Modern means of spacecraft attitude control

Mashtakov Yaroslav,
junior researcher

Keldysh Institute of Applied Mathematics of RAS

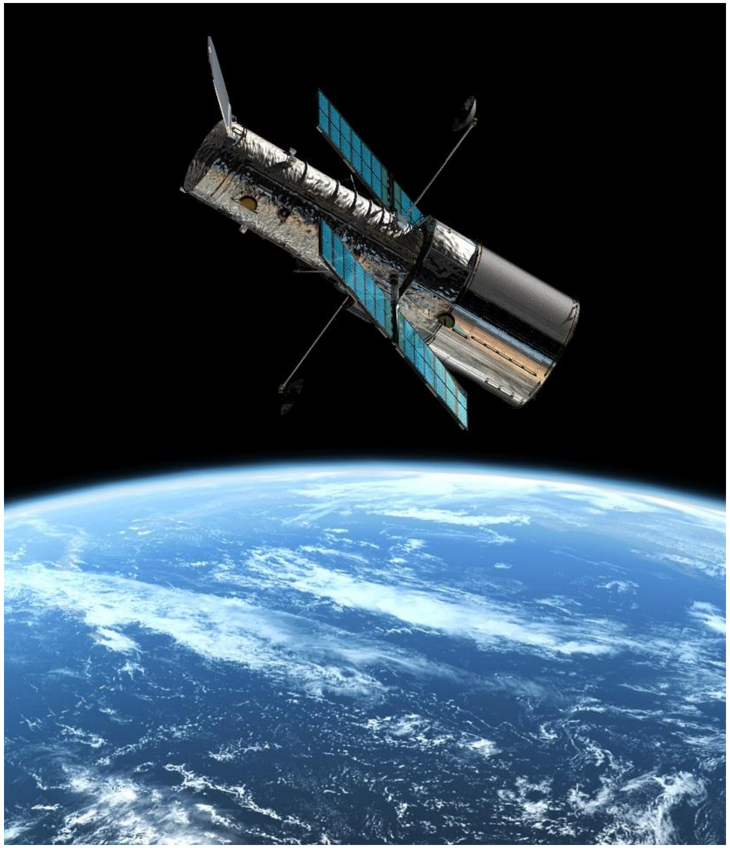
Contents

- Introduction
- Different means of attitude control
  - Gravity-gradient stabilization
  - Thrusters
  - Magnetic torquers
  - Reaction wheels
- Control moment gyros
  - Control algorithms
  - Singularity problem
  - Avoidance algorithms

Introduction

A lot of different missions require to maintain a certain attitude in space:

- Astronomical satellite should be pointed to a certain star
- One of communication satellite’s axis must be directed along the local vertical
- Antenna of interplanetary spacecraft must be pointed at Earth to provide information

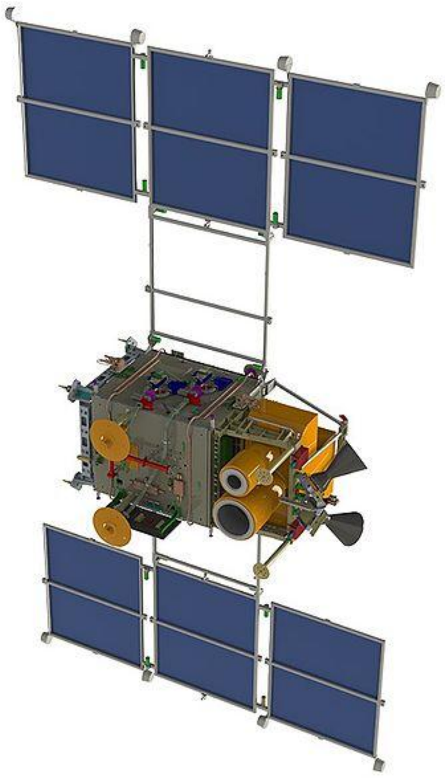


Introduction

Different missions – different accuracy

- Weather observation $\pm 1^o$
- Astronomical observation up to 0,1 arc sec

To provide this extreme accuracy we have to know well current attitude of the satellite and provide even very small control torques.



