DISTRIBUTION OF CORRECTION THRUSTERS UNDER DELTA-V CONSTRAINTS IN LOCAL HORIZONTAL PLANE

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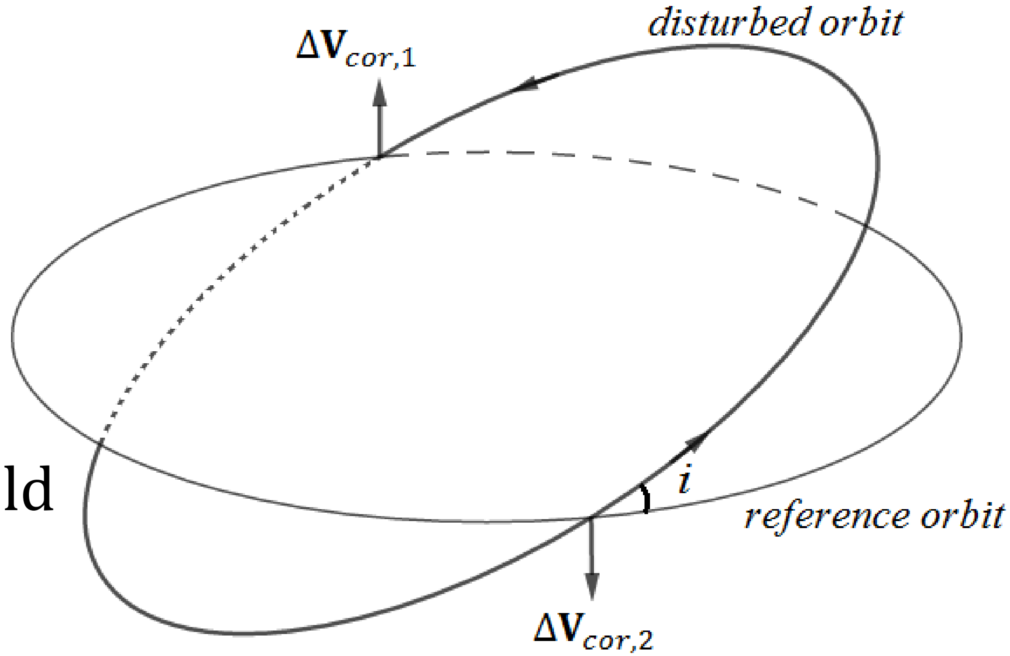
Plan

- Introduction
- Problem statement
- Method of finding minimum required amount of thrusters
- Optimization of the thrusters’ location
- Thruster failure

Introduction

External disturbances

- the Moon
- the Sun
- the non-centrality of the Earth's gravitational field



Correction thrusters produce correction impulses in the nodes of the orbit.

Main goals

- Simultaneously correct orbital elements and provide reaction wheels desaturation

We need:

- Determine the minimum required amount of thrusters,
- Select the optimal direction of thrust axes and the thrusters’ location

