Skoltech Space Center Seminar

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Satellite formations control: approaches and algorithms

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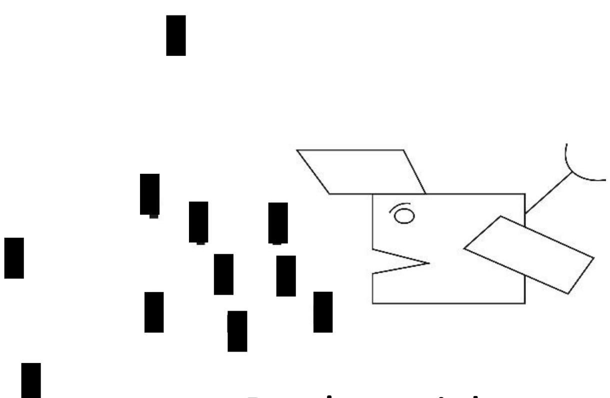
Content

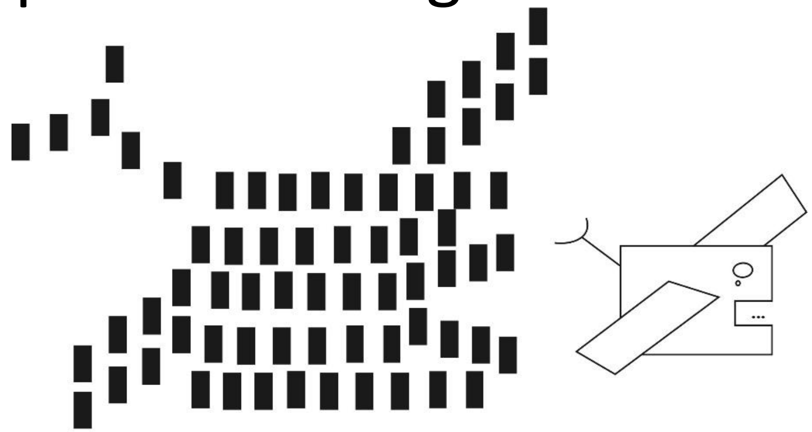
- Introduction
- Distributed space systems control approaches
- Fuelless satellite formation flying control and algorithms
- Conclusions

What is distributed space system?

- A space system consisting of multiple space elements that can communicate, coordinate and interact in order to achieve a common goal.
  – Tolerance for failure of individual systems
  – Scalability and flexibility in design and deployment of system
  – New capabilities compared to a single satellite



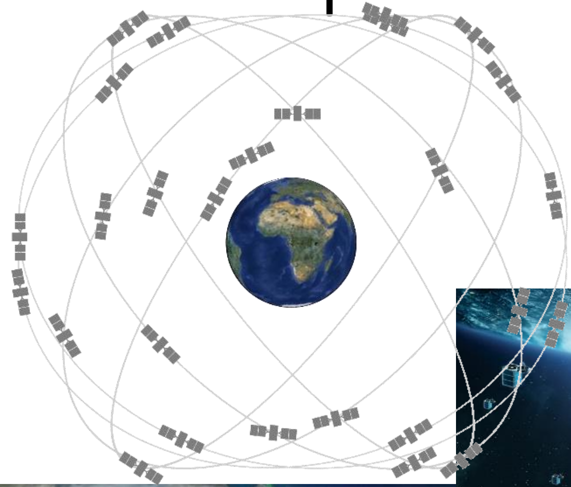
Don’t panic!

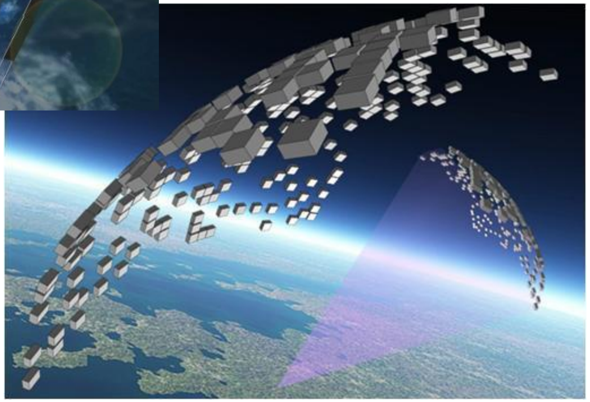


ORGANIZE!

Definitions for distributed space systems

- **Constellation**: similar trajectories without control for relative position; coordination from a control center
- **Formation**: closed-loop control on-board in order to preserve topology in the group and to control relative distances
- **Cluster**: distributed heterogeneous system of satellites to achieve in cooperation a joint objective
- **Swarm**: a group of similar (homogenous) satellites cooperating to achieve a joint goal without fixed positions; Each member determines and controls relative positions in relations to others

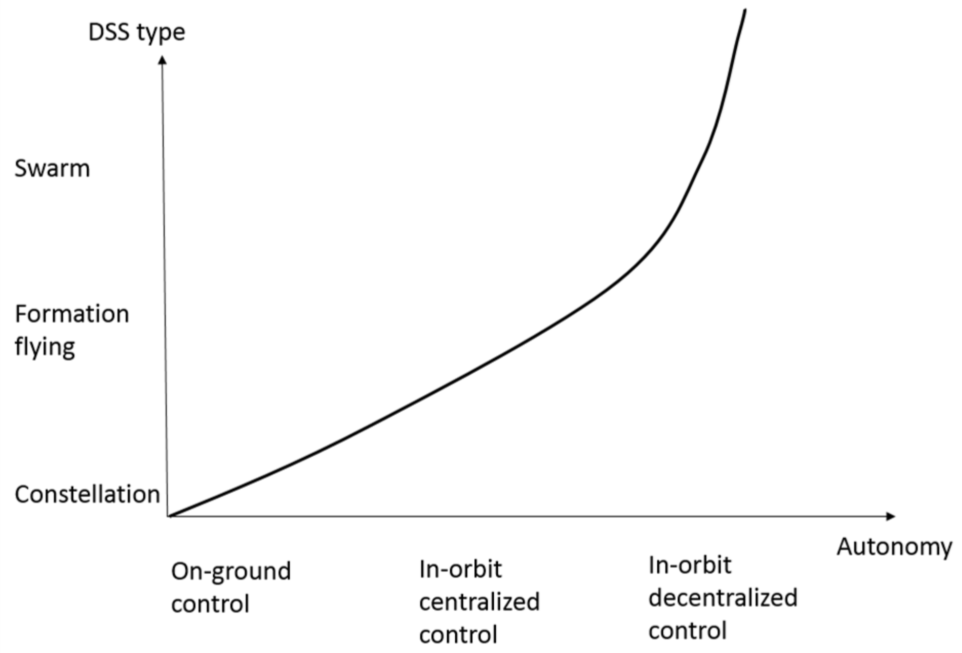


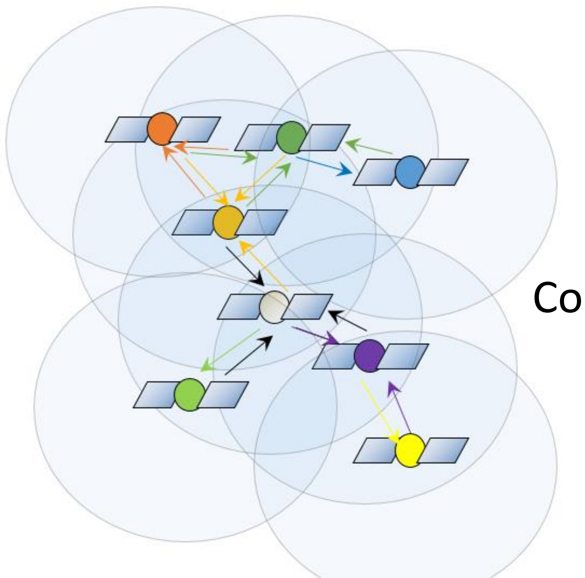


Main parameters of distributed SS

- A number of satellites
- A degree of autonomy
- Communication links between satellites
- Relative trajectory types

Autonomy in DSS





Communication spheres

Natural distributed systems

School of fishes

Flock of birds

Swarm of bees

Herd of animals

Satellite formation flying features

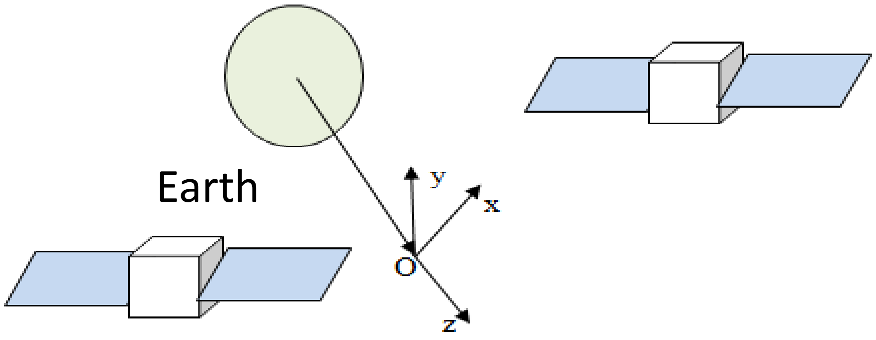
- Relatively small number of satellites
- Centralized control:
    - Mother-daughter relationship: mother knows the best for her children and command them
    - Leader-follower relationship: leader moves everywhere it wants, the followers pursue it
    - Chaser-target relationship: chaser follow the target, that could be noncooperative
- Communication with all the group members
- Motion along predefined trajectories

Equations of relative motion: linear model, near circular orbit

Hill-Clohessy-Wiltshire model:

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = \begin{bmatrix} x & y & z \end{bmatrix}^T$$

$$\begin{cases} \ddot{x} + 2\omega\dot{z} = 0 \\ \ddot{y} + \omega^2 y = 0 \\ \ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0 \end{cases}$$

