An Analysis of the Correction Problem for the Near-Earth Asteroid (99942) Apophis=2004 MN4

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A hazard mitigation problem for the Apophis-Earth possible collision is analyzed in the paper. Taking into account the asteroid size, its velocity and energy (~ 10^3 megatons TNT) of the impact, this collision would give a great hazard for the Earth. Because of this, it is important to analyze the characteristics of the possible prevention for this collision. The orbit correction is studied in the paper. The paper describes firstly a method to determine the asteroid trajectories that are close to the nominal one but have the impacts on the Earth. This has allowed the determination of a set of these asteroid's trajectories that collide with the Earth in 2036. An analysis of this set of the trajectories with the Earth-Apophis impact is performed; its characteristics are determined and presented. The necessary correction of the asteroid velocity to deflect it from the Earth in 2036 is analyzed. This analysis is performed for two options of the correction strategy. Firstly, this is the correction of a hitting trajectory to deflect only it from the Earth. Secondly, this is such a correction of the nominal trajectory, which deflects the whole "tube" of possible (for some probability) trajectories taking into account the errors of the trajectory determination from the observations performed. The two-impulse correction is analyzed, too. It is shown that if the correction of the Apophis orbit is performed before the asteroid-Earth approach in 2029, so the necessary velocity impulse for the correction will be less considerably than for the correction after the approach. Possibilities to use thermo-nuclear effect and impact-kinetic one for this correction are estimated.

I. Introduction

According to the asteroid Apophis observations performed to this time and the asteroid orbit calculated on the base of these observations (see, e.g., [1]), the asteroid Apophis will fly in 2029 at ~ 40,000 km distance from the Earth center without its collision, and this nominal trajectory of the asteroid does not result in any Apophis-Earth collision during long enough following time.

However, for the present level of the Apophis orbit knowledge, there is any probability that some its trajectories, which are close to the nominal one, will hit on the Earth in 2036. Taking into account the asteroid size, its velocity and energy (~ 10^3 megatons TNT) of the impact, this collision gives a great enough hazard for the Earth.

Because of this, it is important for the modern astronomy and space flight dynamics to do more effective observations of the asteroid and define more exactly the asteroid orbit as well as to analyze the characteristics of the possible prevention of this collision. We analyze the orbit correction in this paper.

II. Nominal Trajectory of Asteroid Apophis

Calculation of the asteroid trajectory is a base for an analysis of its orbit correction. This trajectory goes very close to the surface of the Earth in 2029, and very exact calculation of the asteroid Apophis trajectory has to be performed.

At a present step of the analysis, a following model for the heliocentric motion of the asteroid is used to do this trajectory calculation. It is precise enough since it takes into account besides the Sun attraction also the

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main perturbations: all major planets and Moon (as particles), Earth's oblateness and the solar-radiation pressure.

The following differential equation of the asteroid motion is taken:

$$\ddot{\mathbf{r}} = -\frac{\mu_S}{|\mathbf{r}|^3}\mathbf{r} - \sum_i \mu_i \left(\frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} + \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3} \right) + \Delta_1 + \Delta_2 \, ; \tag{1}$$

where: $\mathbf{r}$ is heliocentric radius-vector of the asteroid; $\mathbf{r_i}$ is heliocentric radius-vector of the i-th celestial body (major planet, Moon), they are taken from the JPL-tables of the planets and Moon coordinates JPL DE-405; μ_S and μ_i are the gravity parameters of the Sun and i-th celestial body; additional terms Δ_1 and Δ_2 take into account the Earth oblateness and the solar-radiation pressure.

Influence of small planets, relativistic effect and the Yarcovsky effect are not taken into account in this model. It is possible to do a present analysis of the correction characteristics by this way, but these effects have to be taken into account to do the final analysis of the asteroid-Earth collision problem for 2036.

An integration method that has been developed in the Keldysh Institute of Applied Mathematics [2] is used to integrate these equations (1). A final accuracy of the calculating algorithm for the asteroid Apophis trajectory is tested by comparison with the calculation results of the Institute for Applied Astronomy (IAA) of RAS (V.A. Shor, O.M. Kochetova, and E.I. Yagudina). A good fit with the IAA results is received.

