Алгоритмы непрерывного управления относительным движением спутниковой формации для демонстрации изображений из космоса

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XLV Академические чтения по космонавтике, посвященные памяти академика С. П. Королева и других выдающихся отечественных ученых - пионеров освоения космического пространства

Апрель, 2021

Space Advertisement

1. Canady Jr, John E., and John L. Allen Jr. "Illumination from space with orbiting solar-reflector spacecraft." (1982);

2. Лавренов, А. Н., М. В. Палкин, and Р. А. Петухов. "Технология космической рекламы." Реутов: АО" ВПК" НПО машиностроения (2016);

3. Start Rocket, Orbital display, https://startrocket.me;

Past studies:

- Feasibility study on small satellite formation flying mission for space advertisement;
- Decentralized atmospheric drag based control in the task of initial deployment and maintenance of the satellite formations;
- Mission design and Impulsive control algorithms for deployment, maintenance, and reconfiguration of FF for graphic image demonstration in the sky;

Artist’s impression on graphic demonstration in the sky

Case Study

- As a case study we design a mission with the aim to make two demonstrations above Moscow on Yuris' Day, 12th of April 2021;
- The first demo will be scheduled for morning time and MIPT logo is to be demonstrated, the second demo will be scheduled for evening time and Skoltech logo is to be deployed above Moscow;

Artist's impression on graphic demonstration in the sky

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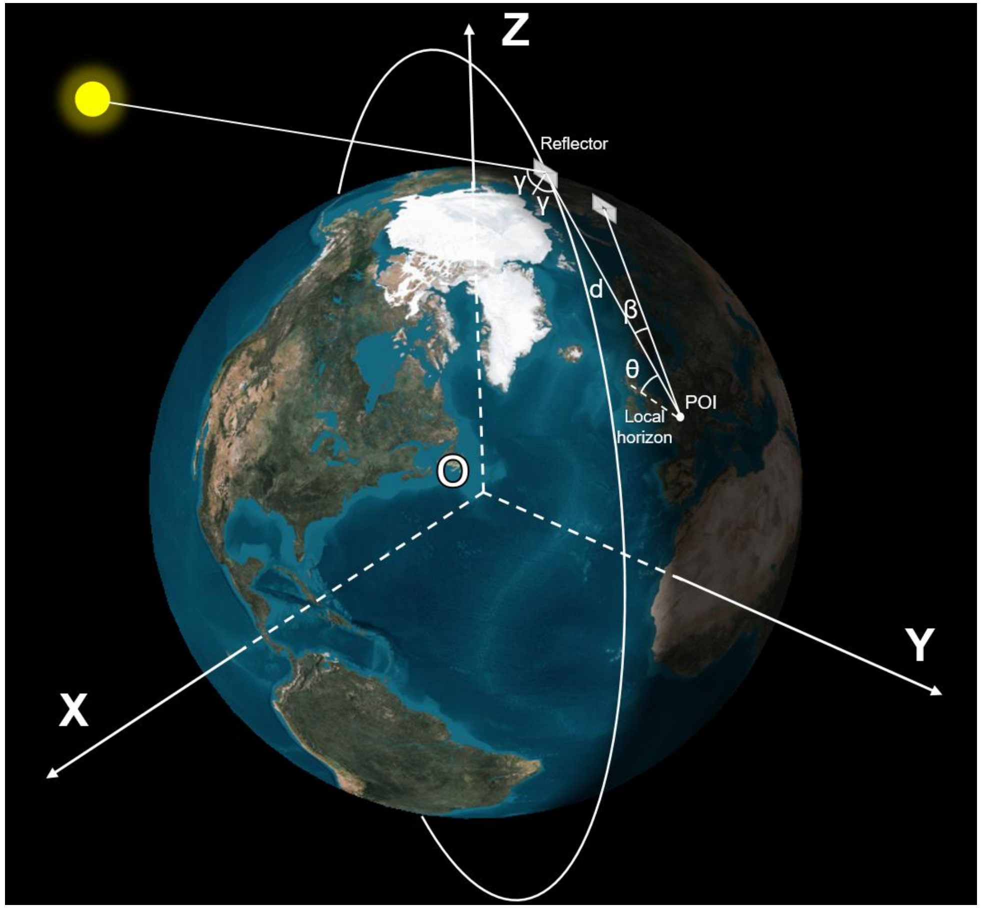
Mission Design Requirements

Image demonstration requirements :

1. Pixel visibility
   - Line of sight POI ↔ Satellite & Sun ↔ Satellite;
   - The Sun elevation angle $\Upsilon_{Sun} < \Upsilon_{sun}^{*}$;
   - Elevation angle of satellite $\theta > \theta^{*}$;
   - Single pixel magnitude $m < m^{*}$;

2. Formation's configuration
   - A pair of satellites should be distinguished by a naked human eye, $\beta > 1'$ [1];



Reflection geometry

1. Yanoff, Myron, and Jay S. Duker. "Ophthalmology." (2009).
m^{*} - maximum admissible pixel magnitude
Υ_{Sun}^{*} - maximum Sun elevation at demo
θ^{*} minimum sat elevation at demo

Spacecraft Visibility Model

- We define spacecraft's visibility in terms of magnitude:

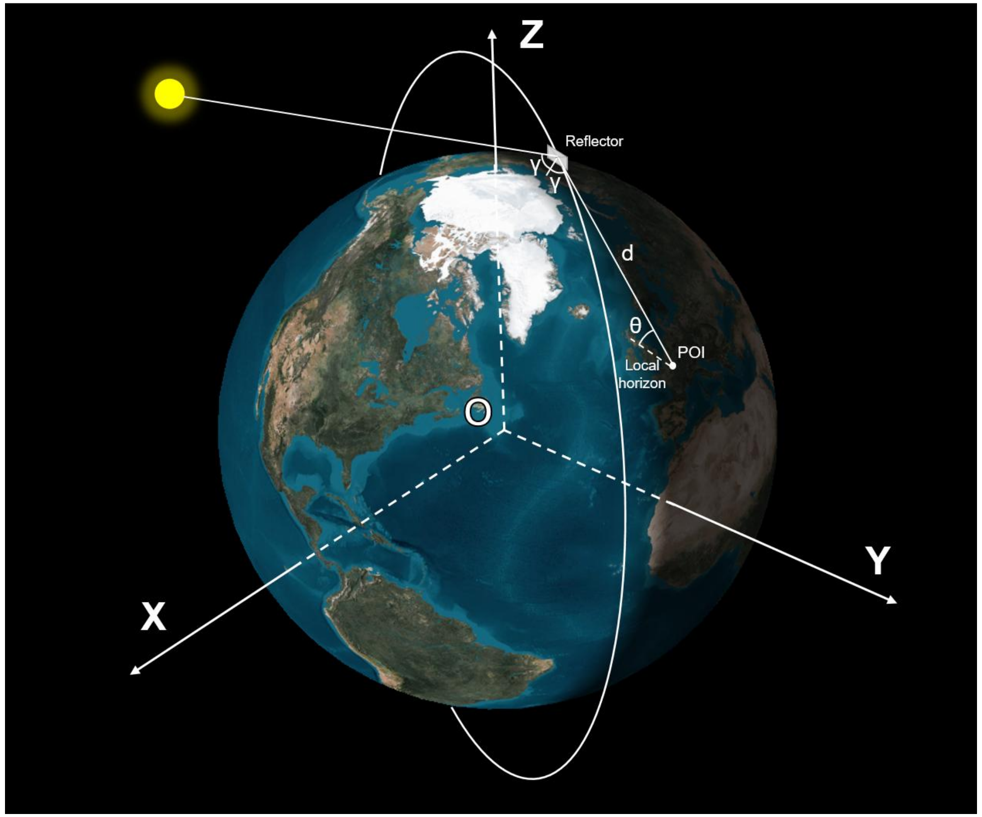
$$m = -2.5\ \mathrm{Log}\left(\frac{I}{I_{ref}}\right);$$