Introduction

To determine current attitude different sensors can be used:

- Gyroscopes to determine angular velocity
- Sun sensor to find direction to the sun
- Star tracker to determine the attitude
- Magnetometers
- Earth sensors
- Etc.

Star tracker

Gravity-gradient stabilization

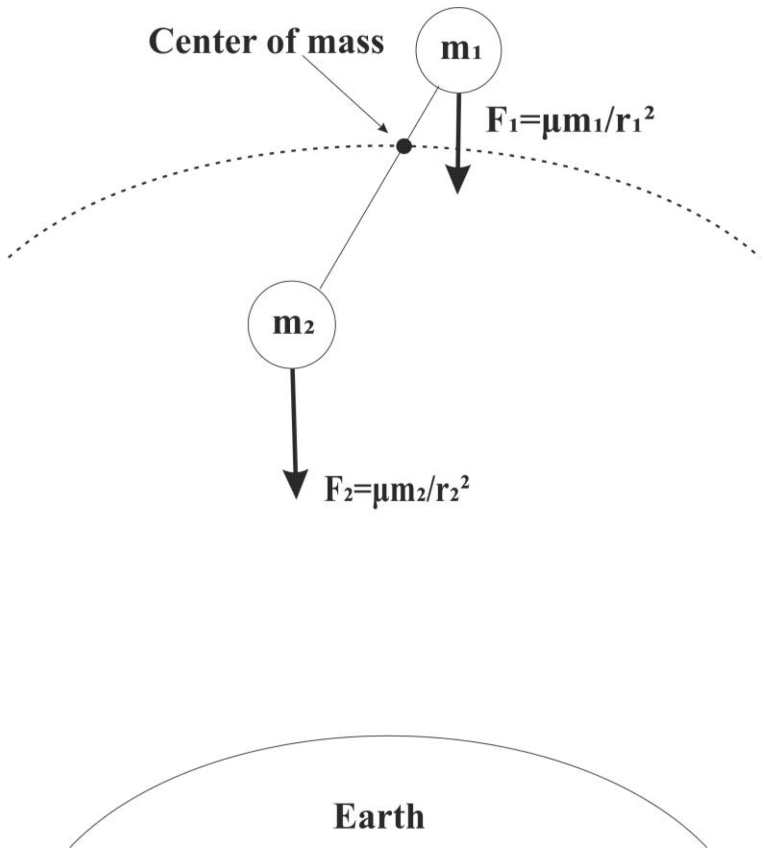
Based on “Tidal Force”: the upper end of satellite feels less gravitational pull than the lower end, so satellite’s axis of minimum moment of inertia is aligned with the local vertical

Advantages:

+ Reliable
+ Simple
+ Doesn’t consume any fuel/energy

Drawbacks:

- Low accuracy
- Satellite’s orientation is fixed



Thrusters

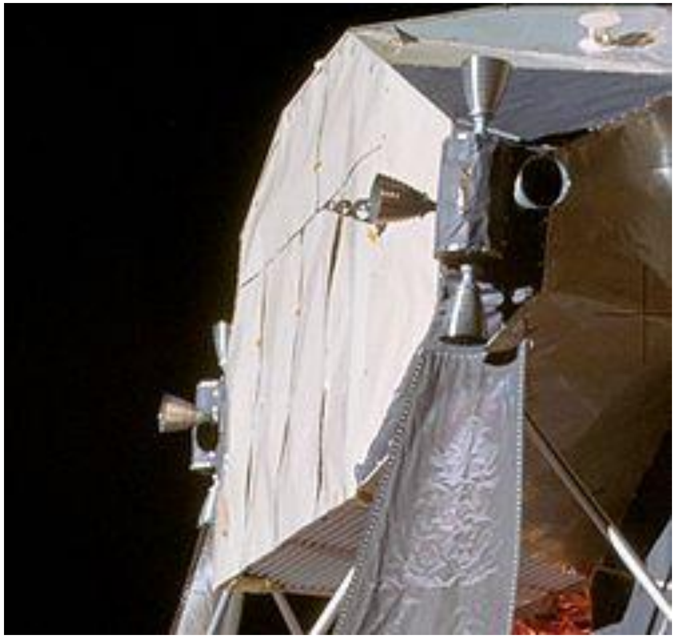
The most common mean of attitude control. They are often used with reaction wheels and CMG to minimize fuel consumption

