Satellite attitude control systems: problems and solutions

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Was founded in 1953

Mathematical modeling for USSR
Space program
Atomic and fusion energetics program
 +
Providing software for modeling

Mstislav Keldysh

Mathematical modeling of physical processes
From the approaches and tools development to the result visualization

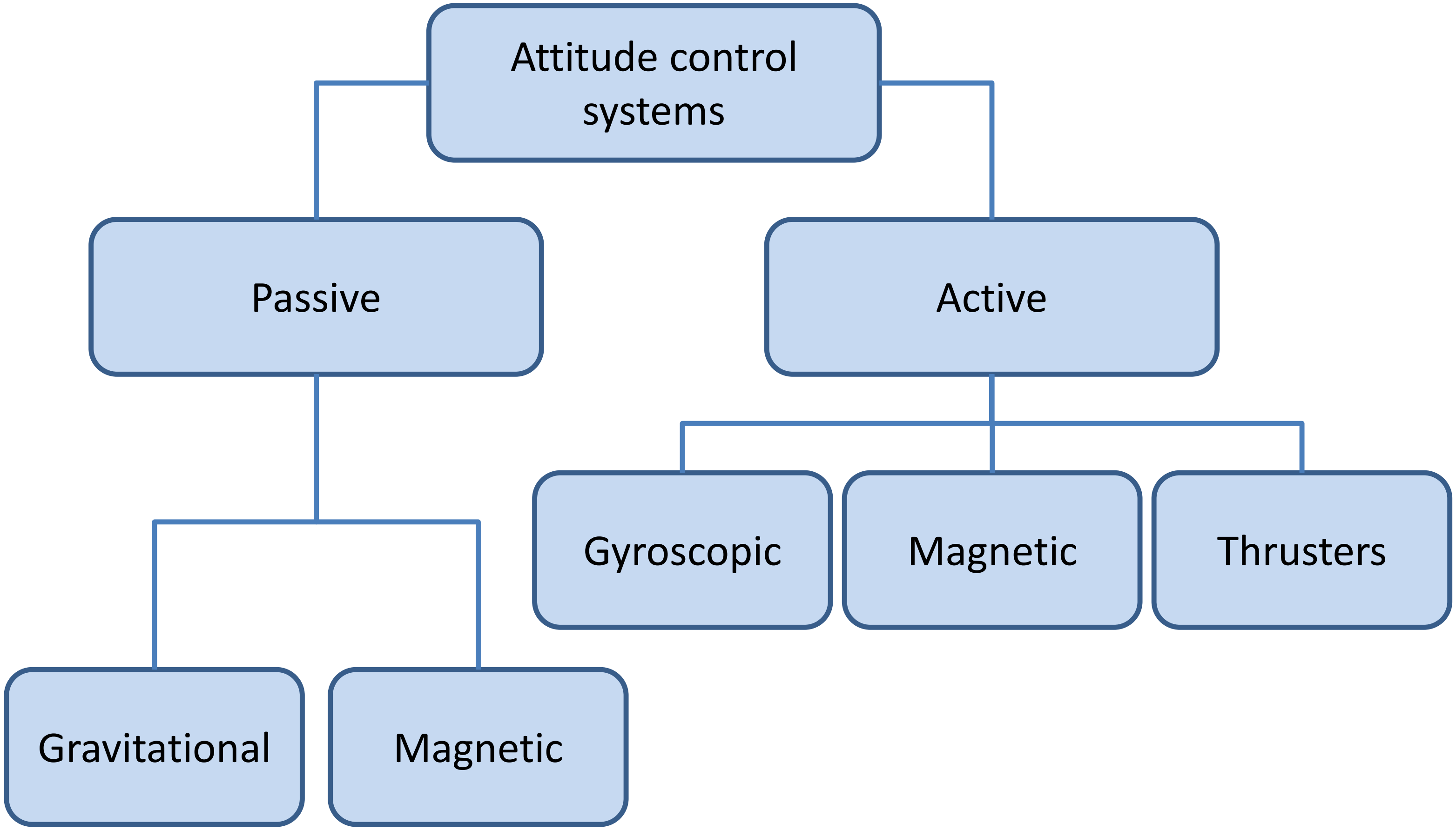
Space systems dynamics study and control development

Why control?

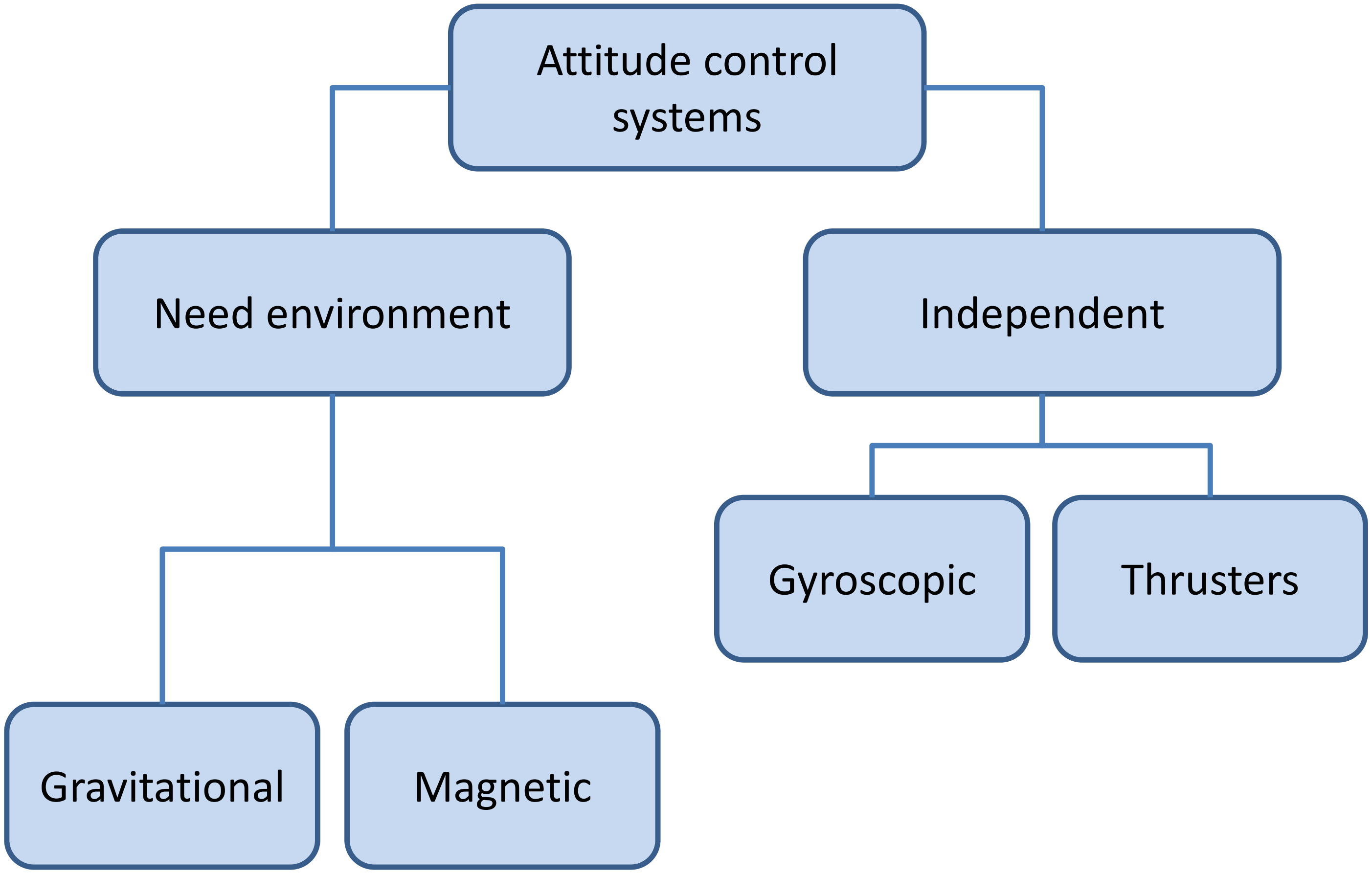
- Solar panels acquisition
- Payload and antennas pointing
- Thrust direction change

Flock (Planet Labs)

