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DISTRIBUTION OF CORRECTION THRUSTERS UNDER DELTA-V CONSTRAINTS IN LOCAL HORIZONTAL PLANE

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This paper is devoted to the problem of orbit correction thrusters distribution on-board the geostationary satellite. It is necessary to simultaneously correct orbital elements and provide reaction wheels desaturation. This problem is complicated by the fact that one of the thrusters might fail but in spite of this the system must maintain its efficiency. Besides, Delta-V Constraints in Local Horizontal Plane should be considered. In this work necessary and sufficient conditions for ensuring the desaturation of the reaction wheels are found. In addition, numerical solution of the optimization problem of the thrusters' location is given.

INTRODUCTION

One of the most attractive classes of orbits is geostationary orbits that are used for applied and commercial purposes. Geostationary satellites "hover" over a point on the Earth surface. This is the feature of geostationary satellites that makes them so attractive. For example, satellite TV receiving antennas on Earth may not change their orientation all the time to track a transmitting satellite, because it is always in the same place in the sky. In addition, geostationary satellites are also often used to provide communications.

It should be noted that the geostationary orbit is not stable. Under the influence of external disturbances from the Moon, the Sun and the non-centrality of the Earth's gravitational field, the orbit parameters change. In order to maintain the orbit correction thrusters are usually used: they produce correction impulses in the nodes of the orbit (Figure 1).

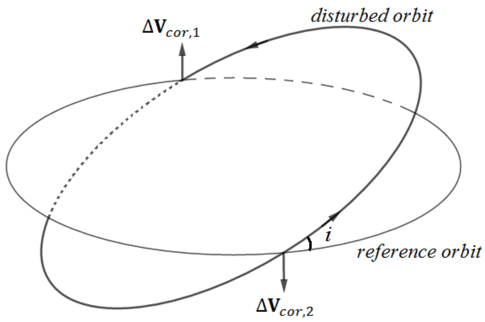

Figure 1. Reference (geostationary) orbit and disturbed orbit.

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In general case, the thrust axes do not pass through the satellite's center of mass. Hence, apart from changing the orbital motion, they also affect the angular motion by creating additional torques which must be compensated by the attitude control system. We can use these torques for ensuring the desaturation of the reaction wheels.

The main goal of this paper is to determine the minimum required amount of thrusters and to develop a method for selecting the optimal direction of thrust axes and the thrusters' location on the satellite, which provides the possibility of simultaneous desaturation of reaction wheels and orbit maintenance. In addition, it is necessary to take into account that one of the thrusters could fail.

It should be noted that this problem has already been covered in literature. For example, in papers [1,2] the problem of simultaneous unloading of the excess angular momentum and issuing the required correction impulse is considered. In work [3] it is proposed to use magnetic coils and thrusters for desaturation of the reaction wheels. The main difference of this work is the consideration of the restrictions on correction impulse in local horizontal plane.

PROBLEM STATEMENT

In the paper it is supposed that satellite has several correction thrusters that are located on the one side of satellite. We use the following coordinate systems:

a) *OXYZ* is the Orbital Frame. The origin *O* is located at the satellite center of mass, the axis *OZ* is parallel to the orbit normal, the axis *OX* is parallel to the satellite radius vector,
b) *Oxyz* is the Body Frame. The axis *Oz* is perpendicular to the plane where the correction thrusters are installed, the axes *Ox* and *Oy* complement it to the right-hand triple.

The satellite is stabilized in the Orbital Frame so that the Body Frame axes coincide with the Orbital Frame ones.

Under the influence of external disturbances the satellite orbit changes its parameters, in particular, it ceases to be geostationary. In order to correct the inclination of the disturbed orbit, the satellite receives the required impulse along the axis *OZ* in the nodes of orbit. Thus, the inclination of the orbit is reduced to zero. In addition, for the correction of the eccentricity and satellite phasing, it is necessary to produce correction impulses along the tangent to the orbit, that is, along the *OY* direction. Usually, corrections along the radius vector direction are rare, so we consider only the restrictions on the required correction impulse in the local horizontal plane *OYZ*.

