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Performance-Invariant Scaling of Square Solar Sails

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- Concept of performance-invariant scaling
- Mass scaling laws for the components of a square sail system
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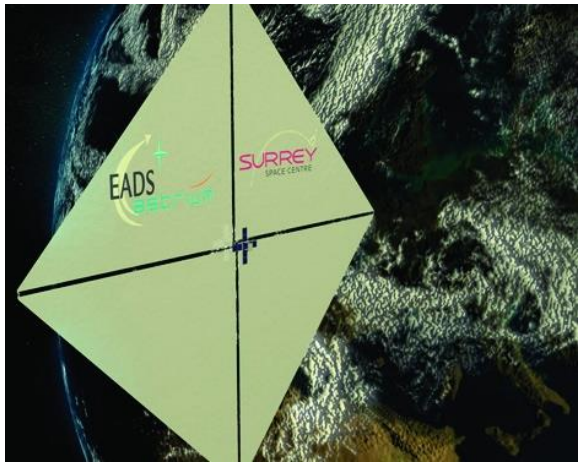
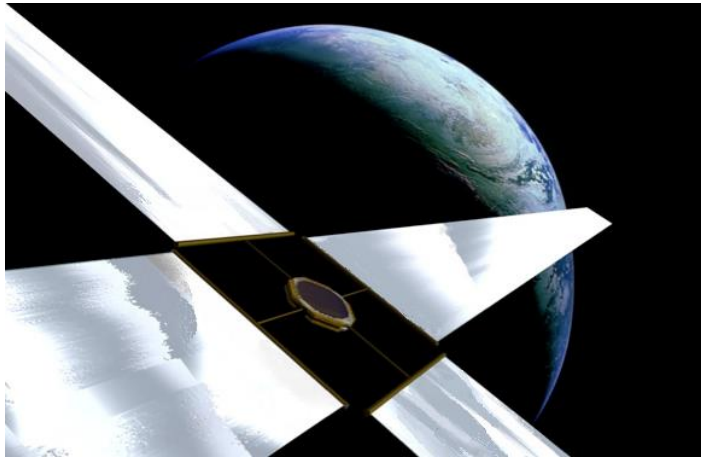
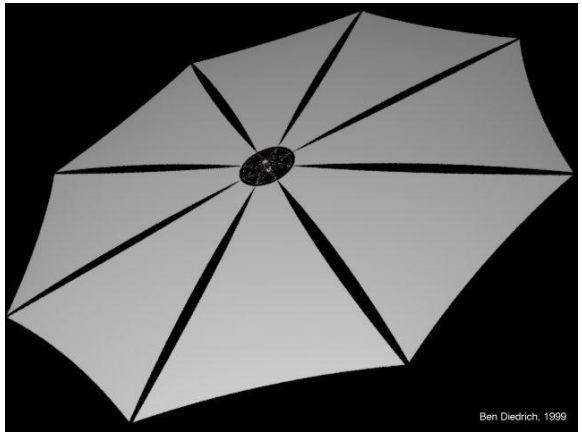
Primary designs of solar sails



Credit: Surrey Space Centre



Credit: R. Blomquist



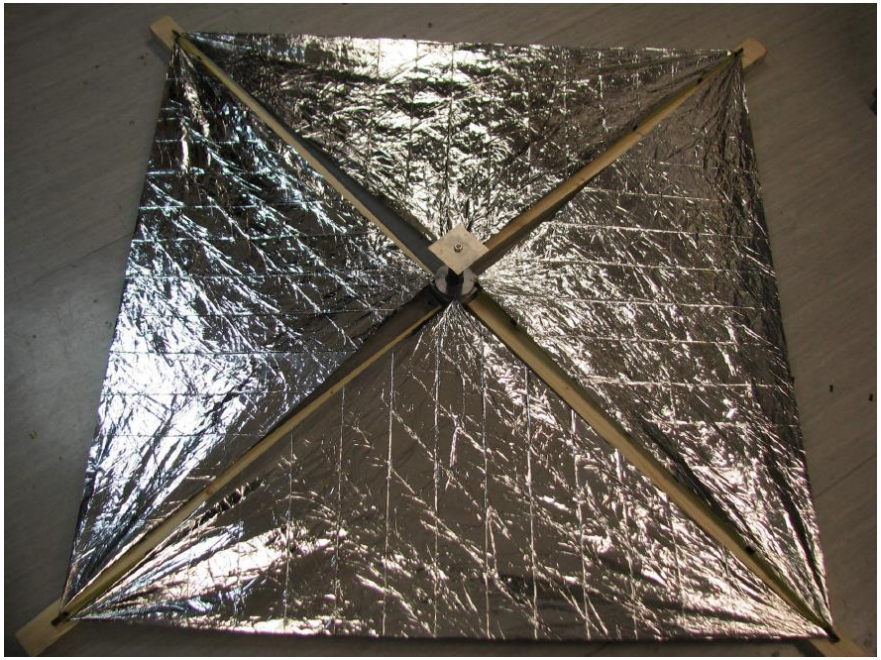
Credit: B. Diedrich

Primary types of solar sails:

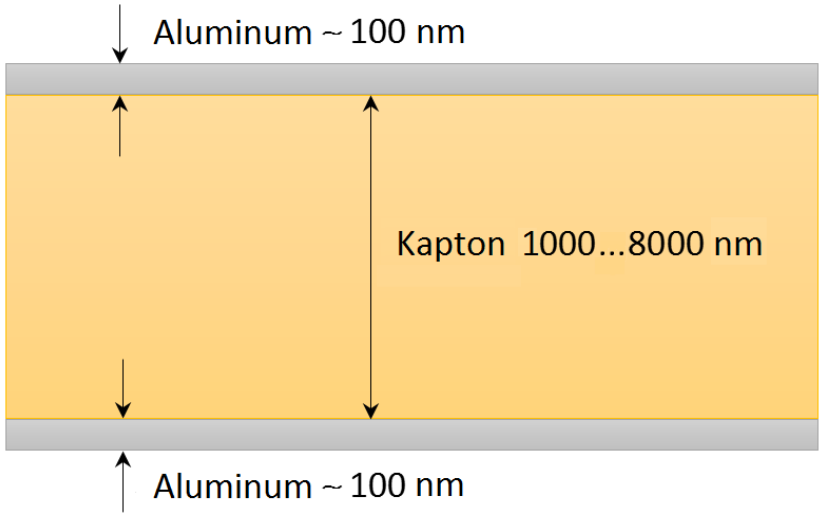
1. Square sail
2. Disk sail
3. Heliogyro

Sail membrane structure

- A solar sail membrane is usually made of some polyimide (Kapton or Mylar).
- The front surface is aluminized in order to better reflect light.



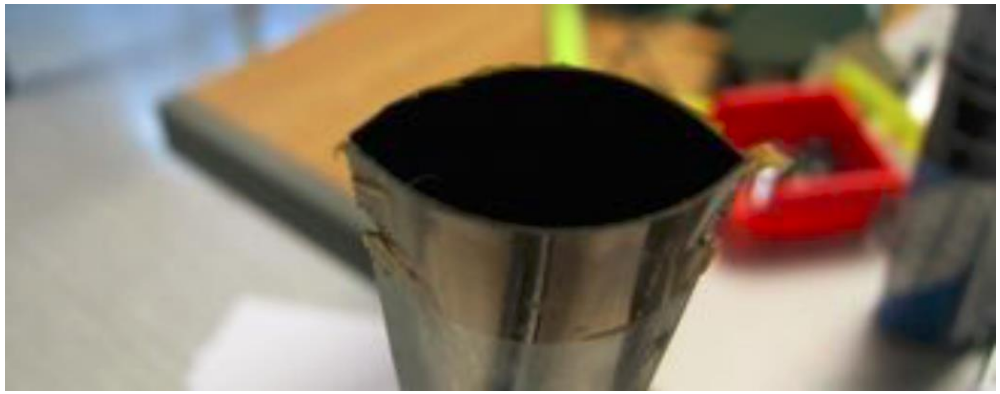
Source: Surrey Space Centre



- The back surface is usually coated with some high-emissivity layer or also aluminized—for LEO sails and if we do not know a priori what surface will be sunlit.

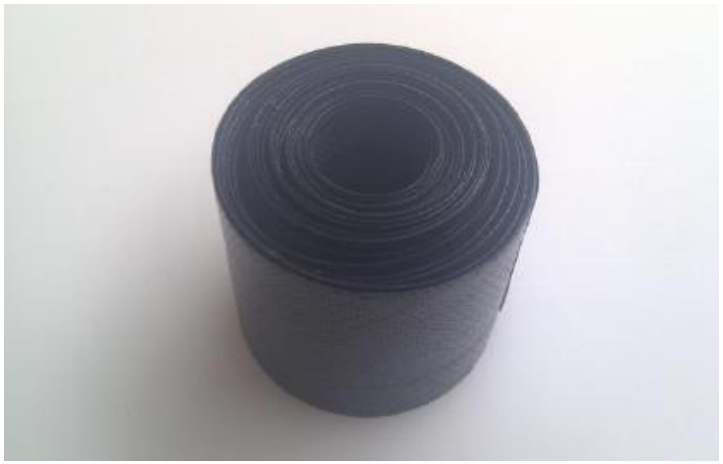
Types of deployable booms

Metallic (e.g., copper-beryllium)