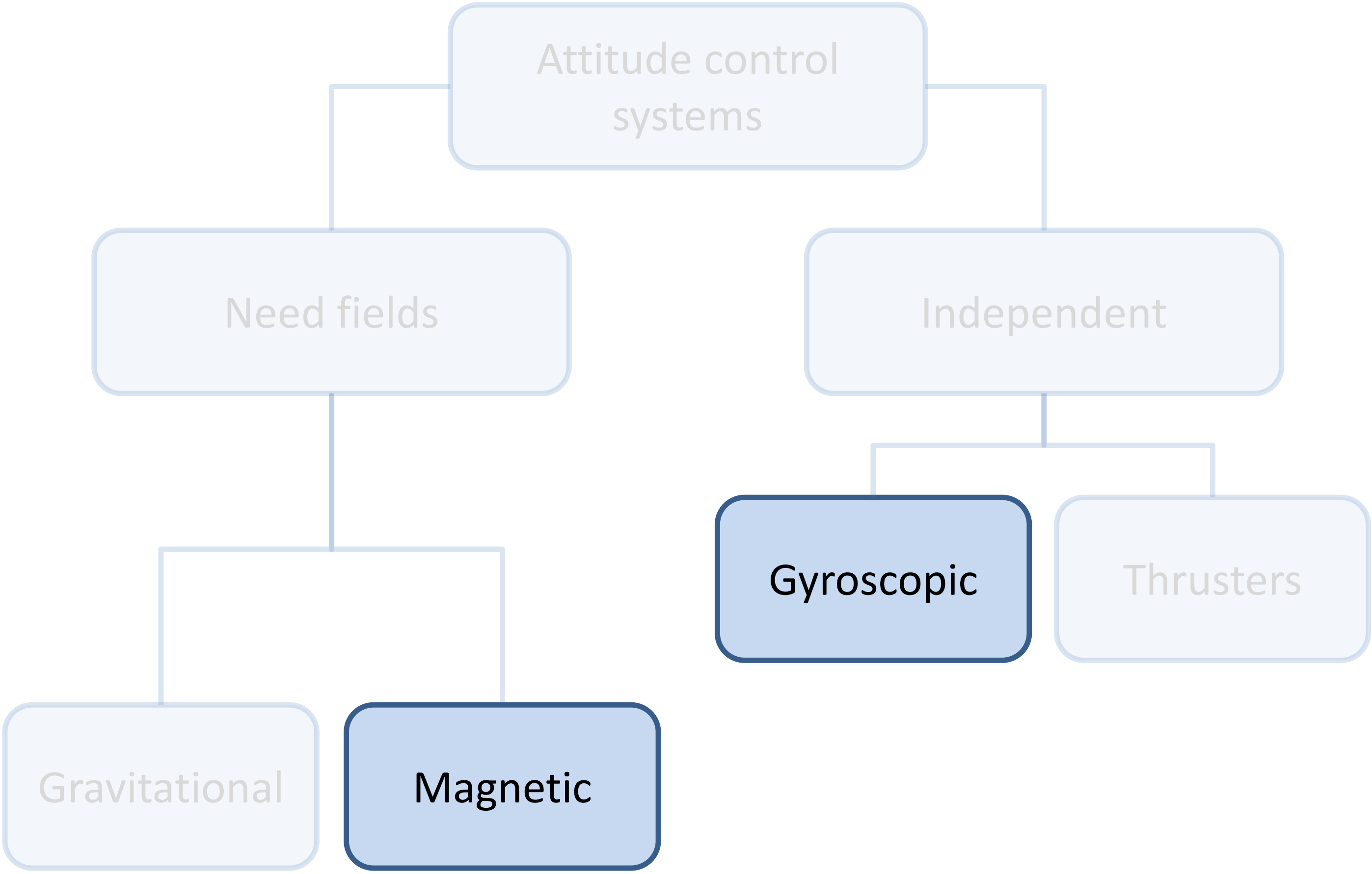
How control?


Attitude control systems
Passive
Active
Gravitational
Magnetic
Gyroscopic
Magnetic
Thrusters

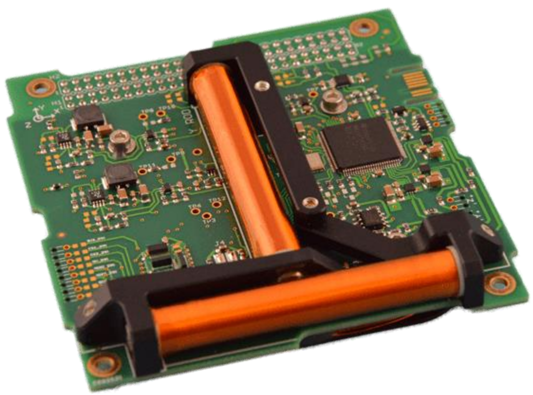
How control?


Attitude control systems
Need environment
Independent
Gyroscopic
Thrusters
Gravitational
Magnetic

How control?

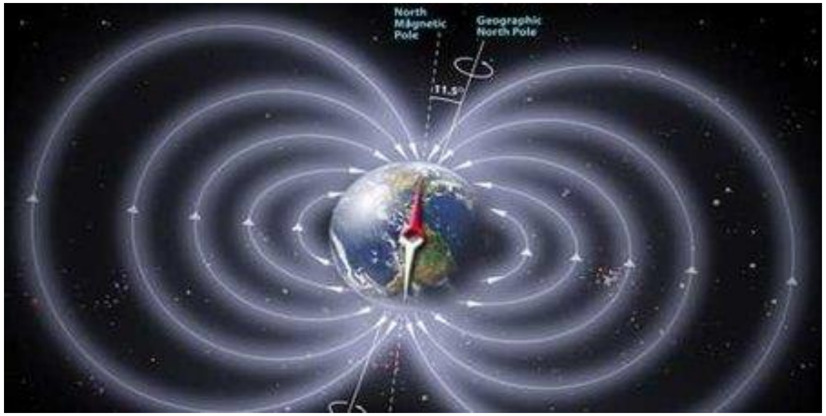

Attitude control systems
Need fields
Independent
Gyroscopic
Thrusters
Gravitational
Magnetic

Magnetic attitude control



CubeSat magnetorquer

+



Geomagnetic field

1. Take magnetorquers 1-3 pieces
2. Take geomagnetic field 1 piece
3. ??????
4. PROFIT

Mathematical model

Dynamics

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M}_{grav} + \mathbf{M}_{ctrl}, \quad \mathbf{M}_{grav} = 3\frac{\mu}{R^3}\mathbf{e}_3 \times \mathbf{J}\mathbf{e}_3$$

Kinematics

$$\dot{\lambda}_0 = -\frac{1}{2}\left(\boldsymbol{\lambda}, \boldsymbol{\omega}\right)$$

$$\dot{\boldsymbol{\lambda}} = \frac{1}{2}\left(\lambda_0\boldsymbol{\omega} + \boldsymbol{\lambda} \times \boldsymbol{\omega}\right)$$

Magnetic control torque

$$\mathbf{M}_{ctrl} = \mathbf{m} \times \mathbf{B} \Rightarrow \mathbf{M}_{ctrl} \perp \mathbf{B}$$ underactuation problem!

The problem is having all of these make satellite move the way we want to!

Angular velocity damping

Angular velocity damping

$$\mathbf{m} = k\boldsymbol{\omega} \times \mathbf{B}$$

$$\mathbf{M}_{ctrl} = -k\boldsymbol{\omega}B^2 + k\mathbf{B}\left(\boldsymbol{\omega}, \mathbf{B}\right)$$

$$\mathbf{V} = \frac{1}{2}\left(\boldsymbol{\omega}, \mathbf{J}\boldsymbol{\omega}\right) \geq 0$$ Lyapunov function

$$\dot{\mathbf{V}} = -k\left|\boldsymbol{\omega} \times \mathbf{B}\right|^2 \leq 0$$

As **B** rotates then $\dot{\mathbf{V}} < 0$

Here we neglect gravitational torque!

$$\Downarrow$$

$$\boldsymbol{\omega} \to 0$$

-Bdot

$$\left(\frac{d\mathbf{B}}{dt}\right)_{abs} = \left(\frac{d\mathbf{B}}{dt}\right)_{rel} + \boldsymbol{\omega} \times \mathbf{B} \approx 0$$

