Satellite in Inertial CS

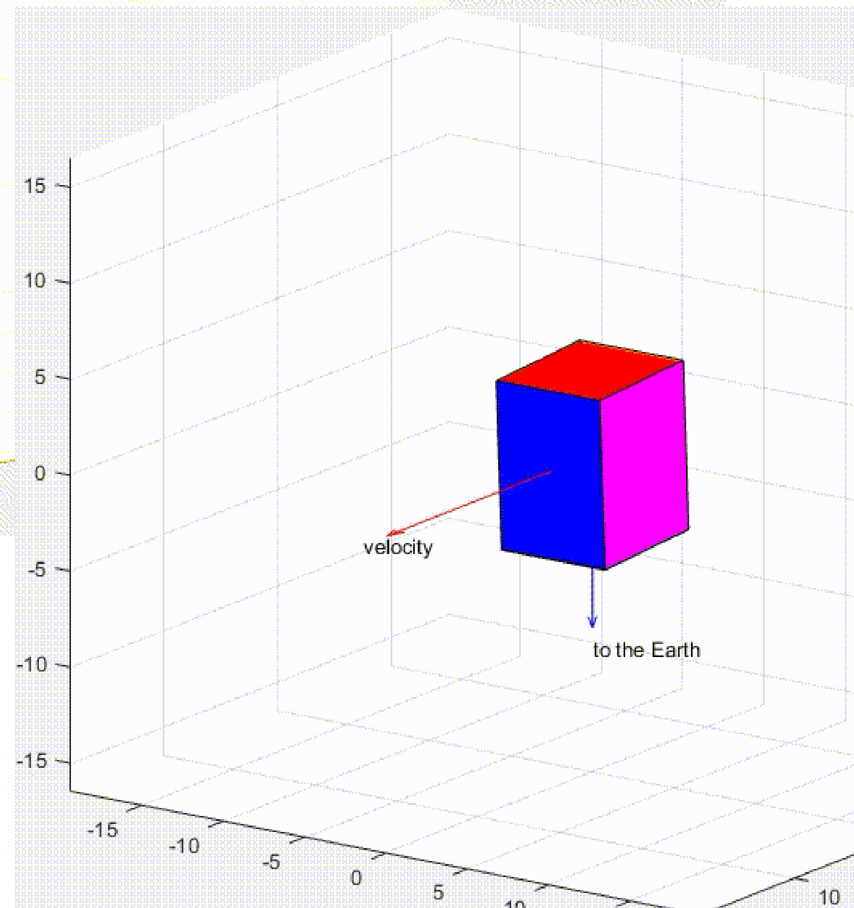
Satellite in Orbital CS



Motion in general case



Satellite in Orbital CS



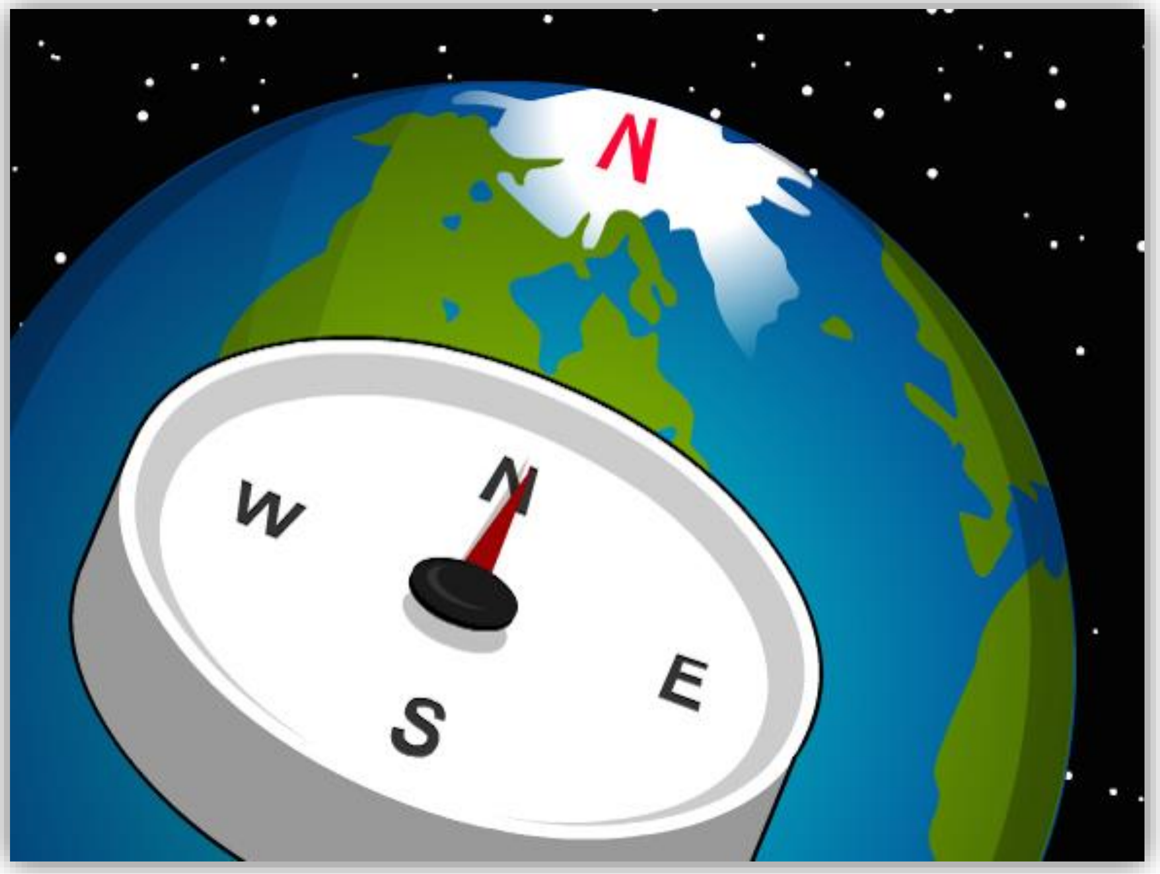
Satellite in Inertial CS

Geomagnetic field

$$\mathbf{M}_{magn} = \mathbf{m} \times \mathbf{B}$$

B - vector of magnetic induction

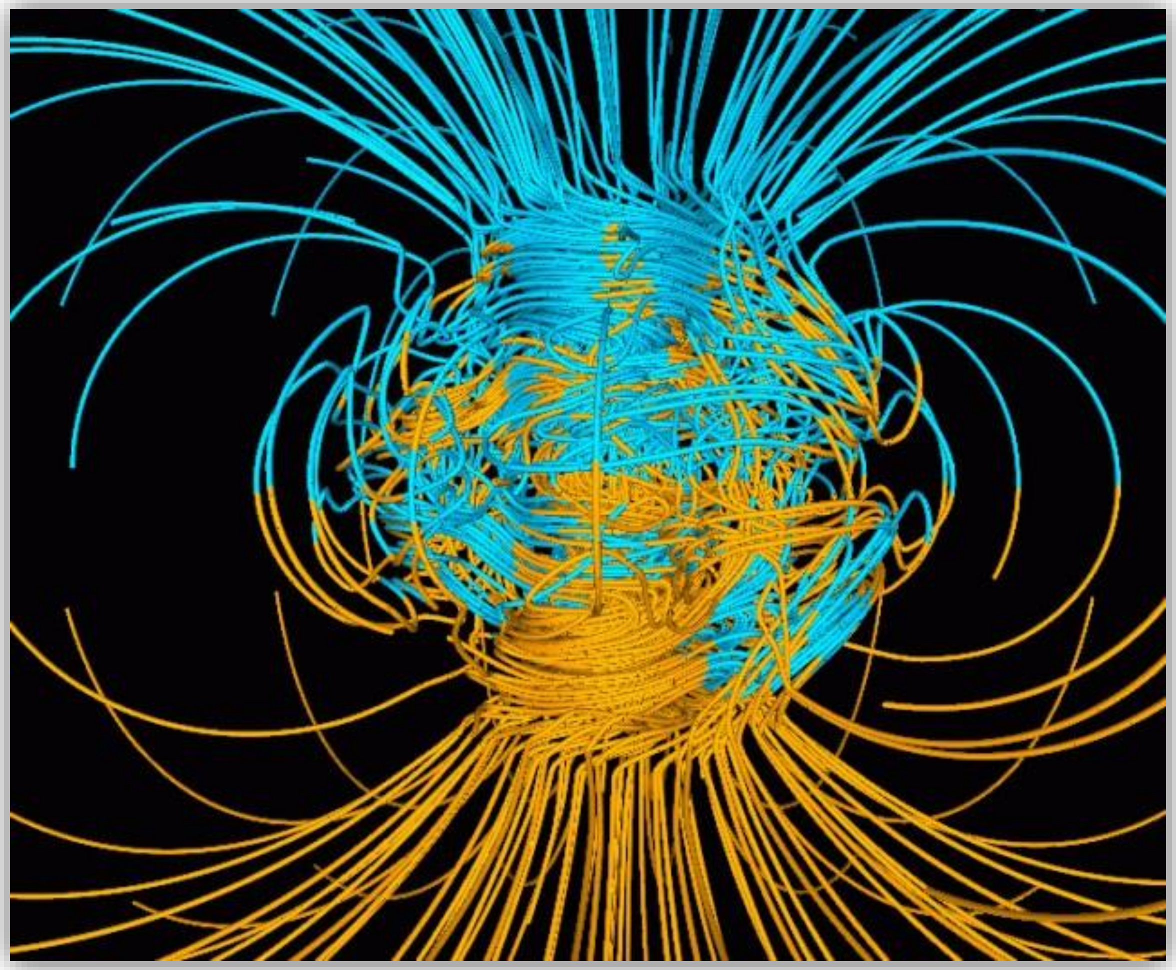