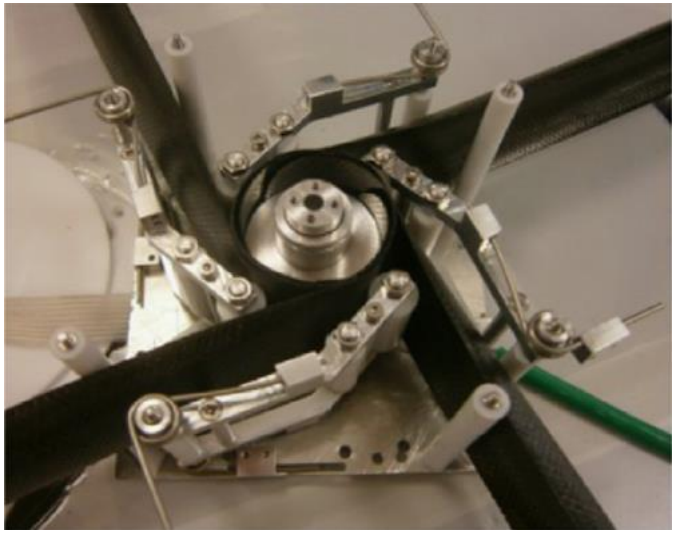


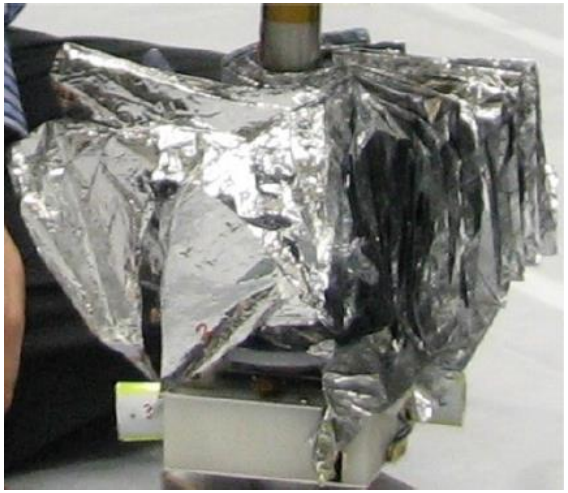
Source: Surrey Space Centre

Carbon fiber reinforced plastic



Sail deployment mechanism





Source: Surrey Space Centre

Solar sail performance characteristics

$$m = m_d + m_m + m_b + m_p$$

Sail loading $\sigma = \frac{m}{A}$

Sail assembly loading $\sigma_s = \frac{m_d + m_m + m_b}{A}$

Characteristic acceleration $a_c = \frac{2\eta P}{\sigma}$ ($\eta = 1$ for an ideal sail)

$$P = \frac{W}{c} = \frac{L_\odot}{4\pi r^2 c} \left(\approx 4.56 \cdot 10^{-6} \text{ N/m}^2 \text{ at 1 a.u.} \right)$$

$L_\odot \approx 4\pi R_\odot^2 \cdot \sigma T_\odot^4 \approx 3.85 \cdot 10^{26}$ J/s is solar luminosity

Performance-invariant scalability vs. design scalability

The design scalability is the ability to manufacture a workable solar sail system of the same design as the test prototype, but of different size.

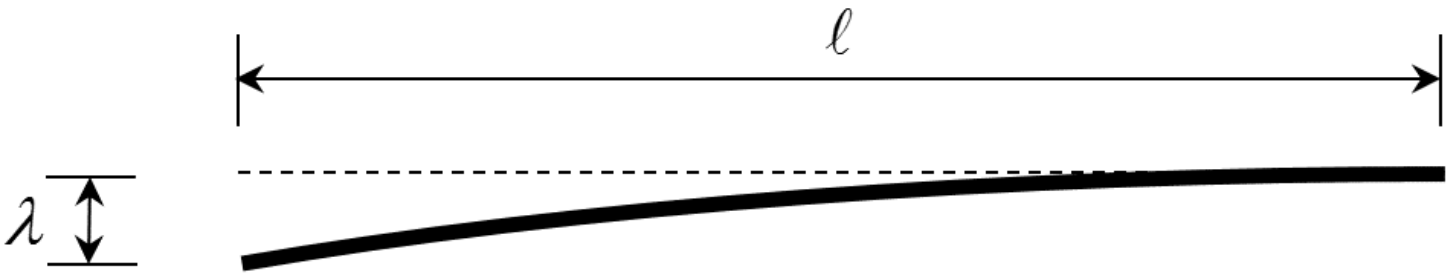
The performance-invariant scalability is the ability to adapt a given sail prototype to the spacecraft of larger or smaller mass so that the characteristic acceleration (i.e., the orbital dynamics) remains the same.