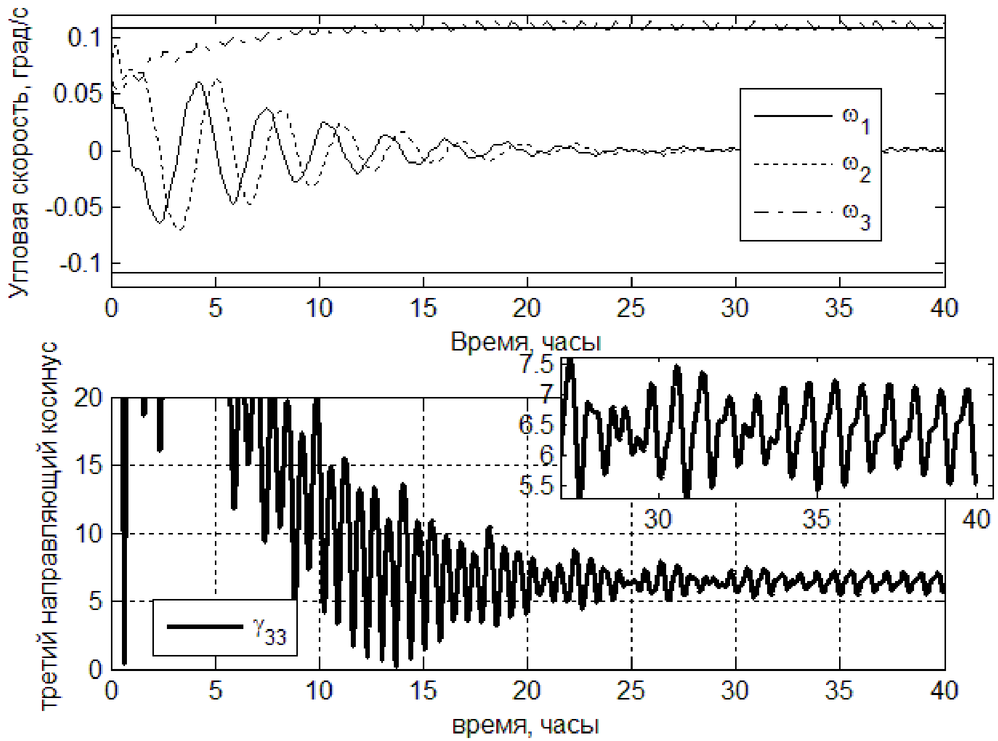
$$\boldsymbol{\omega} \times \mathbf{B} = -\left(\frac{d\mathbf{B}}{dt}\right)_{rel}$$

$$\mathbf{m} = -\left(\frac{d\mathbf{B}}{dt}\right)_{rel}$$

$$\Downarrow$$

$$\boldsymbol{\omega} \to 1.8\boldsymbol{\omega}_o$$

Numerical modeling



M.Yu. Ovchinnikov, D.S. Roldugin, S.S. Tkachev, V.I. Penkov. B-dot algorithm steady-state motion performance // Acta Astronautica, 2018, V. 146, pp. 66-72.

One-axis stabilization

S-dot algorithm

$$\mathbf{m} = kB_0^2 \cos\alpha\left(\boldsymbol{\omega}\times\mathbf{S}\right)$$

Averaged equations

$$\frac{d\rho}{du} = \varepsilon\frac{\cos\rho\cos\gamma - S_{3Z}}{\sin\rho}\left[p\cos\gamma + S_{3Z}\cos\rho\left(q-p\right)\right]\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\gamma}{du} = -\varepsilon\sin\gamma\left[p\cos\gamma + S_{3Z}\cos\rho\left(q-p\right)\right]\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

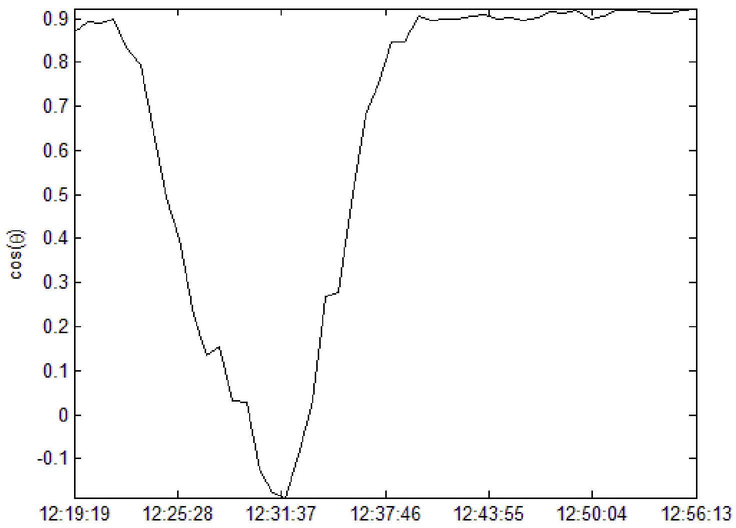
$$\frac{dl}{du} = -\varepsilon l\left[p\sin^2\gamma + (q-p)(S_{3Z}^2 - S_{3Z}\cos\rho\cos\gamma)\right]\left(\cos^2\theta + \frac{C}{A}\sin^2\theta\right),$$

$$\frac{d\theta}{du} = \frac{1}{2}\varepsilon\lambda\left[2\left(\left(1-S_{3Z}^2\right)p + S_{3Z}^2 q\right) - p\sin^2\gamma + \left(q-p\right)S_{3Z}\left(\cos\gamma\cos\rho - S_{3Z}\right)\right]\sin\theta\cos\theta,$$

$$\lambda = \frac{1}{2}\left(1-\frac{C}{A}\right),\ p = \frac{1}{2}\sin^2\Theta,\ q = \cos^2\Theta,\ \varepsilon = \frac{kB_0^2}{\omega_0 C}$$

Result: The axis of the maximum moment of inertia coincides with the angular momentum vector in the asymptotically stable equilibrium

Flight results (Chibis-M)

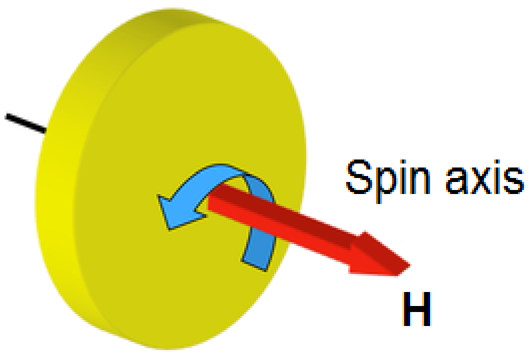


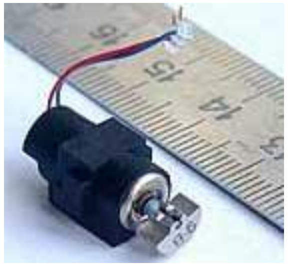
Angular momentum attitude

Chibis-M

S.O.Karpenko, M.Yu.Ovchinnikov, D.S.Roldugin, S.S.Tkachev. New one-axis one-sensor magnetic attitude control theoretical and in-flight performance // Acta Astronautica, V. 105, 2014, N 1, pp. 12-16

Gyroscopic systems

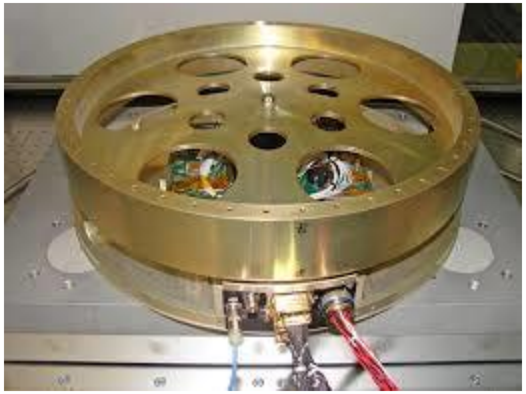




Cell phone vibration motor

$$\mathbf{H} = \mathbf{I}\boldsymbol{\Omega}$$

Spacecraft reaction wheel

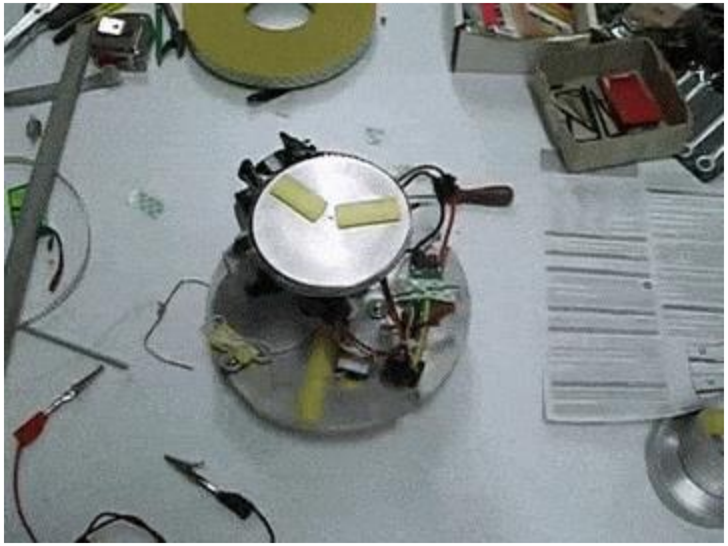


Principle of operation

Angular momentum exchange

$$\mathbf{K} = \mathbf{H} + \mathbf{J}\boldsymbol{\omega} = const$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = -\dot{\mathbf{H}} - \boldsymbol{\omega} \times \mathbf{H}$$



Control torque

$$\mathbf{M}_{ctrl} = -\dot{\mathbf{H}} - \boldsymbol{\omega} \times \mathbf{H}$$