Besides of the conditions for issuing the required velocity change, it is also necessary to ensure desaturation of the reaction wheels. The expression for the nominal torque generated by each thruster can be calculated as

$$\tilde{\mathbf{m}}_i = \left(\tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}_c\right) \times \tilde{\mathbf{e}}_i f = \tilde{\mathbf{r}}_{ic} \times \tilde{\mathbf{e}}_i f, \tag{1}$$

here $\tilde{\mathbf{r}}_i$ is the point of thrust application, $\tilde{\mathbf{r}}_c$ is the position of the satellite center of mass, $\tilde{\mathbf{e}}_i$ is the unit axes, f is the amount of thrust, $\tilde{\mathbf{r}}_{ic} = \tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}_c$.

Let us obtain the necessary equations and set the given constraints. Firstly, consider residual angular momentum. In general case, it can take arbitrary values. Therefore, in order to ensure desaturation of the reaction wheels, it is necessary that the range of possible changes in the angular momentum contains some neighborhood of zero. In other words, it must be possible to change the angular momentum in any direction. Formally, this can be written as follows:

$$\forall \Delta\mathbf{h}_\Sigma \in U(\mathbf{0}) \quad \exists t_i \geq 0 \;\; (i = \overline{1,k}): \quad \sum_{i=1}^{k} \tilde{\mathbf{m}}_i t_i = \Delta\mathbf{h}_\Sigma, \tag{2}$$

here t_i is the operating time of the i-th thruster, k is the number of correction thrusters installed on the satellite.

Secondly, the required impulse along the axis *OZ* and *OY* can be specified using the following expressions in the projections on the on the corresponding axes:

$$\sum_{i=1}^{k}\tilde{e}_{zi}t_i = \frac{\Delta V_z m_s}{f},\qquad \sum_{i=1}^{k}\tilde{e}_{yi}t_i = \frac{\Delta V_y m_s}{f}, \tag{3}$$

here $\tilde{e}_{yi}$, $\tilde{e}_{zi}$ are the projections of the vectors $\tilde{\mathbf{e}}_i$ on the axes *OY* and *OZ* respectively, ΔV_z is the required velocity change along the axis *OZ*, $\Delta V_y \in \left[-V_{y,\max 1},\ V_{y,\max 2}\right]$ is the delta-V constraints along the *OY* axis (as a rule, it is several times less than the required change along the axis *OZ*), and m_s is the satellite mass.

Thus, we get the system

$$\begin{gathered}\forall \Delta\mathbf{h}_\Sigma \in U(0) \quad \exists t_i \ge 0 \ \ (i=\overline{1,k}): \quad \sum_{i=1}^{k}\tilde{\mathbf{m}}_i t_i = \Delta\mathbf{h}_\Sigma,\\ \sum_{i=1}^{k}\tilde{e}_{zi}t_i = \frac{\Delta V_z m_s}{f},\\ \sum_{i=1}^{k}\tilde{e}_{yi}t_i = \frac{\Delta V_y m_s}{f},\end{gathered} \tag{4}$$

that describes all the necessary equations and given constraints.

CONVEX HULL AND MINIMUM REQUIRED AMOUNT OF THRUSTERS

So, the problem is to ensure the consistency of system (4). We can choose thrusters' location and thrust axes that, in turn, determine $\tilde{\mathbf{m}}_i$ ($i=\overline{1,k}$).

Let's try to reduce the problem to the one that we considered in [4], that is, we analyze the convex hulls. To do this we consider the equation

$$\sum_{i=1}^{k}\tilde{e}_{zi}t_i = \frac{\Delta V_z m_s}{f}$$

and make the following substitution:

$$\frac{\tilde{e}_{zi} f}{\Delta V_z m_s} t_i = \tau_i$$

or

$$\frac{\tilde{e}_{zi}}{p} t_i = \tau_i \quad \left(p = \frac{\Delta V_z m_s}{f}\right). \tag{5}$$

So, we actually move from physical operating time to normalized value, the sum of which is equal to 1:

$$\sum_{i=1}^{k}\tau_i = 1.$$

We express t_i from (5) and make following substitution: $\mathbf{m}_i = \tilde{\mathbf{m}}_i \dfrac{p}{\tilde{e}_{zi}}$, $\ e_i = \tilde{e}_{yi}\dfrac{p}{\tilde{e}_{zi}}$.