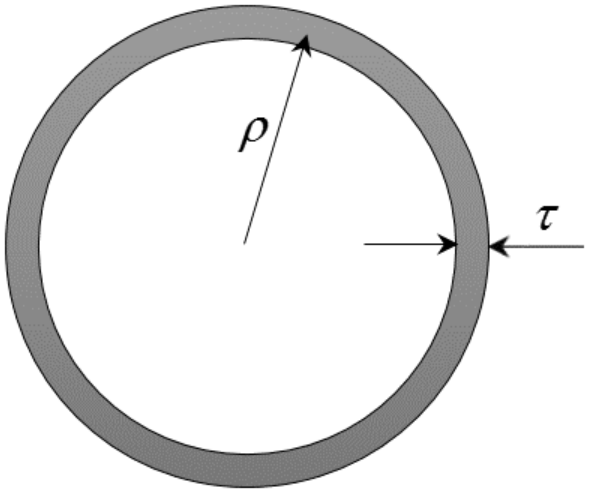
$$m \to k^2 m \qquad A \to k^2 A$$

$$m_m \to k^2 m_m \qquad m_d \approx \text{const}$$

$$m_b \to ? \qquad m_p \to ?$$

Boom flexural deflection (bending)





λ – boom tip deflection

$\lambda = \frac{\ell^3}{EI}\kappa F_\perp$, E – Young's modulus

$I \propto \rho^3 \tau$ – area moment of inertia

$\kappa \leq \frac{1}{3}$ – coefficient defining the distribution of the lateral load F along the boom

$\varepsilon = \frac{\lambda}{\ell} \propto \frac{\ell^2}{E\rho^3\tau}\kappa F_\perp \propto \frac{\ell^4}{\rho^3\tau}$ since $F_\perp \propto \ell^2$

Safe scaling of sail system components

From the Euler-Bernoulli beam theory: the scaling procedure will be safe (no buckling or plastic bending deformations) if we keep the relative boom tip deflection ε.

To scale the boom thickness is hardly feasible due to violation of the strain limit Δ by a boom in the coiled state.

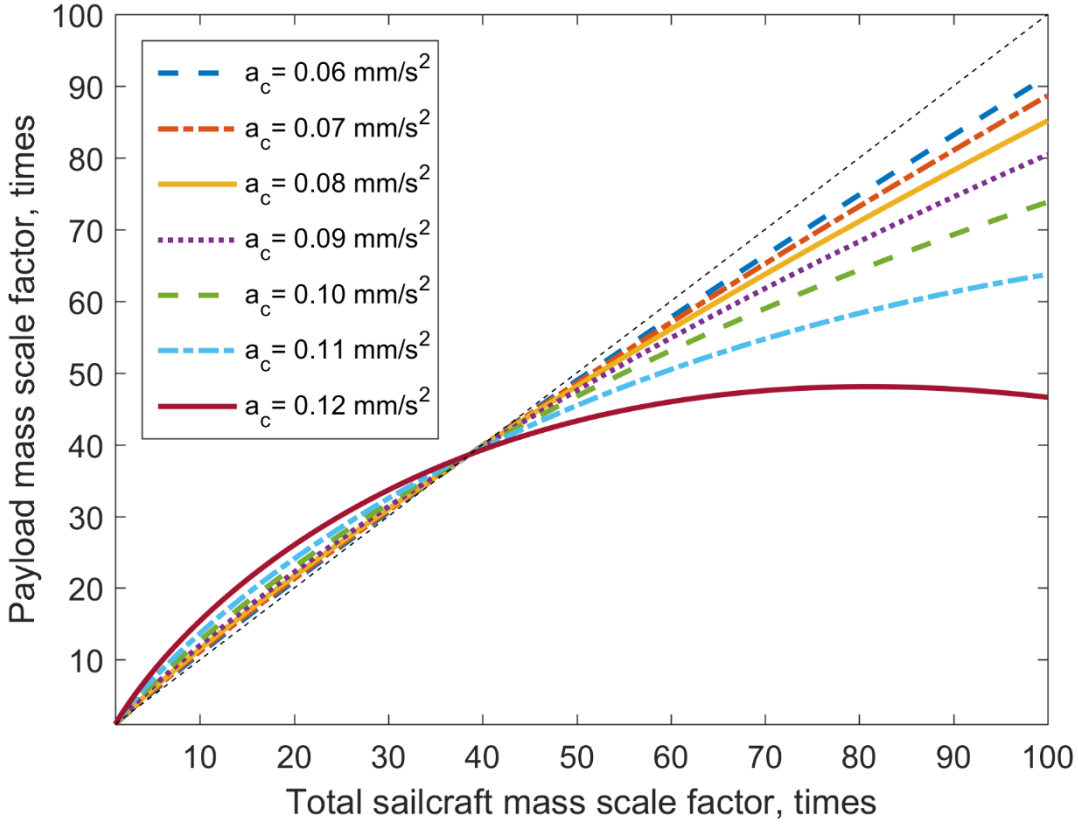
$$\tau / 2r_{drum} \le \Delta \quad \Rightarrow \quad \tau \simeq \text{const}$$

