An Analysis of the Correction Problem for the Near-Earth Asteroid (99942) Apophis=2004 MN4

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A hazard mitigation problem for the Apophis-Earth possible collision is analyzed in the paper. Taking into account the asteroid size, its velocity and energy (~10^3 megatons TNT) of the impact, this collision would give a great hazard for the Earth. Because of this, it is important to analyze the characteristics of the possible prevention for this collision. The orbit correction is studied in the paper. The paper describes firstly a method to determine the asteroid trajectories that are close to the nominal one but have the impacts on the Earth. This has allowed the determination of a set of these asteroid’s trajectories that collide with the Earth in 2036. An analysis of this set of the trajectories with the Earth-Apophis impact is performed; its characteristics are determined and presented. The necessary correction of the asteroid velocity to deflect it from the Earth in 2036 is analyzed. This analysis is performed for two options of the correction strategy. Firstly, this is the correction of a hitting trajectory to deflect only it from the Earth. Secondly, this is such a correction of the nominal trajectory, which deflects the whole “tube” of possible (for some probability) trajectories taking into account the errors of the trajectory determination from the observations performed. The two-impulse correction is analyzed, too. It is shown that if the correction of the Apophis orbit is performed before the asteroid-Earth approach in 2029, so the necessary velocity impulse for the correction will be less considerably than for the correction after the approach. Possibilities to use thermo-nuclear effect and impact-kinetic one for this correction are estimated.

I. Introduction

According to the asteroid Apophis observations performed to this time and the asteroid orbit calculated on the base of these observations (see, e.g., [1]), the asteroid Apophis will fly in 2029 at ~ 40,000 km distance from the Earth center without its collision, and this nominal trajectory of the asteroid does not result in any Apophis-Earth collision during long enough following time.

However, for the present level of the Apophis orbit knowledge, there is any probability that some its trajectories, which are close to the nominal one, will hit on the Earth in 2036. Taking into account the asteroid size, its velocity and energy (~10^3 megatons TNT) of the impact, this collision gives a great enough hazard for the Earth.

Because of this, it is important for the modern astronomy and space flight dynamics to do more effective observations of the asteroid and define more exactly the asteroid orbit as well as to analyze the characteristics of the possible prevention of this collision. We analyze the orbit correction in this paper.

II. Nominal Trajectory of Asteroid Apophis

Calculation of the asteroid trajectory is a base for an analysis of its orbit correction. This trajectory goes very close to the surface of the Earth in 2029, and very exact calculation of the asteroid Apophis trajectory has to be performed.

At a present step of the analysis, a following model for the heliocentric motion of the asteroid is used to do this trajectory calculation. It is precise enough since it takes into account besides the Sun attraction also the

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main perturbations: all major planets and Moon (as particles), Earth’s oblateness and the solar-radiation pressure.

The following differential equation of the asteroid motion is taken:

\[ \dot{r} = -\frac{\mu_s}{|r|} - \sum_i \mu_i \left( \frac{r - r_i}{|r| r - r_i^2} \right) + \Delta_1 + \Delta_2 ; \]

where: \( r \) is heliocentric radius-vector of the asteroid; \( r_i \) is heliocentric radius-vector of the i-th celestial body (major planet, Moon), they are taken from the JPL-tables of the planets and Moon coordinates JPL DE-405; \( \mu_s \) and \( \mu_i \) are the gravity parameters of the Sun and i-th celestial body; additional terms \( \Delta_1 \) and \( \Delta_2 \) take into account the Earth oblateness and the solar-radiation pressure.

Influence of small planets, relativistic effect and the Yarcovsky effect are not taken into account in this model. It is possible to do a present analysis of the correction characteristics by this way, but these effects have to be taken into account to do the final analysis of the asteroid-Earth collision problem for 2036.

An integration method that has been developed in the Keldysh Institute of Applied Mathematics [2] is used to integrate these equations (1). A final accuracy of the calculating algorithm for the asteroid Apophis trajectory is tested by comparison with the calculation results of the Institute for Applied Astronomy (IAA) of RAS (V.A. Shor, O.M. Kochetova, and E.I. Yagudina). A good fit with the IAA results is received.

Let’s give some main characteristics of the asteroid orbit.

Figure 1 shows three heliocentric orbits in projections on the ecliptic plane. Here III: the Earth orbit; I: the asteroid orbit before asteroid’s approaching to the Earth in 2029; II: a nominal orbit of the asteroid after its approaching to the Earth in 2029. A circle shows a point of the asteroid approaching to the Earth. It is seen that semi-major axis of the asteroid orbit is increased after this approaching in 2029.