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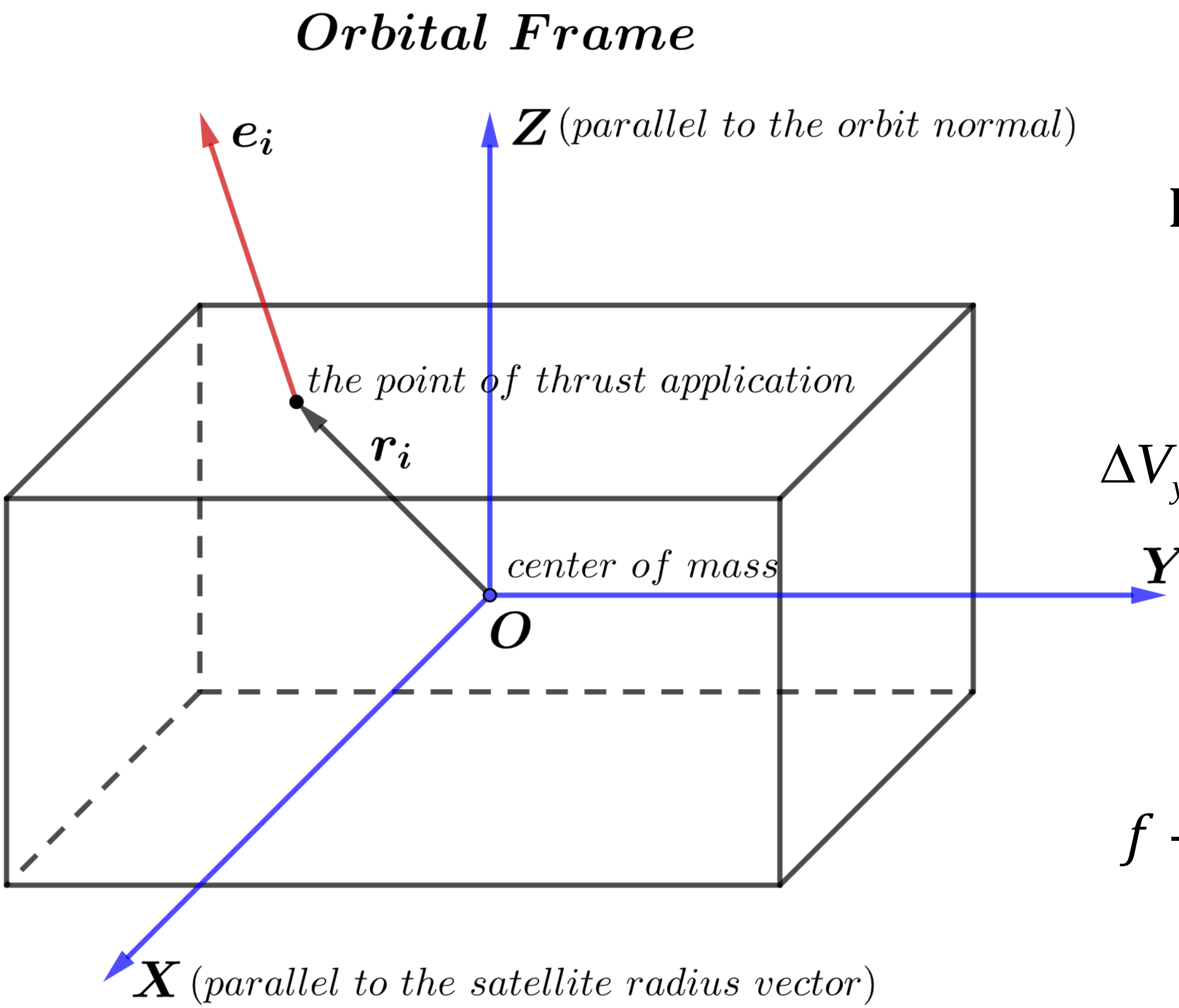
System must maintain its efficiency even when one thruster fails

Problem statement



Required velocity change:

$$\Delta V_z - fixed$$

$$\Delta V_y \in \left[-V_{y,\max 1},\ V_{y,\max 2}\right]$$

$$\mathbf{m}_i = \mathbf{r}_i \times \mathbf{e}_i f$$

f – amount of thrust

Problem statement

Necessary equations and given delta-V constraints:

$$\sum_{i=1}^{k} \mathbf{m}_i t_i = \Delta \mathbf{h}_\Sigma, \quad (t_i \geq 0 - \textit{operating time})$$

$$\sum_{i=1}^{k} e_{i,z} t_i = \frac{\Delta V_z m_s}{f}$$

$$\sum_{i=1}^{k} e_{i,y} t_i = \frac{\Delta V_y m_s}{f}$$

$\Delta \mathbf{h}_\Sigma$ – angular momentum change

m_s – mass of the satellite

$\Delta V_y, \ \Delta V_z$ – delta-V constraints

$\Delta V_y \in \left[-V_{y,\max 1}, \ V_{y,\max 2} \right]$

f – amount of thrust

Minimum required amount of thrusters

Substitution

$$\frac{e_{i,z} f}{\Delta V_z m_s} t_i = \tau_i \qquad (\tau_i \geq 0 - normalized\ value\ of\ operating\ time)$$

$$\sum_{i=1}^{k} \mathbf{m}_i \tau_i = \Delta \mathbf{h}_\Sigma$$

$$\sum_{i=1}^{k} e_i \tau_i = \frac{\Delta V_y m_s}{f}$$

$$\sum_{i=1}^{k} \tau_i = 1, \ \tau_i \geq 0$$

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Introduce 4-dimensional vectors

$$\boldsymbol{\psi}_i = \left(m_{i,x}, \ \ m_{i,y}, \ \ m_{i,z}, \ \ e_i \right)^T, \quad i = \overline{1, k}$$

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That describes convex hull of $\boldsymbol{\psi}_i$