m - dipole moment



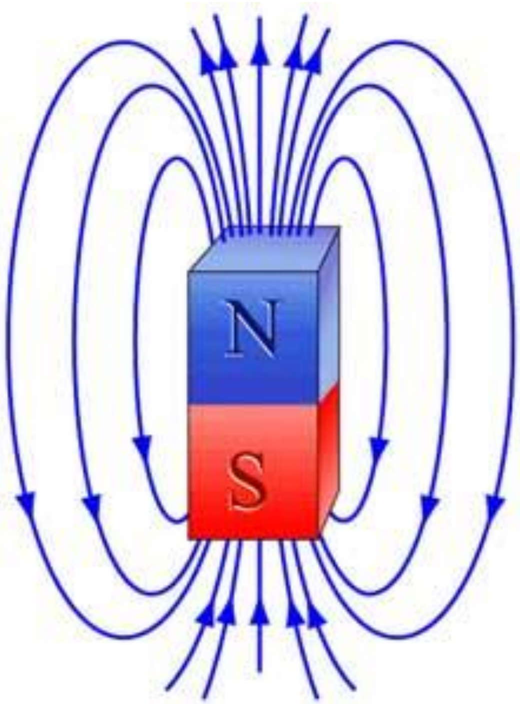
Real geomagnetic field

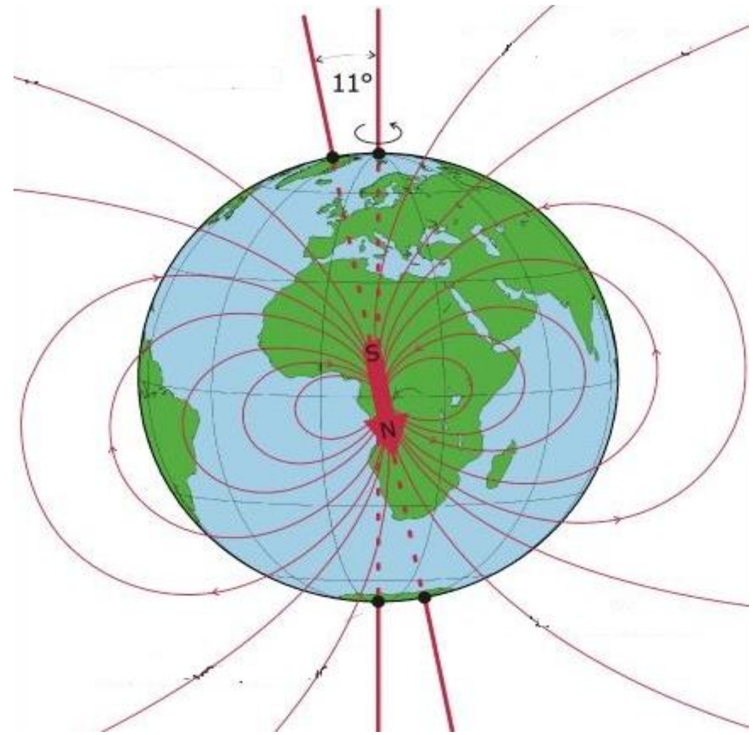
$\mathbf{B} = \mu_0 \nabla V$ — expression for vector of magnetic induction

$$V = -R\sum_{i=1}^{k}\left(\frac{R}{r}\right)^{i+1}\sum_{n=0}^{m}\left(g_n^m(t)\cos m\lambda_0 + h_n^m(t)\sin m\lambda_0\right)P_n^m(\cos\vartheta_0)$$