Advantages:

+ Reliable
+ Simple in use

Drawbacks:

- Consume fuel (and a lot of energy if we use ion thrusters)



RCS blocks on the Apollo Lunar Module

Magnetic torquers

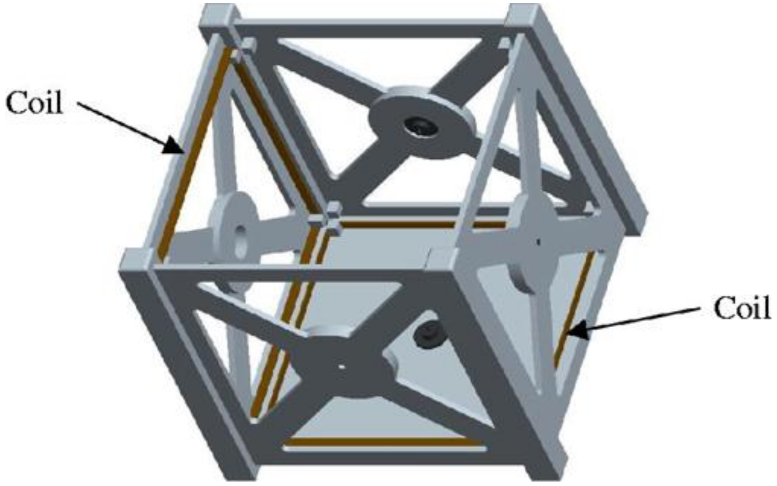
Magnetic torquers include magnetic coils, permanent magnets and hysteresis rods. Usually used as a damper or with CMG and reaction wheels

Advantages:

+ Very light (important for small satellites)
+ Consume small amount of energy and no fuel

Drawbacks :

- Dependence on magnetic field (can be used only on low orbits)
- Provided torques are very limited
- Accuracy is far from satisfactory



Reaction Wheels

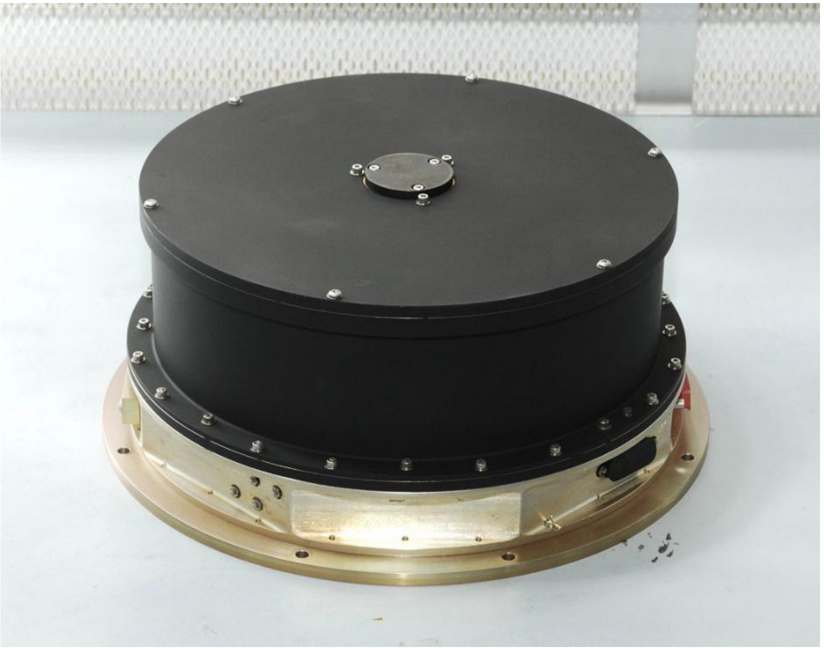
Very common mean of attitude control. Consist of a symmetrical rotor. We can change its spin velocity and thus change its angular momentum.

