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Coupled Motion Determination and Stabilization of a Satellite Equipped with Large Flexible Elements Using ADCS Only

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Abstract

The paper considers the rigid satellite with a set of flexible elements attached to its body. The flexible motion determination along with attitude motion of the main body is estimated in real time using measurements of the star tracker and angular velocity sensor only. The damping of the oscillations and attitude stabilization in the orbital reference frame is performed by reaction wheels installed on the main body. The dependence of the flexible motion determination accuracy on the number of estimated vibration modes is studied. A set of control algorithms based on LQR are proposed in the paper. Its convergence rate and final stabilization accuracy taking into account inaccuracy in motion estimation and unknown disturbances are studied. The problem of the isolated vibration modes that have small influence on the main body motion is discussed.

1. Introduction

Satellites with flexible elements are used to solve a variety of applied problems. Such satellites includes telecommunication satellites with large-sized antennas, deep-space research satellites with solar sails, and satellites with robotic manipulators and external rods. During orbital and angular satellite maneuvering the vibrations inevitably excite due to the large dimensions of flexible elements, which are often made of light materials. These vibrations can not only degrade the accuracy of attitude of the entire satellites, but even lead to instability of the required motion [1,2]. The installation of special damping actuators is desirable for damping low-frequency oscillations in the flexible elements. Usually piezoelectric devices installed on the non-rigid elements are used for this task. However, the case when the satellite is controlled using only the ADCS located on the main satellite body is of practical interest [3].

The attitude control problem of satellite with flexible elements is usually solved using standard approaches (for example, proportional-differential (PD) or proportional-integral-differential controllers, linear-quadratic regulator (LQR), robust control, etc.), but taking into account the features of dynamic spacecraft models with flexible elements. These features can be taken into account either in the synthesis of the control algorithm, or in estimation of the stabilization accuracy, but, as a rule, both options are considered. One of the most common approaches to the synthesis of the attitude stabilization algorithm is not to take into account the flexibility of the structure and form a control law based on the dynamics of a solid body. Such an approach is considered, for example, in [4], where a linear PD controller with a gyroscopic term is proposed. In [5] a PID controller is used. In [6] stability issues are considered and it is shown that if the system is stable as a solid body, then the presence of flexibility cannot make it unstable. In [7] control is based on a linear-quadratic controller. The motion of the satellite with a 122-meter antenna in a geostationary orbit in the vicinity of the position when the antenna is directed to nadir is considered. In [8] attitude control is carried out using an algorithm based on the sliding mode, and vibration damping is carried out using piezoelectric actuators. The work [9] is devoted to the synthesis of robust control, which is implemented using reaction wheels. Each of these approaches can be applied both to controlling only the attitude of the main body, and in conjunction with actuators located directly on flexible elements. As a rule, external disturbances are not included in the control loop, and the considered control algorithms are sufficiently rude with respect to external disturbances. In [10,11] an approach is developed to the analysis of the quality of the stabilization system work in the presence of disturbances. In [12] the influence of external disturbances on the stabilization accuracy of a space shuttle with an antenna extended on a long rod is studied.

The current work primarily solves the problem of the state vector determination of a spacecraft with flexible elements. It is necessary to determine the current attitude and angular velocity of the satellite body and vector of deviations of flexible elements and its derivatives to use it for calculation of the control. The problem of state vector estimation in real time is solved using extended Kalman filter. Star tracker and angular velocity sensors installed on the main satellite body are used for this task. Satellite with flexible elements is controlled by reaction wheels. So, attitude determination and control system of the satellite main rigid body is used for the whole flexible system stabilization in the orbital reference frame only. Developed control algorithm is based on a linear-quadratic controller and its modification - reduced LQR.

2. Satellite with solar panel and antenna motion equations

Consider a satellite consisting of a solid body and two flexible elements – a solar panel attached to the body with a one axis hinge, and an antenna has fixed connection (Fig. 1).