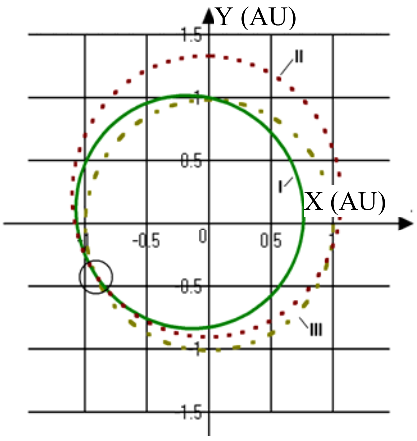

Figure 1. Earth's and Asteroid's heliocentric orbits.

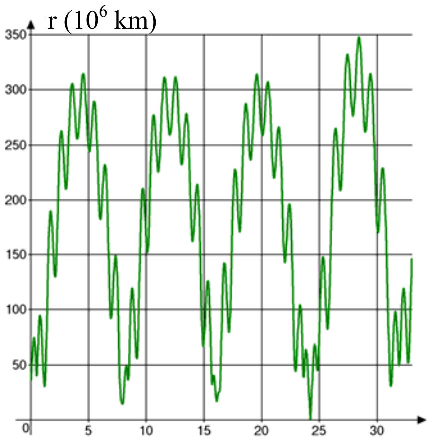

Figure 2. Asteroid-Earth nominal distance in 2005-2038.

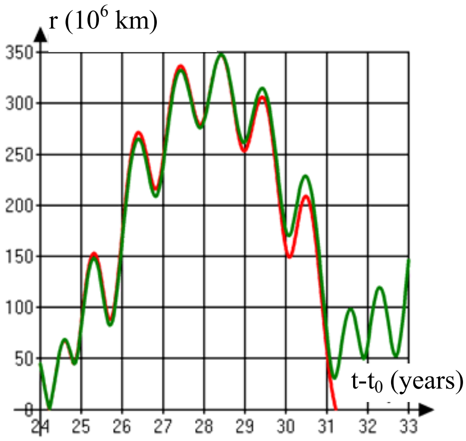

Figure 2a. Asteroid-Earth distance in 2029-2036 for nominal and collision orbits.

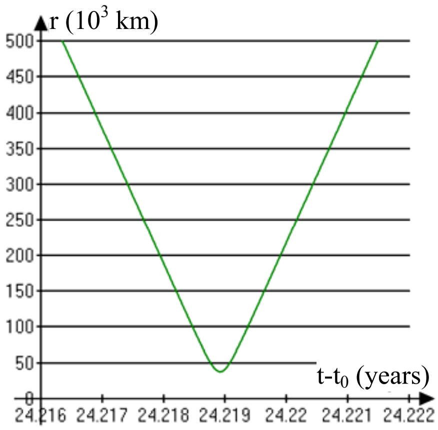

Figure 3. Asteroid-Earth distance for approaching in 2029.

Let's give some main characteristics of the asteroid orbit.

Figure 1 shows three heliocentric orbits in projections on the ecliptic plane. Here III : the Earth orbit; I: the asteroid orbit before asteroid's approaching to the Earth in 2029; II: a nominal orbit of the asteroid after its approaching to the Earth in 2029. A circle shows a point of the asteroid approaching to the Earth. It is seen that semi-major axis of the asteroid orbit is increased after this approaching in 2029.

The orbit **I** before asteroid's approaching to the Earth in 2029 has the elements: orbital period P= 0.89 y., distance to the Sun in aphelion r_α=1.10 AU; distance to the Sun in perihelion r_π=0.75 AU; semi-major axis a=0.92 AU; inclination I=3°.3.

The nominal orbit **II** after asteroid's approaching to the Earth in 2029 has the elements: orbital period P=1.17 y., distance to the Sun in aphelion r_α=1.33 AU; distance to the Sun in perihelion r_π=0.9 AU; semi-major axis a=1.11 AU; inclination I=2°.1. Asteroid's geocentric velocity for approaching to the Earth "at infinity" V_∞

≈ 5.5 km/s. In a case of the Apophis-Earth collision, the collision velocity of about 12.6 km/s corresponds to this approaching velocity.