Advantages:

+ Accuracy
+ Consume no fuel
+ Fast reorientation

Drawbacks:

- Limited rotation speed (we need to cancel stored momentum)
- Other attitude control systems are necessary



Control moment gyros

Consist of reaction wheel with tilting spin axis. There are different types of CMG:

- Single-gimbal
- Dual-gimbal
- Variable speed

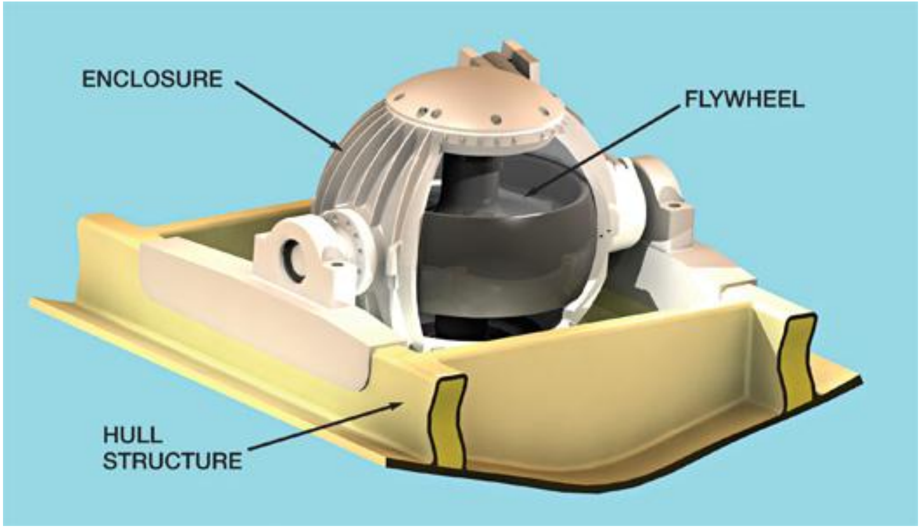
Advantages:

\+ More power efficient than RW

\+ All advantages of RW

Drawbacks:

\- Geometric singularity



Control algorithms

Changing of angular momentum of the satellite:

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}_d, \ \ \mathbf{K} = \mathbf{K}_{\mathbf{sat}} + \mathbf{h}_{CMG}, \ \ \mathbf{h}_{CMG} = \sum_{i=1}^{N} \mathbf{h}_i(\theta_i(\mathrm{t}))$$

After substitution:

$$\frac{d\mathbf{K}_{\mathbf{sat}}}{dt} + \boldsymbol{\omega} \times \mathbf{K}_{\mathbf{sat}} = \mathbf{M}_{dist} + \mathbf{M}_{control}, \ \ \mathbf{M}_{control} = -\frac{d\mathbf{h}_{CMG}}{dt} - \boldsymbol{\omega} \times \mathbf{h}_{CMG}$$

$\mathbf{M}_{control}$ can be obtained, for example, from Lyapunov function:

$$\mathbf{M}_{control} = -\mathbf{M}_{dist} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + \mathbf{J}\mathbf{W}_{rel}\mathbf{A}\boldsymbol{\omega}_0 + \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_0 + k_a\mathbf{S} - k_\omega \boldsymbol{\omega}_{rel}$$

Control algorithms

Control law:

$$\mathbf{M}_{control} = -\frac{d\mathbf{h}_{CMG}}{dt} - \boldsymbol{\omega} \times \mathbf{h}_{CMG}, \frac{d\mathbf{h}_{\mathbf{CMG}}}{dt} = \mathrm{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$$

$$\mathrm{J} = \begin{pmatrix} \frac{\partial \mathbf{h}_1}{\partial \theta_1} & \cdots & \frac{\partial \mathbf{h}_N}{\partial \theta_N} \end{pmatrix}, \dot{\boldsymbol{\theta}} = \begin{pmatrix} \frac{d\theta_1}{dt} & \cdots & \frac{d\theta_N}{dt} \end{pmatrix}^T$$