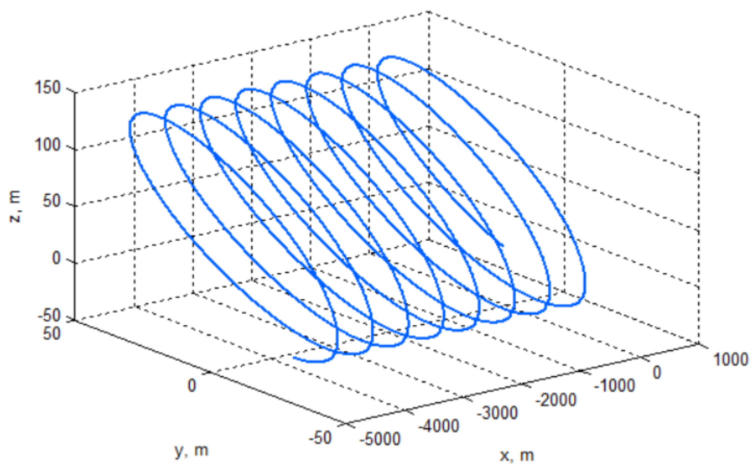


Solution is :

$$\begin{cases} x = -3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t + C_4 \\ y = C_5\sin\omega t + C_6\cos\omega t \\ z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t \end{cases}$$

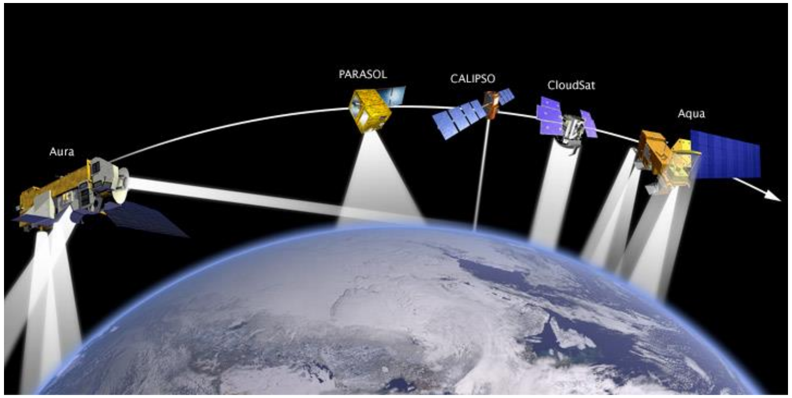
$-3C_1\omega t$ - Relative drift

Common case of free motion



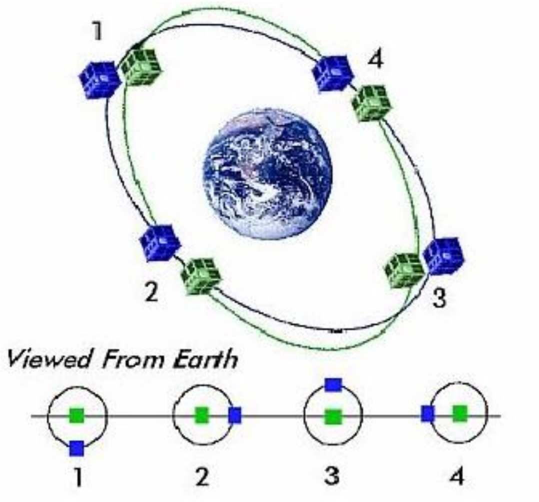
Formation flying specific relative trajectories

- Train formation
- Relative circular orbit
- Projected circular orbit
- Docking trajectories



A-train formation flying

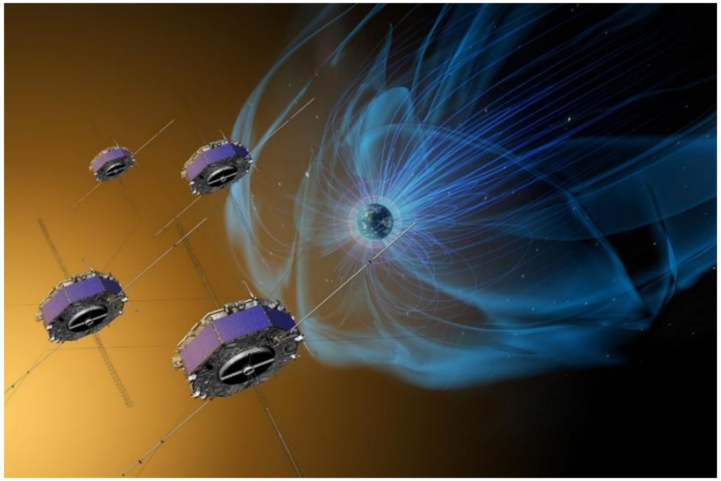
KIKU-7 mission



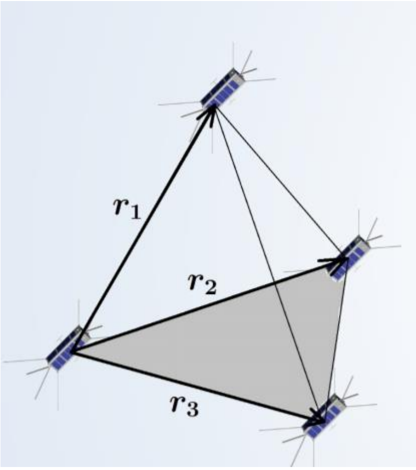
CanSat4&5 mission

Tetrahedral formation flying

- For an experimental study of the spatial distribution of the Earth magnetosphere parameters it is necessary to conduct simultaneous measurements at several points
- At least four satellites are required to carry out spatial measurements
- In the ideal case the satellites should fly so that they are always at the vertices of the regular tetrahedron
- To construct and maintain such a configuration the relative motion control must be applied



Magnetospheric Multiscale Mission



$$V_a = \frac{1}{6}|\vec{r_1} \cdot (\vec{r_2} \times \vec{r_3})|.$$

$$V_r = \frac{\sqrt{2}}{12}L^3$$

$$L = \frac{1}{6}\sum_{i=1}^{6} l_i$$

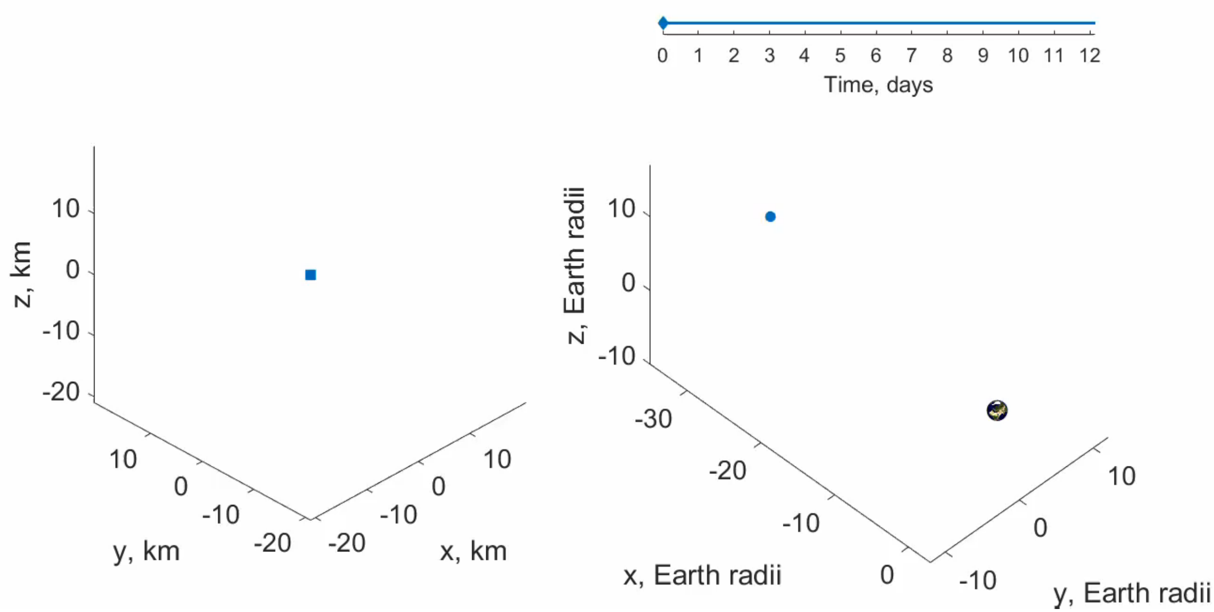
$$Q_v = \frac{V_a}{V_r}$$

Optimization problem

- Find optimal orbits for each of the four spacecraft near the reference orbit to maximize the number of orbital revolutions with acceptable formation quality (Q>0.7)

$$\overline{Q}_{int}(x) = \frac{1}{N_{rev}} \sum_{i=1}^{N_{rev}} \hat{Q}^{i}_{int}(x) \longrightarrow max$$

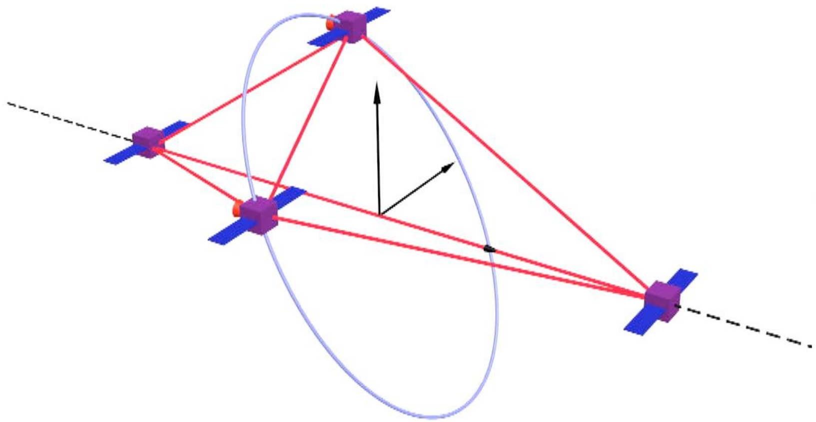
- Unknown vector x: 6 orbital parameters for each of the four deputy satellites (24 variables in total)

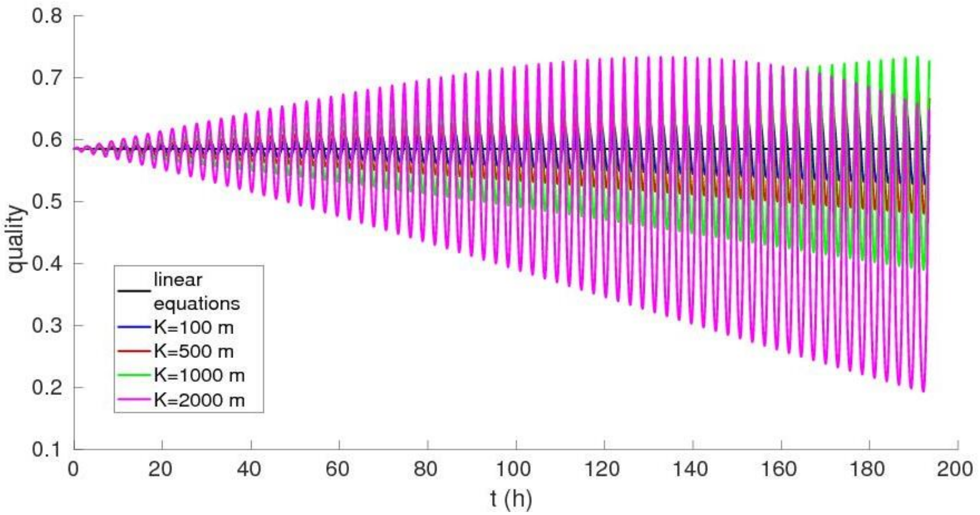


Koptev M.D., Trofimov S.P. Design, deployment and keeping of nanosatellite-based tetrahedral highly elliptical orbit formation// Preprints of Keldysh Institute for Applied Mathematics. 2018. № 97. 28 p.