- The intensity of the light at the POI is given by [2]:

$$I = \frac{I_0 A_r \rho \tau \cos(\gamma) \sin(\theta)}{4d^2 \left(\tan\left(\frac{\alpha}{2}\right)\right)^2},$$

where atmospheric transmissivity τ is given by [2]:

$$\tau = 0.1283 + 0.7559 e^{-0.3778 \sec(\pi/2-\theta)}$$



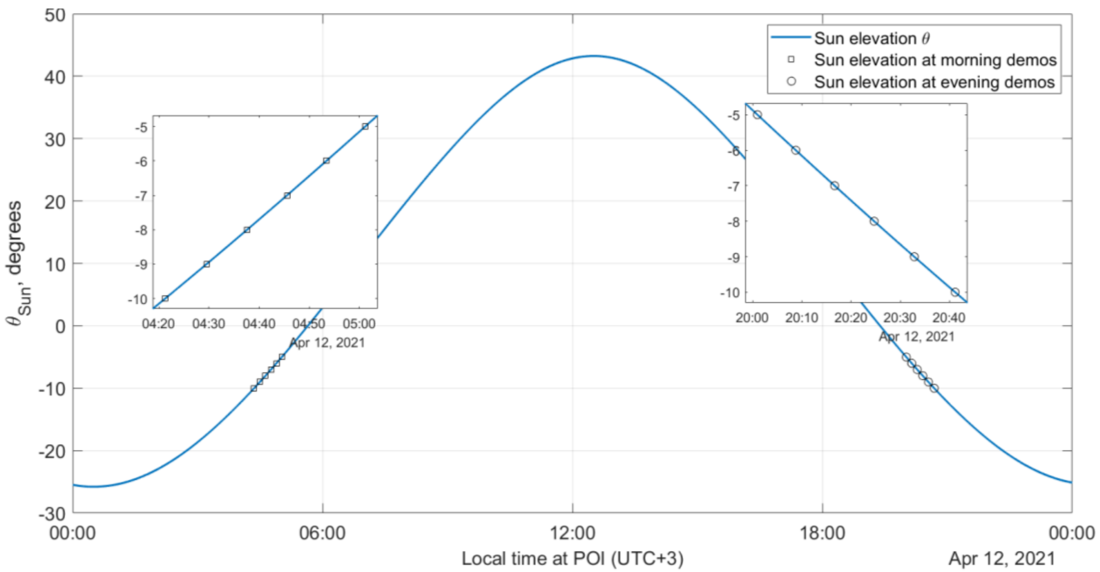
Reflection geometry

- α – included angle of the Sun measured from the Earth;
- d – spacecraft to ground spot distance;
- A_r – area of the CubeSat reflector;
- A_{gs} – area of the ground spot;
- θ – the elevation angle of the spacecraft;
- γ – the incident angle of solar rays;
- I_0 – average intensity of solar energy at the Earth distance;
- ρ – reflectivity coefficient;

2. Canady Jr, John E., and John L. Allen Jr. "Illumination from space with orbiting solar-reflector spacecraft." (1982).

Mission Design

- Circular Sun-synchronous orbits passing near to the terminator line are considered;
- Orbits have to pass above Paris twice a day at the same lighting conditions;
- The sun elevation θ_{sun} should not be greater than -5°;
- Demonstration starts when satellites' elevation θ > 10°;
- Magnitude m of the reflector shall be smaller than -6 or -8



Sun elevation at Moscow on 12th of April

Trade-off matrix for target orbit selection

θ_{sun}, deg	h, km	i, deg	RAAN, deg	TA, deg	Demo duration, min	ISD, m	refl length (-8), m	refl length (-6), m
-5	646.25	97.96	290.24	17.00	8.47	596.98	10.01	3.99
-6	726.78	98.29	290.24	63.93	9.23	654.83	10.97	4.37
-8	408.67	97.05	290.24	49.60	4.31	438.17	8.22	3.27
-9	486.08	97.34	290.24	98.79	5.04	491.87	11.12	4.43
-10	564.39	97.64	290.24	147.13	5.87	544.30	9.61	3.83

