Basics of orbital dynamics and formation flying

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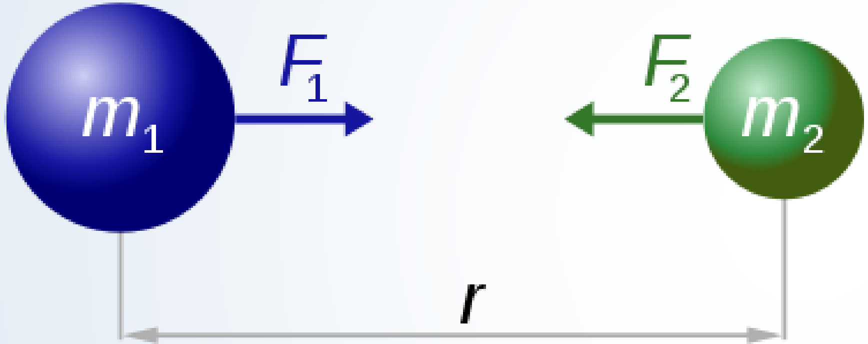
Newton’s gravitation law

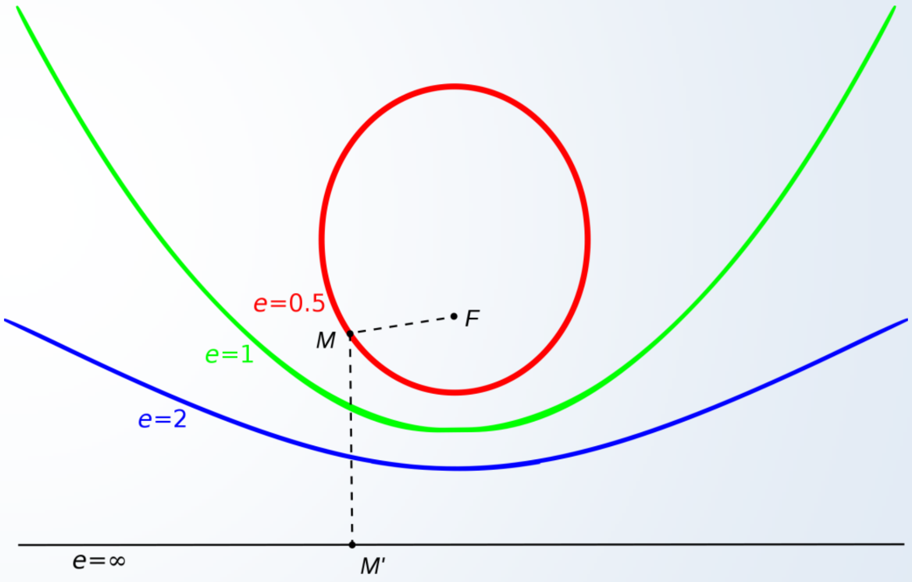


$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

- Objects are attracted by each other
- Attraction force depends on object’s masses and distance between them

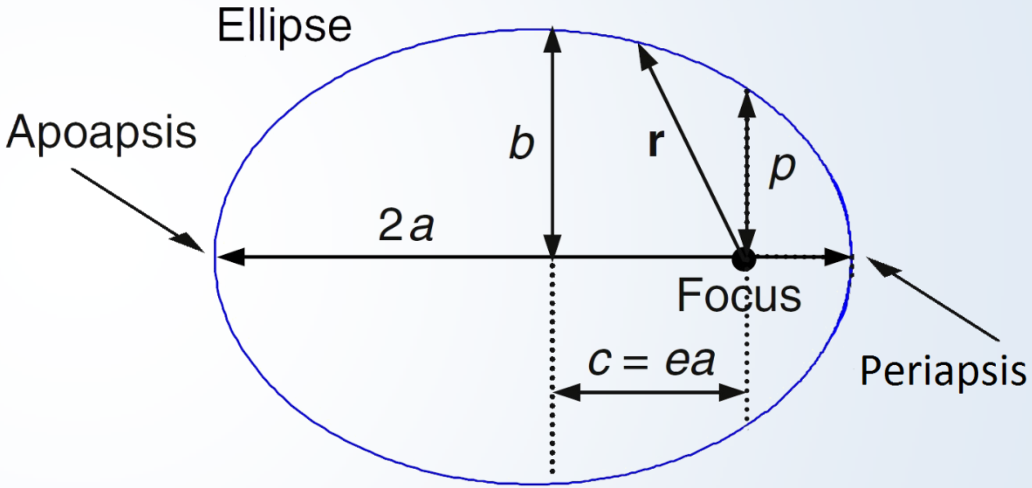
Two-body problem

- Two objects in space – spacecraft and Earth
- The Earth attracts spacecraft
- Spacecraft doesn't attract the Earth
- Orbits without collisions are possible
- Orbits of different bodies in Solar system are conic sections (ellipse, parabola, hyperbola)