Hence, $\varepsilon = \text{const} \Rightarrow \ell^4 \propto \rho^3$

$$m_b \propto \rho \tau \ell \propto \ell^{7/3} \quad \Rightarrow \quad m_b \to k^{7/3} m_b$$

Source: Surrey Space Centre

Payload mass scaling



$$\frac{k^2 m_m + k^{7/3} m_b + m_d + \tilde{m}_p}{m_m + m_b + m_d + m_p} = k^2$$

$$\frac{\tilde{m}_p}{m_p} = k^2 \left[1 + \frac{m_d \left(1 - k^{-2}\right) - m_b \left(k^{1/3} - 1\right)}{m_p} \right]$$

$$m_p = \frac{2\eta P A}{a_c} - m_m - m_b - m_d$$

$\frac{\tilde{m}_p}{m_p} = k_1^2$ independent of a_c when

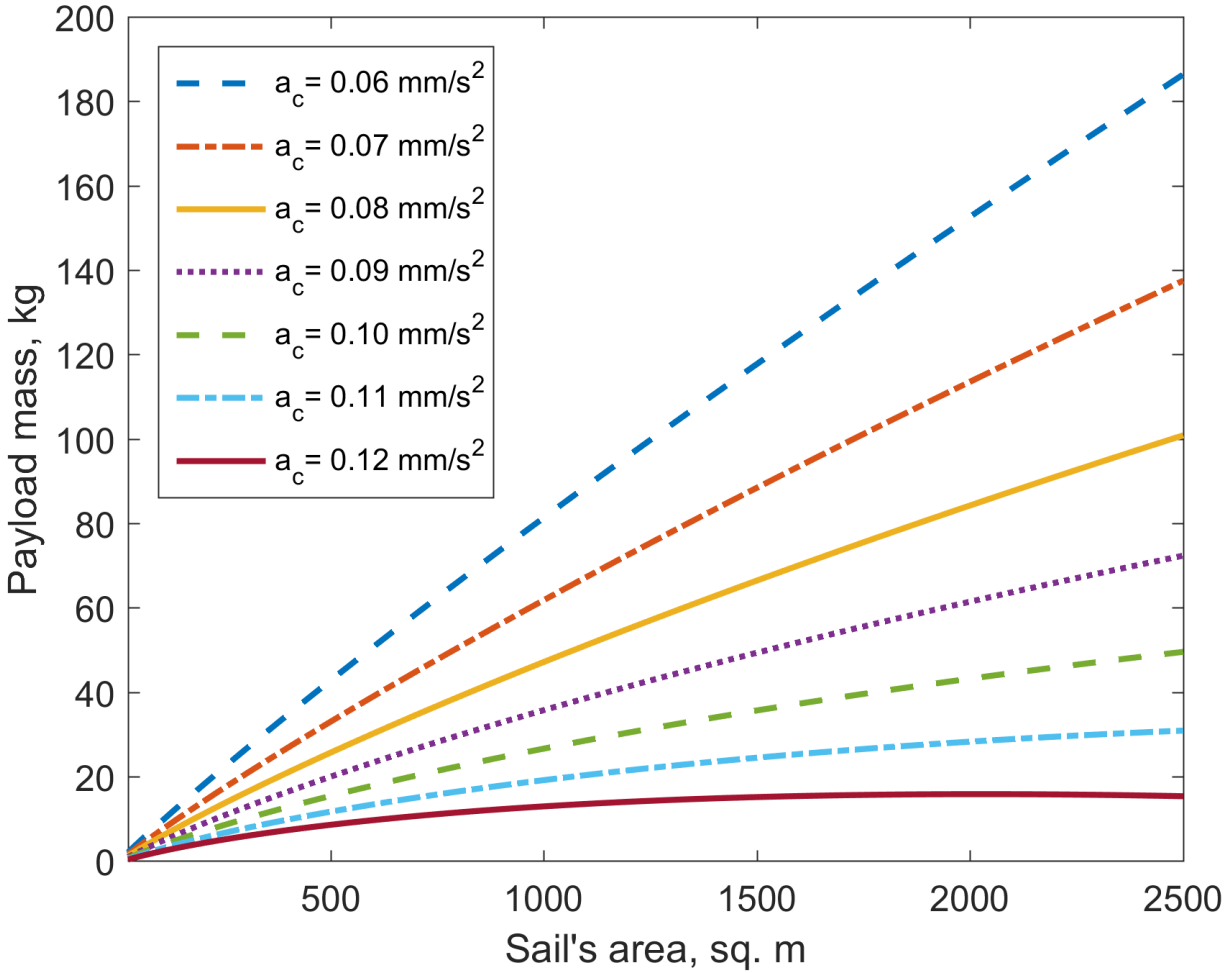
$$m_d \left(1 - k_1^{-2}\right) = m_b \left(k_1^{1/3} - 1\right)$$