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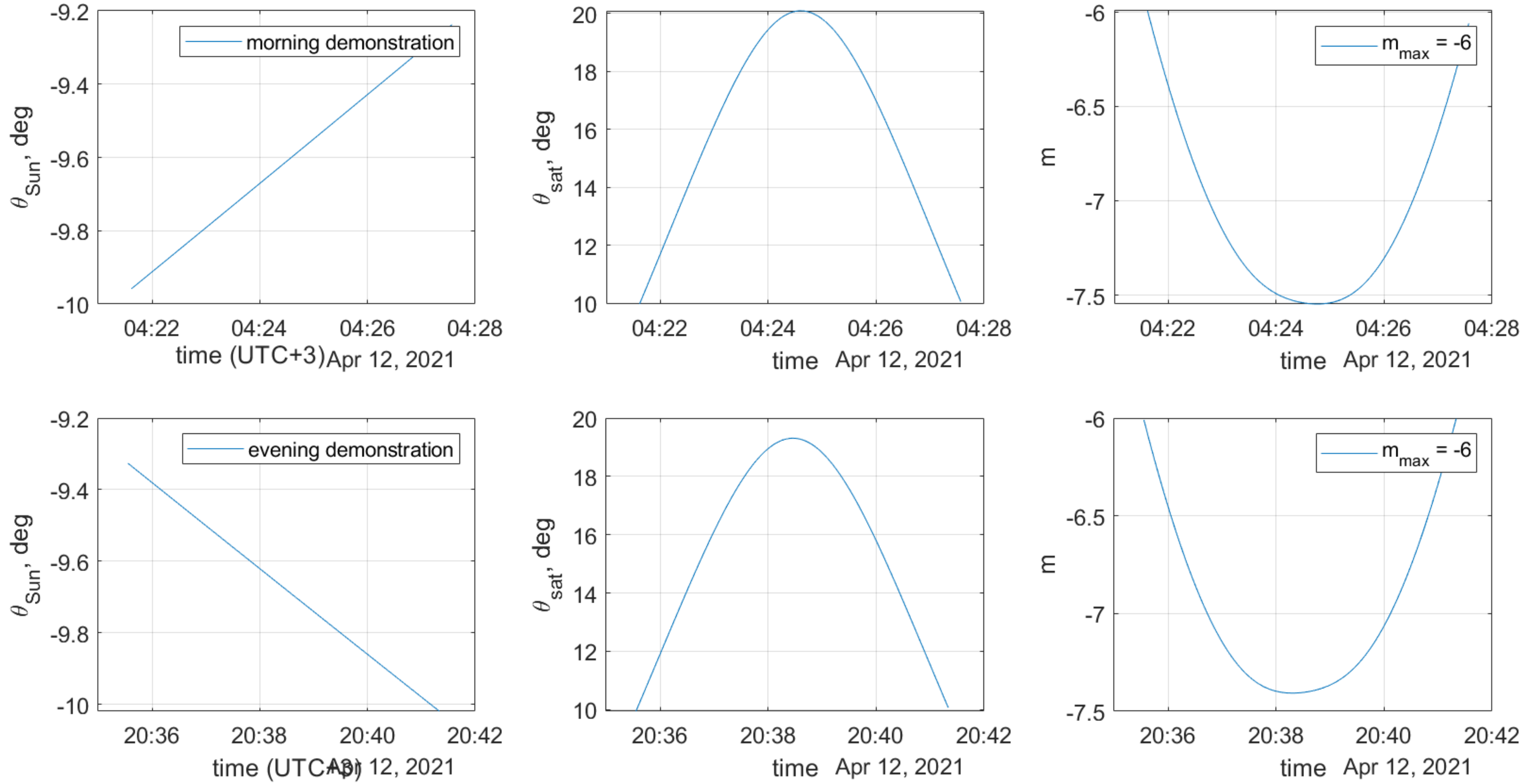
Mission Design: Demonstration Parameters



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Target Relative Trajectories

- Hill-Clohessy-Wiltshire (HCW) equations for relative motion dynamics;

$$\begin{cases} \ddot{x} + 2n\dot{z} = 0; \\ \ddot{y} + n^2 y = 0; \\ \ddot{z} - 2n\dot{x} - 3n^2 z = 0\,. \end{cases}$$

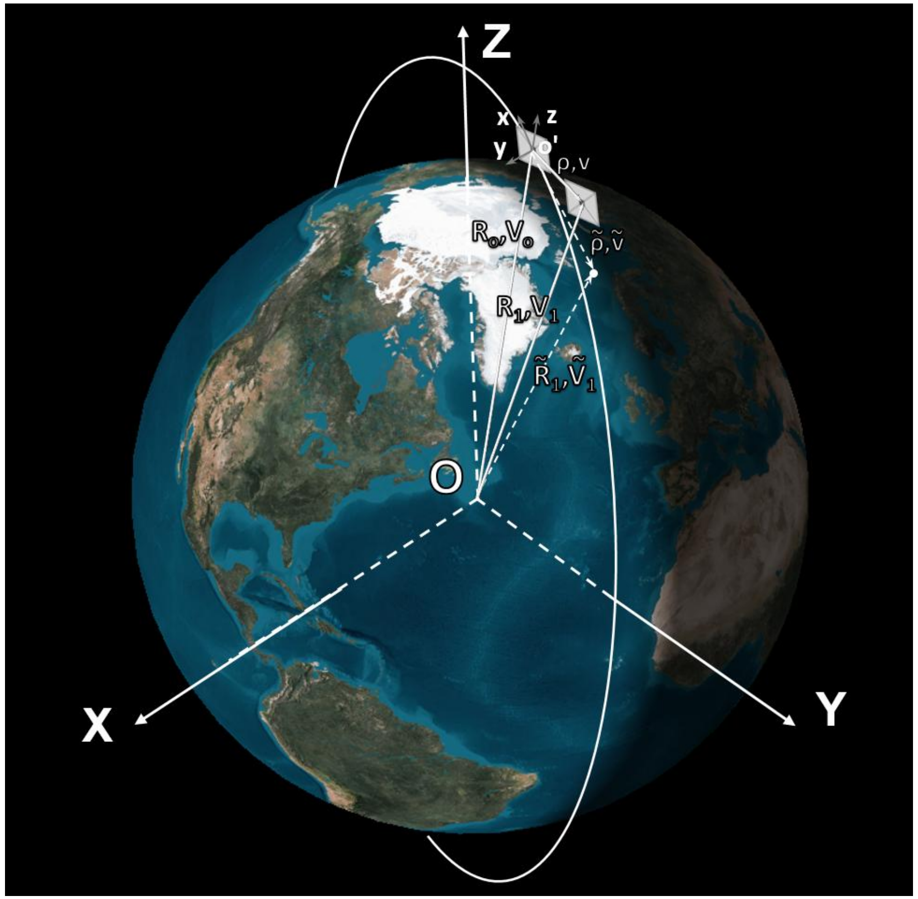
- Analytical solution to HCW equations in case of zero drift;

$$\begin{cases} x(t) = C_1\cos(nt + \alpha_0) + C_3; \\ y(t) = C_2\sin(nt + \alpha_0); \\ z(t) = \frac{C_1}{2}\sin(nt + \alpha_0)\,. \end{cases}$$

- Closed relative trajectories:

Projected circular orbit (PCO)

$$C_1 = r\,;$$
$$C_2 = r\,;$$
$$C_3 = 0\,;$$



Orbital reference frame notation

- **x** - along track
- **y** - normal to the orbital plane
- **z** - local vertical

