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Computer technologies of engineering mechanical problems

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Review of direct low-thrust trajectory optimization methods

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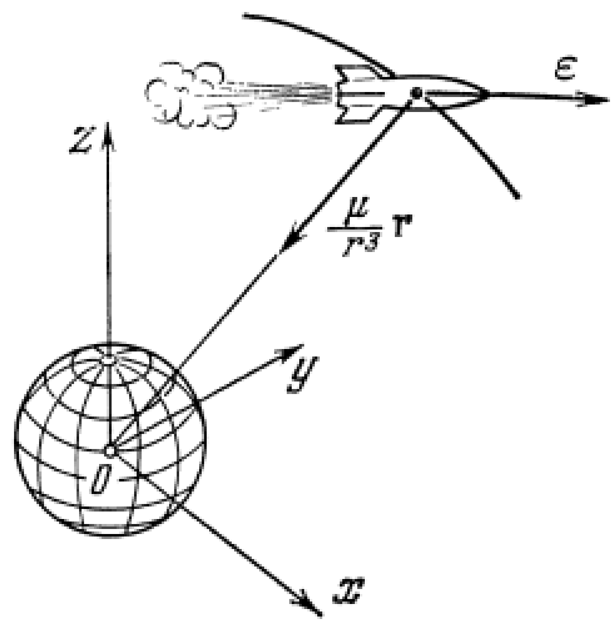
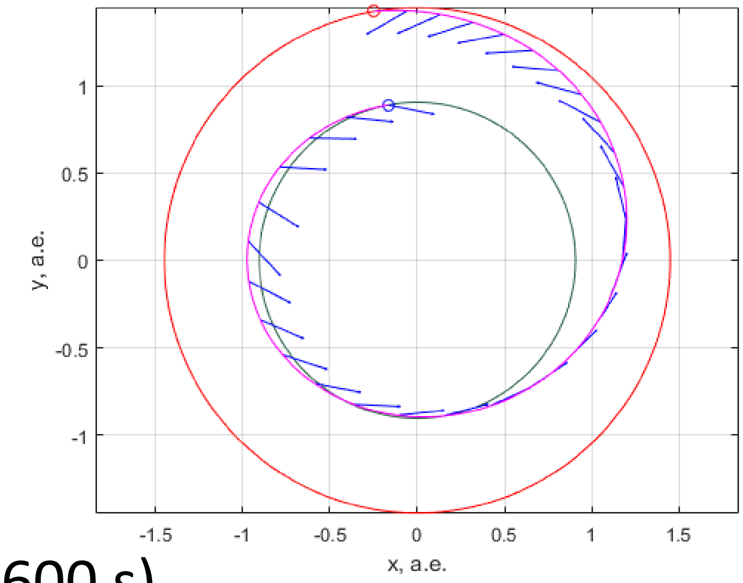


Problem statement

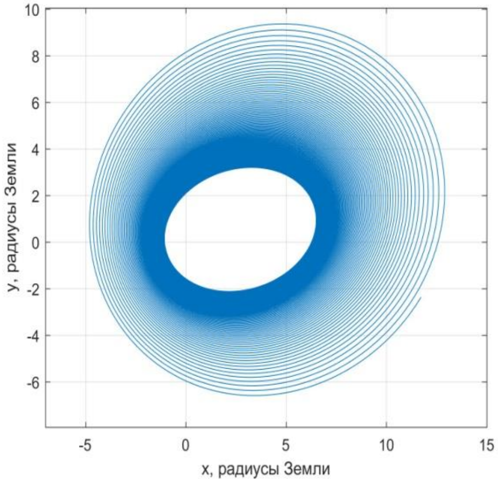
To obtain the fuel optimal Earth-Mars trajectory in the central gravitational field of the Sun



- Low thrust
- Limited thrust acceleration
- Constant exhaust velocity
- Russian engine SPT 100-V (80 mN, 1600 s)
- Ephemeris DE-432
- The mass of the spacecraft is 300 kg
- Time of flight varies from 432 to 711 days

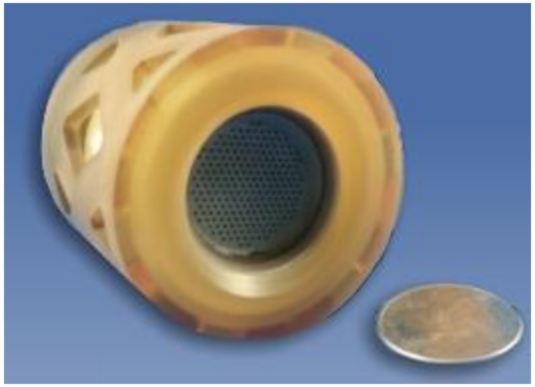
Low thrust

The thrust is small if $\dfrac{a_\tau}{g} < 10^{-4}$



EDB Fakel: SPT-100V

Tethers Unlimited: HYDROS



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Low-thrust trajectory optimization