To simplify let's assume:

$$\boldsymbol{\tau} = \mathrm{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$$

Where $\boldsymbol{\tau}$ is a 3-vector, $\mathrm{J}(\boldsymbol{\theta})$ 3xN matrix and $\dot{\boldsymbol{\theta}}$ is N-vector. The question is: how we can find $\dot{\boldsymbol{\theta}}$ if we know $\boldsymbol{\tau}$?

Singularity problem

If J is a square matrix and $\det(\mathrm{J}) \neq 0$ we can find $\dot{\boldsymbol{\theta}}$:

$$\dot{\boldsymbol{\theta}} = \mathrm{J}^{-1}(\boldsymbol{\theta})\boldsymbol{\tau}$$

But usually there are 4 or more CMGs, so we can't do that.

If $Rg(\mathrm{J}) = 3$ we can use so called pseudoinverse:

$$\dot{\boldsymbol{\theta}}_p = \mathrm{J}^{+}(\boldsymbol{\theta})\boldsymbol{\tau},\ \ \mathrm{J}^{+} = \mathrm{J}^{T}(\mathrm{J}\mathrm{J}^{T})^{-1}$$

But there is a problem when Rg(J) < 3: $\det(\mathrm{J}\mathrm{J}^{T}) = 0$

To avoid it we can use different methodic, for example null motion:

$$\dot{\boldsymbol{\theta}}_{Null} : \ \ \mathrm{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}_{Null} = 0$$

Singularity problem

SVD: any matrix can be decomposed in

$$M_{m\times n} = U_{m\times m}\mathbf{\Sigma}_{m\times n}V^T_{n\times n}, U^{-1} = U^T, V^{-1} = V^T,$$
$$\mathbf{\Sigma} = diag(\sigma_1, \ldots \sigma_m), \sigma_i \ge \sigma_{i+1}, \sigma_i \ge 0$$

So we can obtain J^+ :

$$\mathrm{J}_{3\times N} = U_{3\times 3}\mathbf{\Sigma}_{3\times N}V_{N\times N}{}^T$$
$$\mathrm{J}_{3\times N}{}^+ = V\mathbf{\Sigma}^+U^T, \mathbf{\Sigma}^+ = diag_{N\times 3}(\sigma_1{}^{-1}, \sigma_2{}^{-1}, \sigma_3{}^{-1}).$$

If $\sigma_3 = 0 \Leftrightarrow Rg(\mathrm{J}) < 3$, we cant calculate J^+.

Avoidance algorithms

But by adding some error in control law we can avoid that obstacle:

$$\Sigma^{+} \rightarrow \Sigma^{*} = diag((\sigma_1 + \alpha)^{-1}, (\sigma_2 + \alpha)^{-1}, (\sigma_3 + \alpha)^{-1})$$

Which is equal to

$$\dot{\boldsymbol{\theta}} = \mathrm{J}^{T}(\mathrm{J}\mathrm{J}^{T} + \alpha I_3)^{-1}\boldsymbol{\tau}$$

Even when we are near the singularity (equal to $\det(\mathrm{J}\mathrm{J}^{T}) \rightarrow 0$), we can calculate $(\mathrm{J}\mathrm{J}^{T} + \alpha I_3)^{-1}$ and find .

It can be shown that this expression solves the following minimization problem:

$$\min_{\dot{\boldsymbol{\theta}}}\{\frac{1}{2}\alpha \| \dot{\boldsymbol{\theta}} \| + \frac{1}{2} \| \mathrm{J}\dot{\boldsymbol{\theta}} - \boldsymbol{\tau} \| \}$$

Conclusion

- Different missions require their own set of sensors and actuators: it doesn't necessary install precision star trackers and powerful CMGs on weather observation satellites
- All actuators have drawbacks and advantages