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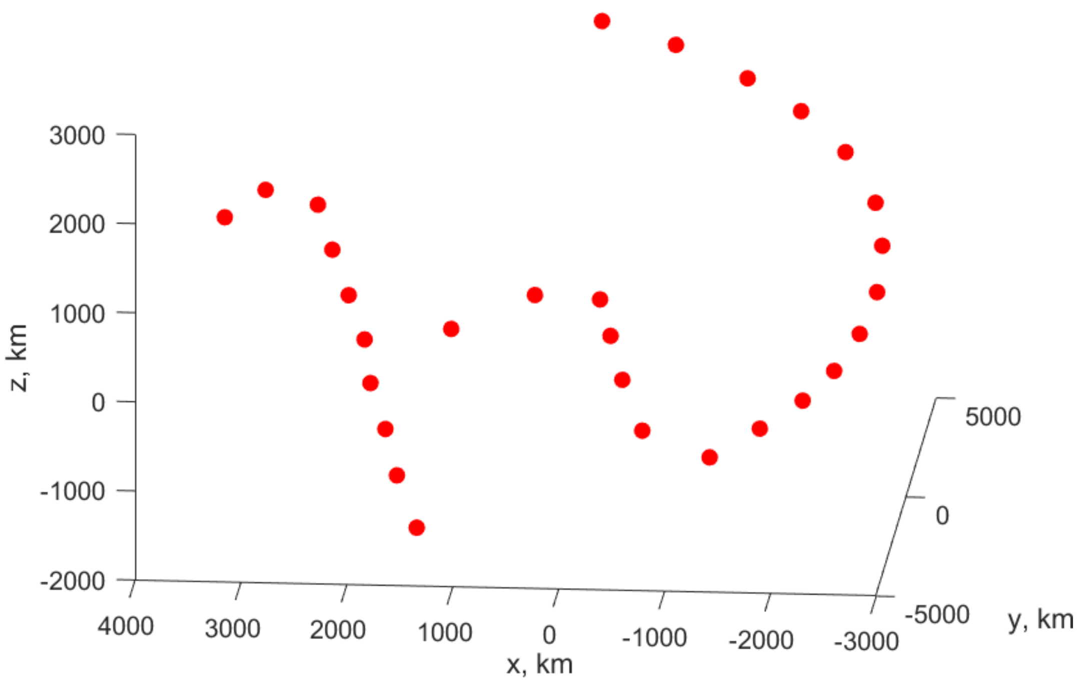
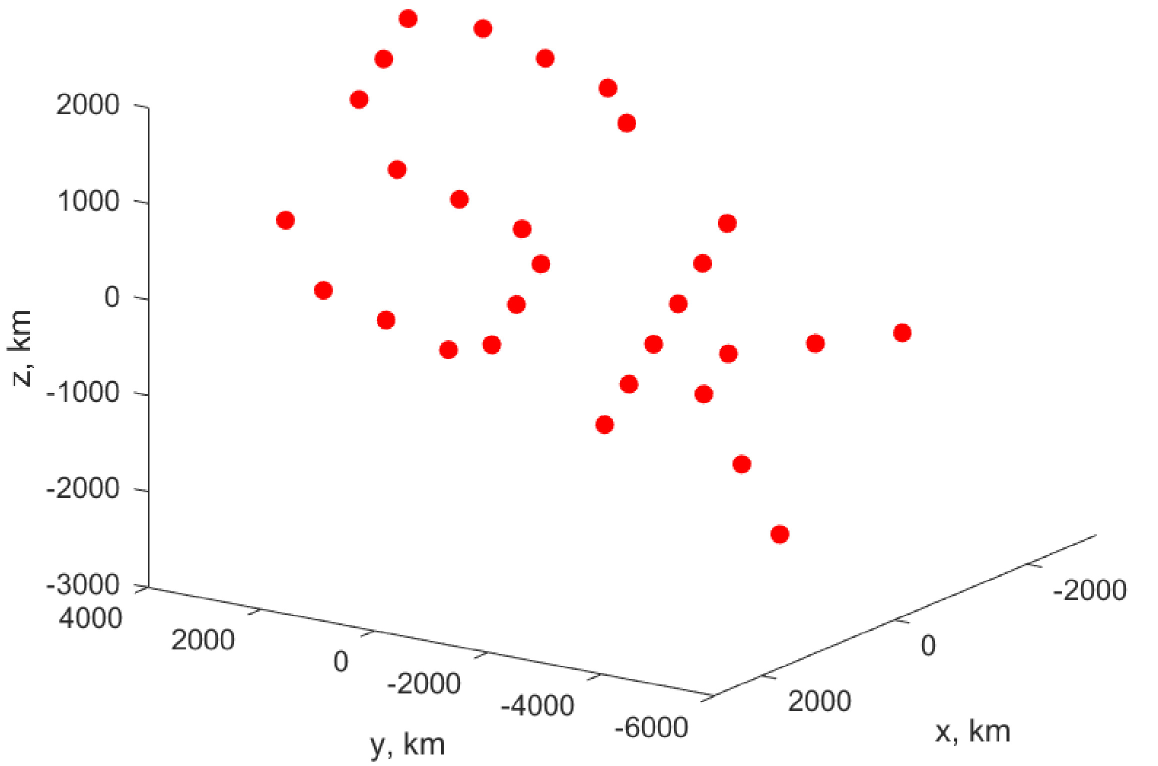
Orbital Configuration

- Minimum intersatellite distance ISD_{min} = 550 meters;
- Position of all satellite are assigned according to PCO solutions of HCW equations;
- Formation consists 29 satellites;



Orbital configuration 1 – MIPT logo



Orbital configuration 2 – Skoltech logo

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Relative Motion Control

- Let's consider a pair satellites in circular LEO orbits with state vectors *(R_0, V_0)* and *(R_1, V_1)* at time *t* given in O**XYZ** inertial;

- Relative position and velocity $(\boldsymbol{\rho}, \boldsymbol{v})$ of the follower satellite wrt leader can be derived as follows:

$$\begin{cases} \boldsymbol{\rho} = A^{-1}(\boldsymbol{R_1} - \boldsymbol{R_0}); \\ \boldsymbol{v} = A^{-1}(\boldsymbol{V_1} - \boldsymbol{V_0}) - \boldsymbol{\omega} \times \boldsymbol{\rho}. \end{cases}$$

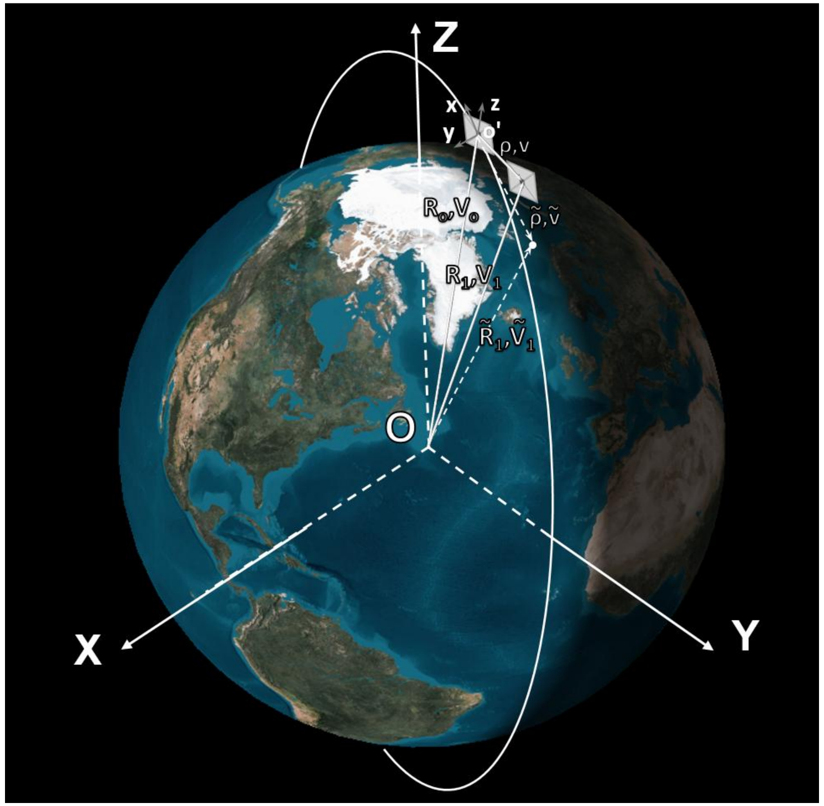
Where A - transition matrix from o'**xyz** to O**XYZ**, $\boldsymbol{\omega}$ = [0; n; 0], where n - mean motion of target satellite in o'**xyz** frame;