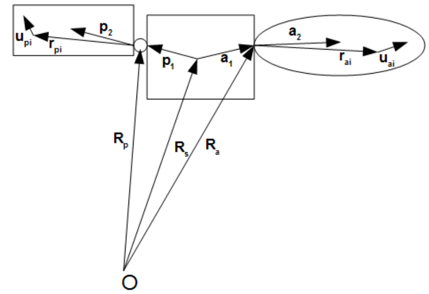

Fig. 1. Scheme of satelllite with flexible elements

In Fig. 1 $\mathbf{r}_{pi}$, $\mathbf{r}_{ai}$ are radius-vectors of i-th point of panel and antenna relative to the body-fixed reference frame with origins indicated in Fig.1; $\mathbf{u}_i$ is deviation of the i-th points from equilibrium positions caused by flexible motion.

The motion equations are derived using approach described in [13–15]. Equations for absolute angular velocity $\boldsymbol{\omega}$, amplitudes of antenna and panel $\mathbf{q}_a$, $\mathbf{q}_p$ are as follows:

$$\mathbf{S}\begin{pmatrix}\dot{\boldsymbol{\omega}}\\ \ddot{\mathbf{q}}_a\\ \ddot{\mathbf{q}}_p\end{pmatrix}=\begin{pmatrix}-\boldsymbol{\omega}\times\mathbf{J}_s\boldsymbol{\omega}+\mathbf{T}_s-\mathbf{N}_{\omega a}-\mathbf{f}_{\omega a}-\mathbf{N}_{\omega p}-\mathbf{f}_{\omega p}\\ -\mathbf{f}_a-\mathbf{N}_a\\ -\mathbf{f}_p-\mathbf{N}_p\end{pmatrix}. \quad (1)$$

Here matrix $\mathbf{S}$ is definitely-positive symmetric, which depends on system parameters and phase variables [15], $\mathbf{T}_s$ is the sum of the external torques acting on the satellite relative to the center of mass of rigid body of satellites (including control torque), $\mathbf{J}_s$ is the inertia tensor, values $\mathbf{N}_{\omega a}$, $\mathbf{N}_{\omega p}$, $\mathbf{N}_a$, $\mathbf{N}_p$ are nonlinearly dependent of system parameters and phase variables terms, $\mathbf{f}_{\omega a}$, $\mathbf{f}_{\omega p}$, $\mathbf{f}_a$, $\mathbf{f}_p$ are determined by the forces acting on the satellite.

These equations are supplemented by kinematic relations for the quaternion $\Lambda=[\lambda_0\ \boldsymbol{\lambda}]$:

$$\begin{aligned}\dot{\lambda}_0 &= -\frac{1}{2}(\boldsymbol{\omega},\boldsymbol{\lambda}),\\ \dot{\boldsymbol{\lambda}} &= \frac{1}{2}(\lambda_0\boldsymbol{\omega}+\boldsymbol{\lambda}\times\boldsymbol{\omega}).\end{aligned} \quad (2)$$

This equations does not include the equation for the variable that defines the angular position of the panel, since it is assumed that the rotation of this element is set independently. The described nonlinear model is used for modeling. The control is based on the model linearized in the vicinity of the required position of the model:

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}. \quad (3)$$

Here

$$\mathbf{x} = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{V}_q \\ \boldsymbol{\lambda} \\ \mathbf{q} \end{pmatrix},$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times n} & \mathbf{0}_{3\times3} & \mathbf{J}^{-1}\mathbf{S}_q\left(\mathbf{M}-\mathbf{S}_q^T\mathbf{J}^{-1}\mathbf{S}_q\right)^{-1}\boldsymbol{\Omega} \\ \mathbf{0}_{n\times3} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times3} & -\left(\mathbf{M}-\mathbf{S}_q^T\mathbf{J}^{-1}\mathbf{S}_q\right)^{-1}\boldsymbol{\Omega} \\ \frac{1}{2}\mathbf{E}_{3\times3} & \mathbf{0}_{3\times n} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times n} \\ \mathbf{0}_{n\times3} & \mathbf{E}_{n\times n} & \mathbf{0}_{n\times3} & \mathbf{0}_{n\times n} \end{pmatrix}$$