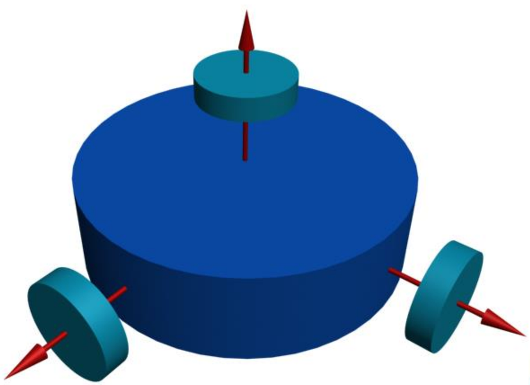
You can create torque by

accelerating or decelerating

changing the angular momentum direction

Reaction wheels

Axis of rotation is fixed



Main dynamical properties:

maximum torque

maximum angular momentum or spin rate

Control algorithm

$$V_0 = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_q\left(1 - q_0\right)$$

$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs} - \mathbf{A}\boldsymbol{\omega}_{ref}$$

$$\mathbf{M}_{ctrl} = -\mathbf{M}_{ext} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} + \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_{ref} - \mathbf{J}\left[\boldsymbol{\omega}_{rel}\right]_{\times}\mathbf{A}\boldsymbol{\omega}_{ref} - k_q\mathbf{q} - k_\omega\boldsymbol{\omega}_{rel}$$

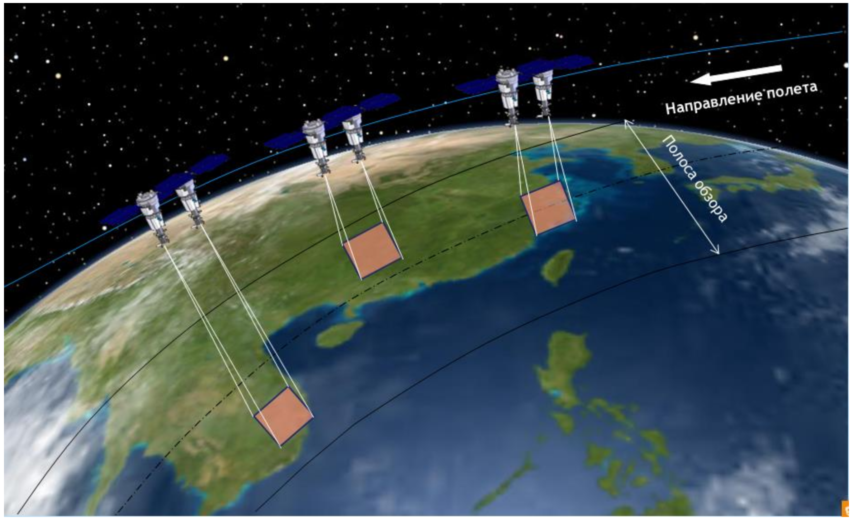
Global asymptotic stability of the *reference motion*

$$\left|\delta\alpha_i\right| \le \begin{cases} \max\left(\left|\mathbf{M}_{dist}\right|\right)\dfrac{1}{2k_a}, & k_\omega^2 - 8k_a J_{ii} \ge 0, \\ \max\left(\left|\mathbf{M}_{dist}\right|\right)\coth\left(\dfrac{\pi k_\omega}{2\sqrt{8k_a J_{ii} - k_\omega^2}}\right)\dfrac{1}{2k_a}, & k_\omega^2 - 8k_a J_{ii} < 0, \end{cases}$$

$$\left|\delta\omega_i\right| \le \begin{cases} \max\left(\left|\mathbf{M}_{dist}\right|\right)\dfrac{2}{\sqrt{2J_{ii}k_a}}\left(\dfrac{k_\omega + \sqrt{k_\omega^2 - 8k_a J_{ii}}}{k_\omega - \sqrt{k_\omega^2 - 8k_a J_{ii}}}\right)^{\frac{-k_\omega}{2\sqrt{k_\omega^2 - 8k_a J_{ii}}}}, & k_\omega^2 - 8k_a J_{ii} \ge 0, \\ \dfrac{\max\left(\left|\mathbf{M}_{dist}\right|\right)}{\sqrt{2J_{ii}k_a}}\exp\left[-\dfrac{k_\omega}{\sqrt{8k_a J_{ii} - k_\omega^2}}\arccos\left(\dfrac{k_\omega}{\sqrt{8k_a J_{ii}}}\right)\right]\left(1 + \coth\left(\dfrac{\pi k_\omega}{2\sqrt{8k_a J_{ii} - k_\omega^2}}\right)\right), & k_\omega^2 - 8k_a J_{ii} < 0. \end{cases}$$

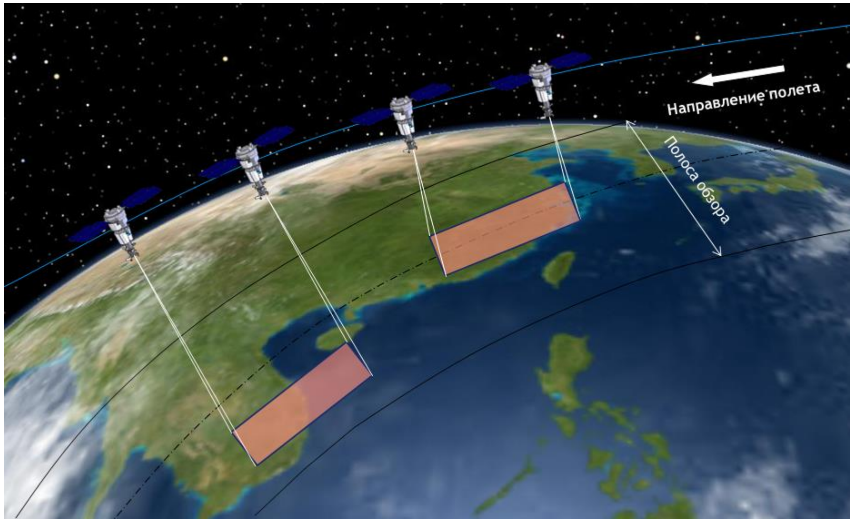
We need reference motion to calculate!

Remote sensing problem

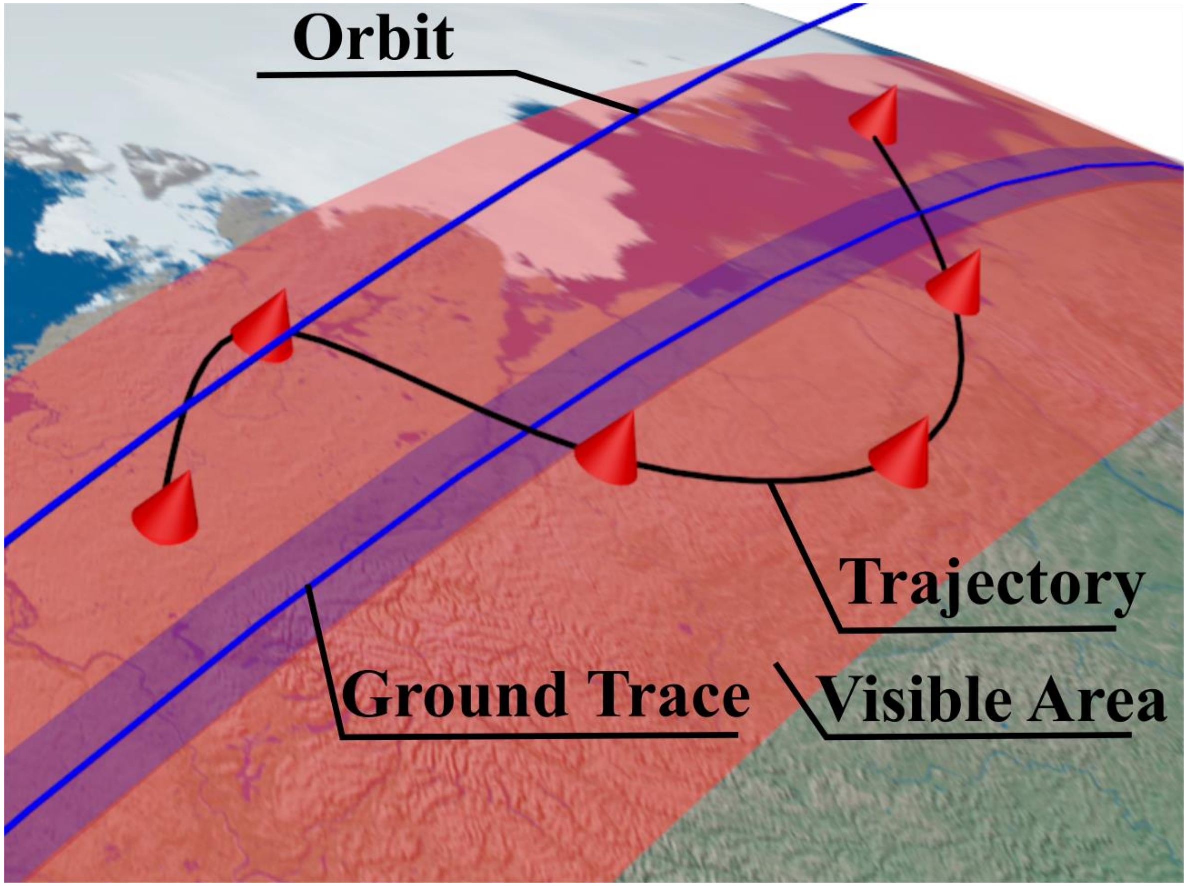


Object «Resurs-P»