Introduce 4-dimensional vectors

$$\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ e_i\right)^T, \quad i = \overline{1, k}$$

Optimization

$$\sum_{i=1}^{k} \mathbf{m}_i \, \tau_i = \Delta \mathbf{h}_\Sigma$$

$$\sum_{i=1}^{k} e_i \tau_i = \frac{\Delta V_y m_s}{f}$$

$$\sum_{i=1}^{k} \tau_i = 1, \;\; \tau_i \geq 0$$

$$\boldsymbol{\psi}_i = \left(m_{i,x}, \;\; m_{i,y}, \;\; m_{i,z}, \;\; e_i \right)^T, \quad i = \overline{1, 5}:$$

- it should be possible to produce $\Delta \mathbf{h}_\Sigma$ in any direction
- also we want $\left| \Delta \mathbf{h}_\Sigma \right| \to \max$ for more efficient desaturation

Optimization

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• it should be possible to produce $\Delta \mathbf{h}_\Sigma$ in any direction

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Convex hull of vectors $\boldsymbol{\psi}_i$: $\mathbf{Ax} \leq \mathbf{d}$ (intersection of half-spaces)

+ constraints on delta-V

$$\Delta V_y \in \left[-V_{y,\max 1}, \ V_{y,\max 2} \right]$$

Optimization of the thrusters' location

Convex hull $\mathbf{Ax} \le \mathbf{d}$

We should find the hyperplane section of this convex hull with the following hyperplane:

$$\Omega_\alpha = \left\{ \mathbf{x} \mid x_4 = \frac{-V_{y,\max 1} m_s}{f} = \alpha \right\} - left\ delta - V\ constraint$$

$$\Omega_\beta = \left\{ \mathbf{x} \mid x_4 = \frac{V_{y,\max 2} m_s}{f} = \beta \right\} - right\ delta - V\ constraint$$

Fix fourth component: $\boldsymbol{\psi}_i = \left(m_{i,x},\ m_{i,y},\ m_{i,z},\ \alpha \right)^T$

Rewrite inequalities in this form: $\left(\frac{\boldsymbol{a}_i}{\|\boldsymbol{a}_i\|},\ \mathbf{x} \right) \le \frac{d_i}{\|\boldsymbol{a}_i\|}$

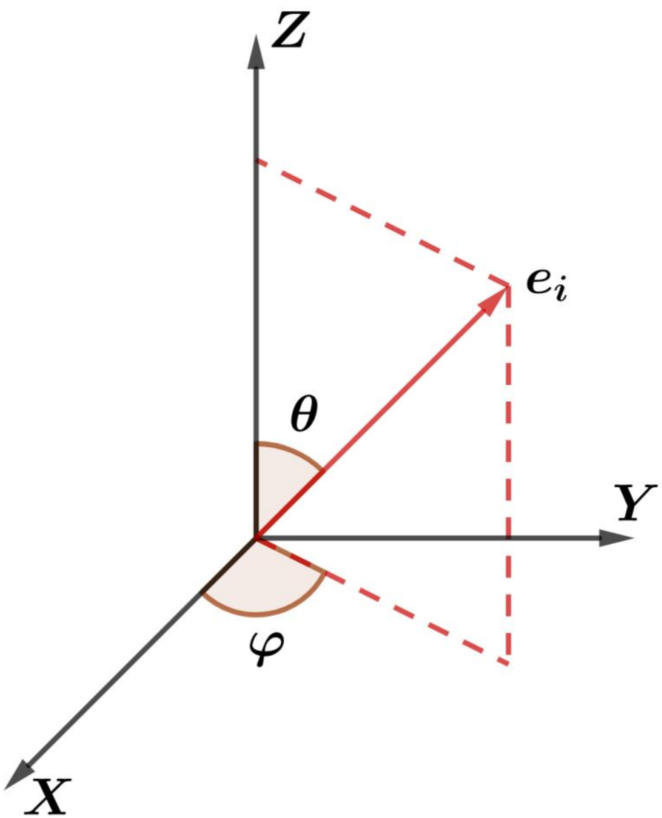
The distance from the origin to the *i*-th plane: $R_i = \frac{|d_i|}{\|\boldsymbol{a}_i\|}$

$$R_\alpha = \min(R_1, R_2, R_3, \ldots)$$

Optimization of the thrusters' location

Optimization problem: $\mathbf{\Phi} = \min\left(R_\alpha, R_\beta\right) \to \max$