,

$$\mathbf{B} = \begin{pmatrix} \mathbf{J}^{-1}\left(\mathbf{E}_{3x3}+\mathbf{S}_q\left(\mathbf{M}-\mathbf{S}_q^T\mathbf{J}^{-1}\mathbf{S}_q\right)^{-1}\mathbf{S}_q^T\mathbf{J}^{-1}\right) \\ -\left(\mathbf{M}-\mathbf{S}_q^T\mathbf{J}^{-1}\mathbf{S}_q\right)^{-1}\mathbf{S}_q^T\mathbf{J}^{-1} \\ \mathbf{0}_{3\times3} \\ \mathbf{0}_{n\times3} \end{pmatrix}.$$

Two sources of disturbances are considered: the gravitational torque and a constant torque of magnitude $8\cdot10^{-4}$ N·m, which can appears due to the operation of the low-thrust propulsion and the torque of solar pressure forces.

3. Control algorithms

The control algorithms are based on the linear-quadratic control. They provide the asymptotic stability of the required motions and at the same time, they can limit the control by the proper selection of the algorithms parameters. The control is derived using a linearized unperturbed model. Note that the problem of satellite with flexible elements attitude after preliminary angular velocity damping is considered. The gravitational torque in the control calculation is not taken into account, since the motion of the satellite in a geostationary orbit is considered where it is rather small with respect to the control effort.

Linear-quadratic regulator minimizes the following cost function

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x}+\mathbf{u}^T\mathbf{R}\mathbf{u}\right)dt .$$

The algorithm allows to minimize a state vector deviation from required one $\mathbf{x}$ and control actuation value $\mathbf{u}$ as well. Positive-definite matrices $\mathbf{Q}$ and $\mathbf{R}$ are the parameters of the algorithm. The control is calculated as follows:

$$\mathbf{u} = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}\mathbf{x}, \quad (4)$$

where $\mathbf{P}$ is the solution of the algebraic Riccatti equation

$$\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}-\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}+\mathbf{Q}=0 .$$

Control algorithm (4) provides asymptotical stability of the zero point taking into account flexible variables and it allows to limit the control values by choosing matrix $\mathbf{R}$. Main disadvantage of this control approach is demanding of the linear approximation, i.e. the algorithm effectively works with linear system only. However, in the case of the system under consideration, this is acceptable assumption, since it is assumed that the initial angular velocity is small and dynamic equations (1) are actually linear.

Considered algorithm required the knowledge of current flexible amplitudes. This information usually is difficult to obtain. That is why a modification of LQR is considered. Equations (1) can be written in the following form [16,17]:

$$\begin{aligned} \dot{\mathbf{y}} &= \mathbf{A}_\omega\mathbf{y}+\mathbf{A}_{\omega q}\mathbf{z}+\mathbf{B}_\omega\mathbf{u}, \\ \dot{\mathbf{z}} &= \mathbf{A}_{qq}\mathbf{z}+\mathbf{B}_q\mathbf{u}, \end{aligned} \quad (5)$$

where variable $\mathbf{y}$ is set by satellite body attitude motion, $\mathbf{z}$ characterizes flexible motion. The functional of system is written in the following form:

$$\mathbf{J} = \int_0^\infty \left(\mathbf{y}^T\mathbf{Q}_y\mathbf{y}+\mathbf{z}^T\mathbf{Q}_z\mathbf{z}+\mathbf{u}^T\mathbf{R}\mathbf{u}\right)dt$$