- The control is needed to keep follower's required periodic reference trajectory obtained with the aid of HCW equations (**ρ*, v***) in a certain position error box;

- The required reference trajectory corresponds to the following state vector in the inertial rf;

$$\begin{cases} \boldsymbol{R}_1^* = \boldsymbol{R_0} + A\boldsymbol{\rho}^*; \\ \boldsymbol{V}_1^* = \boldsymbol{V_0} + A[\boldsymbol{\omega} \times \boldsymbol{\rho}^*] + A\boldsymbol{v}^*. \end{cases}$$

- The difference between $(\boldsymbol{R_1}, \boldsymbol{V_1})$ and $(\boldsymbol{R}_1^*, \boldsymbol{V}_1^*)$ state vectors can be represented in terms of orbital elements difference;
- The control actions targets to adjust the difference via hybrid control algorithm utilizing



Relative motion geometry

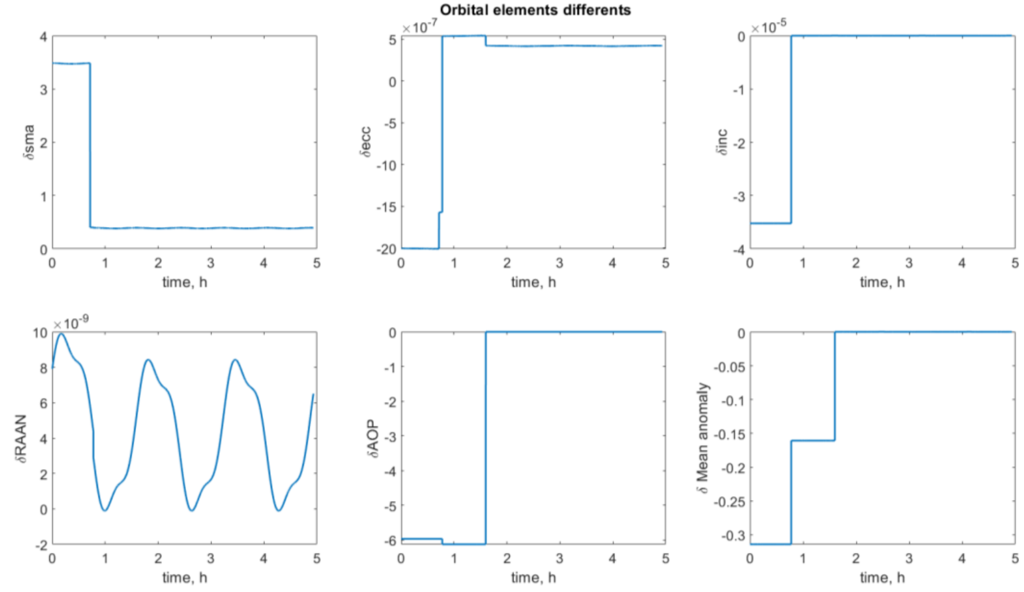
Impulsive Control

- The impulsive control is utilized during deployment and reconfiguration stages for coarse correction of formation satellites' relative orbits within a short time (~2 orbital periods);

- The analytical impulsive control scheme is derived from Gauss Variational Equations and consists of three maneuvers aiming at adjusting the difference between current and required orbital elements;

$$dV_1 = \begin{cases} \dfrac{n\,\delta a\,\eta}{2} \\ 0 \\ 0 \end{cases} \quad dV_2 = \begin{cases} 0 \\ v_{orb}\sqrt{\delta i^2 + \delta\Omega^2 sin^2(i)} \\ -v_{orb}\dfrac{\sqrt{\delta q_1^2 + \delta q_2^2}}{2} \end{cases} \quad dV_3 = \begin{cases} 0 \\ 0 \\ v_{orb}\dfrac{\sqrt{\delta q_1^2 + \delta q_2^2}}{2} \end{cases}$$

where dV_1 can be applied at both apogee or perigee, dV_2 and dV_3 should be applied at argument of latitude $u_1 = \theta_{crit} = tan^{-1}\left(\frac{d\Omega \sin(i)}{di}\right)$ and $u_2 = \theta_{crit} + \pi$; {a, $q_1 = e^*\cos(w)$, $q_2 = e^*\sin(w)$, I, Ω, λ} – equinoctial orbital elements, $\eta = (1 - e^2)^{1/2}$;



Difference between current and required orbital elements as a function of time

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[1] Vaddi S., et al. "Formation establishment and reconfiguration using impulsive control." *Journal of Guidance, Control, and Dynamics* 28.2 (2005): 262-268.

[2] Schaub H., et al. "Impulsive feedback control to establish specific mean orbit elements of spacecraft formations." Journal of Guidance, Control, and Dynamics 24.4 (2001): 739-745.

Continuous LQR-Based Control

- The continuous control is utilized during deployment and reconfiguration stages fine correction of formation satellites' relative orbits and for orbit maintenance;

- The linear dynamic model of a satellite relative motion is described in vector-matrix form as follows;

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}$$

where **x** = $[x, y, z, v_x, v_y, v_z]^T$ is state vector, A is the dynamic matrix incorporating J2 effect, **u** – control vector