Geomagnetic field





$$\mathbf{B} = -\frac{\mu_e}{R^5}\left(\mathbf{k}R^2 - 3(\mathbf{kR})\mathbf{R}\right)$$

$\mu_e = 7.812 \cdot 10^{15} m^3 \cdot kg \cdot s^{-2} \cdot A^{-1}$ — Earth's magnetic constant

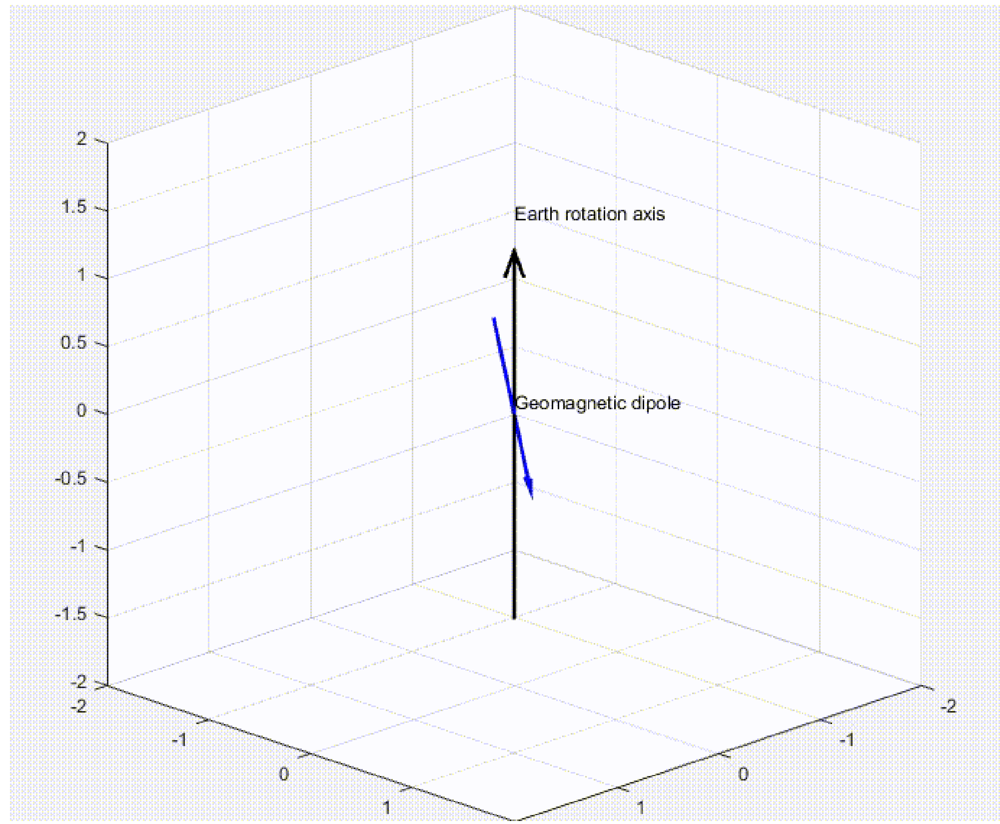
There are two ways of calculating the vector **k** (unit vector co-directional with the magnet)