If the asteroid diameter is D_A=320 m, and its density is d_A=2.5 g/sm^3, so its mass is $m_A \approx 4.3 \cdot 10^{10}$ kg. In this case, energy for the asteroid-Earth collision is of about 800 MT TNT. This is large enough energy that can result in noticeable regional destructions on the Earth. For comparison, Tunguska's explosion had energy released of about 12 MT [3].

Let's give once more an important property of the Apophis trajectory, its approaching to Earth. Figure 2 shows asteroid's nominal distance to the Earth versus the time that is calculated from t_0=30.0 January 2005. It is seen that there will be approaching at 2013 and 2021, then very close approaching in 2029. Figure 3 gives the Apophis -Earth distance for this last approach. It is seen that this distance decreases to small value of about 38,000 km, i.e. the asteroid will fly inside the geostationary orbit. For 2029-2036, Fig.2a gives asteroid's distance to the Earth not only for the nominal orbit but also for the Apophis hitting orbit with the collision in 2036 (a red line).

III. "Tube" of Asteroid's Trajectories

Let's consider a "tube" of asteroid's trajectories, i.e. a set of possible trajectories of the asteroid Apophis according to the dispersion ellipsoid. The IAA results in the measurements processing, in calculation of the asteroid nominal trajectory as well as in analysis of possible deviation for initial data are used for this. According to these results [1], it is assumed that for the initial moment t_0=2005 JAN. 30.0 the confidence set *D* for the deviations

$$\Delta \mathbf{r}_0 = \mathbf{r}_0 - \mathbf{r}_{0n},\ \Delta \mathbf{V}_0 = \mathbf{V}_0 - \mathbf{V}_{0n} \tag{2}$$

of asteroid's initial radius-vector $\mathbf{r}_0$ and velocity vector $\mathbf{V}_0$ from their nominal values $\mathbf{r}_{0n}$, $\mathbf{V}_{0n}$ is:

$$D = \{|\Delta \mathbf{r}_0| \le 3 \text{ km};\ |\Delta \mathbf{V}_0| \le 2 \text{ m/s}\} \tag{3}$$

If the initial data in $\mathbf{r}_0, \mathbf{V}_0$ are given from this set:

$$\mathbf{r}_0 = \mathbf{r}_{0n} + \Delta \mathbf{r}_0\ ;\ \mathbf{V}_0 = \mathbf{V}_{0n} + \Delta \mathbf{V}_0\ ,\ (\Delta \mathbf{r}_0, \Delta \mathbf{V}_0) \in D, \tag{4}$$

a set of the asteroid trajectories, so-called the "tube" of the trajectories is received. In particular, there is received a set of the points in an aim plane (ζ, ξ) that is perpendicular to a nominal geocentric velocity of the asteroid for a moment of its closest flyby near the Earth at the minimal distance on April 13, 2029. This set, i.e. the "dispersion ellipse" is presented on Fig. 4. A light small circle here shows the nominal point of the asteroid flyby in the aim plane. "Semi-major axis" of the "tube" is about 3000 km. Then parameters of the asteroid-Earth approaching in 2029 are: 37600 km +/- 3000 км in perigee distance and t_{an} +/- 120 s for the approaching time t_a.

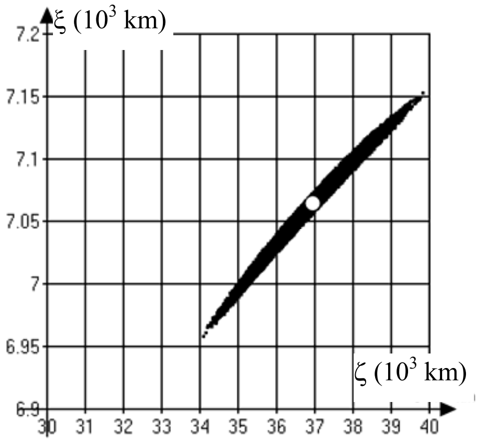

Figure 4. The "dispersion ellipse" for Apophis' orbits in the aim plane near Earth in April 2029: ζ_n ≈36 948 km, $\Delta\zeta$≈3000 km;ξ_n ≈7 064 km, $\Delta\xi$≈100 km.

IV. A Set of Trajectories for Asteroid-Earth Collision in 2036

Between the asteroid trajectories in the "tube" above, there are searched and found the trajectories that lead to the asteroid-Earth collisions in 2036. A following method is developed and used for this.