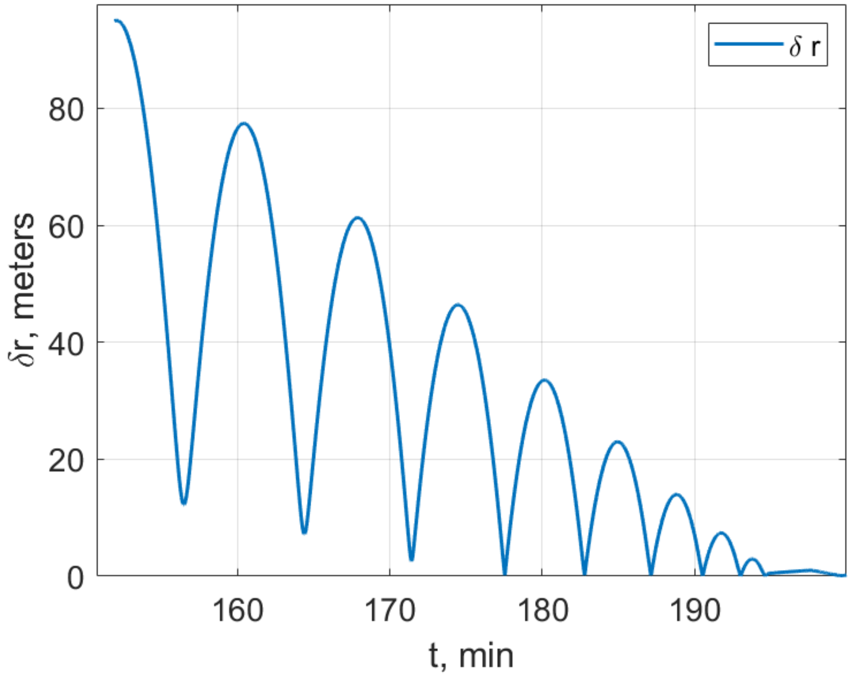
$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ (5c-2)n^2 & 0 & 0 & 0 & 2nc & 0 \\ 0 & 0 & 0 & -2nc & 0 & 0 \\ 0 & 0 & -(3c-2)n^2 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- We utilize the linear quadratic regulator (LQR) which is the feedback control that ensures the minimum of the functional

$$J = \int_0^{\infty} (\boldsymbol{e}^\top Q \boldsymbol{e} + \boldsymbol{u}^\top R \boldsymbol{u}) dt$$

where $\boldsymbol{e} = [\mathbf{x} - \mathbf{x}_d]^T$, $\mathbf{x}_d$ – desired state, $\boldsymbol{u} = -R^{-1}B^TP\boldsymbol{e}$, Q & R are positive definite weight matrices, the matrix P is obtained as a solution of the Riccati equation:

$$A^\top P + PA - PBR^{-1}B^\top P + Q = 0$$



Continuous LQR-based control performance

n – target orbit mean motion;

$$c = \sqrt{1+s},\; s = \frac{3J_2 R_{eq}^2}{2r_{ref}^2}(1 + 3\cos(2i))$$

Numerical study

- Orbital dynamics of formation satellites is described in ECI frame as follows: $$\ddot{R} = -\frac{\mu \boldsymbol{R}}{R^3} + \boldsymbol{f} + \boldsymbol{u};$$

where **R** – satellite formation vector given in ECI, ***f*** stands for external disturbances, ***u*** – control thrust vector, μ is the gravitational parameter of the Earth;

- The target orbit altitude h is about 600 km, therefore we neglect external perturbations except for the J2 effect:

$$\boldsymbol{f}_{\boldsymbol{J_2}} = \frac{3J_2\mu R_{\oplus}^2}{2R^2}\begin{pmatrix} 3sin^2(i)sin^2(u) - 1 \\ -sin^2(i)sin(2u) \\ -sin(2i)sin(u) \end{pmatrix};$$

- Satellites are 12U CubeSat with 18 kg wet mass;
- Propulsion system thrust = 100 mN (referring to SPT-100 thruster);
- Gain matrices: Q = 1e-4 E6x6 , R = E3x3
- Orbit maintenance threshold $\epsilon = \frac{|\boldsymbol{r}-\boldsymbol{r}_d|}{r_d}$ = 0.1;

Table. Mission timeline (April 12, 2021)

Event	Time, UTC+3
Initial epoch	00:00:00
Deployment flag on	00:00:00
Beginning of demo 1	04:21:17
Beginning of demo 1	04:27:40
Reconfiguration flag on	04:37:40
Beginning of demo 2	20:36:45
Beginning of demo 2	20:41:27

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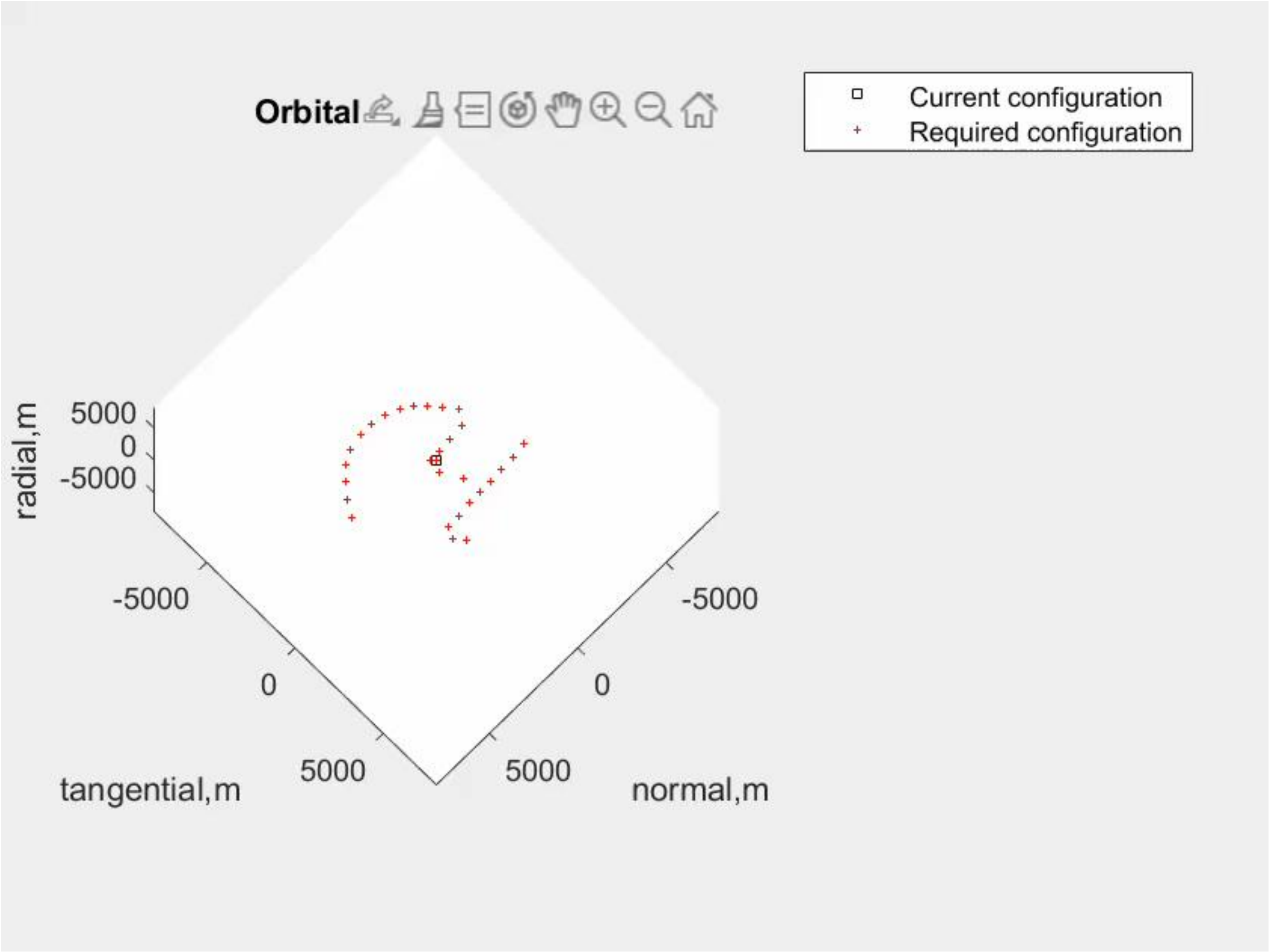
Simulation Results