Sailcraft prototype: CubeSail (3U CubeSat)

$m = 3$ kg $\quad m_d = 500$ g $\quad m_m = 300$ g $\quad m_b = 581$ g

$\frac{\tilde{m}_p}{m_p} > k^2$ if $k < k_1 \approx 6.2$ and vice versa

Payload mass as a function of sail's area



Sailcraft prototype:
CubeSail (3U CubeSat)

$A = 25\ \text{m}^2 \quad m = 3\ \text{kg}$

$m_d = 500\ \text{g}$

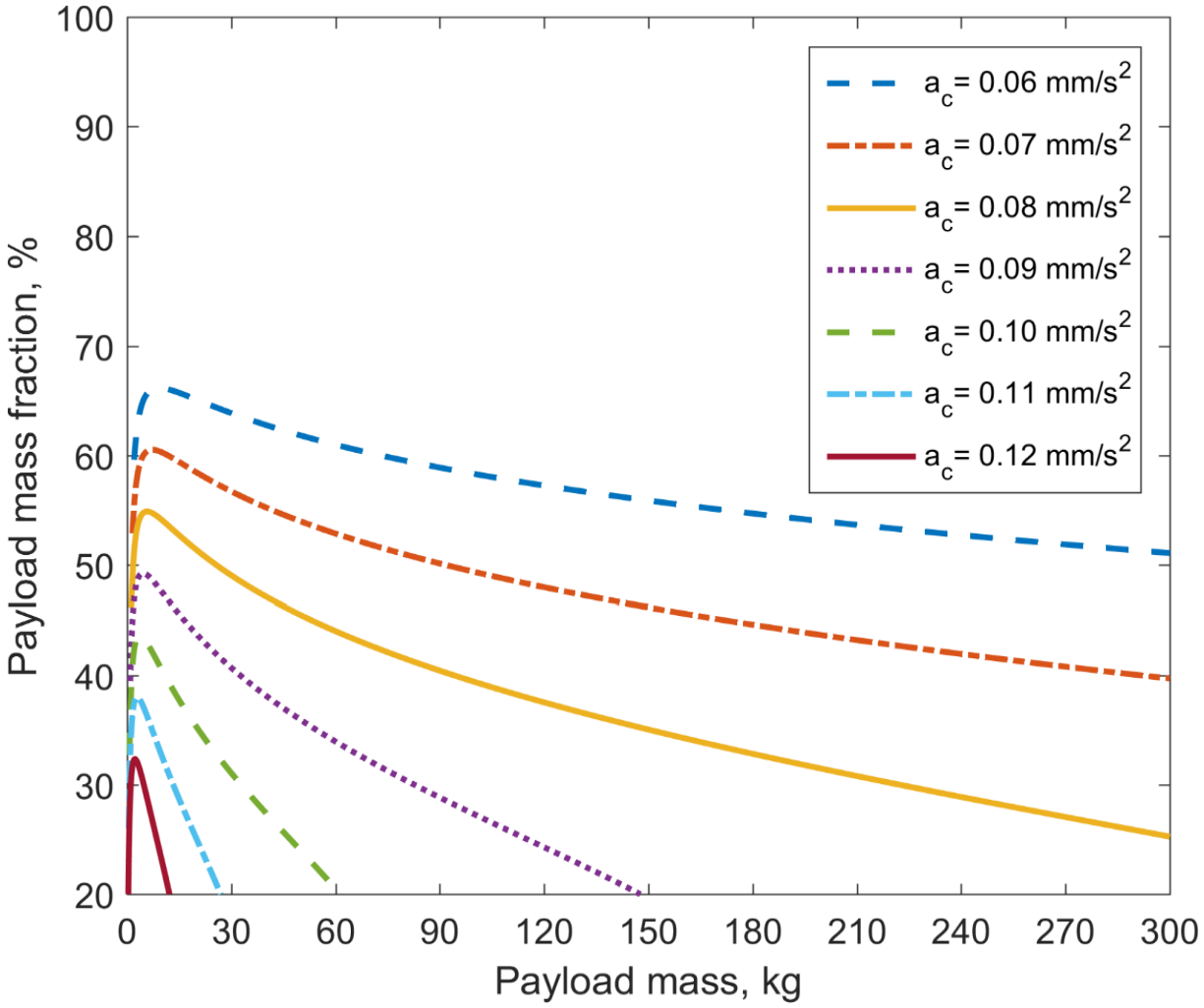
$m_m = 300\ \text{g}$

$m_b = 581\ \text{g}$

The payload mass achieves maximum at

$$k_2 = \frac{216}{343}\left(\frac{2\eta PA}{m_b a_c} - \frac{m_m}{m_b}\right)^3$$

Payload mass fraction scaling



The payload fraction achieves maximum if

$$k = k_3 = \left(6m_d / m_b\right)^{3/7}$$

$k_3 \approx 2$ for CubeSail

In general, if $m_b < 6m_d$, $1 < k_3 < k_1$, whereas if $m_b > 6m_d$, $k_1 < k_3 < 1$

$$m_b = 6m_d \Rightarrow k_1 = k_3 = 1$$

In addition, it is always true that $k_3 < k_2$

Effective specific impulse as a measure of sail system efficiency