Indirect

- Necessary optimality conditions (Pontryagin Maximum Principle)
- Two point boundary value problem (TPBVP)

Direct

- Discretization of the control variables
- Nonlinear programming problem (NLP)

Low-thrust trajectory optimization

Indirect methods: Pontryagin maximum principle

✓ The necessary optimality conditions
✓ High accuracy
✗ The convergence depends strongly on the initial guess
✗ The necessary conditions have to be rederived for the perturbed problem
✗ Costate variables are not physically intuitive

Direct methods: control discretization

✓ The solution is close to the optimal
✗ Medium accuracy
✓ The solution is less sensitive to the initial guess
✓ The methods can be easily applied for the disturbed motion
✓ Unknown control variables are more physically intuitive

Direct optimization techniques

- Optimization in terms of thrust acceleration variables

<u>Tang S., Conway B. A.</u>

<u>1995</u>

- Approximation of the control and state variables by the interpolating polynomials and optimization of the expansion coefficients

<u>Fahroo F., Ross I.M.</u>

<u>2002</u>

- Optimization in terms of impulse variables

<u>Sims J., Flanagan S.</u>

<u>1999</u>

Variables

ε

- Thrust acceleration control (ε)
- Perturbed Keplerian motion on each segment of trajectory

$$\ddot{\mathbf{r}} = -\frac{\mathbf{r}}{r^3} + \varepsilon$$

- Search for optimal thrust acceleration control

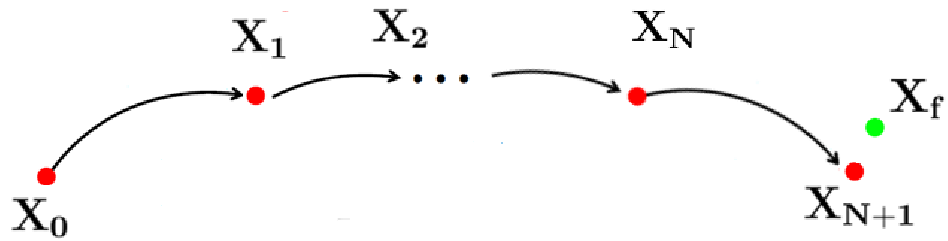
$\Delta \mathbf{V}$

- Thrusting is modeled as a series of impulses ($\Delta \mathbf{V}$)
- Keplerian model on each segment of the trajectory

$$\ddot{\mathbf{r}} = -\frac{\mathbf{r}}{r^3}$$

- Search for optimal impulse control
- Transformation of the impulses to the thrust acceleration

Thrust acceleration control, shooting method



NLP:

$$\begin{cases} J = \sum_{i=1}^{N} |\varepsilon_i| \triangle t \longrightarrow \min \\ |\varepsilon_i| \leq \varepsilon_{max}, i = \overline{1, N} \\ \mathbf{X_{N+1}} - \mathbf{X_f} = \mathbf{0} \end{cases}$$

$3N$ variables: $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \ldots, \boldsymbol{\varepsilon}_N$