- Deployment took 241.3 minutes;
- Reconfiguration took 335.2 minutes;
- The 1-day mission required 9 maintenance corrections between first and second maneuvers;



Formation satellites' dynamics & control simulation visualized in the orbital reference frame

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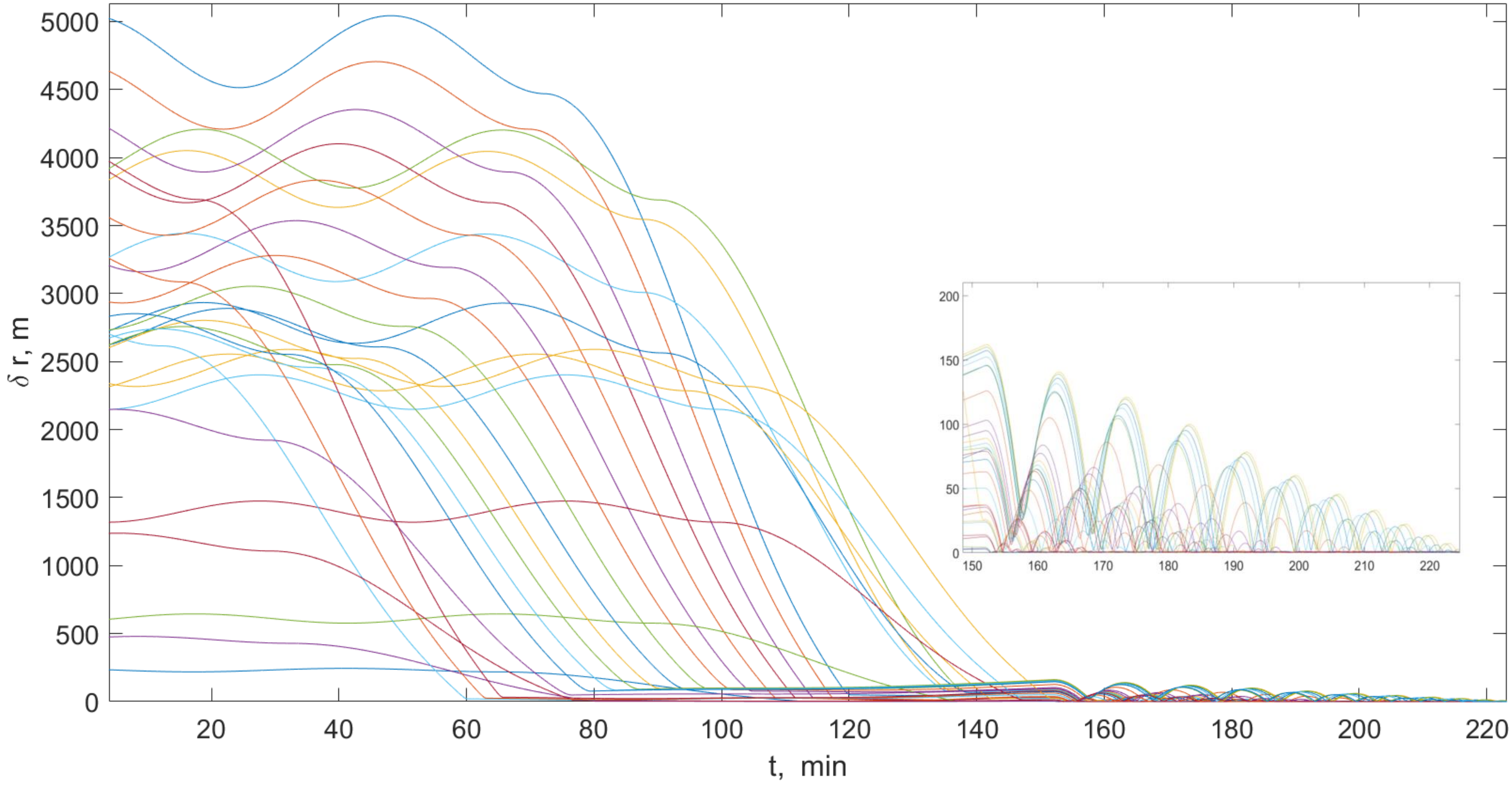
Simulation results: Deployment stage