Reference Trajectories for Tetrahedral Configuration in LEO

- Two of the satellites are moving along the same circular orbit with a constant separation equal to 2D
- The other two satellites are moving along the circular relative trajectories



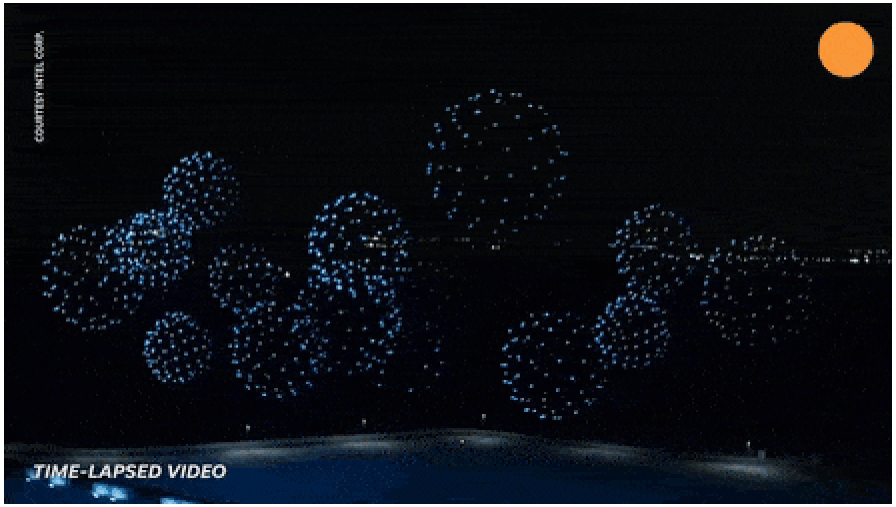


Y. Mashtakov, S. Shestakov Maintenance of the tetrahedral satellite configuration with single-input control // Preprints of Keldysh Institute for Applied Mathematics. 2016. № 95. 27 p.

Satellite swarm features

- A large number of satellites
- Decentralized control
- Communication with limited number of group members
- Motion along occasional trajectories:
  - Random but bounded relative trajectories

Drone light show



Launch of the PlanetLabs 3U CubeSats

Artificial potential control approach

- Collision avoidance

$$U_{ij}^{rep} = -C_{rep} e^{-\frac{d_{ij}}{R_{rep}}}$$

- Alignment

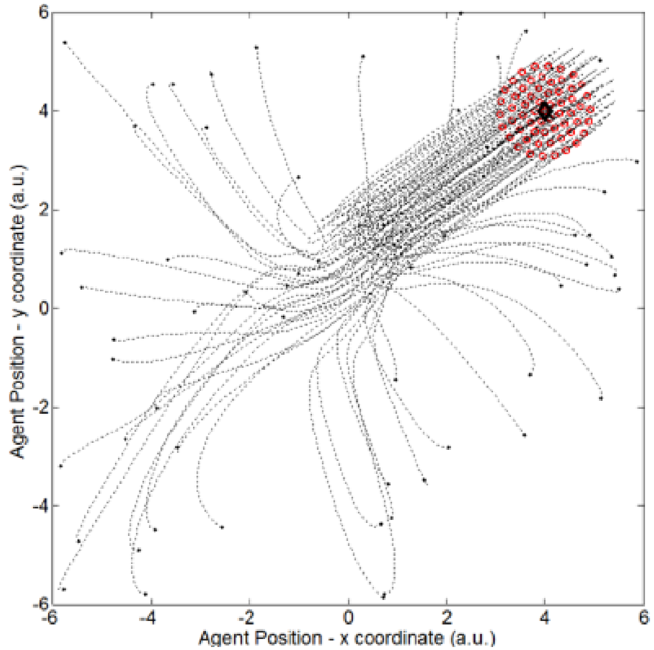
$$\mathbf{d}_i = \sum_{j, j \neq i} C_{al} \left(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij} \right) e^{-\frac{d_{ij}}{R_{al}}} \mathbf{r}_{ij}$$

- Attraction

$$U_{ij}^{at} = -C_{at} e^{-\frac{d_{ij}}{R_{at}}}$$

Equations of motion

$$m_i \mathbf{r}_i = -\nabla_i U(\mathbf{r}_i) + \mathbf{d}_i$$



M. Sabatini, G. B. Palmerini and P. Gasbarri. Control Laws for Defective Swarming Systems// Advances in the Astronautical Sciences, Second IAA DyCoss'2014, V. 153. p. 132-153.

Linear quadratic regulator application

- Collision avoidance

$$\mathbf{u}^{rep} = \sum_{j,\, j\neq i} \mathbf{u}_{ij}^{rep}, when\ d_{ij} < R_{rep}$$

- Alignment

$$\mathbf{u}_i^{al} = \sum_{j,\, j\neq i} \mathbf{u}_{ij}^{al}, when\ R_{al} < d_{ij} < R_{at},$$

$$\mathbf{x}_i^d = [-\frac{\dot{y}}{2\omega_0}\ 0\ 0\ 0\ 0\ 0]$$

- Attraction

$$\mathbf{u}^{rep} = \sum_{j,\, j\neq i} \mathbf{u}_{ij}^{rep}, when\ d_{ij} < R_{rep}$$

Equations of motion

$$\dot{\mathbf{x}}_i = \mathrm{A}\mathbf{x}_i + \mathrm{B}\mathbf{u}_i,$$

Feedback control is

$$\mathbf{u}_i = -\mathrm{R}^{-1} B^T \mathrm{P}\mathbf{e}_i,$$

where $\mathbf{e}_i = \mathbf{x}_i - \mathbf{x}_i^d$,

matrix P is the solution

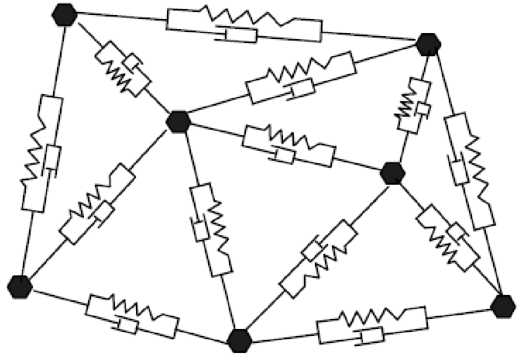
of Riccati equation

$$\mathrm{Q} - \mathrm{PBR}^{-1}\mathrm{B}^T\mathrm{P} + \mathrm{PA} + \mathrm{A}^T\mathrm{P} = 0.$$

M. Sabatini, G. B. Palmerini. Collective control of spacecraft swarms for space exploration// Celest Mech Dyn Astr (2009) 105:229–244

Virtual structure control approach

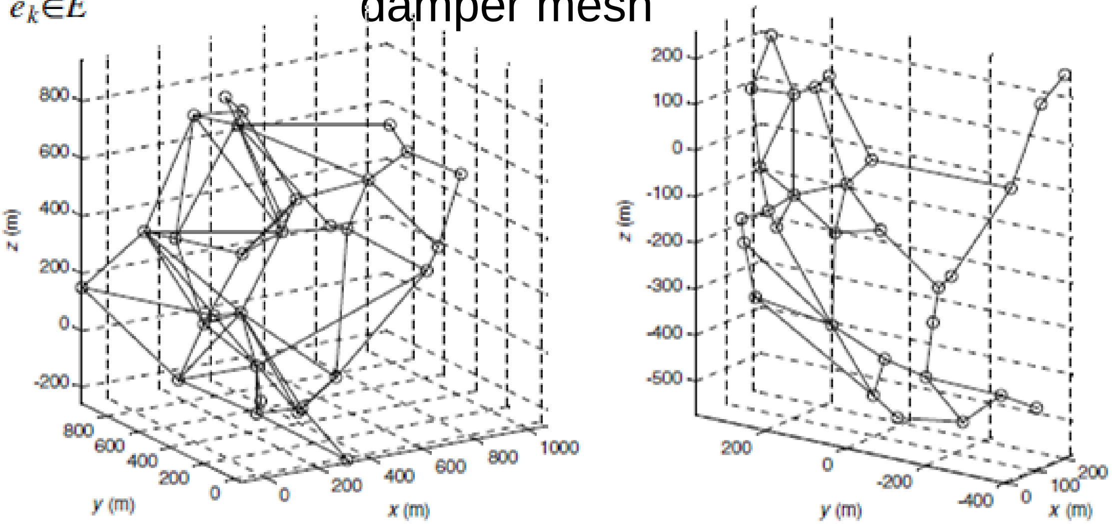
- Imitation of the satellite system by a solid structure model
- Control law



Point masses connected by a spring-damper mesh

$$u_i = -\sum_{e_k \in E} k_s d_{ik}(p_k - p_k^d) - \sum_{e_k \in E} k_d d_{ik} \dot{p}_k$$

Chen Q. et al. Virtual Spring-Damper Mesh-Based Formation Control for Spacecraft Swarms in Potential Fields // J. Guid. Control. Dyn. 2015. Vol. 38, № 3. P. 539–546.