The orbit I before asteroid’s approaching to the Earth in 2029 has the elements: orbital period \( P = 0.89 \text{ y.} \), distance to the Sun in aphelion \( r_\alpha = 1.10 \text{ AU} \); distance to the Sun in perihelion \( r_\pi = 0.75 \text{ AU} \); semimajor axis \( a = 0.92 \text{ AU} \); inclination \( I = 3^\circ.3 \).

The nominal orbit II after asteroid’s approaching to the Earth in 2029 has the elements: orbital period \( P = 1.17 \text{ y.} \), distance to the Sun in aphelion \( r_\alpha = 1.33 \text{ AU} \); distance to the Sun in perihelion \( r_\pi = 0.9 \text{ AU} \); semimajor axis \( a = 1.11 \text{ AU} \); inclination \( I = 2^\circ.1 \). Asteroid’s geocentric velocity for approaching to the Earth “at infinity” \( V_\infty \).
\[ \approx 5.5 \text{ km/s}. \] In a case of the Apophis-Earth collision, the collision velocity of about 12.6 km/s corresponds to this approaching velocity.

If the asteroid diameter is \( D_A = 320 \text{ m} \), and its density is \( \rho_A = 2.5 \text{ g/cm}^3 \), so its mass is \( m_A = 4.3 \times 10^{10} \text{ kg} \). In this case, energy for the asteroid-Earth collision is of about 800 MT TNT. This is large enough energy that can result in noticeable regional destructions on the Earth. For comparison, Tunguska’s explosion had energy released of about 12 MT [3].

Let’s give once more an important property of the Apophis trajectory, its approaching to Earth. Figure 2 shows asteroid’s nominal distance to the Earth versus the time that is calculated from \( t_0 = 30.0 \text{ January 2005} \). It is seen that there will be approaching at 2013 and 2021, then very close approaching in 2029. Figure 3 gives the Apophis-Earth distance for this last approach. It is seen that this distance decreases to small value of about 38,000 km, i.e. the asteroid will fly inside the geostationary orbit. For 2029-2036, Fig.2a gives asteroid’s distance to the Earth not only for the nominal orbit but also for the Apophis hitting orbit with the collision in 2036 (a red line).

### III. “Tube” of Asteroid’s Trajectories

Let’s consider a “tube” of asteroid’s trajectories, i.e. a set of possible trajectories of the asteroid Apophis according to the dispersion ellipsoid. The IAA results in the measurements processing, in calculation of the asteroid nominal trajectory as well as in analysis of possible deviation for initial data are used for this. According to these results [1], it is assumed that for the initial moment \( t_0 = 2005 \text{ JAN. 30.0} \) the confidence set \( D \)

\[ \Delta r_0 = r_0 - r_{0n}, \Delta V_0 = V_0 - V_{0n} \]

of asteroid’s initial radius-vector \( r_0 \) and velocity vector \( V_0 \) from their nominal values \( r_{0n}, V_{0n} \) is:

\[ D = \{ |\Delta r_0| \leq 3 \text{ km}; |\Delta V_0| \leq 2 \text{ m/s} \} \]

If the initial data in \( r_{0n}, V_0 \) are given from this set:

\[ r_0 = r_{0n} + \Delta r_0; V_0 = V_{0n} + \Delta V_0, (\Delta r_0, \Delta V_0) \in D, \]

a set of the asteroid trajectories, so-called the “tube” of the trajectories is received. In particular, there is received a set of the points in an aim plane \((\zeta, \xi)\) that is perpendicular to a nominal geocentric velocity of the asteroid for a moment of its closest flyby near the Earth at the minimal distance on April 13, 2029. This set, i.e. the “dispersion ellipse” is presented on Fig. 4. A light small circle here shows the nominal point of the asteroid flyby in the aim plane. “Semi-major axis” of the “tube” is about 3000 km. Then parameters of the asteroid-Earth approaching in 2029 are: 37600 km +/- 3000 km in perigee distance and \( \tan +/- 120 \text{ s} \) for the approaching time \( t_0 \).

### IV. A Set of Trajectories for Asteroid-Earth Collision in 2036

Between the asteroid trajectories in the “tube” above, there are searched and found the trajectories that lead to the asteroid-Earth collisions in 2036. A following method is developed and used for this. For a given trajectory in the “tube”, its perigee distance \( r_\pi \) for some approaching to the Earth, e.g., in 2036, is considered as a function of the initial data \( Z \):

\[ f = r_\pi(Z); Z = (r_{0n}, V_0). \]

Then the values of this function \( r_\pi(Z) \) are found for a subset formed from some \( 10^4 \) random variables \( Z \) at the possible set \( D \) of initial data and for all the asteroid-Earth approachings till 2113. “Dangerous” trajectories of asteroid with the perigee distance less than \( 10^6 \text{ km} \) are selected at this subset of the Apophis trajectories. These
"dangerous" trajectories are taken as initial approximations for the iterative process that changes the initial data $Z$ and receives the trajectories with smaller perigee distance, and then finds the trajectories with local minimum of the function $r_\pi(Z)$. A method of gradient descent is used for this:

$$Z_{k+1} = Z_k - \theta_k \nabla f(Z_k),$$

(6)

Here $\theta_k (>0)$ is a step in the opposite direction to the gradient $\nabla f(Z_k)$ of the function $f = r_\pi(Z)$.