Route «Resurs-P»



Complex route tracking



Angular Motion Synthesis

Constraints implied by CCD Line Camera:

$$(\mathbf{V}_{rel}, \mathbf{e_3}) = 0$$

$$(\mathbf{V}_{rel}, \mathbf{e_2}) = -\frac{V\rho}{f}$$

$$\mathbf{V}_{rel} = \mathbf{\Omega}_E \times \mathbf{r_p} - \mathbf{V_s} - \boldsymbol{\omega} \times (\mathbf{r_p} - \mathbf{r_s})$$

$$\rightarrow \begin{cases} \omega_2 = -\dfrac{(\mathbf{\Omega}_E \times \mathbf{r_p} - \mathbf{V_s}, \mathbf{e_3})}{\rho} \\ \omega_3 = \dfrac{(\mathbf{\Omega}_E \times \mathbf{r_p} - \mathbf{V_s}, \mathbf{e_2})}{\rho} + \dfrac{V}{f} \end{cases}$$

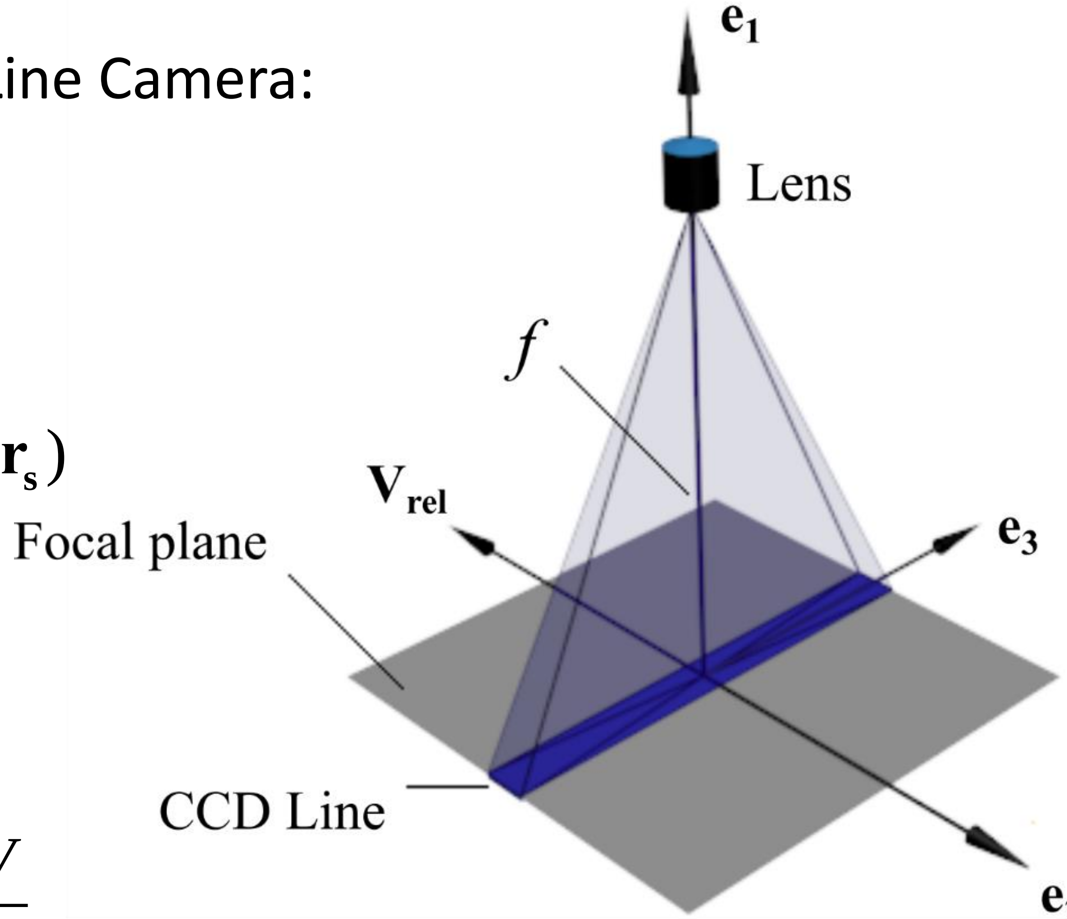


Image Quality

$\delta \mathbf{r}_p$ – aiming point displacement (distance between desirable point the satellite looking at and real one)

$\delta V_{2,3}$ – image velocity error (causes image blur)

$$\begin{pmatrix} \delta \mathbf{r}_p \\ \delta V_2 \\ \delta V_3 \end{pmatrix} = \mathbf{D} \begin{pmatrix} \delta \boldsymbol{\alpha} \\ \delta \boldsymbol{\omega} \\ \delta \mathbf{r}_s \\ \delta \mathbf{V}_s \end{pmatrix} \begin{array}{l} \text{– attitude error} \\ \text{– angular velocity error} \\ \text{– center of mass position error} \\ \text{– center of mass velocity error} \end{array}$$

Y.V.Mashtakov, M.Yu.Ovchinnikov, S.S.Tkachev, Study of the disturbances effect on small satellite route tracking accuracy // Acta Astronautica, 2016, V.129, pp.22-31.

Direction error

$$\delta\mathbf{r}_p = \left\{ \mathbf{e}_1 \frac{(\mathbf{r}_s,\mathbf{e}_1)(\mathbf{r}_s,\mathbf{e}_3)}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2 - (\mathbf{r}_s^2 - R^2)}} + \mathbf{e}_1(\mathbf{r}_s,\mathbf{e}_3) - \mathbf{e}_3 \mathrm{t} \right\} \delta\alpha +$$

$$+ \left\{ \mathbf{e}_2 \mathrm{t} - \mathbf{e}_1 \frac{(\mathbf{r}_s,\mathbf{e}_1)(\mathbf{r}_s,\mathbf{e}_2)}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2 - (\mathbf{r}_s^2 - R^2)}} - \mathbf{e}_1(\mathbf{r}_s,\mathbf{e}_2) \right\} \delta\beta +$$

$$+ \left\{ \mathbf{E}_{3\times 3} - \mathbf{e}_1\mathbf{e}_1{}^T + \frac{\mathbf{e}_1\mathbf{r}_s{}^T}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2 - (\mathbf{r}_s^2 - R^2)}} - (\mathbf{r}_s,\mathbf{e}_1) \frac{\mathbf{e}_1\mathbf{e}_1^T}{\sqrt{(\mathbf{r}_s,\mathbf{e}_1)^2 - (\mathbf{r}_s^2 - R^2)}} \right\} \delta\mathbf{r}_s$$

$$|\delta\mathbf{r}_p| \approx |\delta\mathbf{r}_s| + \rho\,|\delta\boldsymbol{\psi}|$$

Image quality

$$\delta V_{\parallel} = \left| \frac{f}{\rho} \mathbf{e}_3^T \left\{ ([\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,1-3} + [\boldsymbol{\rho}]_\times \mathbf{A}^T [\boldsymbol{\omega}]_\times \mathbf{N}) \delta \boldsymbol{\psi} + [\boldsymbol{\rho}]_\times \mathbf{A}^T \delta \boldsymbol{\omega} + \right.\right.$$

$$\left.\left. + ([\mathbf{A}^\mathbf{T} \boldsymbol{\omega}]_\times + [\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,7-9}) \delta \mathbf{r}_s - \delta \mathbf{V}_s \right\} + \frac{f}{\rho} \left\{ (\mathbf{V}_{rel}, \mathbf{e}_1) \delta \alpha - (\mathbf{V}_{rel}, \mathbf{e}_2) \delta \gamma \right\} \right|$$