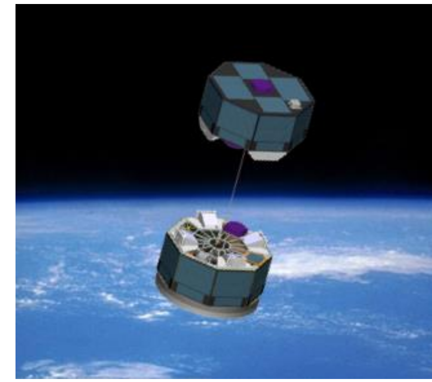


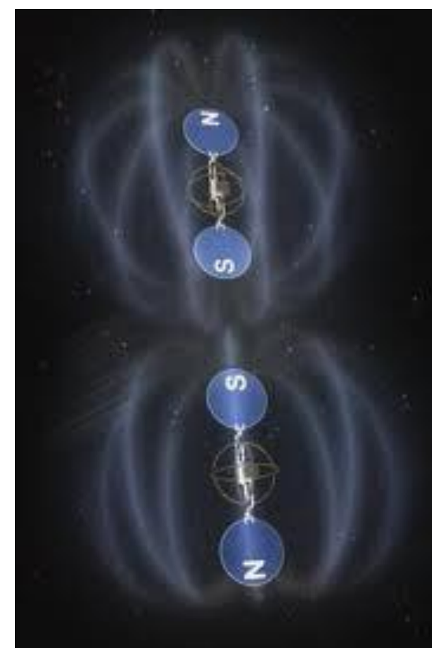
a) Initial time (t = 0 s) b) Steady state (t = 8000 s)

How to control?

Fuelless FF Control Concepts

- Tethered systems
- Aerodynamic drag
- Electro-magnetic interaction
- Solar pressure
- Momentum exchange

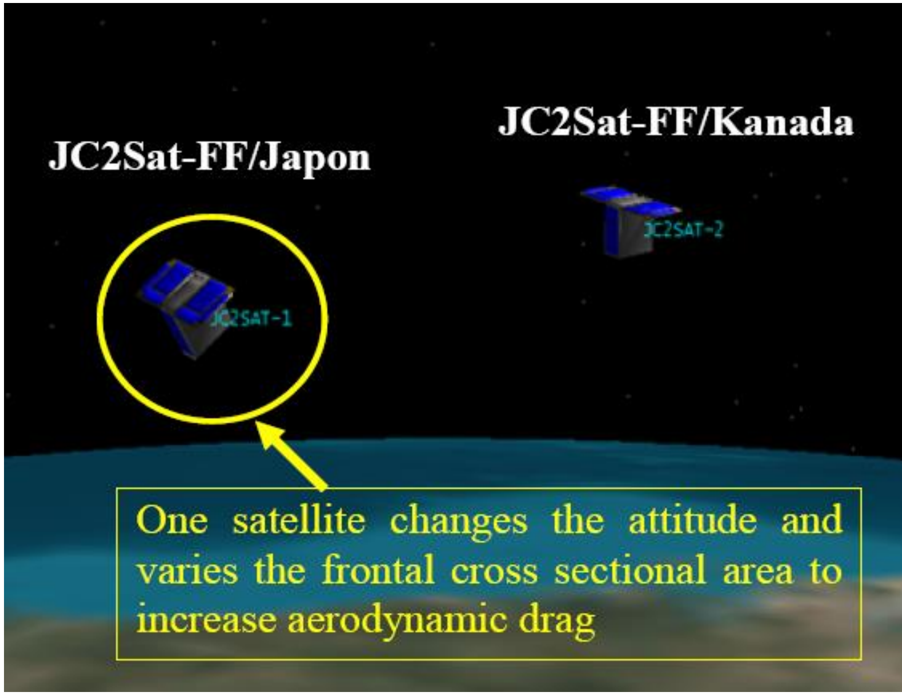




Aerodynamic drug based control

- Features:
  - *Low Earth Orbit*
  - *Satellites with variable cross section area*

- Shortcomings:
  - *Short lifetime*
  - *Attitude control system is needed*



JC2Sat Mission

LQR-based control algorithm

- Aerodynamic drug force

$$\mathbf{f}_i = -\frac{1}{m}\rho V^2 S\{(1-\varepsilon)(\mathbf{e}_V,\mathbf{n}_i)\mathbf{e}_V + 2\varepsilon(\mathbf{e}_V,\mathbf{n}_i)^2\mathbf{n}_i + (1-\varepsilon)\frac{\upsilon}{V}(\mathbf{e}_V,\mathbf{n}_i)\mathbf{n}_i\}^*,$$

$$\mathbf{n} = (\cos\alpha\cos\beta; \sin\beta; \sin\alpha\cos\beta).$$

- Linear-quadratic regulator

$$\dot{\mathbf{x}} = \mathrm{A}\mathbf{x} + \mathbf{b}u,$$

Minimising cost function

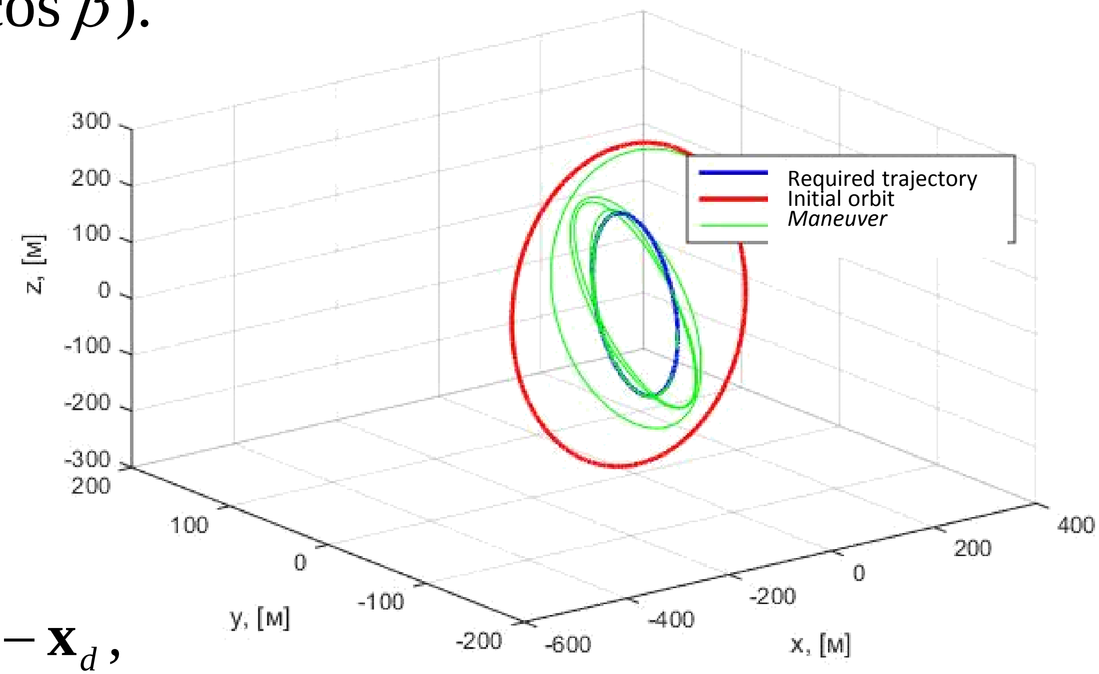
$$J = \int_{\tau}^{\infty} (\mathbf{e}^T \mathrm{Q}\mathbf{e} + \mathbf{u}^T \mathrm{R}\mathbf{u})dt,$$

Feedback control is

$$\mathbf{u} = -\mathrm{R}^{-1}\mathbf{b}^T \mathrm{P}\mathbf{e}, \quad \text{where } \mathbf{e} = \mathbf{x} - \mathbf{x}_d,$$

matrix P is the solution of Riccati equation

$$\mathrm{Q} - \mathrm{PBR}^{-1}\mathrm{B}^T\mathrm{P} + \mathrm{PA} + \mathrm{A}^T\mathrm{P} = 0.$$



Relative trajectories during the maneuver

Swarm control rules for differential drag control

- Most distant satellite drift elimination

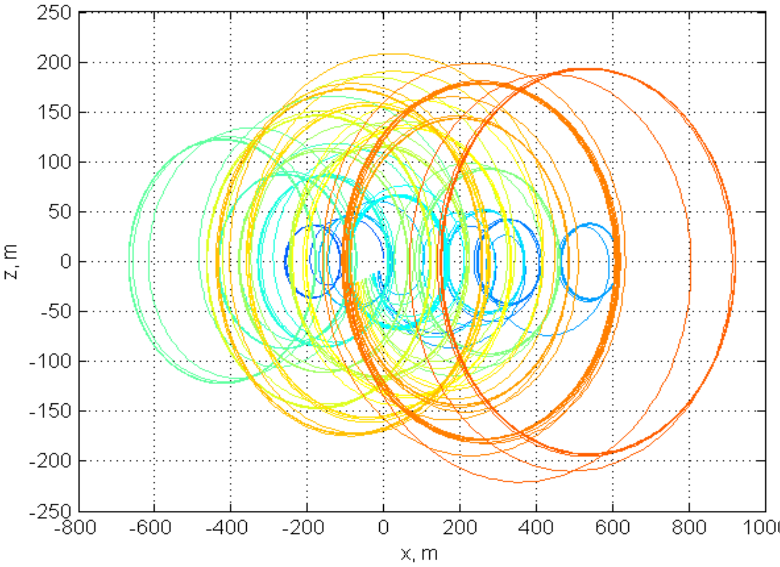
$$u_i^{\max R} = \frac{-\omega C_1^{iJ}}{\Delta T}, J = \arg\left(\max_j\left(R_{ij}\right)\right), j \in [1,\ldots,N_{comm}], j \neq i, R_{ij} \leq R_{comm}$$