Effective specific impulse of a sail system:

$$I_{sp} = \frac{a_c T}{g_0} \frac{1}{\ln(1/\nu)}$$ where ν is the payload mass fraction

For a 2-year flight, CubeSail with nominal and optimal (x2) designs would respectively have the following specific impulse values:

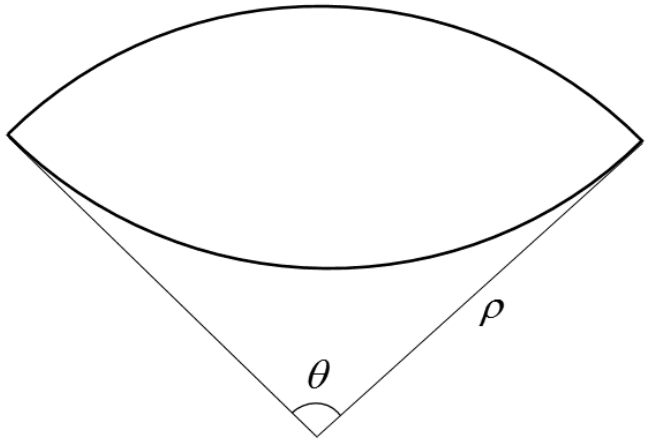
$$I_{sp} = \frac{0.06 \cdot 10^{-3} \cdot 730 \cdot 24 \cdot 3600}{9.81 \cdot \ln(3.42/2.04)} \text{ s} \approx 747 \text{ s}$$

+25%

$$I_{sp} = \frac{0.06 \cdot 10^{-3} \cdot 730 \cdot 24 \cdot 3600}{9.81 \cdot \ln(13.68/9.05)} \text{ s} \approx 934 \text{ s}$$

Accurate consideration of deployment mechanism mass and size scaling

$h = \theta\rho$ – height of the booms in the coiled state $\Rightarrow\ h \propto \ell^{4/3}$



If the scale factor k is not too much, there is an option of changing the subtended angle θ, which increases the cross-sectional moment of inertia.