For a given trajectory in the "tube", its perigee distance r_π for some approaching to the Earth, e.g., in 2036, is considered as a function of the initial data *Z*:

$$f = r_\pi(Z);\ Z = (\mathbf{r}_0, \mathbf{V}_0). \tag{5}$$

Then the values of this function $r_\pi(Z)$ are found for a subset formed from some 10^4 random variables *Z* at the possible set *D* of initial data and for all the asteroid-Earth approachings till 2113. "Dangerous" trajectories of asteroid with the perigee distance less than 10^6 km are selected at this subset of the Apophis trajectories. These "dangerous" trajectories are taken as initial approximations for the iterative process that changes the initial data Z and receives the trajectories with smaller perigee distance, and then finds the trajectories with local minimum of the function $r_\pi(Z)$. A method of gradient descent is used for this:

$$Z_{k+1} = Z_k - \theta_k \, \mathbf{grad} \, f(Z_k), \tag{6}$$

Here θ_k (>0) is a step in the opposite direction to the gradient $\mathbf{grad}\, f(Z_k)$ of the function $f = r_\pi(Z)$.

By this way, there are found over 2000 dangerous trajectories (with r_π <10^6 km) for a time interval 2036-2113, including 131 ones for approaching in April 2036. They give a set of hitting trajectories with the Earth-Apophis collisions in 2036 (r_π <6371 km). Let's give a brief description of this set of the collision trajectories.

Let's consider the plane (Δx_0, Δy_0) for the deviations of asteroid's initial coordinates x_0, y_0 from their nominal values x_{0n}, y_{0n} . Figure 5 gives here the point subset that corresponds to the hitting orbits of the Apophis-Earth collisions in 2036. These trajectories generate a point subset on the aim plane (ζ, ξ) near the Earth in April 2029.

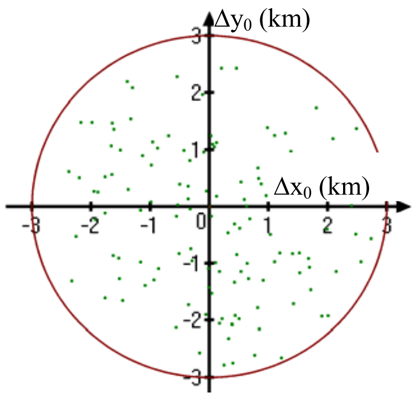

Figure 5. Variations of initial coordinates x_0, y_0 that correspond to the collision orbits in April 2036.

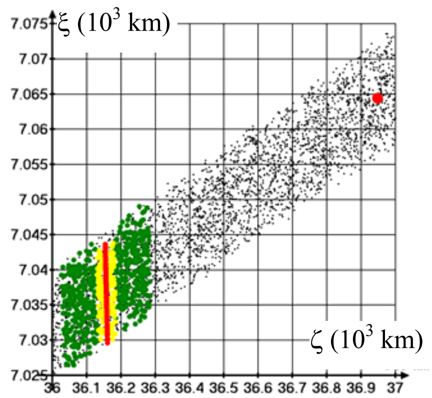

Figure 6. Sets of "dangerous" and collision trajectories (in April 2036) on the aim plane near Earth in 2029.

Figure 6 gives this hitting subset and three other ones on the "dispersion ellipse". A narrow (almost vertical) red strip for ζ≈36160 km (i.e. for about 800 km from the nominal point to the Earth) is generated by points of intersection for the Apophis hitting orbits and the aim plane; for these asteroid's orbits: r_π <6371 km in 2036. Surrounding yellow band is generated by the asteroid orbits with the perigee distance that more than 6371 km and less than $1*10^6$ km in 2036 (6371 km <r_π <$1*10^6$ km). Green point band corresponds to the asteroid orbits with r_π <$5*10^6$ km in 2036, for it ζ≈36000-36300 km.