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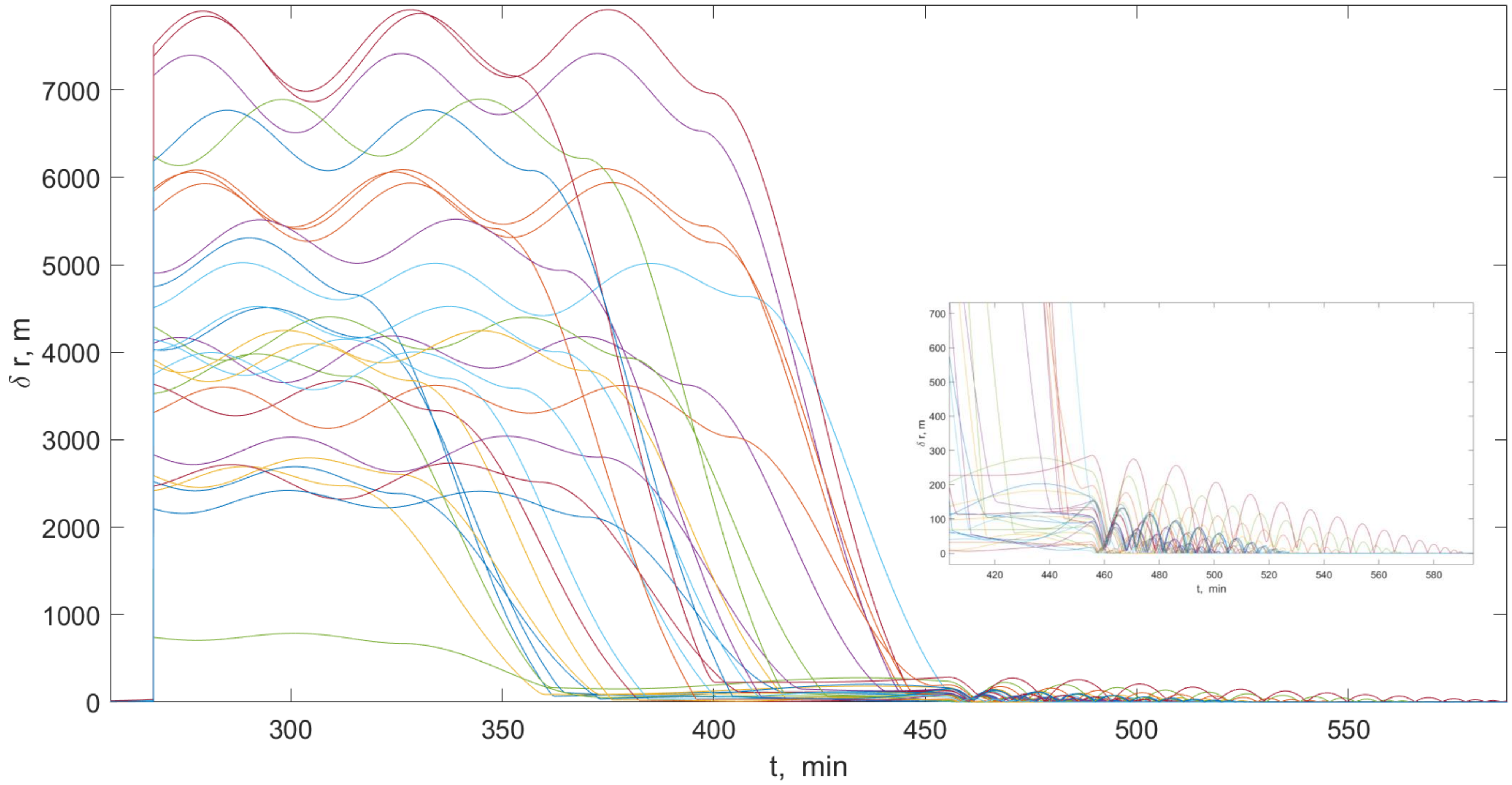
Simulation results: Reconfiguration stage



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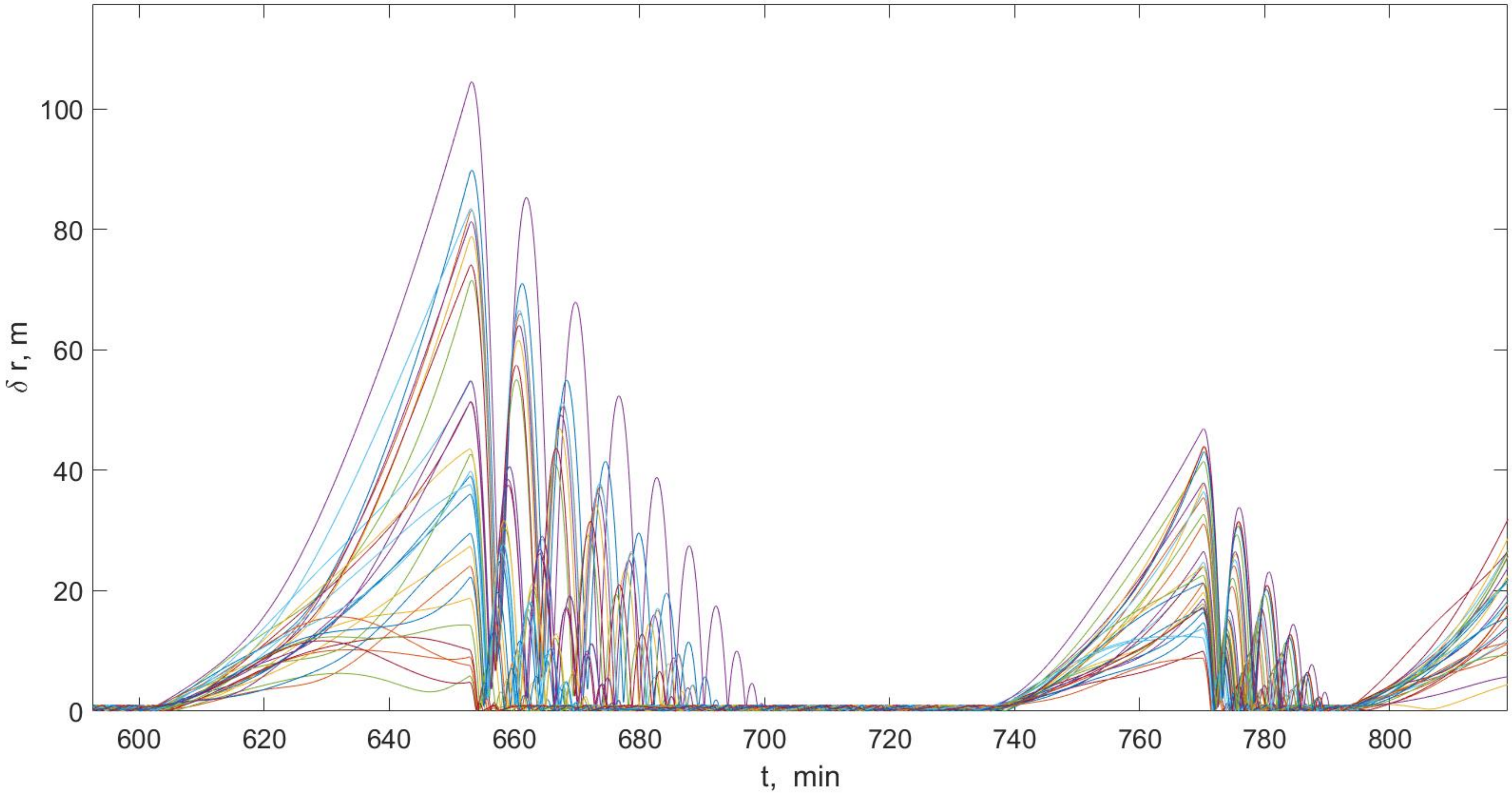
Simulation results: Maintenance stage



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Summary

- The hybrid control algorithm for multi-satellite formation deployment, maintenance, and reconfiguration was developed;
- In the algorithm, the impulsive maneuvers are utilized for coarse correction of relative orbits during deployment and reconfiguration, and continuous LQR-based control is used for the post-correction and maintenance stage;
- The proposed control algorithm allows deploying and reconfiguring a satellite formation within a short time (~ 3-4 orbit periods) and maintain the required orbital configuration with sub-meter precision;
- The control algorithm performance was tested for state-of-the-art thrusters accessible on the market;

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