Geomagnetic field

- **direct dipole model**

Dipole is antiparallel to Earth rotation axis

$$\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$



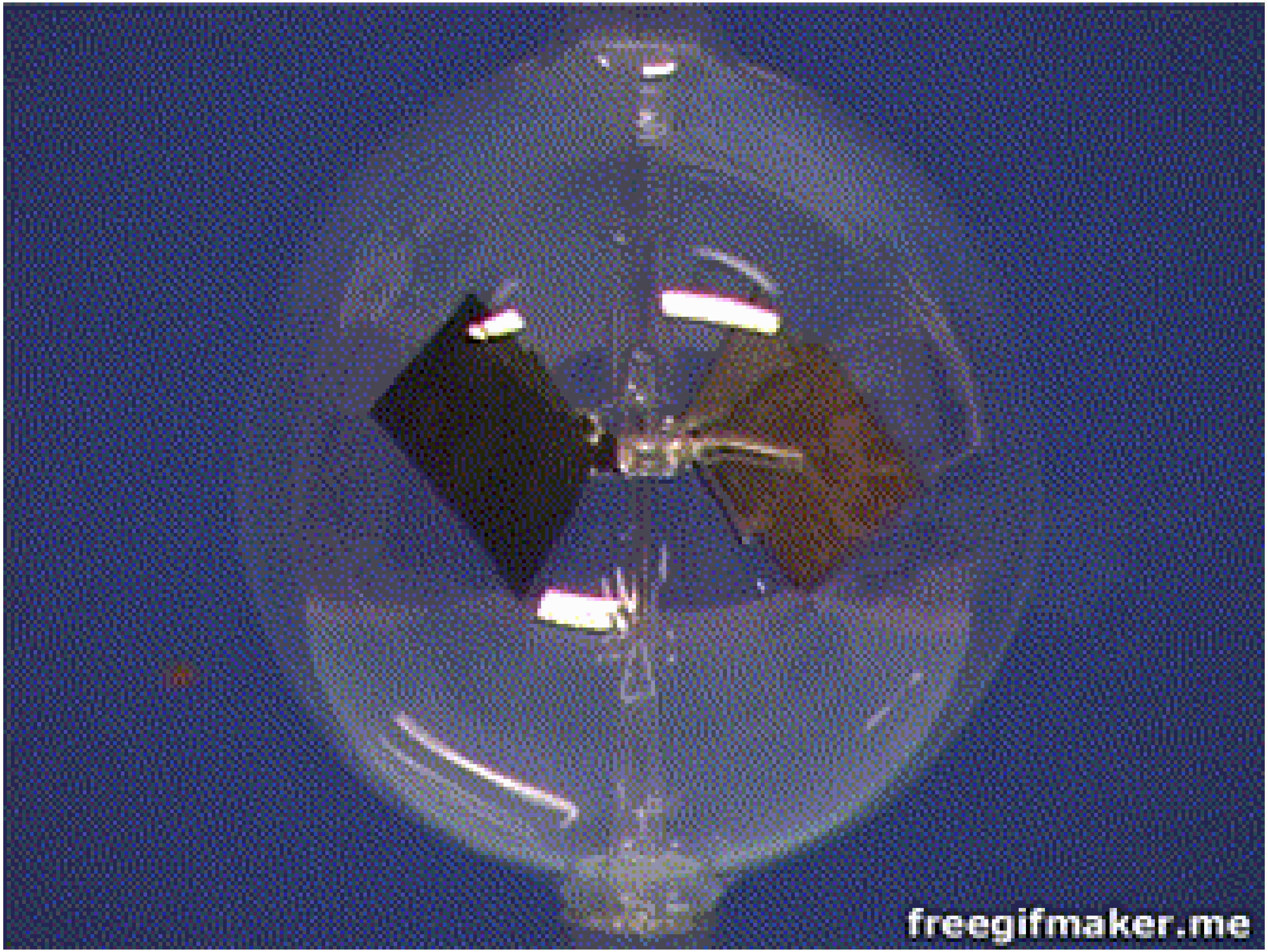
- **inclined dipole model**

Dipole is tilted

$$\mathbf{k} = \begin{pmatrix} \cos\lambda \sin\theta \\ \sin\lambda \sin\theta \\ \cos\theta \end{pmatrix}$$

$\theta \approx 168.3^\circ,\ \lambda_0 \approx -71.88^\circ,\ \lambda = \lambda_0 + \omega_{Earth} t$ – changes with time

Radiation pressure



Solar radiation pressure

The total force of solar radiation pressure acting on flat surface

$$\mathbf{F}_{sun} = -S\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s,\mathbf{n})\mathbf{n} + \alpha(1-\mu)\left(\mathbf{r}_s + \frac{2}{3}\mathbf{n}\right)\right)$$

$\mathbf{r}_s = \frac{\mathbf{R}_s}{|\mathbf{R}_s|}$ – unit vector of the Sun direction