By this way, there are found over 2000 dangerous trajectories (with $r_\pi < 10^6$ km) for a time interval 2036-2113, including 131 ones for approaching in April 2036. They give a set of hitting trajectories with the Earth-Apophis collisions in 2036 ($r_\pi < 6371$ km). Let’s give a brief description of this set of the collision trajectories.

Let’s consider the plane $(\Delta x_0, \Delta y_0)$ for the deviations of asteroid’s initial coordinates $x_0, y_0$ from their nominal values $x_{0n}, y_{0n}$. Figure 5 gives here the point subset that corresponds to the hitting orbits of the Apophis-Earth collisions in 2036. These trajectories generate a point subset on the aim plane ($\zeta, \xi$) near the Earth in April 2029.

Figure 6 gives this hitting subset and three other ones on the “dispersion ellipse”. A narrow (almost vertical) red strip for $\zeta \approx 36160$ km (i.e. for about 800 km from the nominal point to the Earth) is generated by points of intersection for the Apophis hitting orbits and the aim plane; for these asteroid’s orbits: $r_\pi < 6371$ km in 2036. Surrounded yellow band is generated by the asteroid orbits with the perigee distance that more than 6371 km and less than $1*10^6$ km in 2036 ($6371$ km < $r_\pi < 1*10^6$ km). Green point band corresponds to the asteroid orbits with $r_\pi < 5*10^6$ km in 2036, for it $\zeta \approx 36000-36300$ km.

In the aim plane near Earth in 2029, Fig. 7 illustrates some results of analysis for the Apophis-Earth approaches from 2036 till 2113. Here:

A) Black small circle (the nominal orbit) and white surrounding band have no close approach;  
B) Grey bands have $r_\pi = 1-5*10^6$ km;  
C) Dark domain has $r_\pi = 6371$ km - $10^6$ km;  
D) Black vertical line for $\zeta \approx 36160$ km corresponds to the Apophis hitting orbits with the Apophis-Earth collision, $r_\pi < 6371$ km in 2036.

So, almost all the points of the dispersion ellipse correspond to close approaches with the Earth in 2036-2113.

V. Correction of Asteroid Orbit

The fact that near the Apophis’ nominal trajectory there is the set of the asteroid orbits with the impact on the Earth leads to necessity to analyze the problem of the Apophis’ orbit correction. Let’s give some results of this problem study.
A necessary correction of the asteroid orbit to deflect it from the Earth in 2036 and by this way to prevent this asteroid-Earth collision is studied. This analysis is performed for some options of the correction strategy.

Firstly, this is the one-impulse correction of the hitting (in 2036) orbit to deflect the asteroid from the Earth in 2036. A scheme of the one-impulse correction is shown on Fig. 8. For this, some variants of the one-impulse correction are used:
- One-parameter correction of the perigee distance for 2036;
- One-parameter correction of the flyby point distance on the aim plane near the Earth in 2036;
- Two-parameter correction of the flyby point coordinates on the aim plane near the Earth in 2036.

Characteristics of these one-impulse corrections are similar. Depending on the time of the correction $t_c$, Fig. 9 gives a value of the correction velocity impulse $\Delta V_C$ to deflect the Apophis from the Earth at the distance $\Delta r_\pi = 10^6$ km in April 2036. This velocity impulse is small enough if the correction is performed before the Apophis-Earth approaching in 2029. But the correction velocity increases considerably (in about $10^3$ times) for the correction after this approaching. So, for the correction before 2029, the necessary velocity impulse will be less considerably than for the correction after 2029.

Then, there is analyzed the one-impulse correction of the asteroid’s orbit, which deflects the asteroid from the Earth (with the dispersion ellipse) changing its perigee distance in 2029 for $\Delta r_\pi = (5, 10, 15, 20) \times 10^3$ km. Figure 10 gives a value of the correction velocity impulse $\Delta V_C$ for this case.

The two-impulse correction is analyzed, too. Figure 11 gives the scheme of this correction. The first velocity impulse $\Delta V_{C1}$ transfers the asteroid from an initial hitting orbit $T_1$ on a transfer orbit $T_2$. Then the
second velocity impulse $\Delta V_{C2}$ transfers the asteroid to a final orbit $T_f$. This two-impulse correction can improve the characteristics of the correction because in this case there is more possibilities for the final orbit $T_f$ to be taken “well” enough, without close approaching with the Earth for a long enough time.