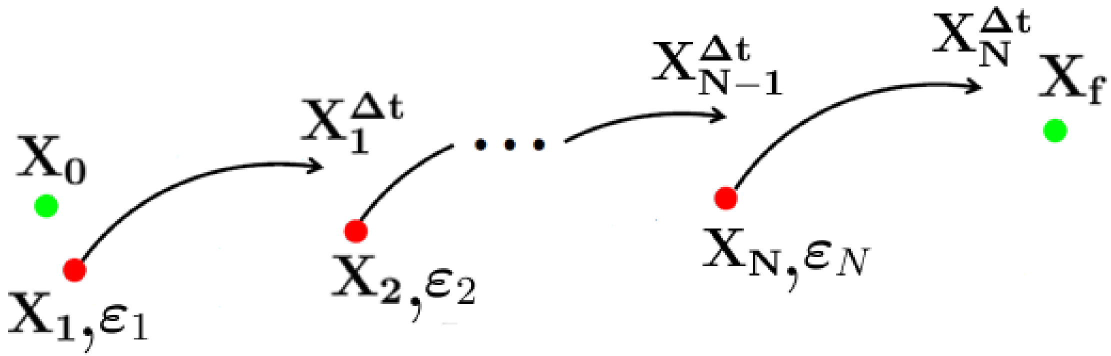
$N + 6$ constraints

where ε_{max} - the maximum value of the thrust acceleration,

$\mathbf{X_0}$, $\mathbf{X_f}$ - the phase vectors of the planets,

$\mathbf{X_{N+1}}$ - the phase vector of the spacecraft at the last node

Thrust acceleration control, multiple shooting method



$$\begin{cases} J_1 = \sum\limits_{i=1}^{N} |\varepsilon_i| \Delta t \longrightarrow \min \\ |\varepsilon_i| \leqslant \varepsilon_{max}, i = \overline{1, N} \\ \mathbf{X}_1 - \mathbf{X}_0 = \mathbf{0} \\ \mathbf{X}_i^{\Delta t} - \mathbf{X}_{i+1} = \mathbf{0}, i = \overline{1, N-1} \\ \mathbf{X}_N^{\Delta t} - \mathbf{X}_f = \mathbf{0} \end{cases}$$

$9N$ variables: $\mathbf{X_1}, \mathbf{X_2}, ..., \mathbf{X_N}$
$\boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}, ..., \boldsymbol{\varepsilon_N}$

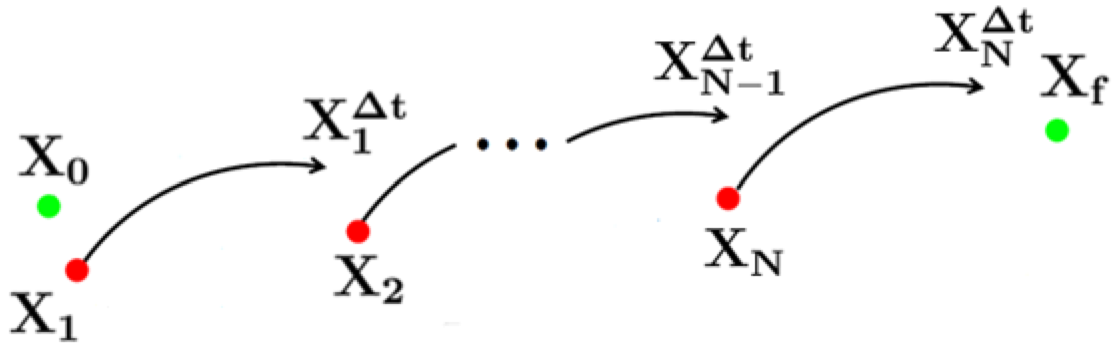
$N + 6(N+1)$ constraints

where $\sum\limits_{i=1}^{N} |\varepsilon_i| \Delta t = \Delta V_{\sum}$ - the relative velocity consumption,

$\mathbf{X}_i$ - the phase vectors of the spacecraft at the control nodes,

$\mathbf{X}_i^{\Delta t}$ -the phase vectors of the spacecraft after integration of the equations of motion on the ith segment

Impulse control, multiple shooting method



$$\begin{cases} J_2 = \sum_{i=1}^{N} |\Delta \ \mathbf{v}_i| \longrightarrow \min \\ |\Delta \mathbf{v}_i| \leqslant \Delta v_{max}, i = \overline{1, N} \\ \mathbf{r}_1 - \mathbf{r}_0 = \mathbf{0} \\ \mathbf{r}_i^{\Delta t} - \mathbf{r}_{i+1} = \mathbf{0}, i = \overline{1, N-1} \\ \mathbf{r}_N^{\Delta t} - \mathbf{r}_f = \mathbf{0} \end{cases}$$