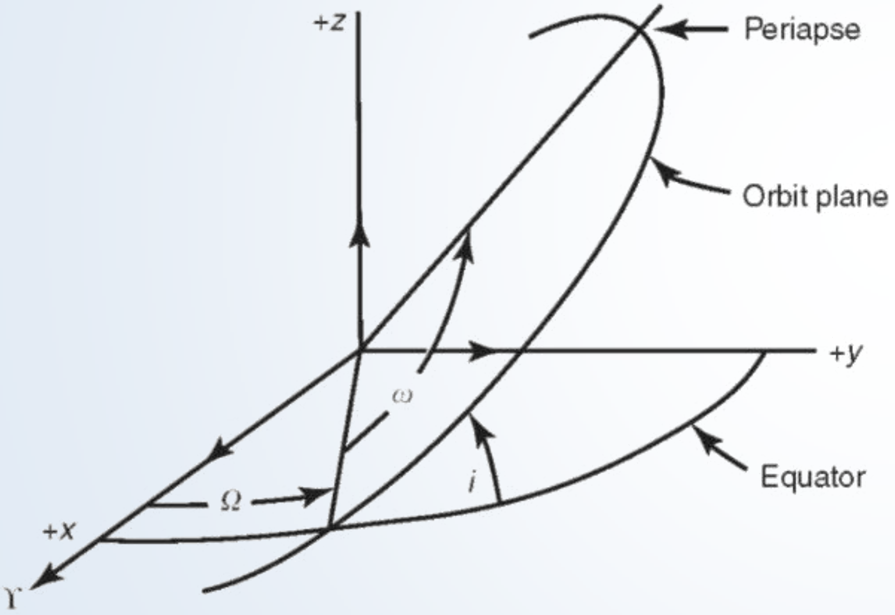


Elliptical orbit

- Periapsis – the point of least distance
- Apoapsis – the point of greatest distance
- Eccentricity *e* - shape of the ellipse, describing how much it is elongated compared to a circle
- Semimajor axis *a* —the sum of the periapsis and apoapsis distances divided by two



Keplerian elements

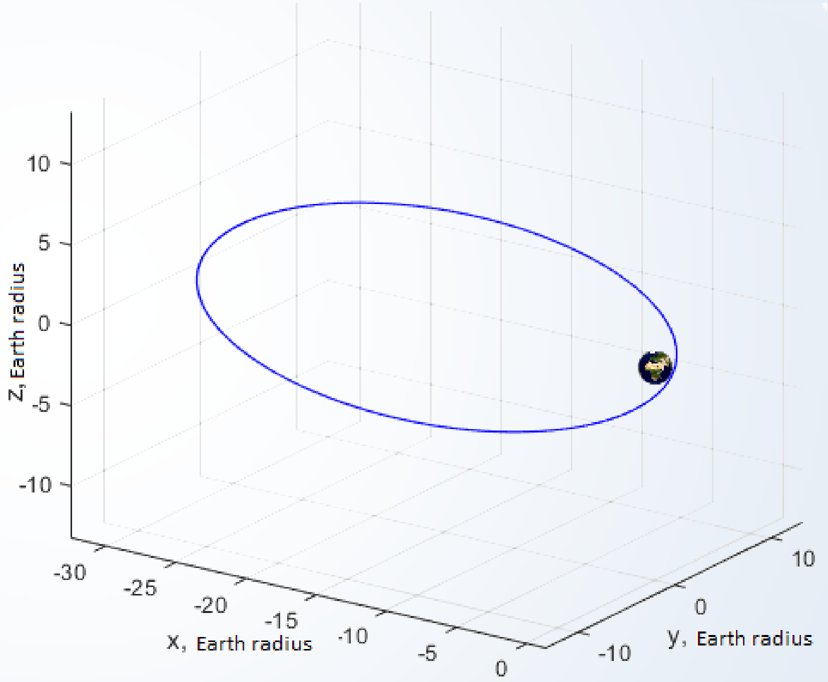


- Inclination i —vertical tilt of the ellipse
- Longitude of the ascending node Ω — horizontally orients the ascending node of the ellipse (where the orbit passes upward through the reference plane)
- Argument of periapsis ω defines the orientation of the ellipse in the orbital plane, as an angle measured from the ascending node to the periapsis.