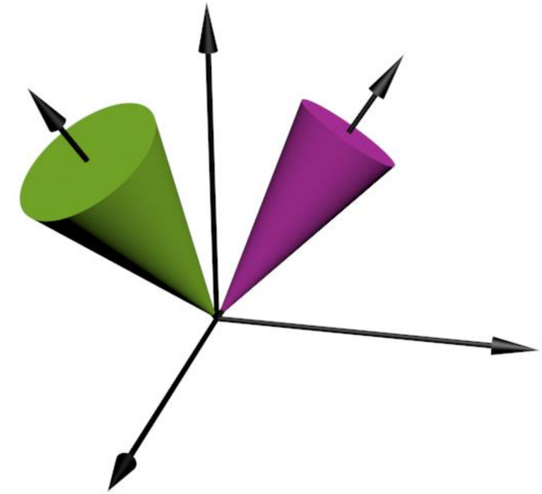
$$\delta V_{\perp} = \left| \frac{f}{\rho} \mathbf{e}_2^T \left\{ ([\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,1-3} + [\boldsymbol{\rho}]_\times \mathbf{A}^T [\boldsymbol{\omega}]_\times \mathbf{N}) \delta \boldsymbol{\psi} + [\boldsymbol{\rho}]_\times \mathbf{A}^T \delta \boldsymbol{\omega} + \right.\right.$$

$$+ ([\mathbf{A}^\mathbf{T} \boldsymbol{\omega}]_\times + [\boldsymbol{\Omega} - \mathbf{A}^T \boldsymbol{\omega}]_\times \mathbf{D}_{1-3,7-9}) \delta \mathbf{r}_s - \delta \mathbf{V}_s \Big\} - \frac{f}{\rho} (\mathbf{V}_{rel}, \mathbf{e}_1) \delta \beta -$$

$$\left. + (\mathbf{V}_{rel}, \mathbf{e}_2) \frac{f}{\rho^3} \boldsymbol{\rho}^T \delta \mathbf{r}_s - (\mathbf{V}_{rel}, \mathbf{e}_2) \frac{f}{\rho^3} \boldsymbol{\rho}^T \mathbf{D} \delta \mathbf{x} \right|$$

$$|\delta V| \approx f\, |\delta \boldsymbol{\omega}|$$

Case of restricted zones



Have one or several restricted directions

Problem: perform slew maneuver and avoid restricted zones

$$V = \frac{1}{2}(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}) + k_a\left(1 - (\mathbf{D}\mathbf{n}_{ref}, \mathbf{n})\right) f$$

$$f = 1 + \sum_{k=1}^{n} f_i, \quad f_i = \begin{cases} H_i & \lambda_i < 0 \\ H_i\left(-3\lambda_i^2 + 2\lambda_i^3 + 1\right) & 0 \le \lambda_i \le 1, \\ 0 & 1 < \lambda_i \end{cases} \quad \lambda_i = \frac{\operatorname{acos}(\mathbf{n}, \mathbf{h}_i) - a_i}{b_i - a_i}$$

$$\mathbf{M}_{ctrl} = -M_{ext} - k_\omega \boldsymbol{\omega}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - k_a\left[\mathbf{n}_{ref} \times \mathbf{n}\right] +$$

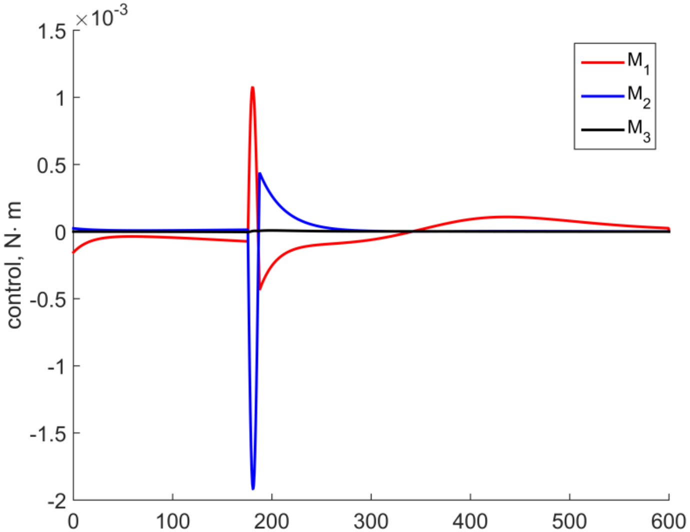
Standard control

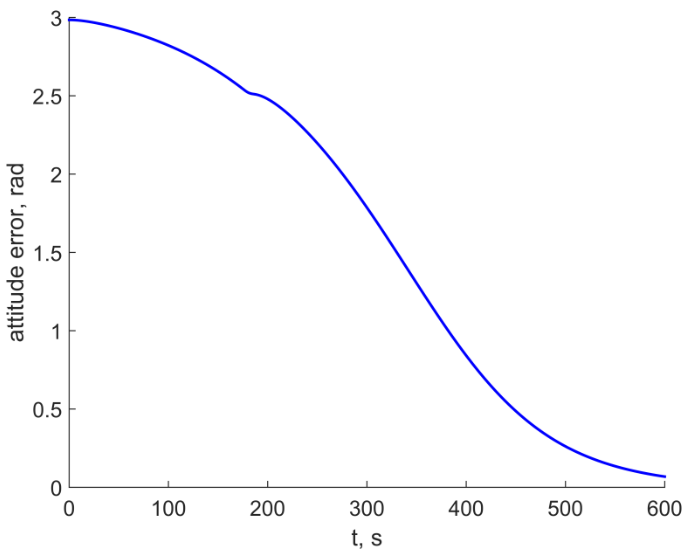
$$+ k_a\left(1 - \left(\mathbf{n}_{ref}, \mathbf{n}\right)\right) \sum_{i=1}^{n} \frac{H_i\left(6\lambda_i^2 - 6\lambda_i\right)}{b_i - a_i} \frac{\mathbf{n} \times \mathbf{h}_i}{\sqrt{1 - \left(\mathbf{n}, \mathbf{h}_i\right)^2}}$$

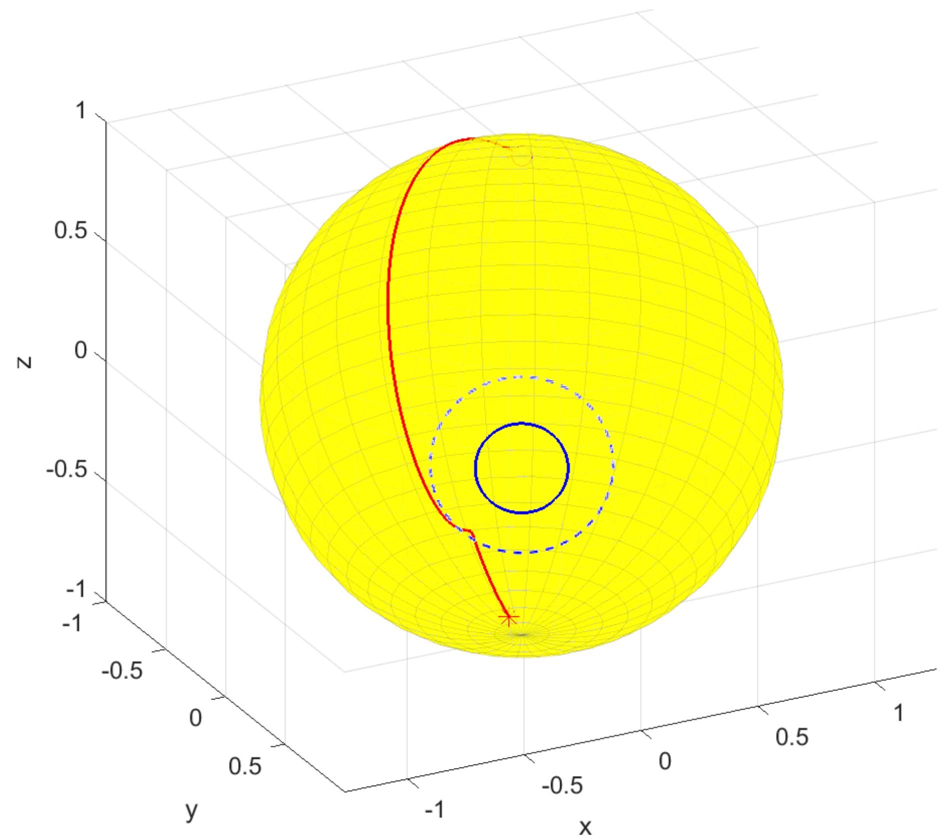
$$- k_a\left[\mathbf{n}_{ref} \times \mathbf{n}\right](f - 1)$$