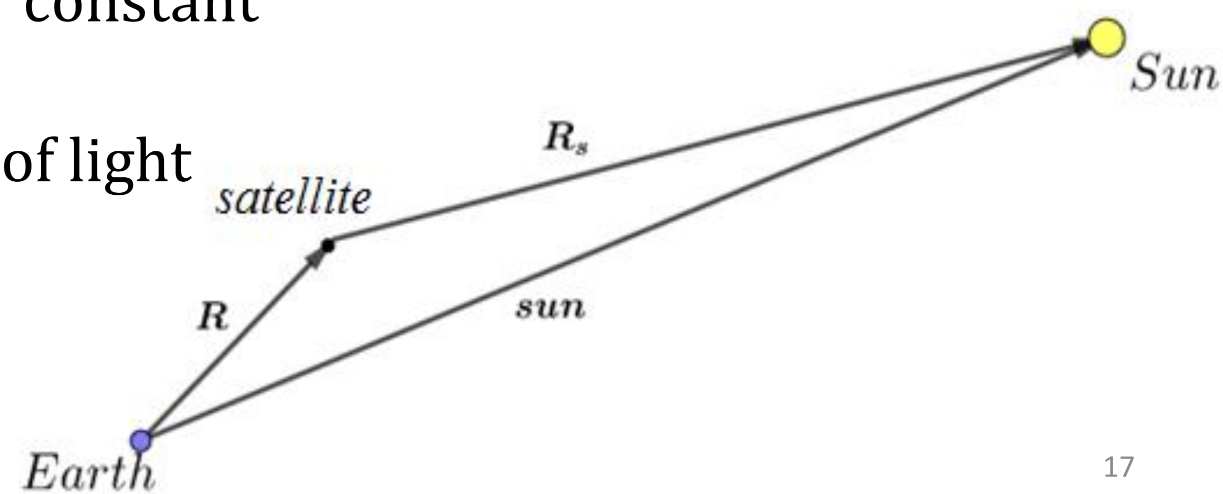
$\alpha,\ \mu$ – the characteristics of the reflecting surface