- Maximum drift elimination

$$u_i^{\max C_1} = \frac{-\omega C_1^{iJ}}{\Delta T}, J = \arg\left(\max_j\left(C_{ij}\right)\right), j \in [1,\ldots,N_{comm}], j \neq i, R_{ij} \leq R_{comm}$$

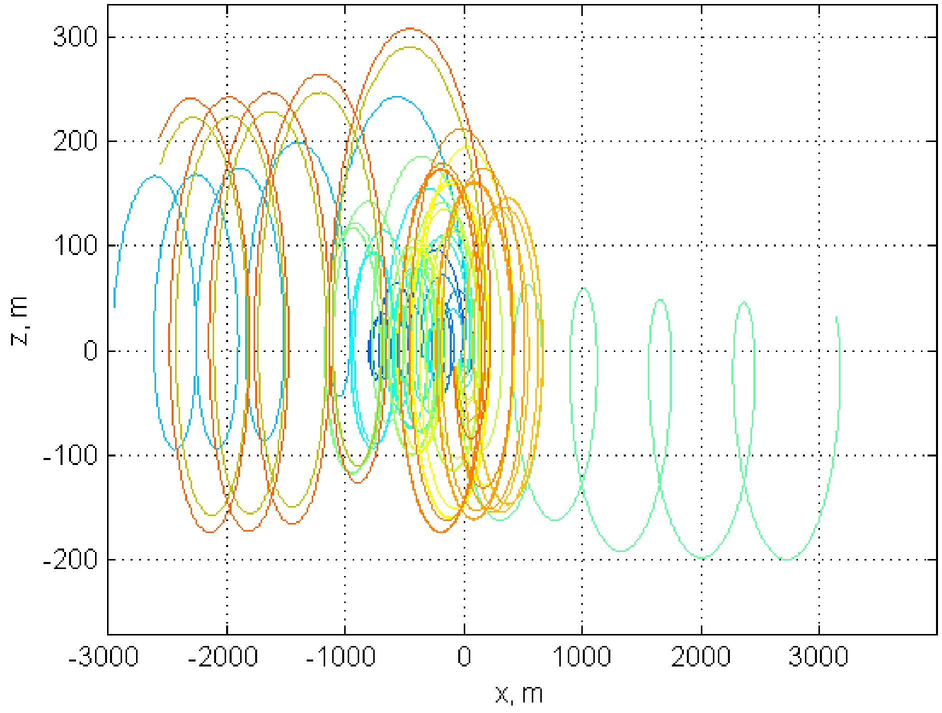
- Average drift elimination

$$\bar{C}_1^i = \sum_{j=1}^{N_{comm}} C_1^{ij} / N_{comm}, \quad \bar{u}_i = \frac{-\omega \bar{C}_1^i}{\Delta T}$$

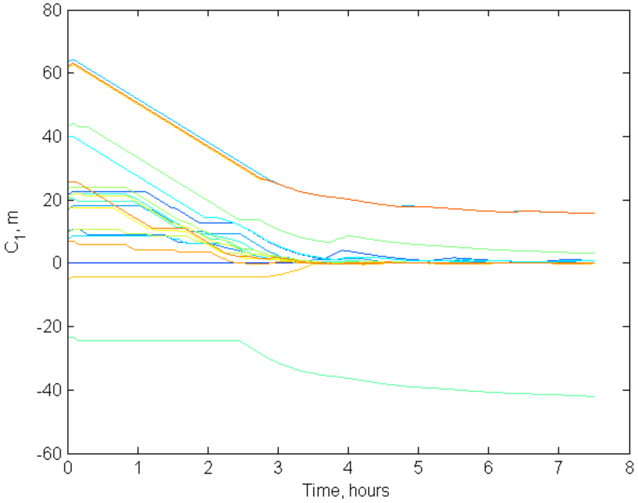


Separation of the swarm

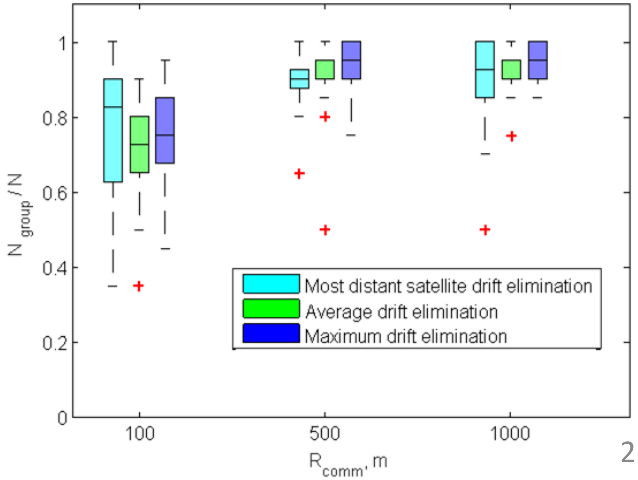
Example of the relative motion trajectories in the case of separation of the swarm



Relative drifts



The probability of separation



Electro-magnetic interaction based control

- Magnetic interaction

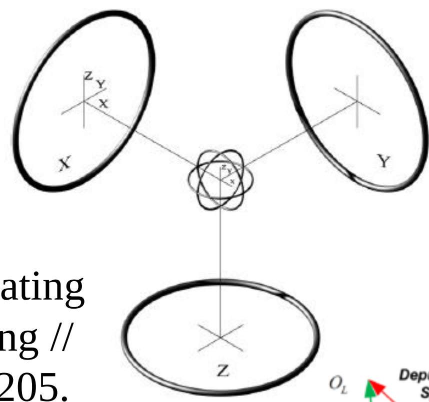
Youngquist R.C., Nurge M.A., Starr S.O. Alternating magnetic field forces for satellite formation flying // Acta Astronaut. Elsevier, 2013. Vol. 84. P. 197–205.

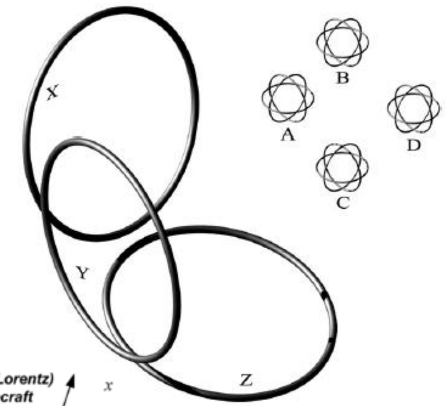
- Lorenz force of charged satellite

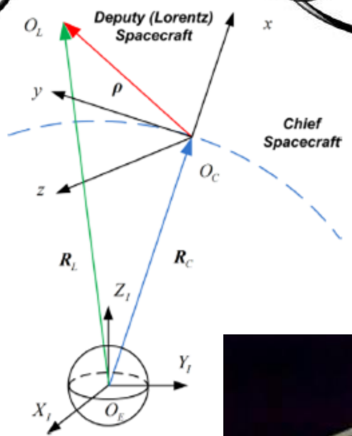
Peck M.A. et al. Spacecrat Formation Flying Using Lorentz Forces // J. Br. Interplanet. Soc. 2007. Vol. 60. P. 263–267.

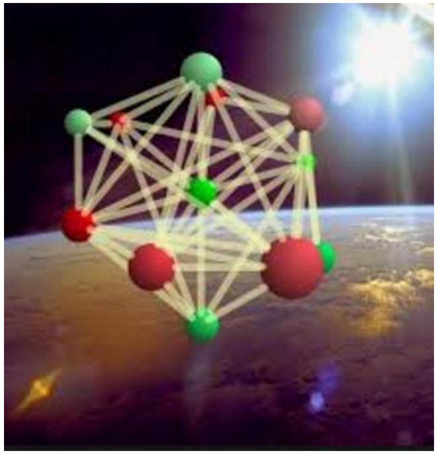
- Coulomb force interaction

Schaub H. et al. Challenges and Prospects of Coulomb Spacecraft Formation Control of the Astronautical Sciences // J. Astronaut. Sci. 2004. Vol. 52. P. 169–193.



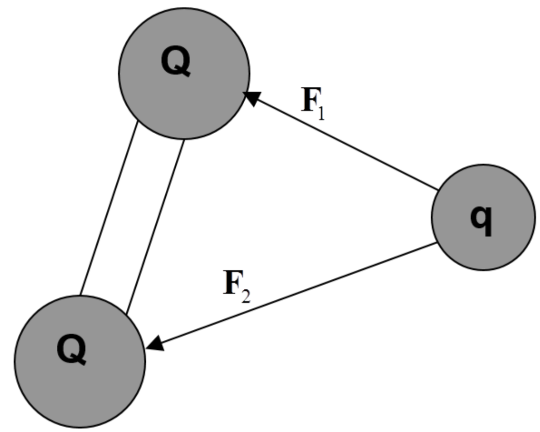




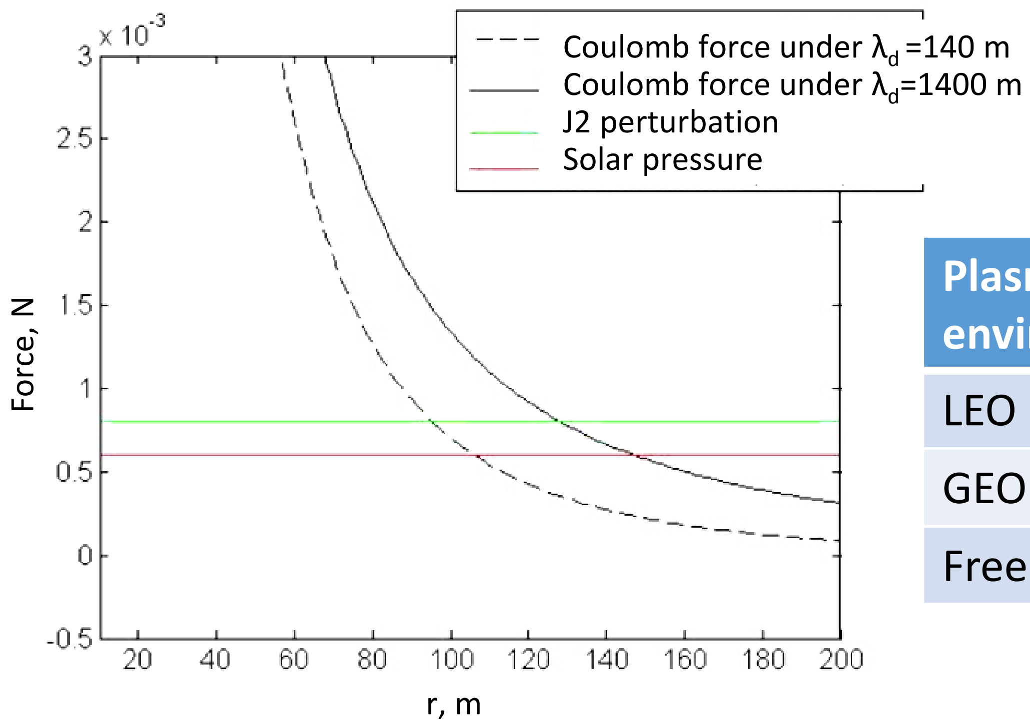


Coulomb force based control algorithm

- Features:
  - *The charging device is required*
  - *Small relative distances*
  - *Charges are eliminating by plasma*



$$f_{12} = k_c \frac{\mathbf{r}_{12}}{r_{12}^3} q_1 q_2 e^{-\frac{r_{12}}{\lambda_d}}$$