Passive motion

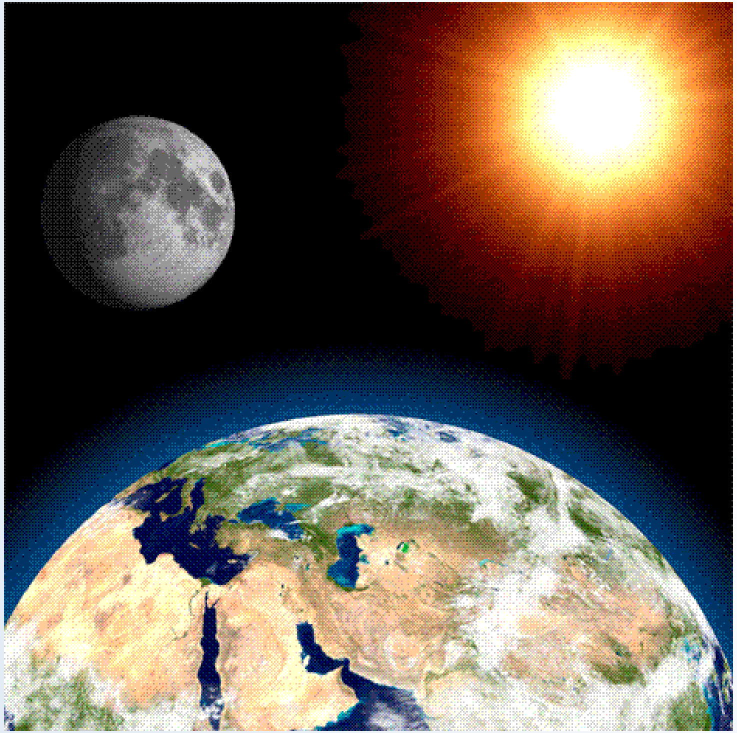
Keplerian setup:

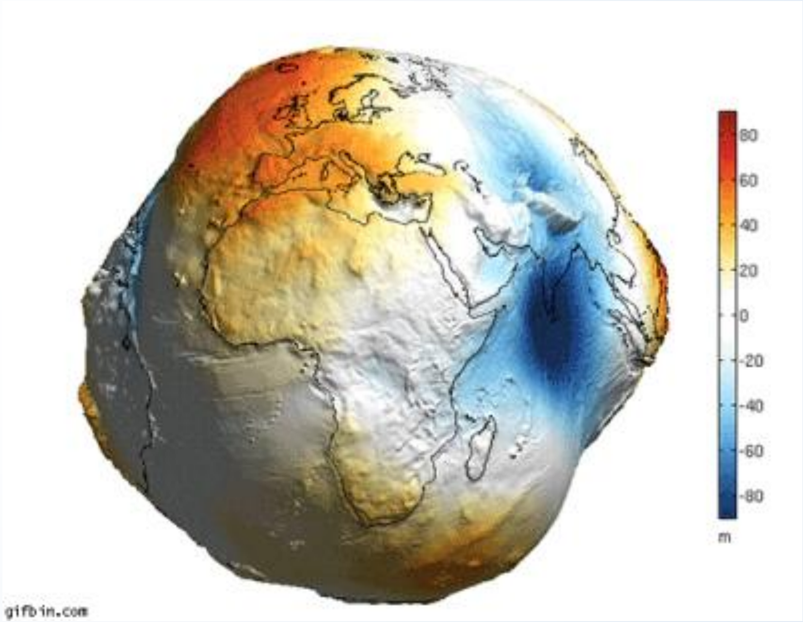
$$
\begin{aligned}
p &= const, \\
e &= const, \\
i &= const, \\
\Omega &= const, \\
\omega &= const, \\
\frac{d\nu}{dt} &= \frac{\sqrt{\mu p}}{r^2}.
\end{aligned}
$$