Then the system (4) can be rewritten in this form:

$$\forall \Delta \mathbf{h}_\Sigma \in U(0) \quad \exists \tau_i \geq 0 \ \ (i = \overline{1,k}): \quad \sum_{i=1}^{k} \mathbf{m}_i \, \tau_i = \Delta \mathbf{h}_\Sigma,$$
$$\sum_{i=1}^{k} \tau_i = 1, \tag{6}$$
$$\sum_{i=1}^{k} e_i \tau_i = \frac{\Delta V_y m_s}{f}.$$

Now we need to provide the compatibility of the system (6). Let us introduce four-dimensional vectors $\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ e_i \right)^T$, $\quad i = \overline{1,k}$. The region of all possible angular momentums $\Delta \mathbf{h}_\Sigma$ and velocity values ΔV_y is the convex hull of the vectors $\boldsymbol{\psi}_i$. Therefore, it is necessary to make sure that the Cartesian product of the sets $U(0) \times \left[\frac{-V_{y,\max 1} m_s}{f},\ \frac{V_{y,\max 2} m_s}{f} \right]$ is contained inside the resulting convex hull.

For this, the convex hull should have at least a non-zero volume. Thus, it is necessary to have $k = 5$ vectors, because the convex hull of $k = 4$ vectors is some subset of the hyperplane, and its volume is equal to zero. Moreover, in order to fulfill the required conditions, the origin should be the inner point of the constructed convex hull. It allows screening out the wrong sets of thrusters' distribution. So, the following system should have a solution:

$$\boldsymbol{\psi}_1 \tau_1 + \boldsymbol{\psi}_2 \tau_2 + \boldsymbol{\psi}_3 \tau_3 + \boldsymbol{\psi}_4 \tau_4 + \boldsymbol{\psi}_5 \tau_5 = \mathbf{0},$$
$$\tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 = 1, \tag{7}$$
$$\tau_i > 0.$$

Strict inequalities on the coefficients τ_i exclude that the origin can be on the face, that is, the origin is an internal point of the convex hull. Thus, there is such a neighborhood of zero, which entirely lies in that convex hull. Using the Cramer's rule, we can find the solution of system (7). It is presented below:

$$\tau_1 = \frac{\Delta_1}{\Delta}, \quad \tau_2 = \frac{\Delta_2}{\Delta}, \quad \tau_3 = \frac{\Delta_3}{\Delta}, \quad \tau_4 = \frac{\Delta_4}{\Delta}, \quad \tau_5 = \frac{\Delta_5}{\Delta}, \tag{8}$$

here,

$$\Delta = \det \begin{Vmatrix} m_{1,x} & m_{2,x} & m_{3,x} & m_{4,x} & m_{5,x} \\ m_{1,\mathrm{y}} & m_{2,\mathrm{y}} & m_{3,\mathrm{y}} & m_{4,\mathrm{y}} & m_{5,\mathrm{y}} \\ m_{1,\mathrm{z}} & m_{2,\mathrm{z}} & m_{3,\mathrm{z}} & m_{4,\mathrm{z}} & m_{5,\mathrm{z}} \\ e_1 & e_2 & e_3 & e_4 & e_5 \\ 1 & 1 & 1 & 1 & 1 \end{Vmatrix} = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5,$$

and Δ_i are corresponding minors.

Two cases are possible: $\Delta > 0$ and $\Delta < 0$. Without loss the generality, we will consider only the first case, since if $\Delta < 0$, we can change the vectors numbering and, therefore, the sign of a determinant. Moreover, in order to satisfy the condition of positivity $\lambda_i > 0$, it is necessary and sufficient that

$$\Delta_i > 0, \quad i = \overline{1,5}. \tag{9}$$

So, when we find some thrusters' distribution, we can check the correctness of it, using ratio (9), and determine the operating time using (8).

OPTIMIZATION OF THE THRUSTERS' LOCATION

Conditions (9) are not sufficient to ensure the maximum angular momentum, because we can only produce some non-zero value of the angular momentum. In order to optimize the thrusters' distribution and to obtain the greatest possible angular momentum for a specific implementation, we will use numerical optimization methods. Moreover at the same time we should produce the necessary value of the velocity change in the local horizontal plane.

Consider five four-dimensional vectors corresponding to each thruster:

$$\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ e_i\right)^T,\ (i = \overline{1,5}) \tag{10}$$

The convex hull of these vectors is a polytope in four-dimensional space. Taking into account delta-V constraints, we should find the hyperplane section of this convex hull with the following hyperplanes:

$$\begin{aligned} \Omega_a &= \{\tilde{\mathbf{x}} \mid \tilde{x}_4 = a\}, \\ \Omega_b &= \{\tilde{\mathbf{x}} \mid \tilde{x}_4 = b\}, \end{aligned} \tag{11}$$

here $\tilde{x}_4$ - the fourth component of the vector $\tilde{\mathbf{x}}$.