Plasma environment	$\lambda_{d\ min}$, m	$\lambda_{d\ max}$, m
LEO	0.02	0.4
GEO	142	1496
Free space	7.4	24

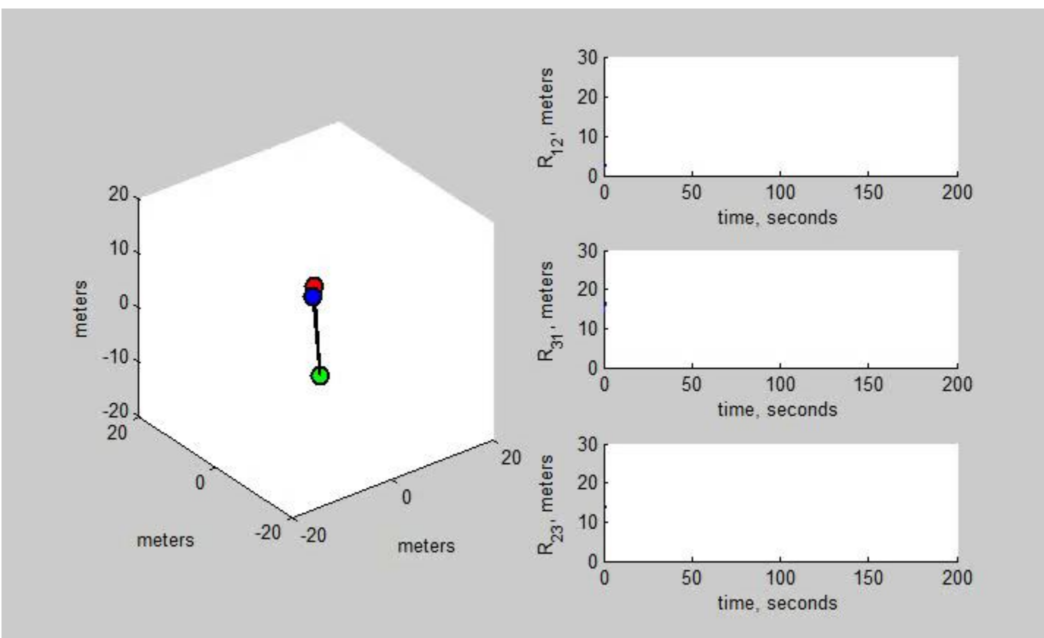
Sliding-mode control for three charged satellites

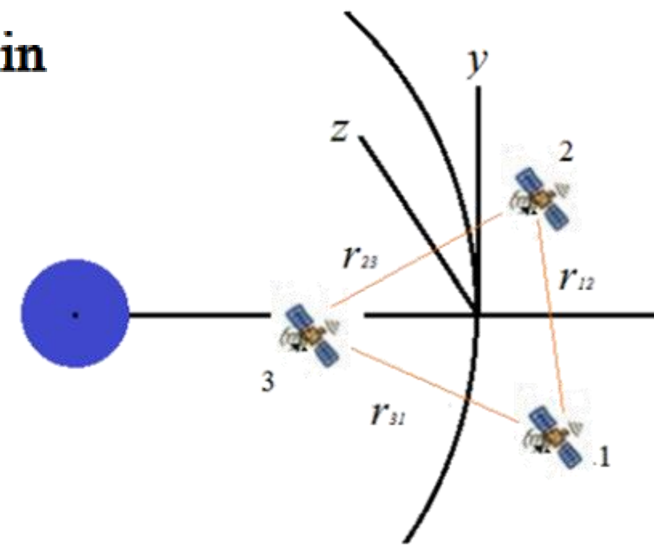
- Lyapunov-candidate function

$$V = \frac{1}{2}\dot{r}_{12}^2 + \frac{1}{2}\dot{r}_{23}^2 + \frac{1}{2}\dot{r}_{31}^2 + \frac{1}{2}k_1(r_{12} - a_1)^2 + \frac{1}{2}k_2(r_{23} - a_2)^2 + \frac{1}{2}k_3(r_{31} - a_3)^2,$$

- For negative sign should be:

$$\Phi = (q_1 q_2 - \alpha_3)^2 + (q_2 q_3 - \alpha_1)^2 + (q_1 q_3 - \alpha_2)^2 \to \min$$





Shestopyorov A.I., Tkachev S.S. Control for Three-Craft Coulomb Formation// Preprints of Keldysh Institute for Applied Mathematics. 2018. № 5. 17 p.

Solar radiation pressure based control

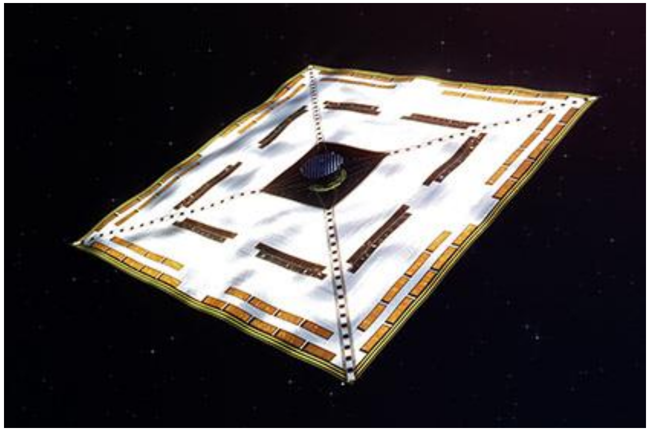
- Solar sail with fixed orientation

Smirnov G.V., Ovchinnikov M.Y., Guerman A.D. Use of solar radiation pressure to maintain a spatial satellite formation // Acta Astronaut. 2007. Vol. 61, № 7-8. P. 724–728.

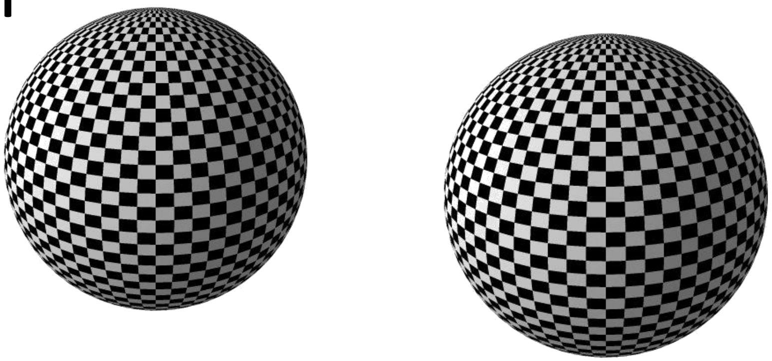
IKAROS Mission

- Solar sail with variable reflection

Mori O. et al. First Solar Power Sail Demonstration by IKAROS // Trans. Japan Soc. Aeronaut. Sp. Sci. Aerosp. Technol. Japan. 2010. Vol. 8, № ists27. P. To_4_25 – To_4_31.

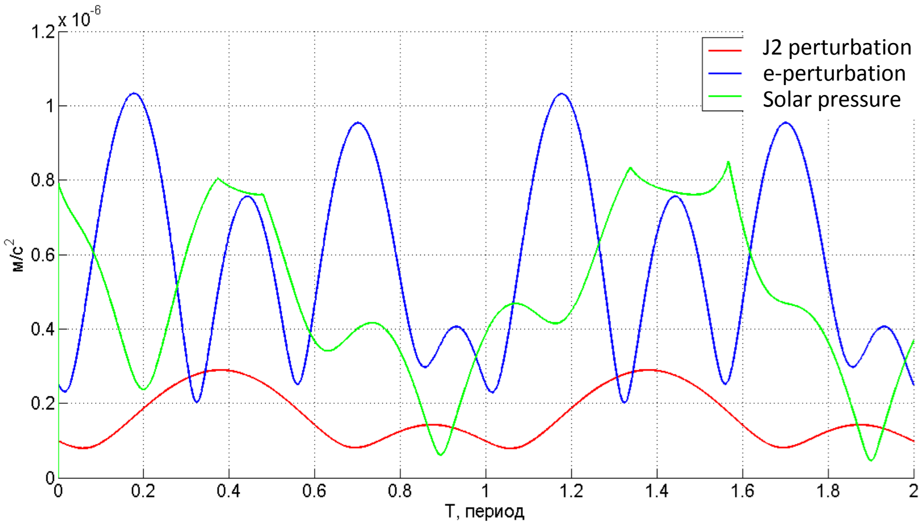


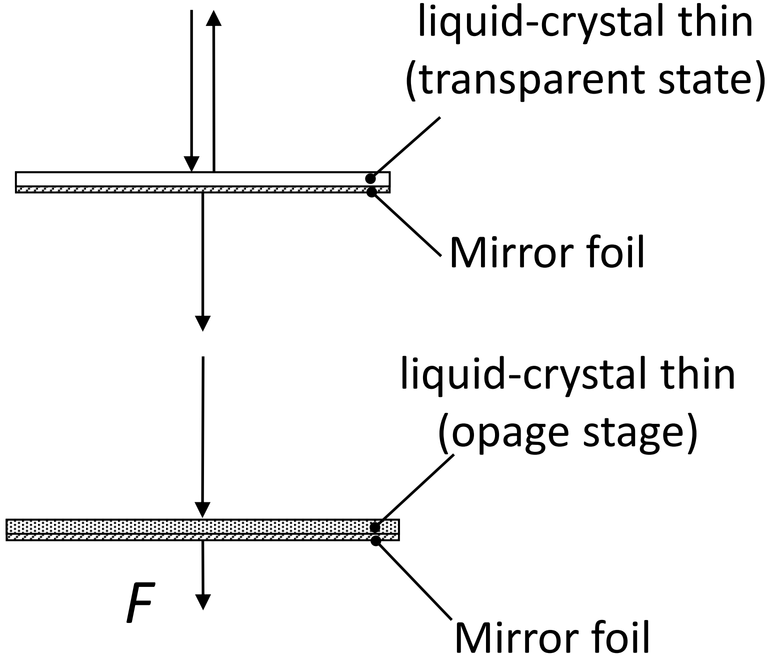
Solar radiation pressure based control



We consider:

- *Spherical satellites*
- *Variable reflection on "pixel" surface*
- *Nearcircular orbits*





PD-controller-based control algorithm

- Motion equations:

$$\begin{cases} \dot{\boldsymbol{\rho}} = \mathbf{v}, \\ \dot{\mathbf{v}} = \mathbf{f}(\boldsymbol{\rho}, \mathbf{v}) + \mathbf{u}. \end{cases}$$

- PD-regulator:

$$\mathbf{u} = -k_\rho(\boldsymbol{\rho} - \boldsymbol{\rho}_{ref}) - k_v(\mathbf{v} - \mathbf{v}_{ref}) + \dot{\mathbf{v}}_{ref} - \mathbf{f}$$

where

$$k_\rho,\ k_v = \text{const} > 0, \text{ choosen that } k_v = \frac{k_\rho^2}{4};$$

Solar pressure model

- The solar pressure force:

$$\mathbf{F} = -P_c \left(\int_{S^+} (1-k)(\mathbf{s},\mathbf{n})\mathbf{s}\,dS + 2\int_{S^+} k\mathbf{n}(\mathbf{s},\mathbf{n})^2\,dS \right)$$

- The reflection function:

$$k(\varphi,\theta) = g(\varphi)\cdot h(\theta),$$

where

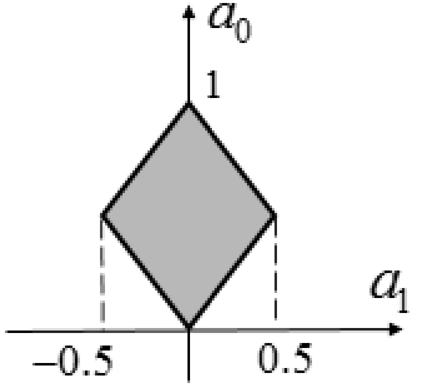
$$g(\varphi) = a_1\cos(\varphi+\alpha) + a_0,\ h(\theta) = \frac{1}{2} + \frac{1}{2}\sin 4\theta.$$

$a_0, a_1, \alpha -$ Variable control parameters

Restrictions are: $0 \le k \le 1.$

$$0 < \left(a_1\cos(\varphi+\alpha)+a_0\right)\left(\frac{1}{2}+\frac{1}{2}\sin 4\theta\right) \le 1$$

$$0 \le a_1\cos(\varphi+\alpha)+a_0 \le 1$$

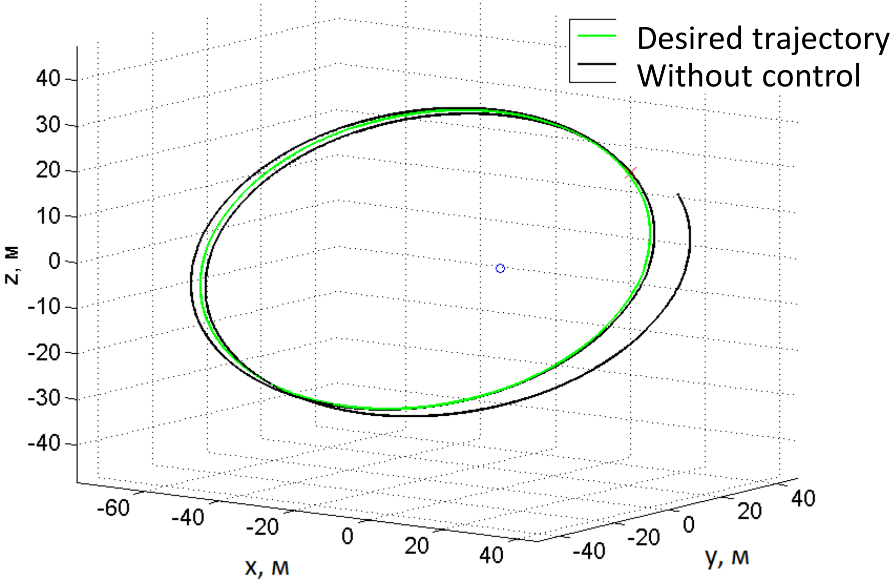


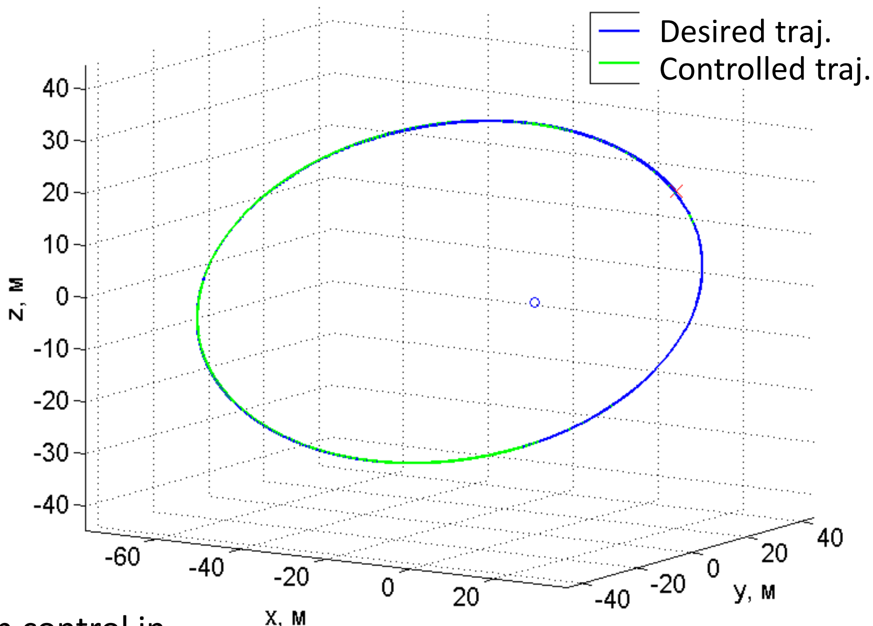
Numerical example

$$F_1 = -\frac{\pi^2}{32} R_d^2 P_c a_1 \cos\alpha, \qquad \alpha = \operatorname{arctg}\left(-\frac{F_2}{F_1}\right),$$

$$F_2 = \frac{\pi^2}{32} R_d^2 P_c a_1 \sin\alpha, \qquad \Rightarrow \qquad a_1 = -\frac{32}{\pi^2 P_c R_d^2} \cdot \frac{F_1}{\cos\alpha},$$

$$F_3 = -\frac{P_c \pi^2 R_d^2}{4} a_0 + P_c \pi (R_p^2 - R_d^2). \qquad a_0 = -\frac{4}{\pi^2 P_c R_d^2} F_3 + \frac{4}{\pi R_d^2}\left(R_p^2 - R_d^2\right).$$





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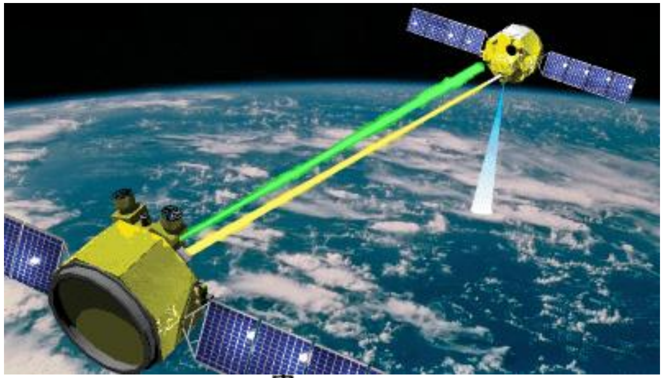
The momentum exchange-based control

- The momentum from lasers for repulsive force

Y. K. Bae. A contamination-free ultrahigh precision formation flying method for micro-, nano-, and pico-satellites with nanometer accuracy. In Space Technology and Applications International Forum-Staif 2006, volume 813, pages 1213–1223, 2006.

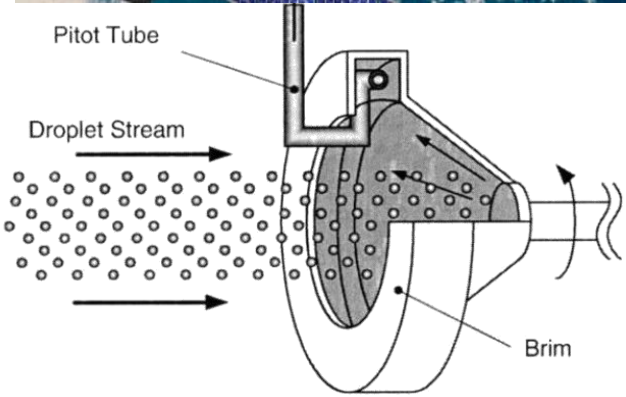
- Continuous stream of mass travelling between the satellites

S. G. Tragesser. Static formations using momentum exchange between satellites. Journal of guidance, control, and dynamics, 32(4):1277 – 1286, 2009.