Restriction areas avoidance

Simulation results



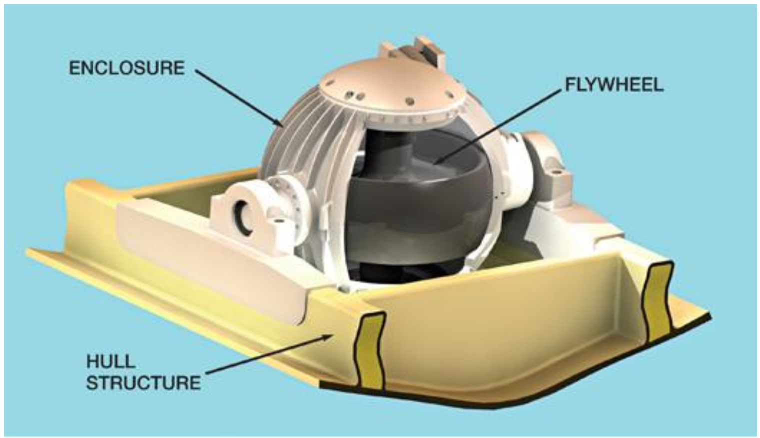




Gyroscopes

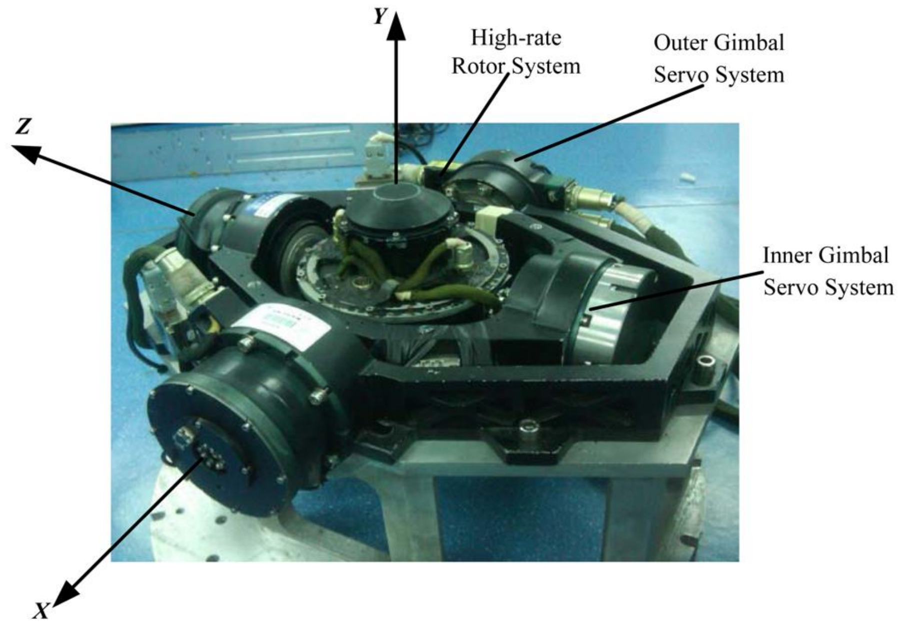
Axis of rotation can change its direction

Single gimbal control momentum gyro (SGCMG)



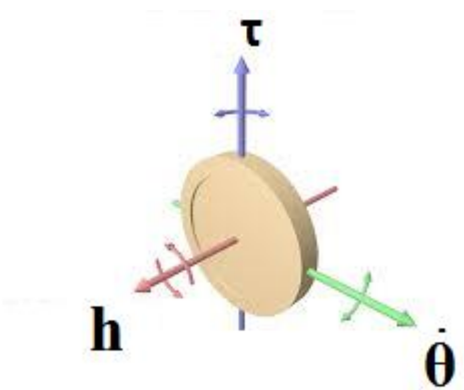
Main dynamical properties:

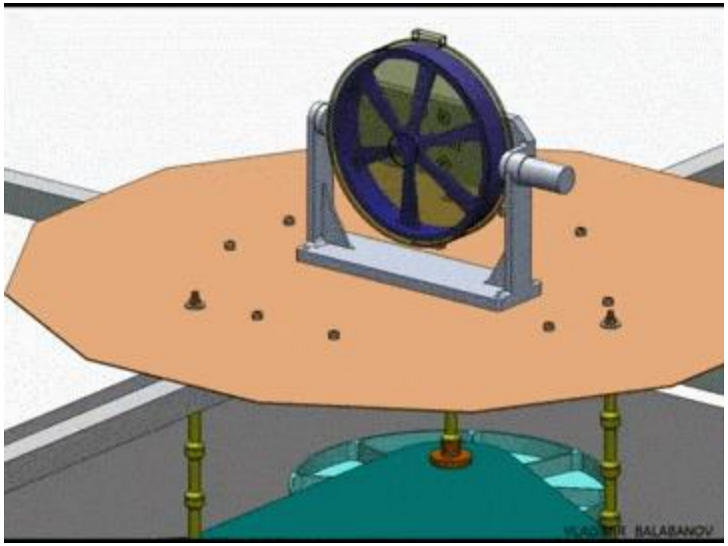
flywheel angular momentum
maximum gimbal spin rate



Dual gimbal control momentum gyro (DGCMG)

Single gimbal CMG



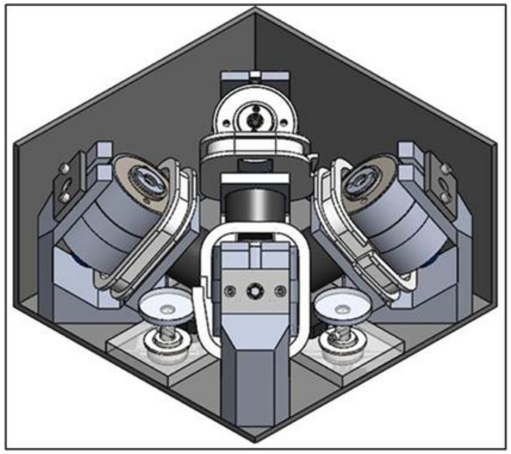


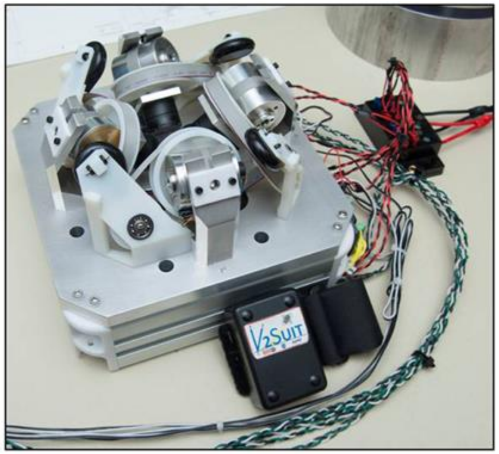
$$\boldsymbol{\tau} = \dot{\boldsymbol{\theta}} \times \mathbf{h}$$