$\mathbf{n}$ – unit vector of the normal to the satellite surface

$\Phi_0 = 1367\ \frac{W}{m^2}$ – solar constant

$c = 3\cdot 10^8\ \frac{m}{s}$ – speed of light

S – surface area

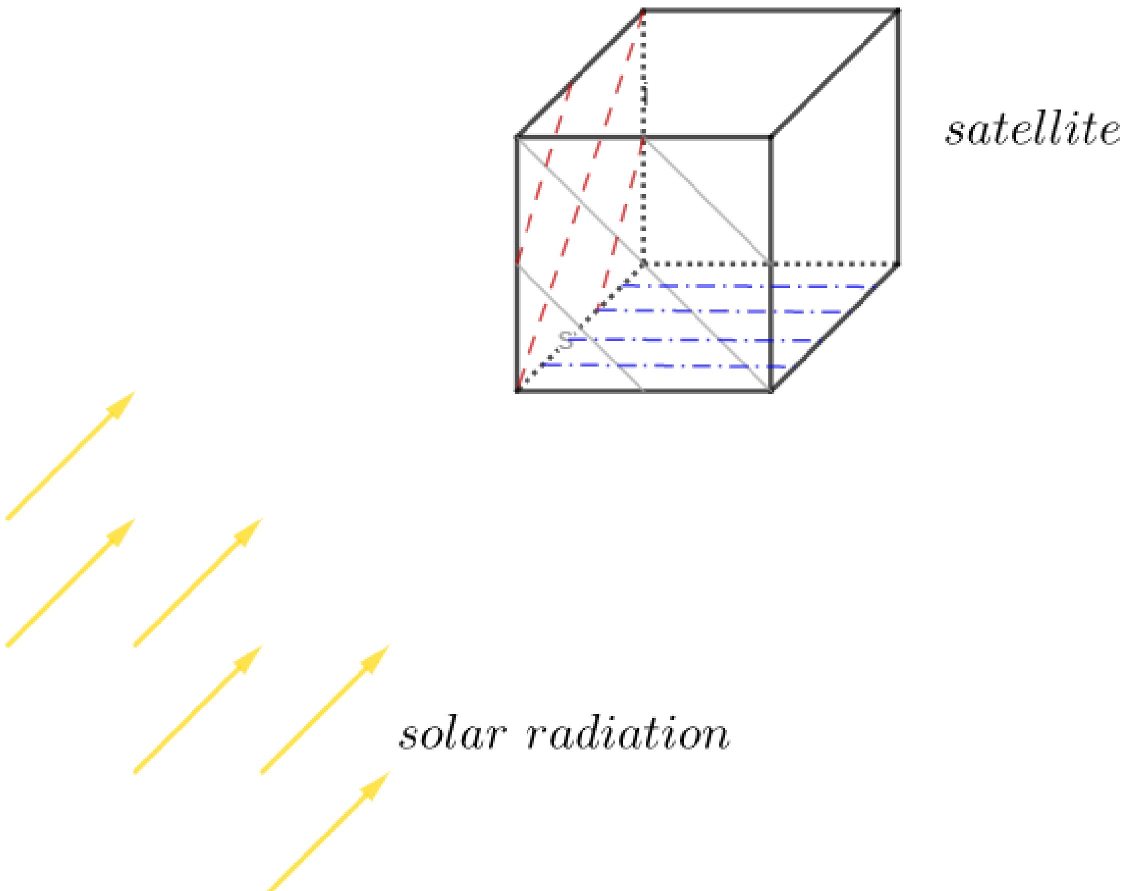


Solar radiation pressure

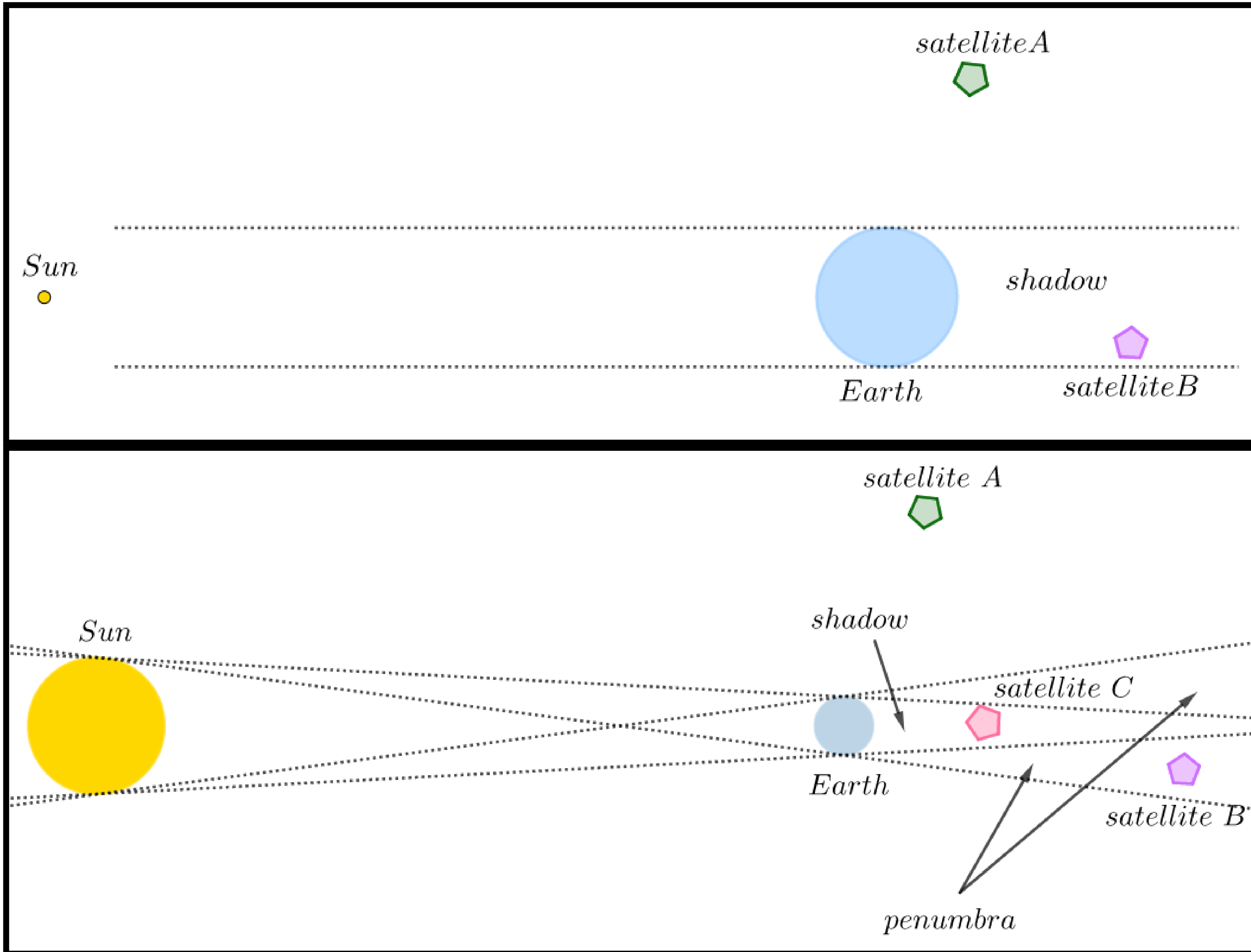
Not all surfaces are lightened in a complex shape satellite