Bounded orbit

Different perturbations

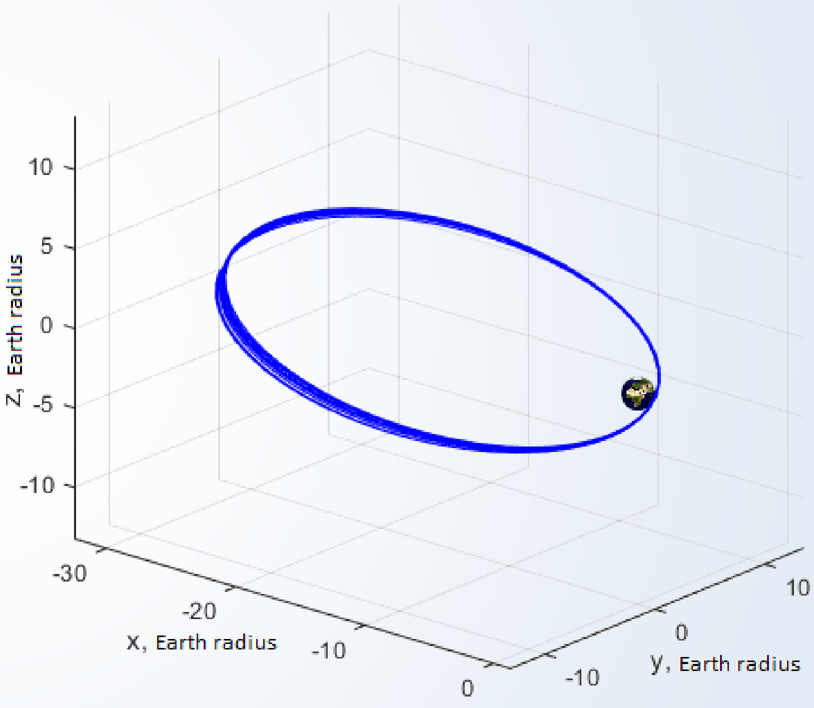




Perturbed motion

"Real" model:

$$
\begin{aligned}
\frac{dp}{dt} &= 2F_y \cdot r\sqrt{\frac{p}{\mu}}, \\
\frac{de}{dt} &= \sqrt{\frac{p}{\mu}}\Big\{F_x \sin\nu + \\
&\quad + F_y\left[\left(1+\frac{r}{p}\right)\cos\nu + e\frac{r}{p}\right]\Big\}, \\
\frac{di}{dt} &= F_z\frac{r}{\sqrt{\mu p}}\cos u, \\
\frac{d\Omega}{dt} &= F_z\frac{r}{\sqrt{\mu p}}\frac{\sin u}{\sin i}, \\
\frac{d\omega}{dt} &= \sqrt{\frac{p}{\mu}}\Big[-F_x\frac{\cos\nu}{e} + \\
&\quad + F_y\left(1+\frac{r}{p}\right)\frac{\sin\nu}{e} - F_z\frac{r}{p}i\sin u\Big], \\
\frac{d\nu}{dt} &= \frac{\sqrt{\mu p}}{r^2} + \frac{p}{e\mu}\left[-F_x\cos\nu - F_y\left(1+\frac{r}{p}\right)\sin\nu\right].
\end{aligned}
$$

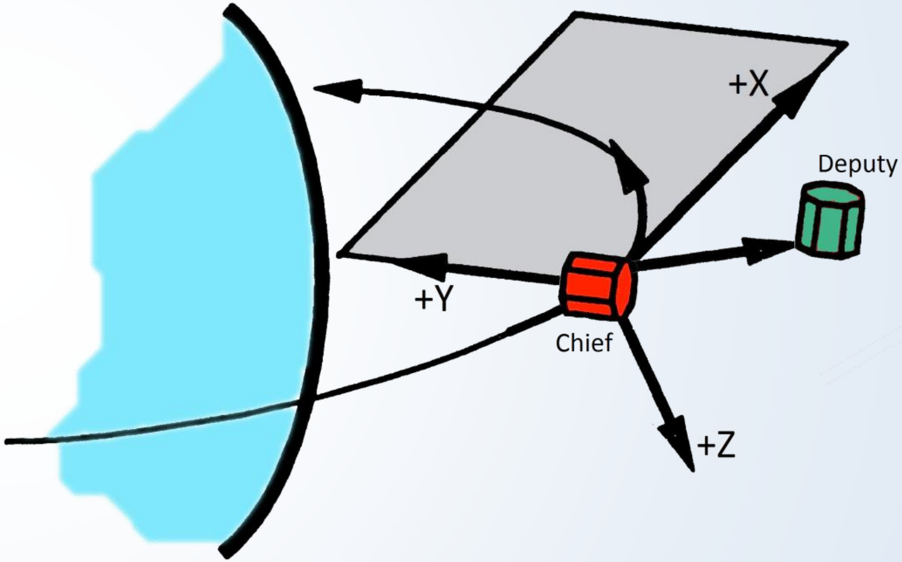


Unbounded orbit

Relative motion

Hill-Clothessy-Wiltshire equations:

$$\ddot{x} - 2n\dot{y} - 3n^2x = 0,$$
$$\ddot{y} + 2n\dot{x} = 0,$$
$$\ddot{z} + n^2z = 0.$$