$6N$ variables: $\mathbf{X_1}, \mathbf{X_2}, ..., \mathbf{X_N}$

$N + 3(N+1)$ constraints

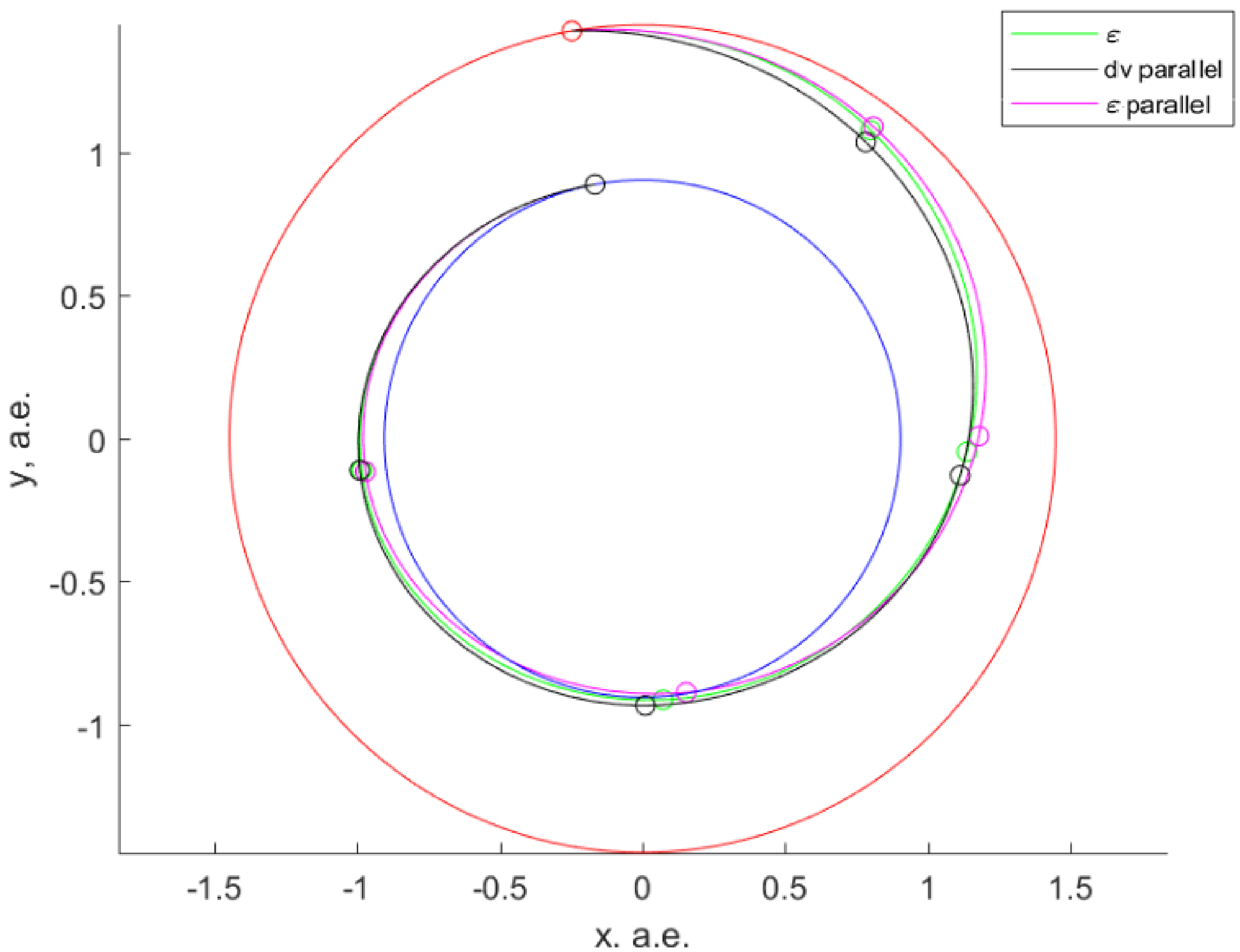
where $\Delta v_i = v_{i+1} - v_i^{\Delta t}$ - the impulse on the i^th segment,

$\sum_{i=1}^{N} |\Delta \mathbf{v}_i| = \Delta V_{\sum}$ - the relative velocity consumption, $\mathbf{r}_0$,$\mathbf{r}_f$ - the radius vectors of the planets,

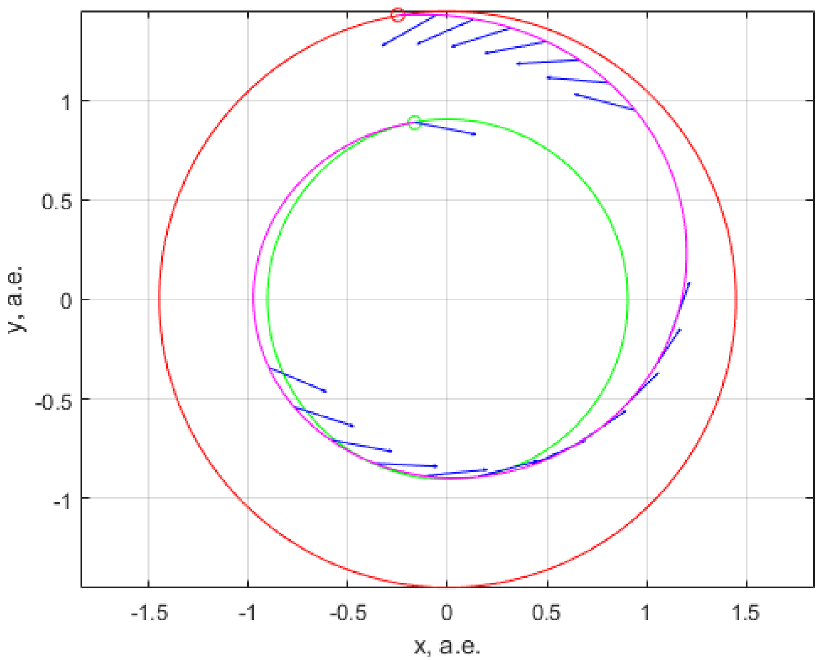
$\mathbf{r}_i$ - the radius vector of the spacecraft at the i^th control node,