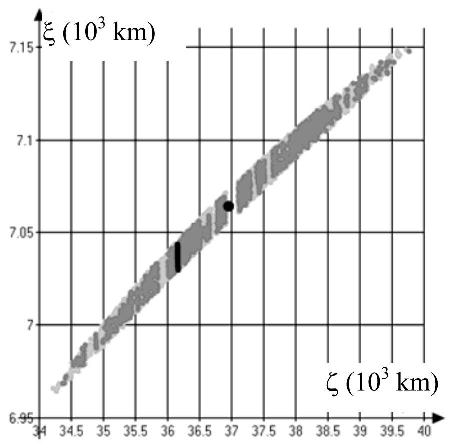

Figure 7. Point subsets in the aim plane near Earth in 2029 for Apophis' orbits, which have at least one close approach with the Earth from 2036 till 2113.

In the aim plane near Earth in 2029, Fig. 7 illustrates some results of analysis for the Apophis-Earth approaches from 2036 till 2113. Here:

A) Black small circle (the nominal orbit) and white surrounding band have no close approach;

B) Grey bands have r_π =1-$5*10^6$ km;

C) Dark domain has r_π =6371 km -10^6 km;

D) Black vertical line for ζ≈36160 km corresponds to the Apophis hitting orbits with the Apophis-Earth collision, r_π < 6371 km in 2036.

So, almost all the points of the dispersion ellipse correspond to close approaches with the Earth in 2036-2113.

V. Correction of Asteroid Orbit

The fact that near the Apophis' nominal trajectory there is the set of the asteroid orbits with the impact on the Earth leads to necessity to analyze the problem of the Apophis' orbit correction. Let's give some results of this problem study.

A necessary correction of the asteroid orbit to deflect it from the Earth in 2036 and by this way to prevent this asteroid-Earth collision is studied. This analysis is performed for some options of the correction strategy.

Firstly, this is the one-impulse correction of the hitting (in 2036) orbit to deflect the asteroid from the Earth in 2036.

A scheme of the one-impulse correction is shown on Fig. 8.

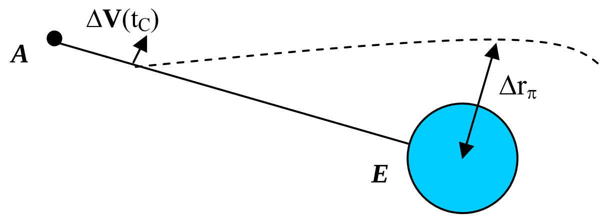

Figure 8. Scheme of one-impulse correction for a collision trajectory.

For this, some variants of the one-impulse correction are used:

- One-parameter correction of the perigee distance for 2036;
- One-parameter correction of the flyby point distance on the aim plane near the Earth in 2036;
- Two-parameter correction of the flyby point coordinates on the aim plane near the Earth in 2036.

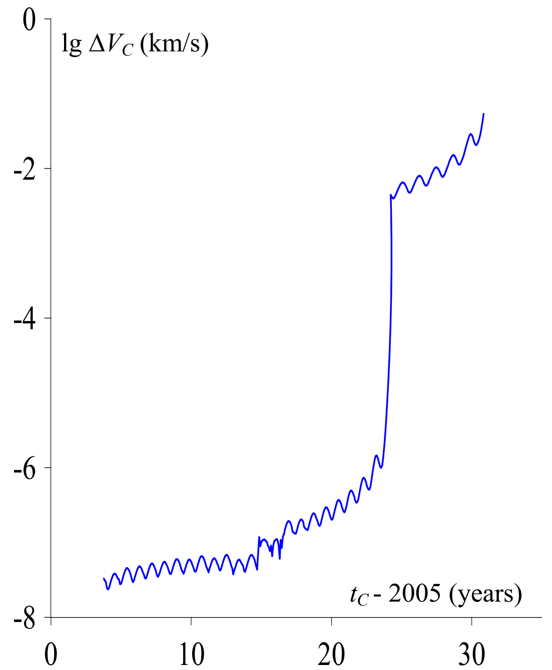

Figure 9. Value of Correction Velocity Impulse for Apophis' deflection from Earth at the distance Δr_{π} =10^6 km in April 2036.