Example: cubic satellite

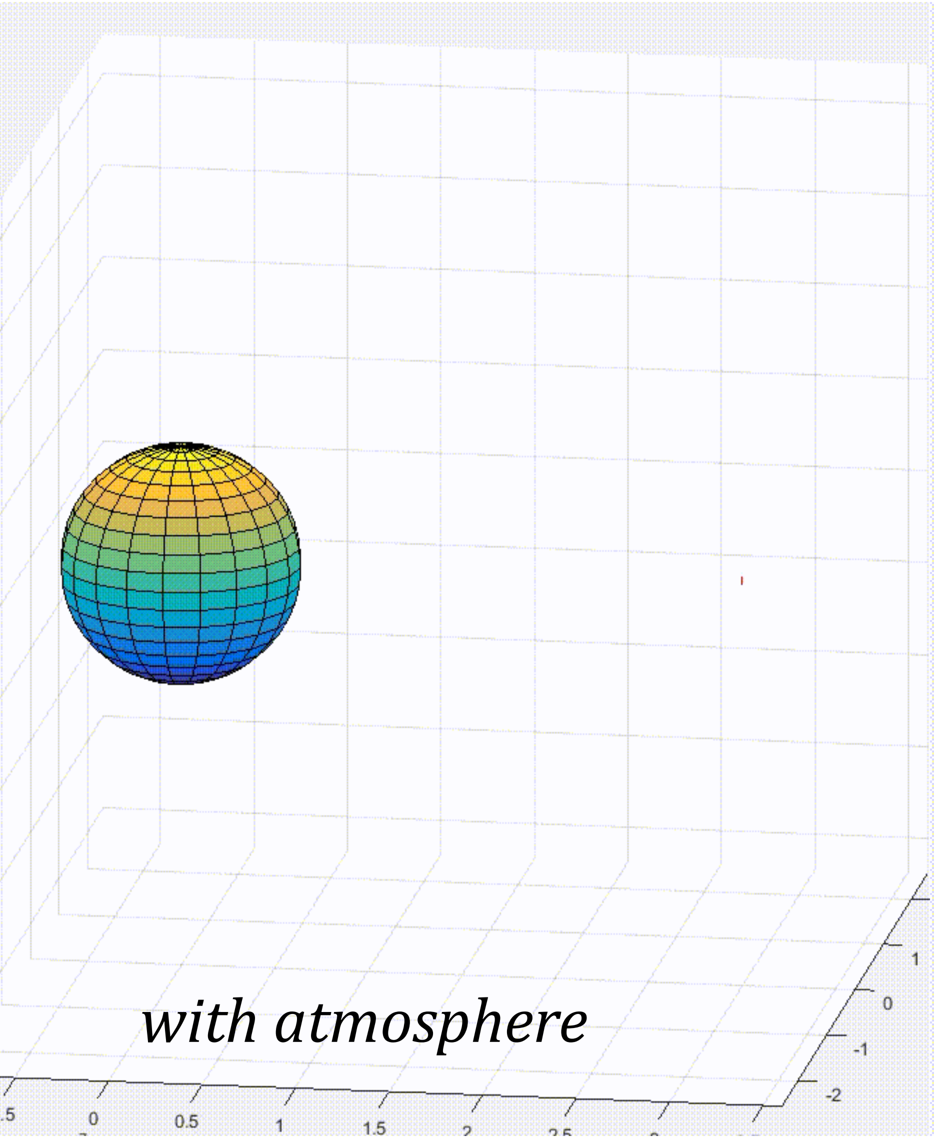
We should consider only three lightened surface



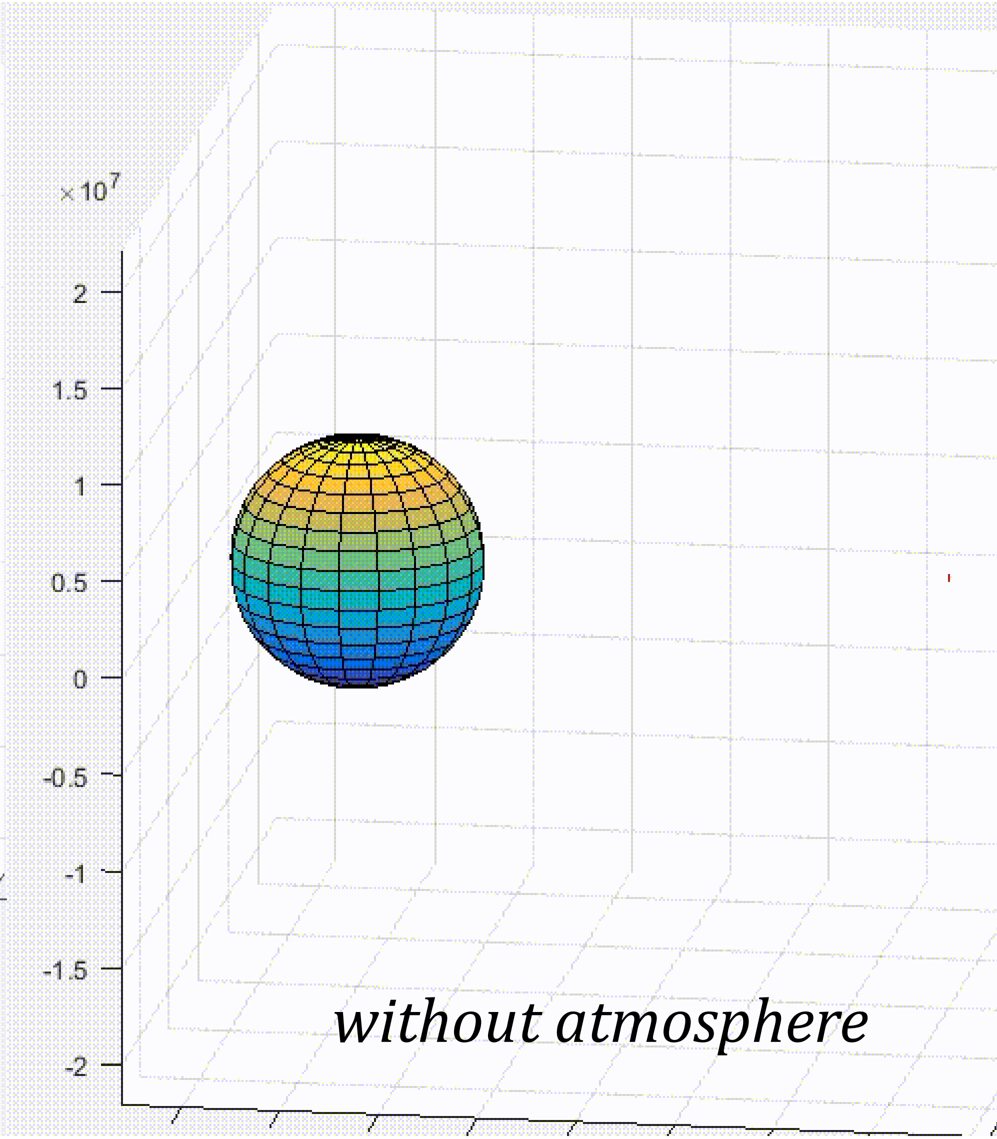
Shadow models



Atmospheric drag



with atmosphere



without atmosphere

Atmospheric drag

1) The air resistance force (frontal drag)