Consider the hyperplane section of a convex hull. We represent the convex hull as a system of inequalities:

$$\tilde{\mathbf{A}}\tilde{\mathbf{x}} \le \tilde{\mathbf{d}}, \tag{12}$$

here $\tilde{\mathbf{A}}$ is the $p \times 4$ matrix, $\tilde{\mathbf{x}}$ is the 4×1 vector and $\tilde{\mathbf{d}}$ is the $p \times 1$ vector (p – the number of inequalities that define considered convex hull). To find the hyperplane section, we can fix the fourth component of vectors $\boldsymbol{\psi}_i$, that is $\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ a\right)^T$ or $\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ b\right)^T,\ (i = \overline{1,5})$. After that the inequality system (12) can be rewritten as follows:

$$\mathbf{A}\mathbf{x} \le \mathbf{d}, \tag{13}$$

here $\mathbf{A}$ is the $p \times 3$ matrix, $\mathbf{x} = \left(m_{i,x},\ m_{i,y},\ m_{i,z}\right)^T$, $\mathbf{d}$ is the $p \times 1$ vector, and besides $\mathbf{A}$ is the first three columns of matrix $\tilde{\mathbf{A}}$, $\mathbf{d} = \tilde{\mathbf{d}} - a\mathbf{A}_4$ or $\mathbf{d} = \tilde{\mathbf{d}} - b\mathbf{A}_4$ – depending on the constraints ($\mathbf{A}_4$ is the fourth column of matrix $\tilde{\mathbf{A}}$). Therefore, inequality system (13) describes the hyperplane section of the convex hull.

In order to find the maximum radius of the sphere, which can be inscribed in the resulting plane section, we need to find the distance from the origin to each of the surfaces. Consider in more detail one of the inequalities (13) (for example, the first one). We write it in this form:

$$\left(\frac{\boldsymbol{a}_1}{\|\boldsymbol{a}_1\|},\ \mathbf{x}\right) \le \frac{d_1}{\|\boldsymbol{a}_1\|} \tag{14}$$

here $\|\boldsymbol{a}_1\|$ is the norm of the first row of matrix $\mathbf{A}$. The distance from the point to the plane given by the inequality (14) is equal to $\frac{|d_1|}{\|\boldsymbol{a}_1\|}$. Further, in order to ensure that zero is an internal point of the hyperplane section of a convex hull, it is necessary to check that $d_1 > 0$. Otherwise, zero lies outside the convex hull. In the same way we can find the distance from the point to the planes given by each inequalities of system (13). From the p obtained values, we choose the minimum value, that is $R_a = \min(R_1, R_2, \ldots, R_p)$. This is the desired radius.

Similarly, we can find radius of the inscribed sphere for the right border. From two obtained radii we choose their minimum, that is $R = \min\left(R_a, R_b\right)$. Thus, we obtain the maximum possible radius of the sphere centered in zero, which can be inscribed in the convex hull of the vectors $\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ e_i\right)^T,\ (i = \overline{1,5})$ for some specific implementation. Since the radius of the resulting sphere is directly related to the angular momentum magnitude, we want to find a thruster's configuration at which this radius is maximum. Therefore, it is necessary to solve the following optimization problem:

$$\mathbf{\Phi} = \min\left(R_a, R_b\right) \to \max. \tag{15}$$

It should also be noted that this task is complicated by the fact that the components of the vectors $\boldsymbol{\psi}_i$ cannot be chosen independently. There is a relationship between them, given by the following expression:

$$\tilde{\mathbf{m}}_i = \left(\tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}_c\right) \times \tilde{\mathbf{e}}_i f = \tilde{\mathbf{r}}_{ic} \times \tilde{\mathbf{e}}_i f, \tag{16}$$

here $\tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}_c = \tilde{\mathbf{r}}_{ic}$ is the thrusters position relative to the center of mass, whence it follows that

$$\mathbf{m}_i = \tilde{\mathbf{m}}_i \frac{p}{\tilde{e}_{zi}} = \tilde{\mathbf{r}}_{ic} \times \tilde{\mathbf{e}}_i f \frac{p}{\tilde{e}_{zi}}. \tag{17}$$

This expression should be taken into account when selecting the position and direction of the thrusters axes.