Restrictions: $\varphi_i \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \cup \left[\frac{4\pi}{3}, \frac{5\pi}{3}\right], \quad \theta_i \in \left[0, \frac{\pi}{3}\right]$



$$\left|r_{i,x}\right|, \left|r_{i,y}\right| < 0.5\text{ m}, \quad r_{i,z} = 0.5\text{ m}, \quad i = \overline{1,5}$$

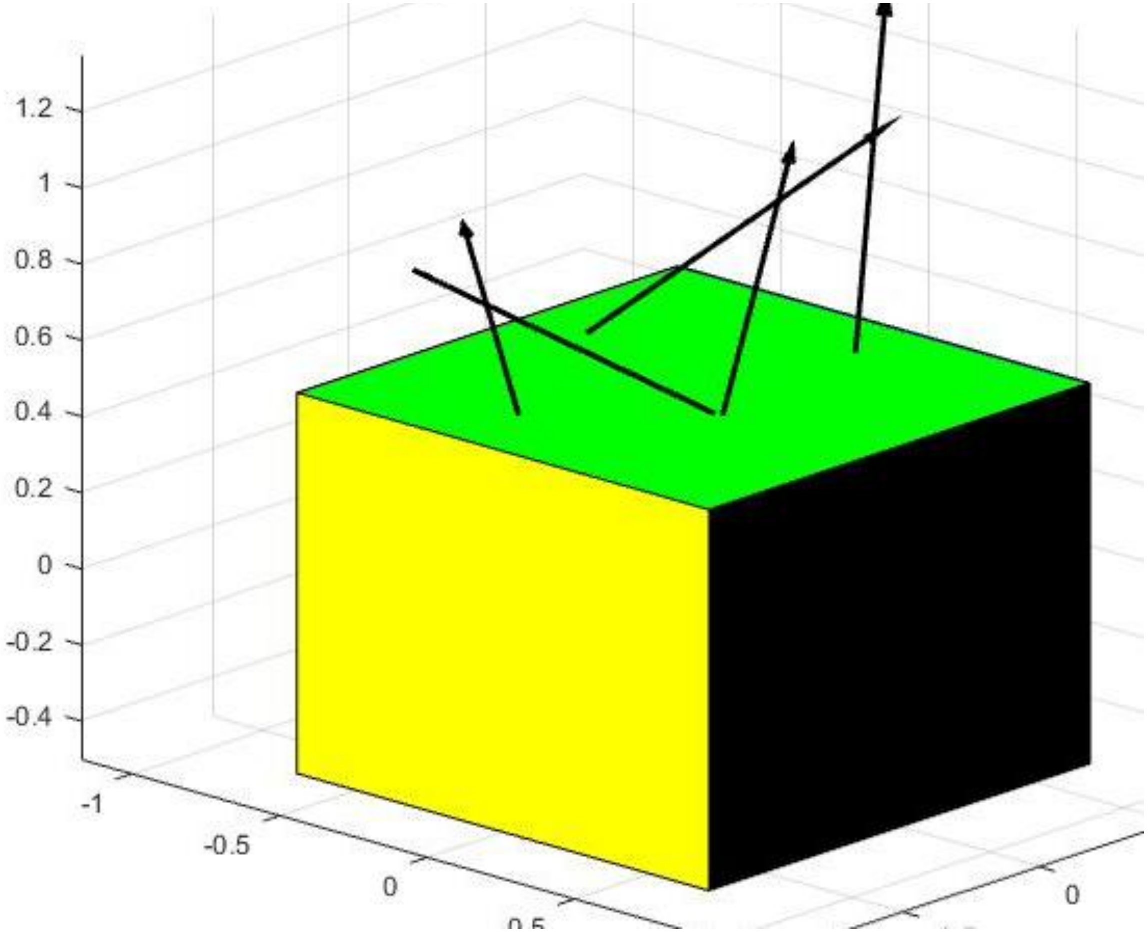
$$m_s = 5000\text{ kg}, \quad f = 0.1\text{ N},$$

$$\Delta V_z = 0.03\,\frac{\text{m}}{\text{s}}, \quad V_{y,\max 1} = V_{y,\max 2} = 0.003\,\frac{\text{m}}{\text{s}},$$

$$\alpha = \frac{-V_{y,\max 1} m_s}{f} = -150\text{ s},$$

$$\beta = \frac{V_{y,\max 2} m_s}{f} = 150\text{ s}.$$

№	φ (deg)	θ (deg)	$\tilde{r}_x$ (m)	$\tilde{r}_y$ (m)	$\tilde{r}_z$ (m)
1	62.24	57.29	-0.5	0.157	
2	120	57.29	0.193	-0.098	
3	118.44	20.07	0.089	0.5	0.5
4	240.74	57.29	0.172	-0.115	
5	299.4	40.97	-0.185	-0.445	