Relative motion solutions

$$x(t) = \left[4x(0) + \frac{2\dot{y}(0)}{n}\right] + \frac{\dot{x}(0)}{n}\sin(nt) - \left[3x(0) + \frac{2\dot{y}(0)}{n}\right]\cos(nt)$$

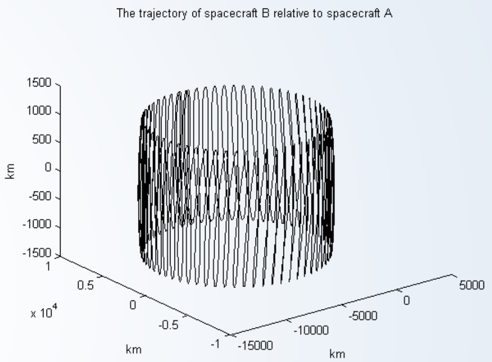
$$y(t) = -[6nx(0) + 3\dot{y}(0)]t + \left[y(0) - \frac{2\dot{x}(0)}{n}\right] + \left[6x(0) + \frac{4\dot{y}(0)}{n}\right]\sin(nt) + \frac{2\dot{x}(0)}{n}\cos(nt),$$

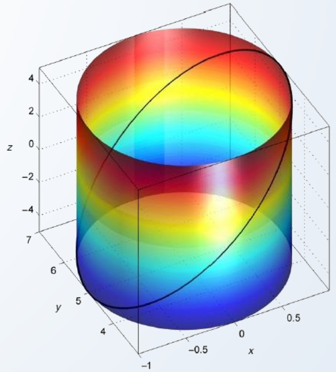
$$z(t) = \frac{\dot{z}(0)}{n}\sin(nt) + z(0)\cos(nt),$$

$$\dot{x}(t) = \dot{x}(0)\cos(nt) + [3x(0)n + 2\dot{y}(0)]\sin(nt)$$

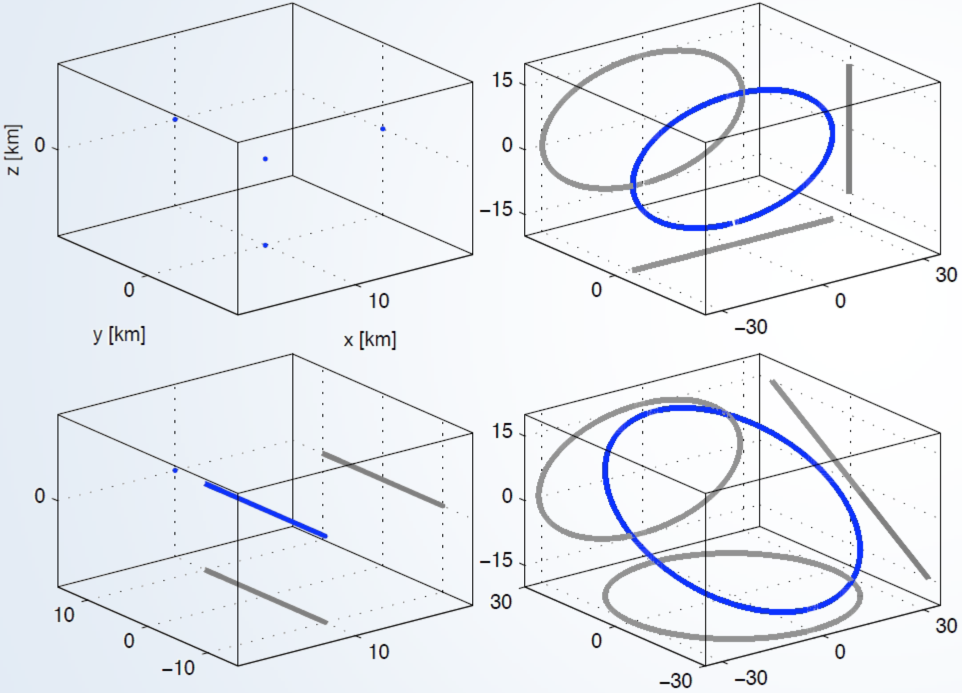
$$\dot{y}(t) = -[6nx(0) + 3\dot{y}(0)] + [6x(0)n + 4\dot{y}(0)]\cos(nt) - 2\dot{x}(0)\sin(nt)$$