It is not possible to obtain an analytical solution of this optimization problem, so it is solved numerically. Coordinates of the thrusters' installation positions $\tilde{\mathbf{r}}_{ic} = \left(\tilde{r}_{xi},\ \tilde{r}_{yi},\ \tilde{r}_{zi}\right)^T$ and direction of the thrust axis $\tilde{\mathbf{e}}_i$ were chosen as varying parameters. Thrusters are installed on the upper part of the satellite, thus coordinate $\tilde{r}_{zi}$ is fixed. Vectors $\tilde{\mathbf{e}}_i$ are parameterized by two angles φ_i (the angle between the projection $\tilde{\mathbf{e}}_i$ on the plane *Oxy* and the axis *Ox*) and θ_i (the angle between $\tilde{\mathbf{e}}_i$ and axis *Oz*). Then vectors $\boldsymbol{\psi}_i$ are uniquely calculated.

We also impose additional restrictions on the selected parameters:

$$\begin{gathered} \varphi_i \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \cup \left[\frac{4\pi}{3}, \frac{5\pi}{3}\right], \quad \theta_i \in \left[0, \frac{\pi}{3}\right], \\ \left|\tilde{r}_{xi}\right|, \left|\tilde{r}_{yi}\right| < 0.5\,\text{m}, \quad \tilde{r}_{zi} = 0.5\,\text{m}, \quad i = \overline{1,5}. \end{gathered} \tag{18}$$

Restrictions on the thrusters' position caused by the limited size of the satellite, the restrictions on the angles θ_i are due to the fact that the main correction impulse must be carried out along the axis *OZ*. Finally, the restrictions on φ_i are empirical.

In addition, the following values of the satellite mass, the characteristic velocity along the *OZ* axis, and the characteristic velocity constraints along the *OY* axis are selected:

$$\begin{gathered} m_s = 5000\,\text{kg}, \quad f = 0.1\,\text{N}, \\ \Delta V_z = 0.03\,\frac{\text{m}}{\text{s}}, \quad \Delta V_y = 0.003\,\frac{\text{m}}{\text{s}}, \\ a = -\frac{\Delta V_y m_s}{f} = -150\,\text{s}, \\ b = \frac{\Delta V_y m_s}{f} = 150\,\text{s}. \end{gathered} \tag{19}$$

The results of the numerical solution of this optimization problem are presented in Table 1 and on Figure 2.

Table 1. Local optimal distribution for 5 thrusters.

№	φ (deg)	θ (deg)	$\tilde{r}_x$ (m)	$\tilde{r}_y$ (m)	$\tilde{r}_z$ (m)
1	62.24	57.29	-0.5	0.157	0.5
2	120	57.29	0.193	-0.098	
3	118.44	20.07	0.089	0.5	
4	240.74	57.29	0.172	-0.115	
5	299.4	40.97	-0.185	-0.445	