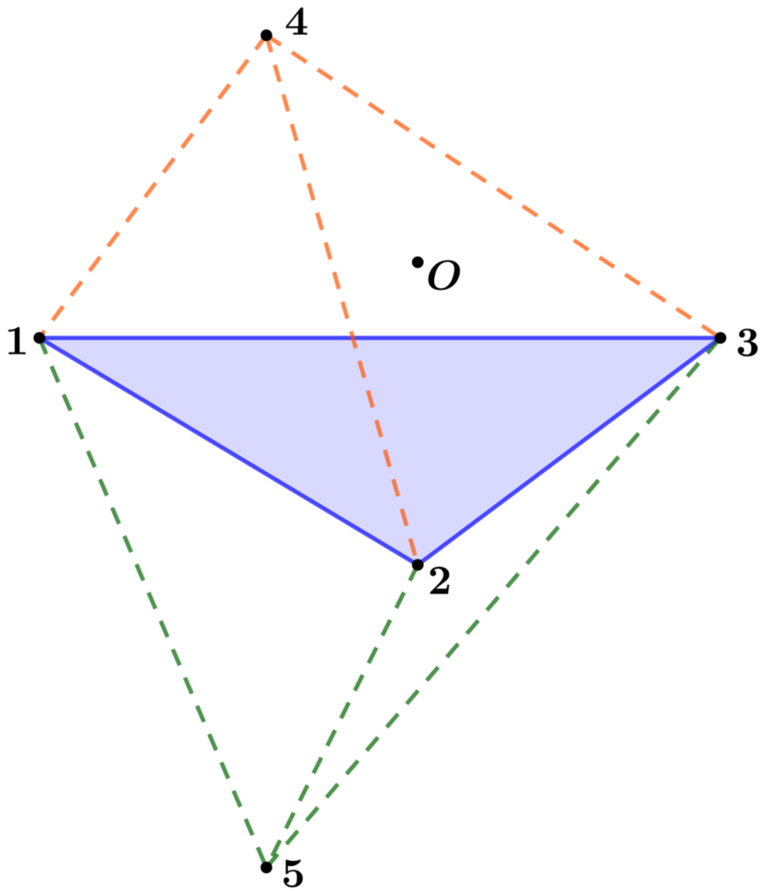


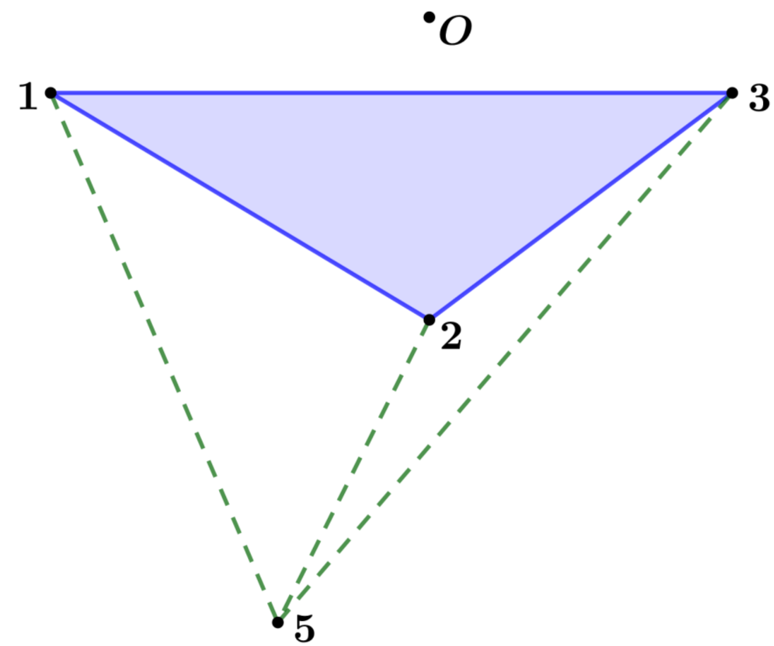
The largest possible angular momentum

$$\left|\Delta \mathbf{h}_{\Sigma}\right| = 23.09 \quad \mathrm{N \cdot m \cdot s}$$