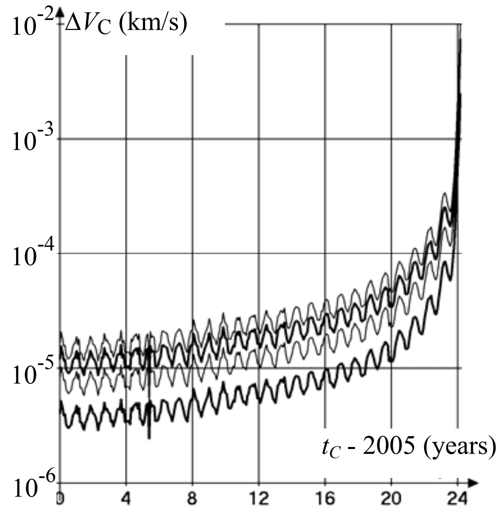

Figure 10. Value of Correction Velocity Impulse for Apophis' deflection from Earth at the perigee distance correction Δr_{π} =(5, 10, 15, 20)*10^3 km in April 2029.

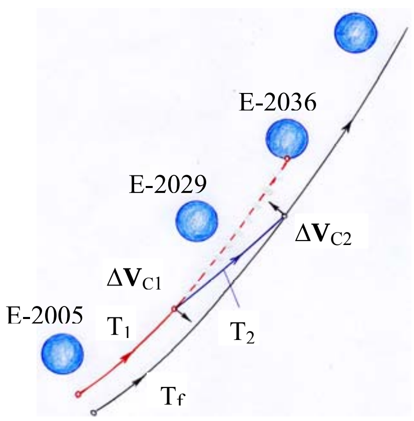

Figure 11. Scheme of two-impulse correction of the hitting trajectory.

Characteristics of these one-impulse corrections are similar.

Depending on the time of the correction t_C, Fig. 9 gives a value of the correction velocity impulse ΔV_C to deflect the *Apophis* from the Earth at the distance Δr_{π} =10^6 km in April 2036. This velocity impulse is small enough if the correction is performed before the Apophis-Earth approaching in 2029. But the correction velocity increases considerably (in about 10^3 times) for the correction after this approaching. So, for the correction before 2029, the necessary velocity impulse will be less considerably than for the correction after 2029.

Then, there is analyzed the one-impulse correction of the asteroid's orbit, which deflects the asteroid from the Earth (with the dispersion ellipse) changing its perigee distance in 2029 for Δr_{π} = (5, 10, 15, 20)*10^3 km.

Figure 10 gives a value of the correction velocity impulse ΔV_C for this case.

The two-impulse correction is analyzed, too. Figure 11 gives the scheme of this correction. The first velocity impulse $\Delta \mathbf{V}_{C1}$ transfers the asteroid from an initial hitting orbit T_1 on a transfer orbit T_2. Then the second velocity impulse $\Delta\mathbf{V}_{C2}$ transfers the asteroid to a final orbit T_f. This two-impulse correction can improve the characteristics of the correction because in this case there is more possibilities for the final orbit T_f to be taken "well" enough, without close approaching with the Earth for a long enough time.

The value of the first velocity impulse $\Delta\mathbf{V}_{C1}$ for the two-impulse correction is similar to the value of the velocity impulse for the one-impulse correction if the perigee distance deflections Δr_π are the same for these two schemes of correction. The second impulse $\Delta\mathbf{V}_{C2}$ of the two-impulse correction increases its total velocity impulse.

Possibilities to use the impact-kinetic effect and the thermo-nuclear one for the asteroid correction are estimated.

Table 1 gives evaluation of the spacecraft mass m_{SC} (near Apophis) and m_0 (on the LEO) for the kinetic-impact effect on Apophis to correct its fly-by near Earth in 2036. Some moments of correction t_C are taken: 2021-2022, 2028, 2029^- (before approaching), and 2029^+ (after approaching).

Table 1. Spacecraft mass evaluation for kinetic-impact effect.

t_C, years	Δr_π, 10^3 km	ΔV_C, m/s	m_{SC}, 10^3 kg A/B	m_0 (LEO), 10^3 kg A/B
2020-2021	1000	$0.6\ 10^{-4}$	1 / 0.4	2 / 0.8
2028	1000	10^{-3}	14 / 5	31 / 11
2029^-	1000	10^{-2}	140 / 50	310 / 110
2029^+	30	0.12	1700 /610	3700 /1300

The perigee distance correction $\Delta r_\pi = 10^6$ km is taken before approaching, and $\Delta r_\pi = 30\cdot10^3$ km is taken after approaching. Two models of the spacecraft-asteroid impact are analyzed.