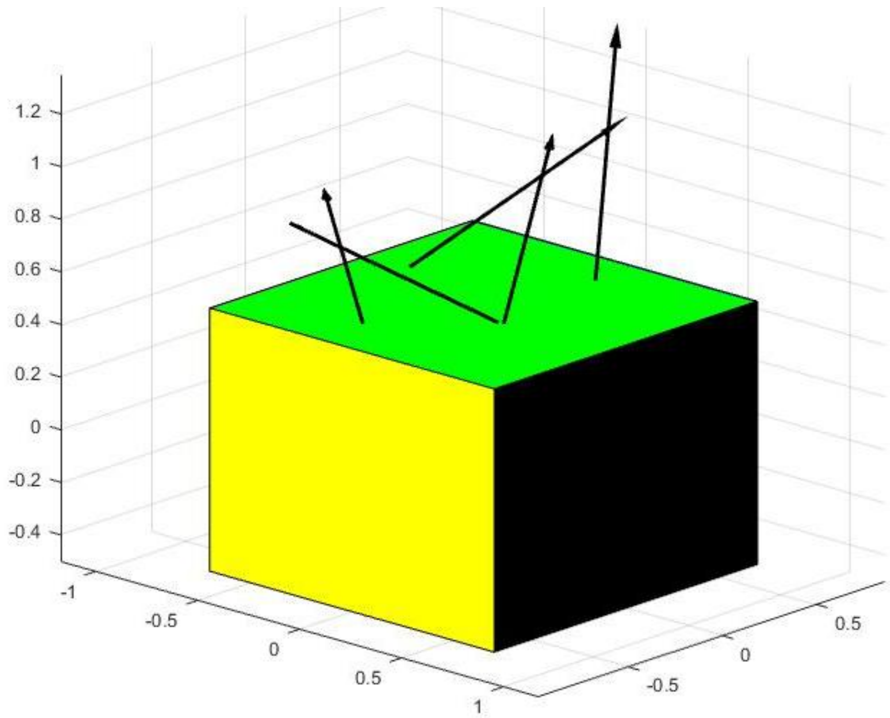

Figure 2. Local optimal distribution for 5 thrusters.

Furthermore, the maximum possible radius of the sphere centered in zero, which can be inscribed in the convex hull of the vectors $\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ e_i\right)^T$, $(i = \overline{1,5})$ for this implementation is equal to $R = 23.09$. Thereby, in this case the module of greatest possible angular momentum equals $|\Delta\mathbf{h}_\Sigma| = 23.09\ \ \mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}$.

THRUSTER FAILURE

To provide the required correction impulse and the reaction wheels desaturation, at least five thrusters must be installed. However, if one of them fails, it would be impossible to satisfy all conditions. Therefore, firstly, it is necessary to find out the minimum number of the installed correction thrusters, which can ensure the system efficiency when one of thrusters fails. Secondly, thrusters should be located in the most optimal way.

Let us show that adding one thruster can't solve the problem. Consider *n*-dimensional space. In order for the convex hull of *k* points in *n*-dimensional space to have a non-zero volume, at least $k = n+1$ points are necessary. Therefore, if we got only $k = n+1$ thrusters and one of them fails, the system would not be able to provide the desaturation and necessary velocity change. Let us show that $k = n+2$ thrusters also cannot solve the problem. If there are $k = n+2$ points in *n*-dimensional space, let us draw an internal hyperplane through *n* arbitrary points. The internal hyperplane means that it is not coinciding with a face of the convex hull. The remaining two points are on the opposite sides of this hyperplane. The hyperplane divides the convex hull into two *n*-dimensional pyramids, which intersect along this hyperplane. Thus, if one of these two points is removed, there is only one pyramid. Thus, it is impossible to guarantee that the zero point always enters the convex hull of the remaining points, sometimes it is an inner point of removed pyramid or lies in the hyperplane, which also does not satisfy the set conditions. Therefore, the addition of one thruster does not solve the problem.

To prove that adding two thrusters is enough for this task, we give examples for four-dimensional space: the arrangement of seven thrusters in which even when one of the thrusters fails system must maintain its efficiency (Table 2 and Figure 3).

Table 2. The distribution of seven correction thrusters.

№	φ (deg)	θ (deg)	$\tilde{r}_x$ (m)	$\tilde{r}_y$ (m)	$\tilde{r}_z$ (m)
1	84.54	54.96	-0.307	0.063	0.5
2	112.17	38.41	0.133	-0.087	
3	75.04	26.81	0.099	0.309	
4	240.29	46.43	0.068	-0.013	
5	290.85	56.49	-0.081	-0.274	
6	288.86	55.85	-0.057	0.310	
7	86.24	20.14	0.032	-0.278	