Thruster failure

3-dimensional space





Fourth thruster fails

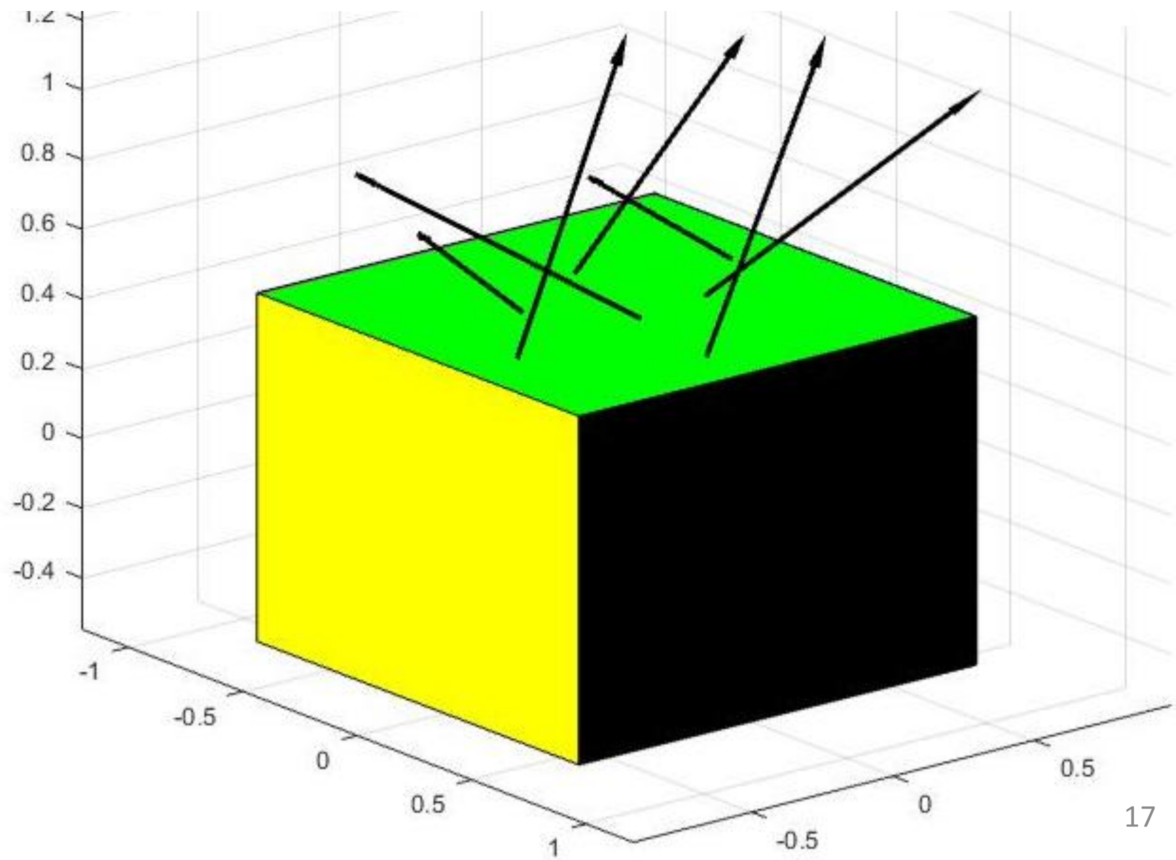
O lies outside the resulted convex hull

№	φ (deg)	θ (deg)	$\tilde{r}_x$ (m)	$\tilde{r}_y$ (m)	$\tilde{r}_z$ (m)
1	106.02	60	-0.295	0.095	0.5
2	98.07	30.71	0.499	-0.087	
3	73.3	58.99	0.089	0.243	
4	240.53	60	0.126	-0.025	
5	300	59.62	-0.118	-0.24	
6	291.52	59.96	-0.125	0.499	
7	97.78	27.78	0.187	-0.499	

Local optimal distribution

The largest possible angular momentum

$$\left|\Delta\mathbf{h}_\Sigma\right| = 2.703\ \ \mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}$$



Summary

- Method of finding minimum required amount of thrusters is suggested
- Optimization of the thrusters' location is considered
- The case of thruster failure is discussed

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