$$\mathbf{F}_{atm} = -\frac{1}{2}\rho_a \left|\mathbf{V}\right| \mathbf{V} C_D S$$

2) The force acting on the elementary part of the surface area *dS*

$$d\mathbf{F}_{atm} = -\rho_a \Big((1-\varepsilon_{atm})(\mathbf{V},\mathbf{n})\mathbf{V} + 2\varepsilon_{atm}(\mathbf{V},\mathbf{n})^2\mathbf{n} + (1-\varepsilon_{atm})\alpha_{atm}(\mathbf{V},\mathbf{n})\mathbf{n} \Big) dS$$

$\varepsilon_{atm},\ \alpha_{atm} \in \left(0,\ 1\right)$ – the characteristics of the surface

Atmospheric drag

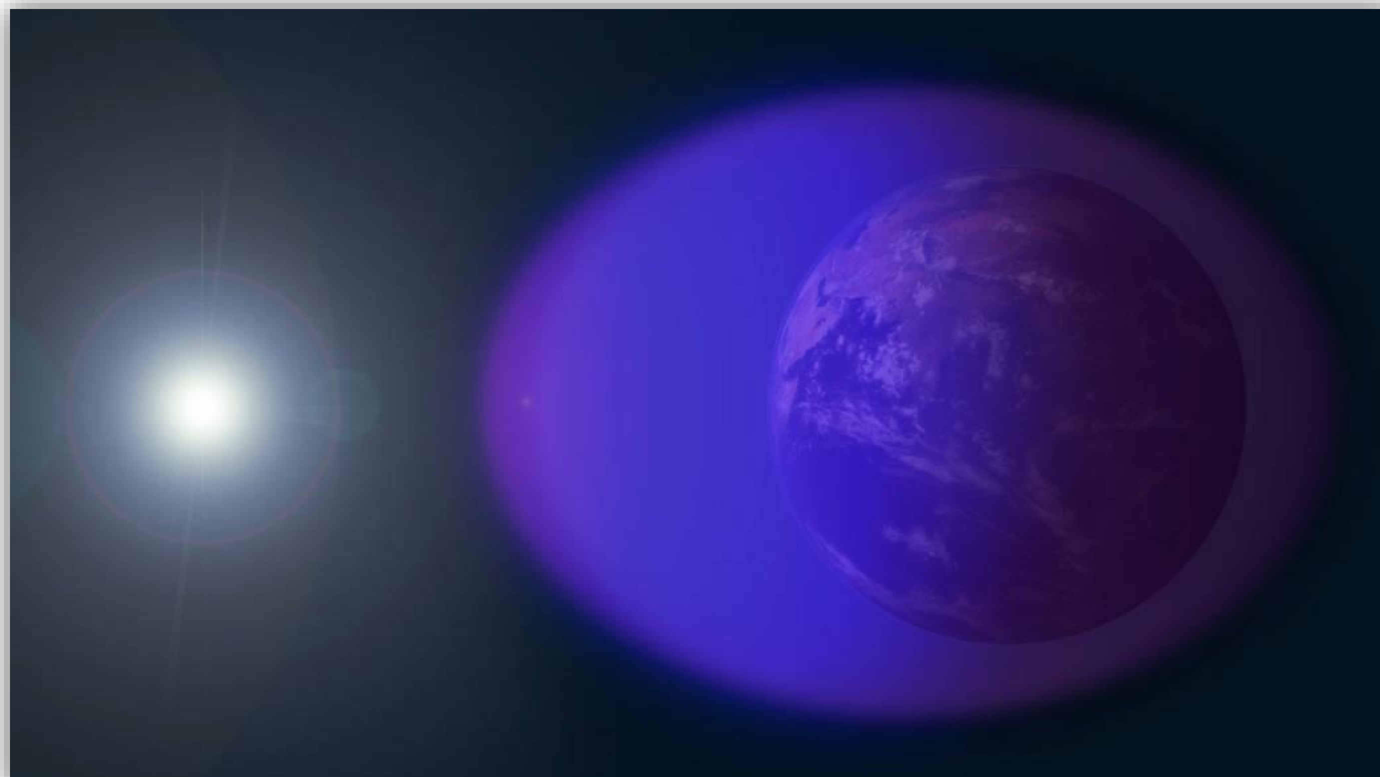
Density distribution models

1) Constant density $\rho_a = const$

2) Exponential density distribution model $\rho_a = \rho_0 \exp\left(\frac{H - h_0}{\xi}\right)$

$\rho_0,\ H,\ h_0,\ \xi$ – some parameters

3) CIRA



Summary

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Thank you for listening!

This work is supported by RFBR (grant no. 16-01-00739)

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