$$\dot{z}(t) = \dot{z}(0)\cos(nt) - z(0)n\sin(nt)$$





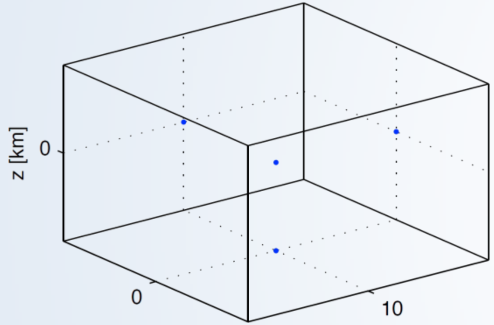
Particular cases



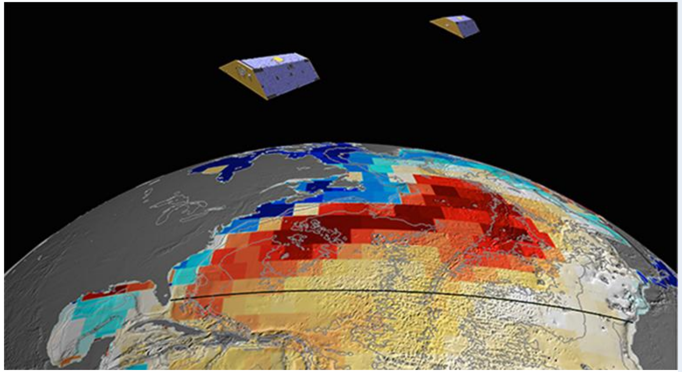
- *Leader-Follower*
- *Pendulum*
- *Cartwheel*
- *General Circular Orbit*

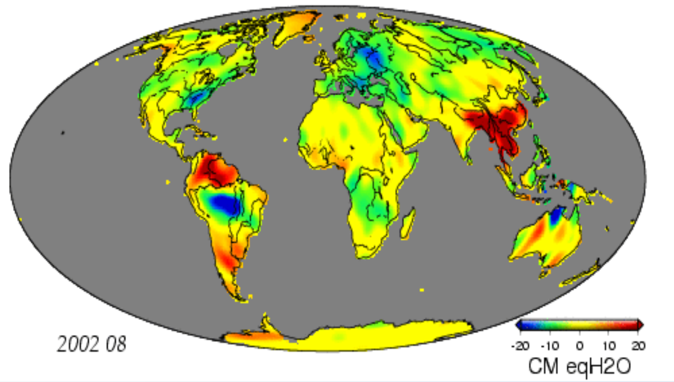
Particular case:
Leader-Follower, mission GRACE

Deputy is not moving relatively to chief:



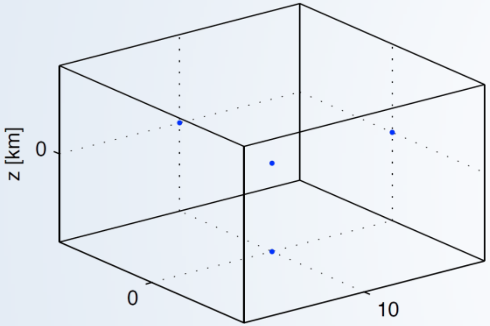
$$x(t) = \rho$$
$$y(t) = 0$$
$$z(t) = 0$$



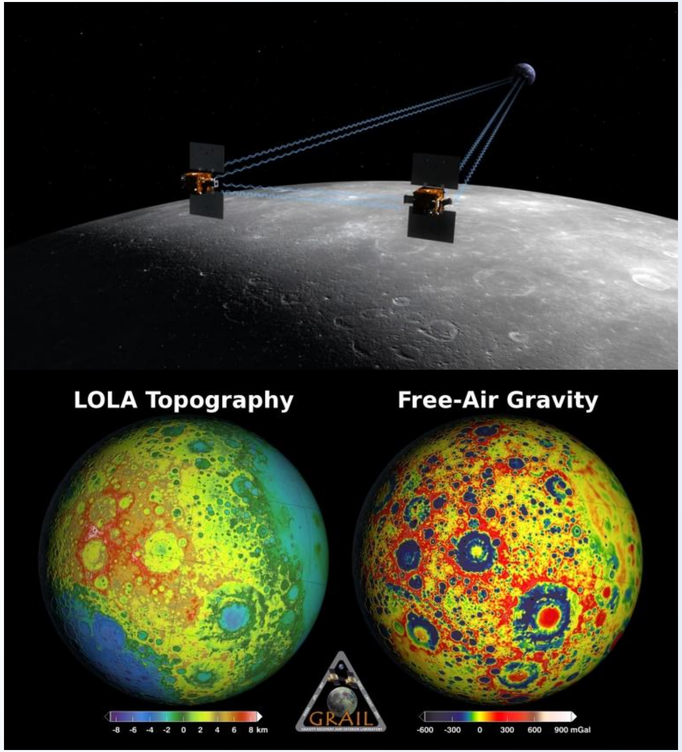


Particular case:
Leader-Follower, mission GRAIL

Deputy is not moving relatively to chief:



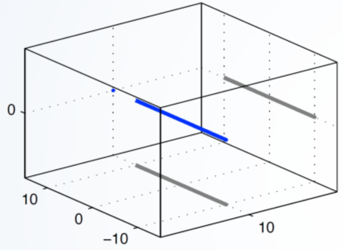
$$x(t) = \rho$$
$$y(t) = 0$$
$$z(t) = 0$$

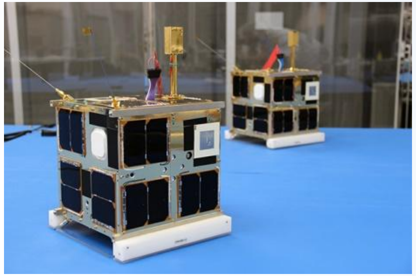


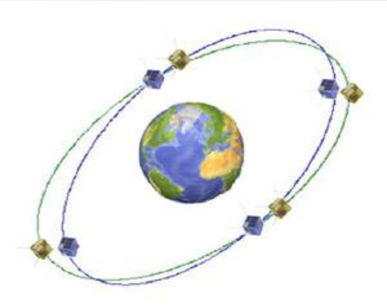
Particular case:
Pendulum&Cartwheel, mission InSAR

Deputy is moving on a line in relation to chief:

$$x(t) = \rho_x$$
$$y(t) = \rho_y \cos(nt + \beta)$$
$$z(t) = 0$$

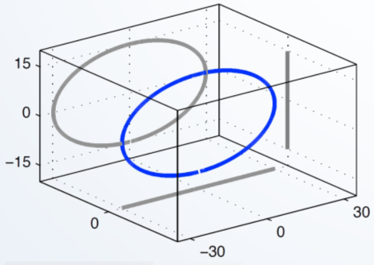


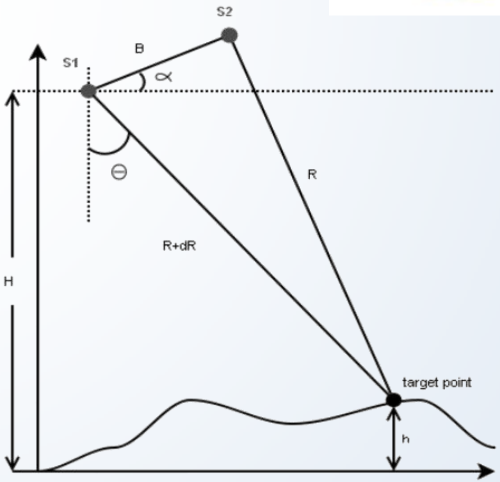




Deputy’s trajectory is a circle:

$$\frac{x^2}{\rho^2} + \frac{y^2}{\frac{3\rho^2}{4}} = 1$$
$$\frac{x^2}{\rho^2} + \frac{y^2}{\frac{\rho^2}{4}} = 1$$
$$y = \sqrt{3}z$$

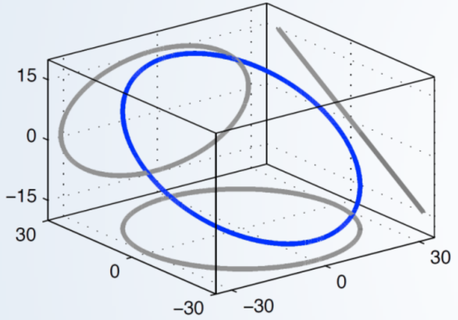




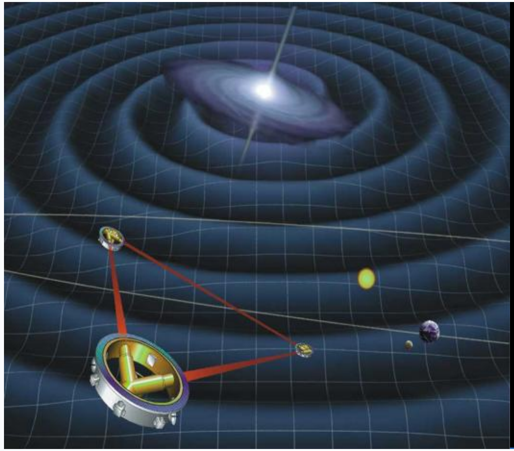
Particular case:

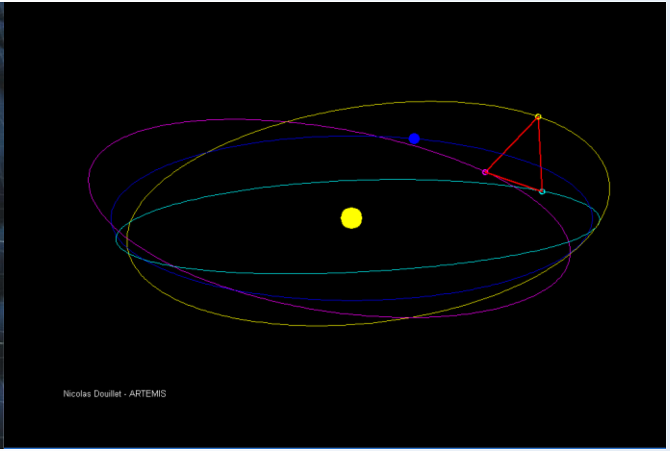
General Circular Orbit (GCO), mission LISA

Deputy is moving around chief:

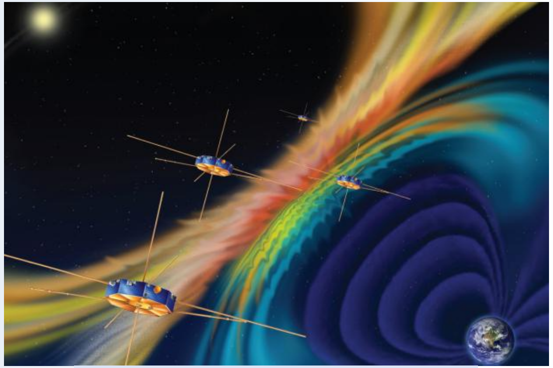


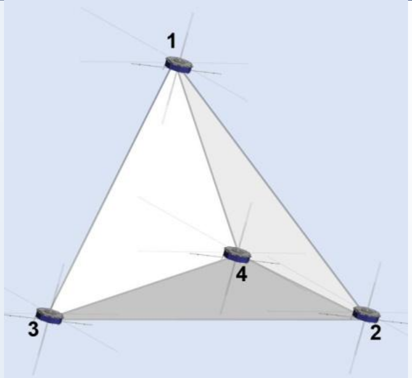
$$\frac{x^2}{4\rho_z^2} + \frac{z^2}{\rho_z^2} = 1$$

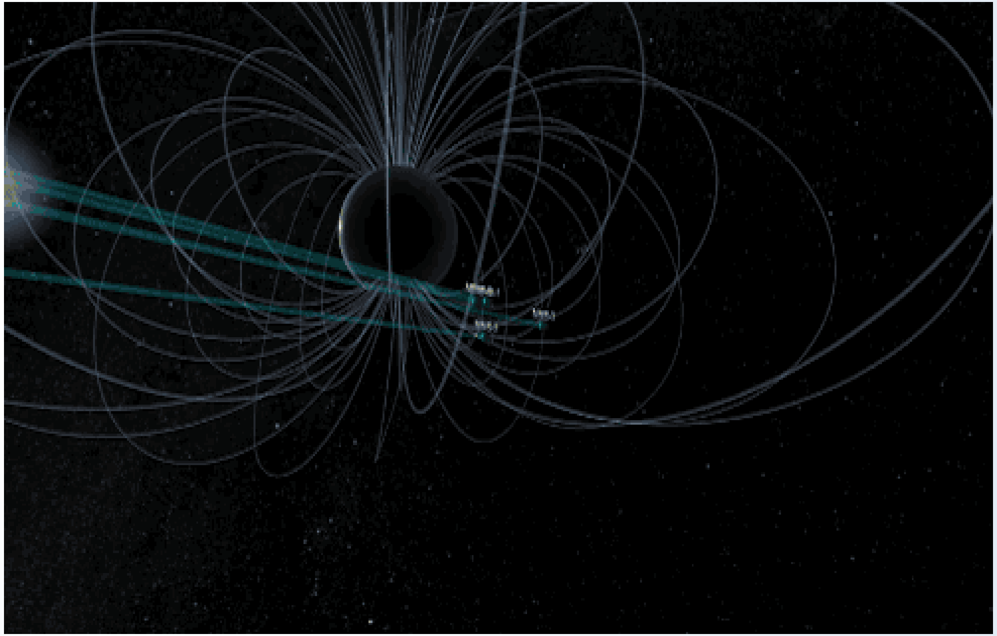




Corresponding missions: MMS







感謝諸位的時間!

Thanks for your attention!

Спасибо за внимание!