Model A is a model of the perfectly inelastic impact when the spacecraft mass is

$$m_{SC} \approx (m_A + m_{SC})\, \Delta V_C / (c\, V_{SC}); \tag{7}$$

here V_{SC} is the spacecraft velocity relative to the asteroid for the impact; $c = \cos\angle(\mathbf{V}_{SC}, \Delta\mathbf{V}_C)$.

Model B is the Stanjukovich K.P. model of high-velocity explosive impact [4-6] when a mass rejected from the asteroid after the impact results in an additional momentum applied to the asteroid :

$$m_{SC} \approx m_A\, \Delta V_C / (c\,(1+k) V_{SC});\ k = 0.6\ V_{SC} / V^*; \tag{8}$$

here V^* is a critical velocity of explosion, it depends on the matters of the impacting bodies, $V^* = 2$ km/s is taken [4].

It is seen from the Table 1 that the spacecraft mass is suitable enough for the correction before the 2029.

Table 2 gives evaluation of the energy E_N (in Megatons of TNT) for the thermo-nuclear effect on the asteroid surface (contact explosive) according to [7, 8].

Table 2. Energy evaluation for thermo-nuclear effect.

t_C, years	2029^-	2029^+	2031	2033	2034	2035(1)	2035(2)
Δr_π, 10^6 km	1	1	1	1	1	1	1
ΔV_C, m/s	10^{-2}	4	6	8	11	21	54
E_N, Megatons	0.004	1.6	2.4	3.2	4.5	8.3	22

It is seen that the energy after approaching in 2029 is considerably larger than before one. Nevertheless, this energy is suitable enough for the correction in 2029-2033.

VI. Conclusion

Analysis of the Apophis motion shows that near its nominal orbit, in a tube of possible (for present level for our knowledge of the Apophis motion) orbits, there is a small set of the Apophis orbits which have collisions with the Earth in 2036. This collision for the impact energy of about 1000 MT TNT could result in big destructions and human victims. Because of this, it is important to analyze the problem to prevent the Apophis – Earth collision if the hitting orbit of the asteroid is realized.

It is also very important to know better the asteroid orbit and develop International Program for both ground measurements (especially during Apophis-Earth close approaching in 2013, 2021) and sending to the Apophis a special spacecraft with some radio-device to solve the question of the Apophis-Earth collision possibility.

Determination of physical characteristics for the Apophis is important, too.

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References

1. Yagudina, E.I., and Shor, V.A. "The orbit determination of (99942) Apophis=2004 MN4 from optical and radar data", *All-Russian Conference "Asteroid-Comet Hazard – 2005*", 3-7 October 2005, St. Petersburg. Materials of the Conference. SPb: Institute of Applied Astronomy of RAS, 2005, pp. 355-358.
2. Stepan'yants, V. A, and L'vov, D. V. "Effective Algorithm for the Motion Differential Equations System Solving", *Mathematical Modeling* (Moscow, Russia), Vol. 12, No. 6, 2000, pp. 9-14.
3. Morrison, D. (ed.) *The Spaceguard Survey: Report of the NASA International Near-Earth-Object Detection Workshop*. January 25, 1992. JPL/California Institute of Technology, Pasadena, California, USA. 1992.
4. Baum, F.A., Stanjukovich, K.P., and Shekhter, B.I. *Physics of Explosion,* GIPhML Publisher, Moscow, 1959. Chap. XIV.
5. Ivashkin, V.V., and Zaytsev, A.V. "Analysis of Possible Changes of the Orbit of an Asteroid Approaching the Earth by Impact of a Spacecraft", *Cosmic Research,* Vol. 37, No. 4, 1999, pp. 382-392.
6. Ivashkin, V.V. "On the asteroid-comet hazard mitigation problem for the Earth," *Journal of Harbin Institute of Technology (New Series)*, Harbin, China. Vol. 10, No. 4, December, 2003, pp. 358-362.
7. Ahrens, T.J., and Harris, A.W. "Deflection and fragmentation of near-Earth asteroids," *Nature*, Vol. 360, 1992, pp. 429-433.
8. Nechay, V.Z., et al. "Nuclear explosion near the surface of asteroids and comets. General description of phenomenon," Chelyabinsk Scientific Center News: *Space Protection of the Earth*, I. Snezhinsk, RFNC-VNIITF, 1997, pp. 178-182.