The value of the first velocity impulse $\Delta V_{C1}$ for the two-impulse correction is similar to the value of the velocity impulse for the one-impulse correction if the perigee distance deflections $\Delta r_\pi$ are the same for these two schemes of correction. The second impulse $\Delta V_{C2}$ of the two-impulse correction increases its total velocity impulse.

Possibilities to use the impact-kinetic effect and the thermo-nuclear one for the asteroid correction are estimated.

Table 1 gives evaluation of the spacecraft mass $m_{SC}$ (near Apophis) and $m_0$ (on the LEO) for the kinetic-impact effect on Apophis to correct its fly-by near Earth in 2036. Some moments of correction $t_C$ are taken: 2021-2022, 2028, 2029 (before approaching), and 2029 (after approaching).

Table 1. Spacecraft mass evaluation for kinetic-impact effect.

<table>
<thead>
<tr>
<th>$t_C$, years</th>
<th>$\Delta r_\pi$, $10^3$ km</th>
<th>$\Delta V_{C}$, m/s</th>
<th>$m_{SC}$, $10^3$ kg</th>
<th>$m_0$ (LEO), $10^3$ kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020-2021</td>
<td>1000</td>
<td>0.6 $10^{-2}$</td>
<td>1 / 0.4</td>
<td>2 / 0.8</td>
</tr>
<tr>
<td>2028</td>
<td>1000</td>
<td>$10^{-3}$</td>
<td>14 / 5</td>
<td>31 / 11</td>
</tr>
<tr>
<td>2029</td>
<td>1000</td>
<td>$10^{-2}$</td>
<td>140 / 50</td>
<td>310 / 110</td>
</tr>
<tr>
<td>2029+</td>
<td>30</td>
<td>0.12</td>
<td>1700 / 610</td>
<td>3700 / 1300</td>
</tr>
</tbody>
</table>

The perigee distance correction $\Delta r_\pi = 10^6$ km is taken before approaching, and $\Delta r_\pi = 30 \cdot 10^3$ km is taken after approaching. Two models of the spacecraft-asteroid impact are analyzed.

Model A is a model of the perfectly inelastic impact when the spacecraft mass is

$$m_{SC} \approx (m_A + m_{SC}) \Delta V_C / (c V_{SC});$$

here $V_{SC}$ is the spacecraft velocity relative to the asteroid for the impact; $c = \cos \angle (V_{SC}, \Delta V_C)$.

Model B is the Stanjukovich K.P. model of high-velocity explosive impact [4-6] when a mass rejected from the asteroid after the impact results in an additional momentum applied to the asteroid:

$$m_{SC} \approx m_A \Delta V_C / (c (1+k)V_{SC}); k = 0.6 V_{SC} / V^*;$$

here $V^*$ is a critical velocity of explosion, it depends on the matters of the impacting bodies, $V^* = 2$ km/s is taken [4].

It is seen from the Table 1 that the spacecraft mass is suitable enough for the correction before the 2029.

Table 2 gives evaluation of the energy $E_N$ (in Megatons of TNT) for the thermo-nuclear effect on the asteroid surface (contact explosive) according to [7, 8].

Table 2. Energy evaluation for thermo-nuclear effect.

<table>
<thead>
<tr>
<th>$t_C$, years</th>
<th>2029</th>
<th>2029+</th>
<th>2031</th>
<th>2033</th>
<th>2034</th>
<th>2035(1)</th>
<th>2035(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r_\pi$, $10^3$ km</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\Delta V_{C}$, m/s</td>
<td>$10^{-2}$</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>21</td>
<td>54</td>
</tr>
<tr>
<td>$E_N$, Megatons</td>
<td>0.004</td>
<td>1.6</td>
<td>2.4</td>
<td>3.2</td>
<td>4.5</td>
<td>8.3</td>
<td>22</td>
</tr>
</tbody>
</table>

It is seen that the energy after approaching in 2029 is considerably larger than before one. Nevertheless, this energy is suitable enough for the correction in 2029-2033.

VI. Conclusion

Analysis of the Apophis motion shows that near its nominal orbit, in a tube of possible (for present level for our knowledge of the Apophis motion) orbits, there is a small set of the Apophis orbits which have collisions with the Earth in 2036. This collision for the impact energy of about 1000 MT TNT could result in big
destructions and human victims. Because of this, it is important to analyze the problem to prevent the Apophis – Earth collision if the hitting orbit of the asteroid is realized.

It is also very important to know better the asteroid orbit and develop International Program for both ground measurements (especially during Apophis-Earth close approaching in 2013, 2021) and sending to the Apophis a special spacecraft with some radio-device to solve the question of the Apophis-Earth collision possibility.

Determination of physical characteristics for the Apophis is important, too.

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References