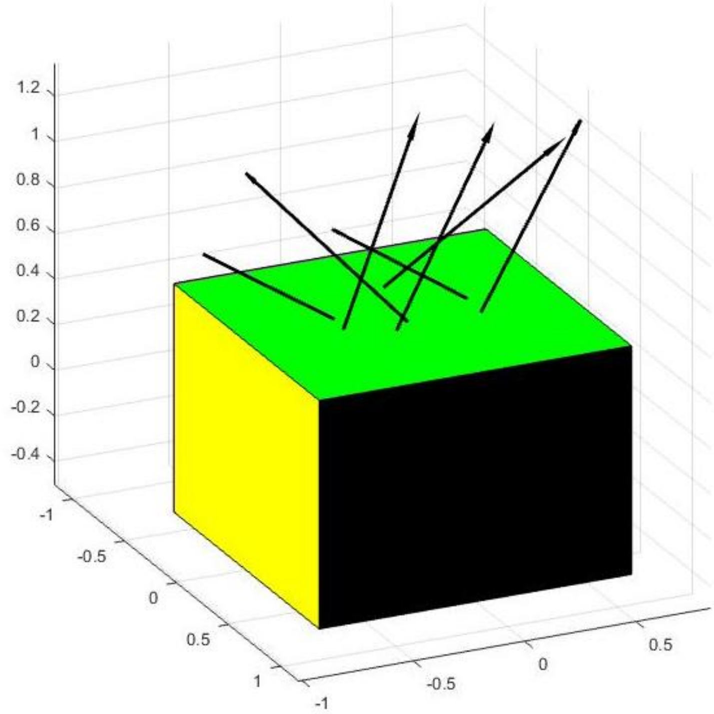

Figure 3. The distribution of seven correction thrusters.

This example is not an optimal thrusters' arrangement. To find the optimal one, we will use numerical optimization methods. As an initial approximation we can take the values which presented in Table 2. We use an approach, which is similar to that described above for five thrusters, but here we check every set of six thrusters separately, imitating the failure of the seventh one. After that, taking into account restrictions (18) and (19) we will get local optimal position of the thrusters (Table 3 and Figure 4).

Table 3. Local optimal distribution for 7 thrusters.

№	φ (deg)	θ (deg)	$\tilde{r}_x$ (m)	$\tilde{r}_y$ (m)	$\tilde{r}_z$ (m)
1	106.02	60	-0.295	0.095	0.5
2	98.07	30.71	0.499	-0.087	
3	73.3	58.99	0.089	0.243	
4	240.53	60	0.126	-0.025	
5	300	59.62	-0.118	-0.24	
6	291.52	59.96	-0.125	0.499	
7	97.78	27.78	0.187	-0.499	

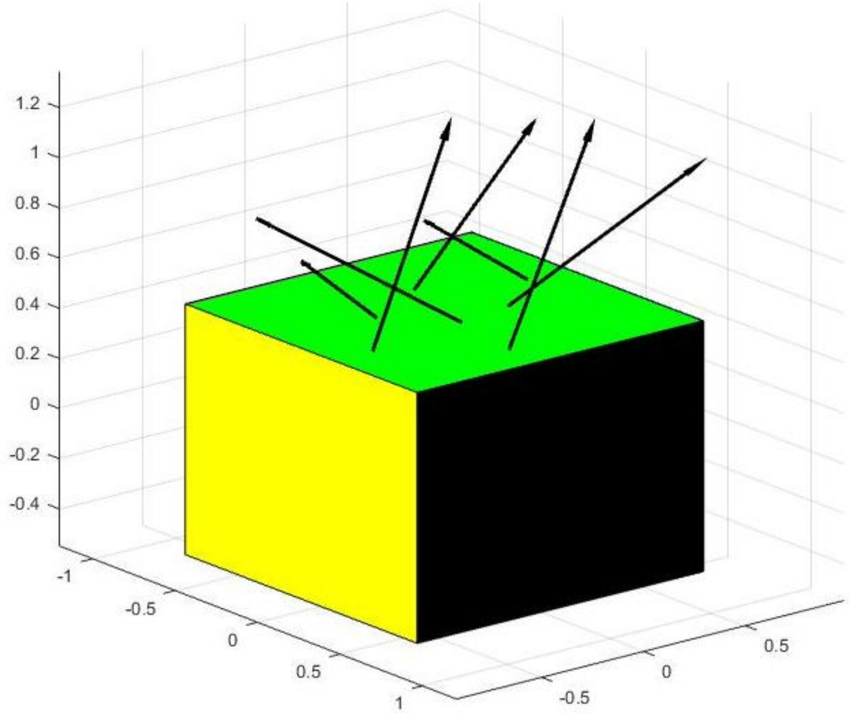

Figure 4. Local optimal distribution for 7 thrusters.

Here we get that the maximum possible radius of the sphere centered in zero, which can be inscribed in the convex hull of every set of six vectors, is equal to $R = 2.703$. Thereby, in this case the module of greatest possible angular momentum equals $|\Delta \mathbf{h}_\Sigma| = 2.703 \ \mathrm{N \cdot m \cdot s}$.

CONCLUSION

The problem of finding the configuration of the correction thrusters' location is considered, in which the required satellite velocity change is guaranteed and the reaction wheels are desaturated. It is shown that in the case when one thruster fails seven thrusters are needed to solve the problem. Numerical solution of the problem is provided.

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