$r = \sqrt{\dfrac{4\tau\ell}{\pi} + r_{drum}^2}$ – radius of the 4-boom coil

(assuming the Archimedes spiral approximation)

Conclusions

- The mass scaling laws are derived for all the components of the square sail system.
- As a result, the payload mass scaling law is obtained. It is non-monotonic (which is intuitively predictable) due to the fact that the booms mass grows faster than the sail area if scaled safely.
- For a given sail prototype, there exist the maximum payload mass we can potentially eject in orbit with the required level of characteristic acceleration.
- The payload mass fraction is also scaled non-monotonically. For a given sail prototype, there exist the optimal scale: the booms should be about 6 times as heavy as the deployment mechanism.

Acknowledgments

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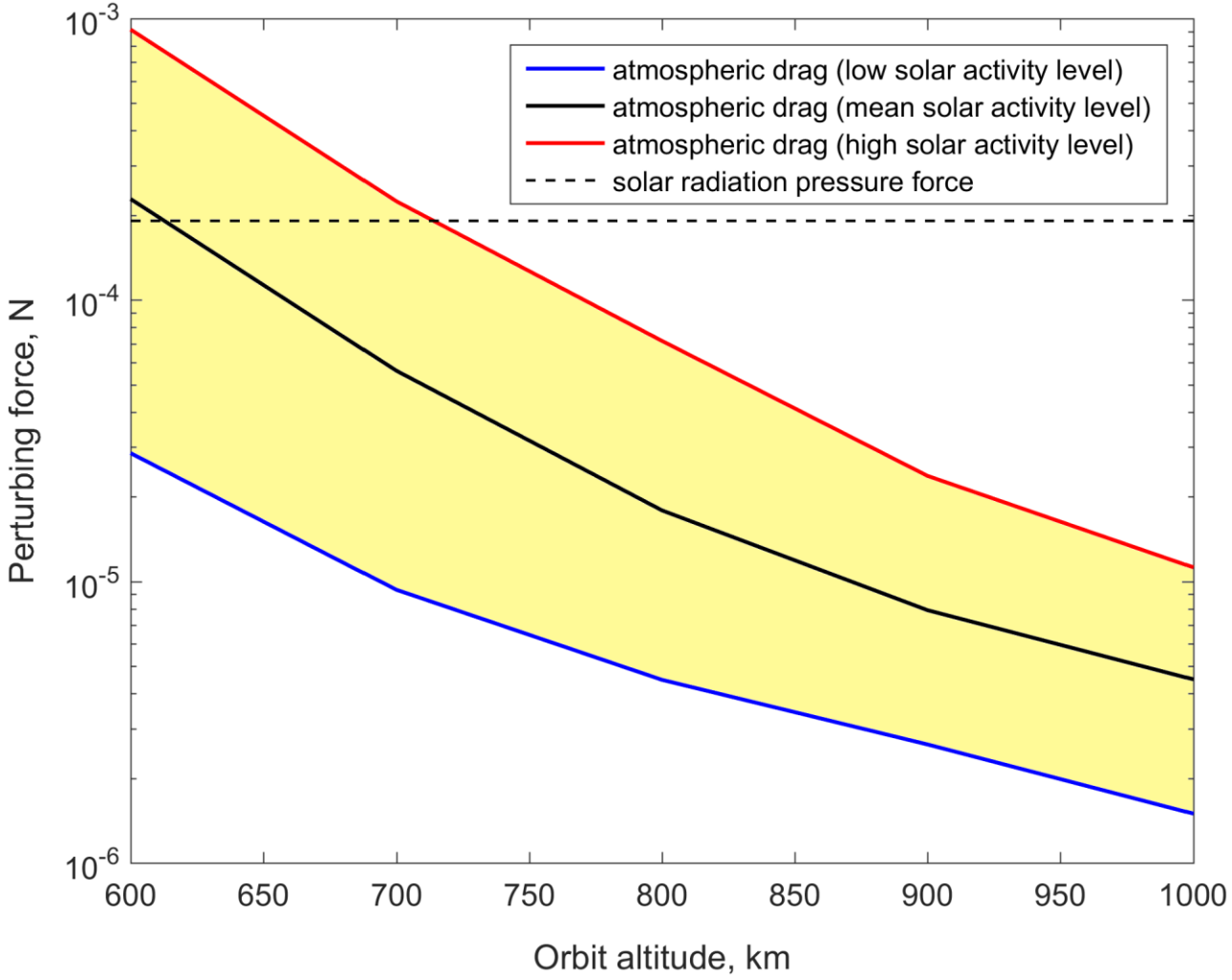
Thank you for your attention

Perturbing forces for LEO sailcraft



Maximum values are depicted for a sailcraft with a sail of 25 m^2

Drag coefficient C_D=2.2