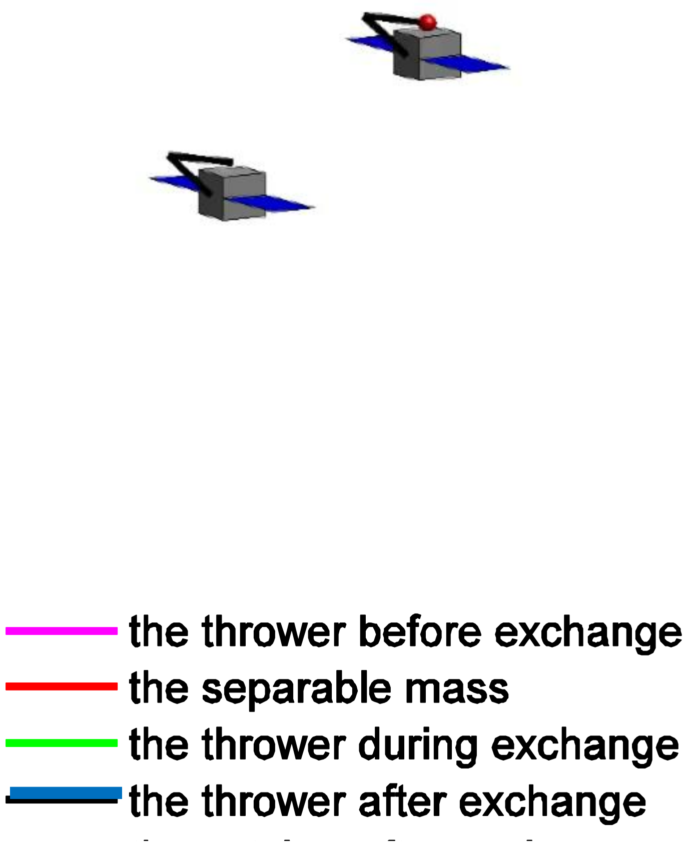
- Liquid droplet streams exchange

T. Joslyn and A. Ketsdever. Constant momentum exchange between microspacecraft using liquid droplet thrusters. In 46th joint Propulsion Conference, volume 6966, pages 25–28, 2010.



Single mass exchange control concept

- At command the single mass separates from the satellite
- The separated mass moves to the other satellite and impacts it absolutely inelastically
- After the whole mass transfer the resulting relative trajectory changes in adjustable way



The Problem Formulation

Boundary problem:

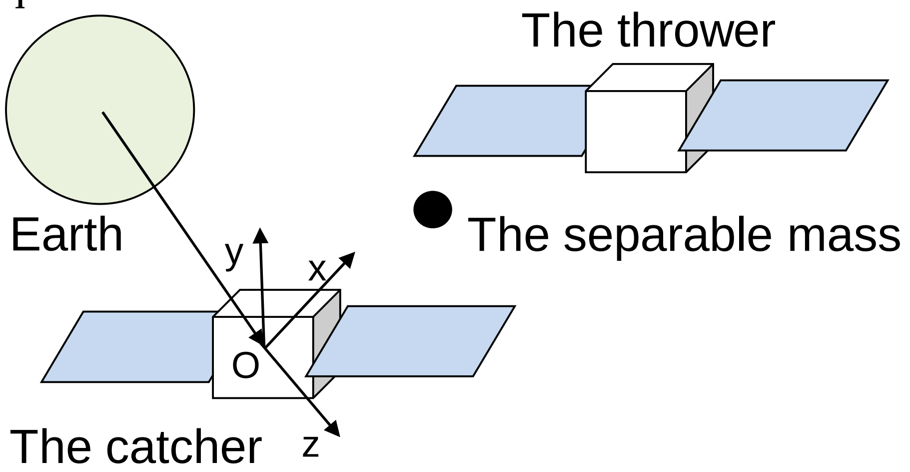
What is the initial relative velocity of the mass required to hit the thrower?

Initial conditions:

$$x_0 = x(t_0),\ y_0 = y(t_0),\ z_0 = z(t_0),$$

The final position:

$$x_1 = x\left(t_1\right) = 0,\ y_1 = y\left(t_1\right) = 0,\ z_1 = z\left(t_1\right) = 0.$$



Hill - Clohessy - Wiltshire equations:

$$\ddot{x} + 2\omega\dot{z} = 0,$$

$$\ddot{y} + \omega^2 y = 0,$$

$$\ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0$$

The exact solution:

$$x = C_4 - 3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t,$$

$$y = C_5\sin\omega t + C_6\cos\omega t,$$

$$z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t$$

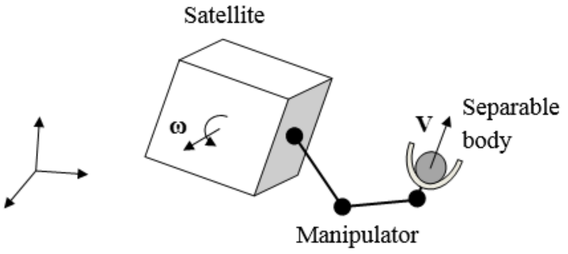
$$C_1 = 2z(t_0) + \frac{\dot{x}(t_0)}{\omega}, C_2 = \frac{\dot{z}(t_0)}{\omega},$$

$$C_3 = -3z(t_0) - \frac{2\dot{x}(t_0)}{\omega}, C_4 = x(t_0) - \frac{2\dot{z}(t_0)}{\omega},$$

$$C_5 = \frac{\dot{y}(t_0)}{\omega}, C_6 = y(t_0).$$

The Analytical Problem Solution

Throwing mass relative velocity:



$$\delta \dot{x} = -\dot{x}_0 - 2z_0\omega + \frac{1}{\Delta}[x_0\omega \sin u + 2z_0\omega(\cos u - 1)],$$

$$\delta \dot{y} = -\dot{y}_0 - y_0\omega \frac{\cos u}{\sin u},$$

$$\delta \dot{z} = -\dot{z}_0 - \frac{1}{\Delta}[2x_0\omega(1 - \cos u) + z_0\omega(3u\cos u - 4\sin u)],$$

where $u = \omega(t_m - t_e),\ \Delta = 3u\sin u - 8(1 - \cos u).$

The resulting thrower satellite velocity after mass throwing

$$\mathbf{v}_t = \mathbf{v}_{t,0} - \frac{m}{M}\delta\mathbf{v}.$$

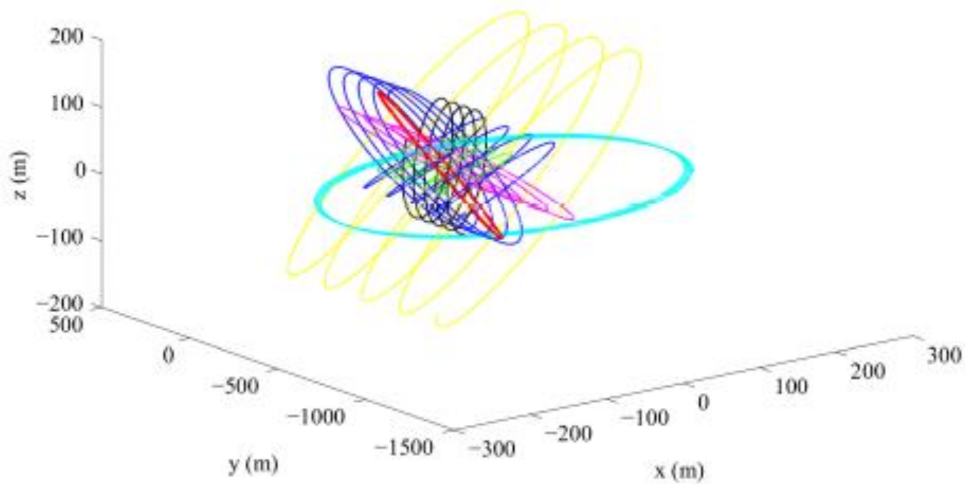
The resulting *catcher* **satellite velocity after mass catching**

$$\mathbf{v}_c(t_m) = \frac{m}{M+m}\mathbf{v}_s(t_m).$$

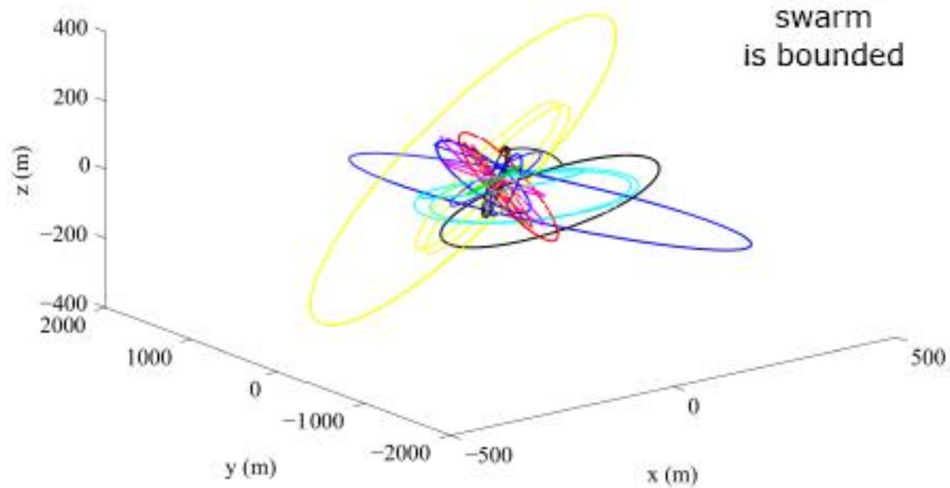
For instance, the final relative trajectory $\tilde{\mathrm{C}}_1$ constant:

$$\tilde{C}_1 = \left(2z_0 + \frac{\dot{x}_0}{\omega}\right) + \frac{k(k+2)}{(k+1)^2} \cdot \frac{x_0\cos s - 2z_0\sin s}{8\sin s - 6s\cos s}.$$

Sequence of mass exchanges for a swarm construction



without control formation degrades



with control swarm is bounded

Conclusion

- The swarm of the satellites is a new paradigm in space systems
- The fuelless control approaches are fitting small satellite restrictions, they are smart but challenging
- We should allow for the distributed system to be autonomous and self-organizing, but we must be watchful

Thank you for your attention!