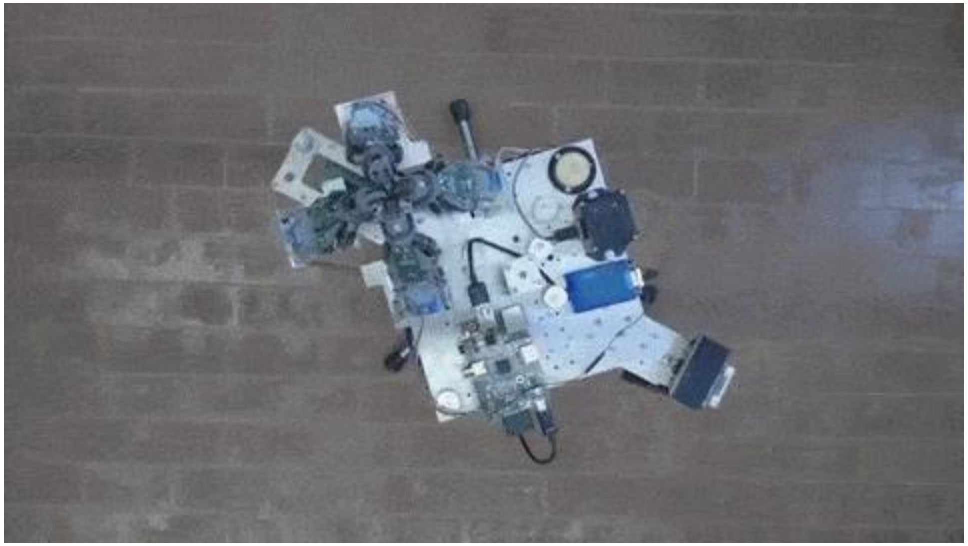
https://www.honeybeerobotics.com

Single gimbal CMG

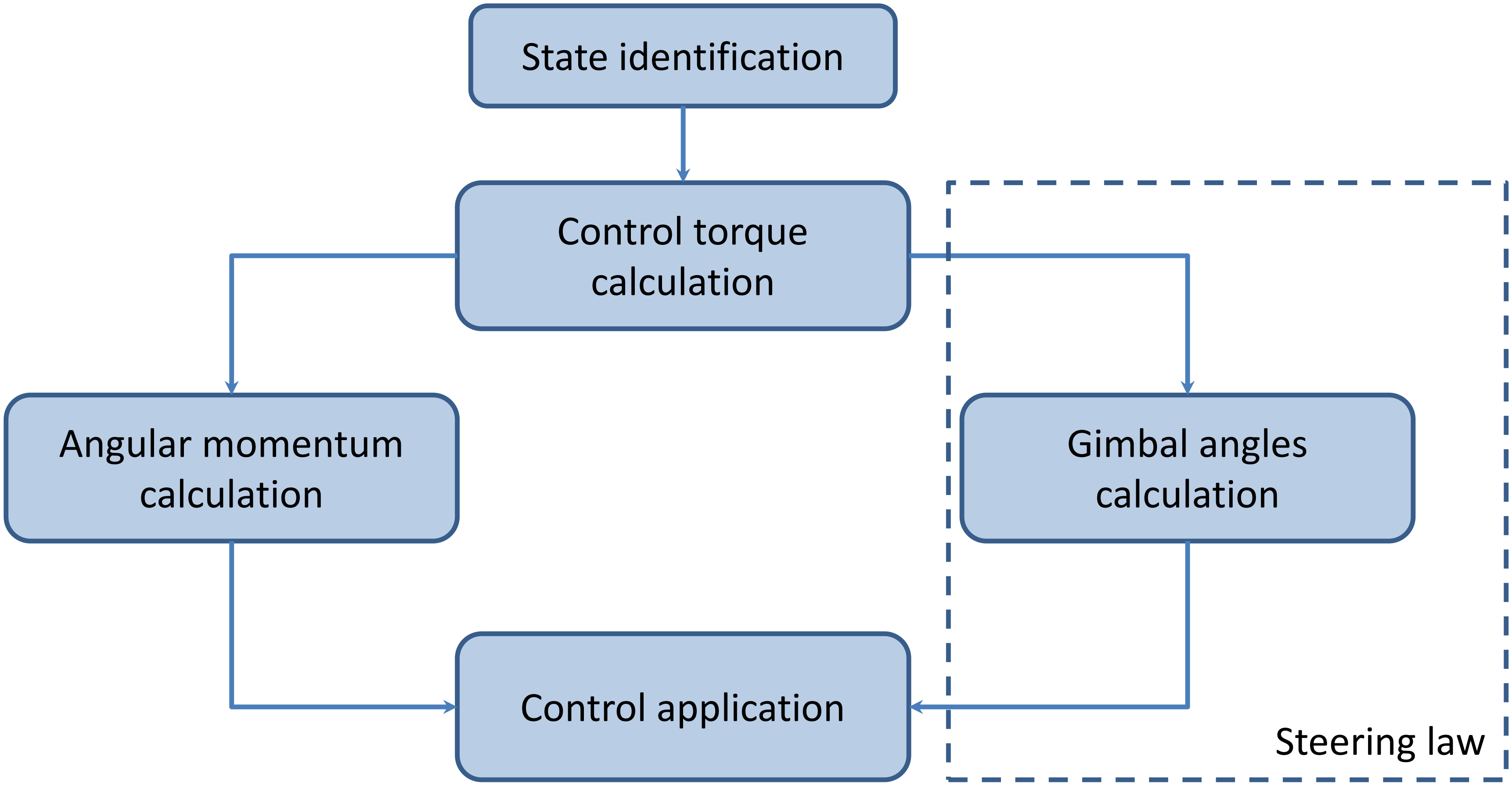
Pyramid type







Attitude control system operation



Steering law & singularities

$$\boldsymbol{\tau} = \dot{\mathbf{H}} = \mathbf{F}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \qquad \dim \mathbf{F}(\boldsymbol{\theta}) = 3 \times n$$

$$rank\,\mathbf{F} \geq 3$$

$$\dot{\boldsymbol{\theta}} = \mathbf{F}^{+}\dot{\mathbf{H}}$$

$$\mathbf{F}^{+} = \mathbf{F}^{T}\left(\mathbf{F}\mathbf{F}^{T}\right)^{-1}$$

$$rank\,\mathbf{F} < 3$$

$$\Downarrow$$

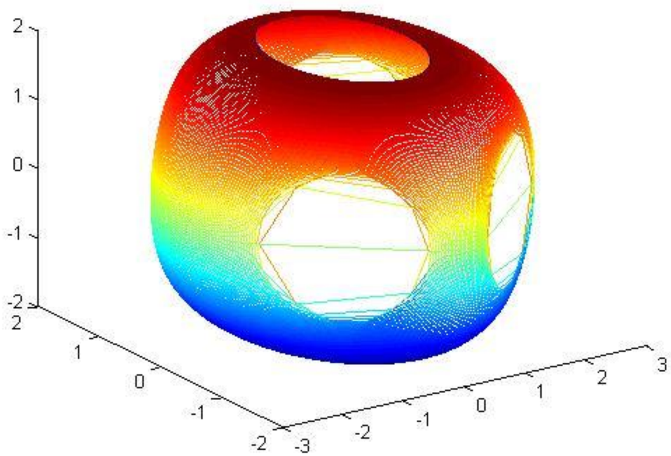
Lost controllability

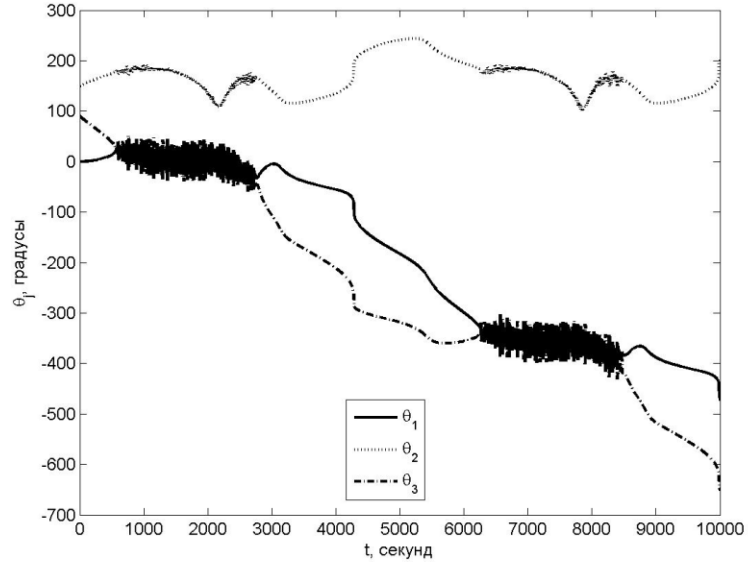
$$\Downarrow$$

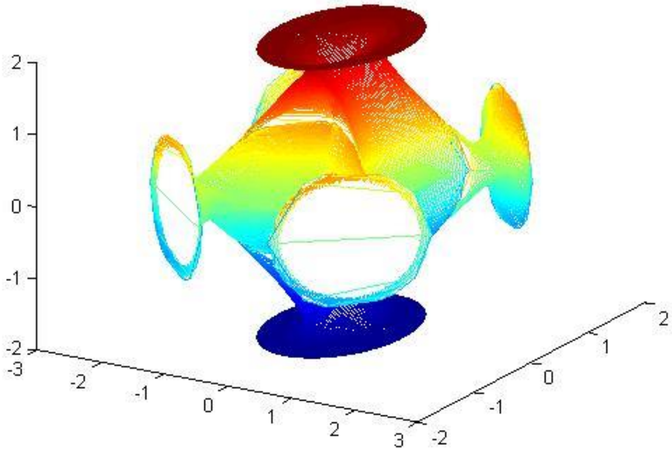
Singularity avoidance algorithm

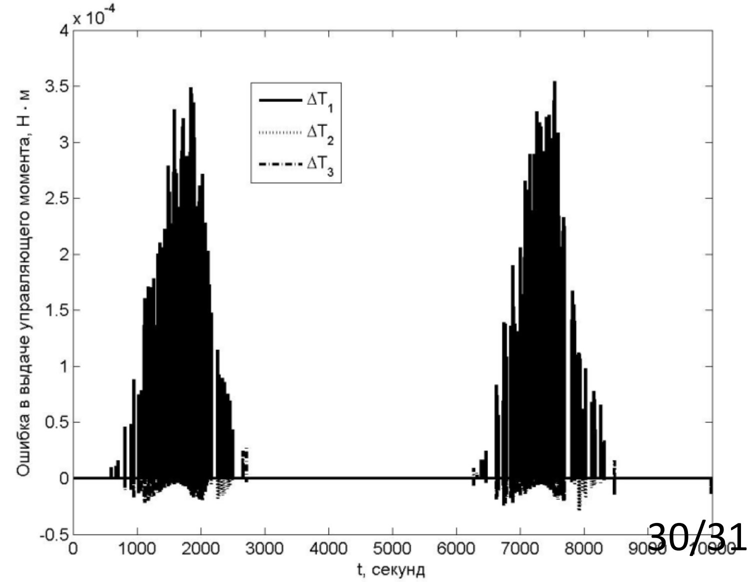
$$\dot{\boldsymbol{\theta}} = \left(\mu\mathbf{E}_3 + \mathbf{F}^{T}\mathbf{F}\right)^{-1}\mathbf{F}^{T}\dot{\mathbf{H}} = \mathbf{F}^{T}\left(\mathbf{F}\mathbf{F}^{T} + \mu\mathbf{E}_3\right)^{-1}\dot{\mathbf{H}}$$

Numerical example









Desaturation

Control can be created only when angular rate is varied

$\Downarrow$

You lost controllability when you can't change the angular rate

$\Downarrow$ Too small spin rate

$\Downarrow$ Too high spin rate
Saturation

$\Downarrow$

Need additional algorithms for desaturation

$\Leftarrow$ Magnetic control system $\mathbf{m} = \mathbf{k}\mathbf{H} \times \mathbf{B}$

$\Leftarrow$ Thrusters