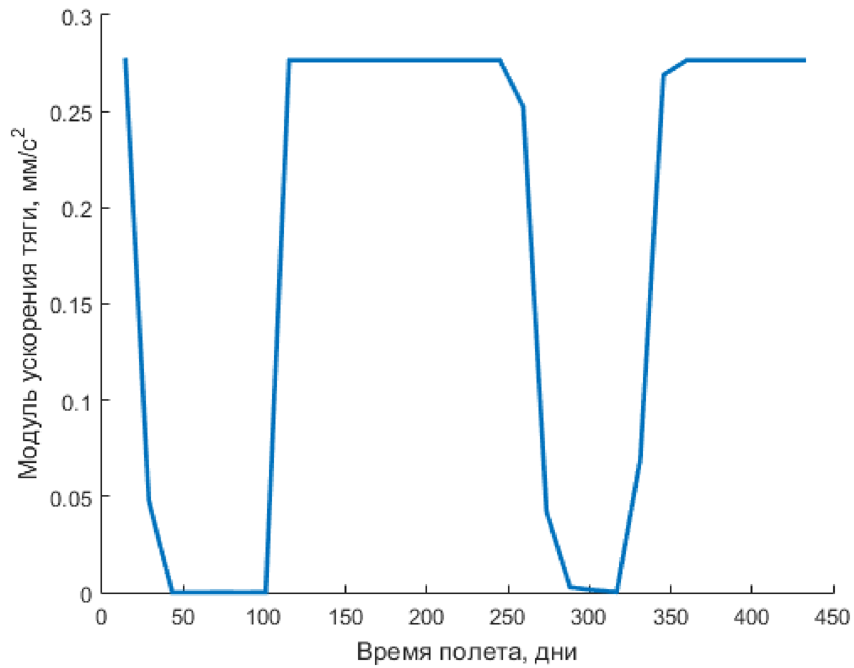
Δv_{max} - the maximum value of the impulse

The optimal Earth-Mars trajectories (earth launch date: 01.01.2020, time of flight: 432 days)



Optimal trajectories





Launch date: 01.01.2020
Arrival date: 08.03.2021
Time of flight: 432 days
Segments per orbit: 30
Relative velocity consumption: 6, 73 km/s
Relative fuel consumption: 34, 75%

Computational comparison of methods

	Convergence region (times when the method converged)	Efficiency (average number of iterations)
Thrust acceleration control, shooting	✗ 239/280 (86%)	✓ High (41 iterations)
Thrust acceleration control, multiple shooting	✗ 259/280 (93%)	✓ High (55 iterations)
Impulse control, multiple shooting	✓280/280 (100%)	✗ Medium (97 iterations)

- **Programming language:** MATLAB R2017a
- **Optimization algorithm:** fmincon, SQP (sequential quadratic programming)
- Time of flight varies from 432 to 711 days, N = 5

Computation time requirements

Thrust acceleration control, shooting	✓ 4.6217 ± 2.1534 s
Thrust acceleration control, multiple shooting	✗ 65.8191 ± 6.2296 s
Impulse control, multiple shooting	✓ 4.6809 ± 2.0532 s

- **Personal computer:** the operating system Windows 7, CPU Intel Core i3 -2377M, frequency 1.5 GHz, RAM 4.0 GB.

Conclusion

- The two direct low-thrust trajectory optimization methods were compared: the one based on optimization of the thrust acceleration and the one that optimizes the impulses that approximate the thrust arcs.
- The method based on optimization of the thrust acceleration and shooting technique is the most fast, but it has a small convergence region.
- The method that optimizes the impulses has the largest convergence region and acceptable computation time requirements.

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