The equations (5) allows to construct the control that minimizes the flexible modes excitement. To do that consider the following:

$$0 = \dot{\mathbf{z}} = \mathbf{A}_{qq}\mathbf{z}+\mathbf{B}_q\mathbf{u} .$$

When this condition is satisfied, the equations take the form

$$\dot{\mathbf{y}} = \mathbf{A}_\omega\mathbf{y}+\left(\mathbf{B}_\omega-\mathbf{A}_{\omega q}\mathbf{A}_{qq}^{-1}\mathbf{B}_q\right)\mathbf{u} ,$$

and a term is added to the functional that is responsible for minimizing the effect on the vibration modes (the second term in the second part):

$$\mathbf{J} = \int_0^\infty \left(\mathbf{y}^T\mathbf{Q}_y\mathbf{y}+\mathbf{u}^T\left(\mathbf{R}+\left(\mathbf{A}_{qq}^{-1}\mathbf{B}_q\right)^T\mathbf{Q}_z\mathbf{A}_{qq}^{-1}\mathbf{B}_q\right)\mathbf{u}\right)dt .$$

Let us designate

$$\mathbf{R}_x = \mathbf{R}+\left(\mathbf{A}_{qq}^{-1}\mathbf{B}_q\right)^T\mathbf{Q}_z\mathbf{A}_{qq}^{-1}\mathbf{B}_q ,$$

$$\mathbf{B}_x = \mathbf{B}_\omega-\mathbf{A}_{\omega q}\mathbf{A}_{qq}^{-1}\mathbf{B}_q ,$$

then the reduced LQR is obtained:

$$\mathbf{u} = -\mathbf{R}_x^{-1}\mathbf{B}_x^T\mathbf{P}\mathbf{y} ,$$

where $\mathbf{P}$ is the Riccati equiation solution:

$$\mathbf{A}_\omega^T\mathbf{P}+\mathbf{P}\mathbf{A}_\omega-\mathbf{P}\mathbf{B}_x\mathbf{R}_x^{-1}\mathbf{B}_x^T\mathbf{P}+\mathbf{Q}_y=0$$

With constant matrices of the system $\mathbf{A}_\omega$ and control $\mathbf{B}_x$, it is enough to solve the Riccati equation once. If the matrices change quasi-stationary, as in the case under consideration, then it is possible to have several matrix values $\mathbf{P}$ for different panel rotation angles.

4. State vector estimation

To estimate the state vector in real time, a star tracker and angular velocity sensor are used. The processing of its measurements in real time is carried out using the extended Kalman filter, which gives the best estimate of the state vector by the mean square criterion [18,19]. The extended Kalman filter can be used for nonlinear dynamic system and the measurement model. The measurement frequency is 4 Hz in this paper. Measurement errors are modeled as an unbiased normally distributed random variable with standard deviations for the star tracker angular velocity sensor $\sigma_\lambda = 2\cdot 10^{-5}$ and $\sigma_\omega = 1\cdot 10^{-4}$ deg / s, respectively. The dynamical model is described by (1) and (2). And the measurement model in the case is as follows:

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{r}_k .$$

where $\mathbf{z} = \left[\boldsymbol{\lambda}^T \quad \boldsymbol{\omega}^T\right]^T$ is the measurement vector, $\mathbf{H}$ is the measurement matrix of the following form:

$$\mathbf{H} = \begin{bmatrix} \mathbf{E}_{3\text{x}3} & 0_{3\text{xn}} & 0_{3\text{x}3} & 0_{3\text{xn}} \\ 0_{3\text{x}3} & 0_{3\text{xn}} & \mathbf{E}_{3\text{x}3} & 0_{3\text{xn}} \end{bmatrix}.$$

It is assumed that perturbations are present in the motion model, i.e. the motion model is not accurate. Let disturbances act both on the satellite body and the flexible elements. As a perturbation model a random normally distributed unbiased value with the following standard deviations is used:

$$\sigma_{\dot{\omega}} = 1\cdot 10^{-8}\ \text{deg/s}^2,\ \sigma_{\ddot{q}} = 1\cdot 10^{-4},$$

where $\sigma_{\dot{\omega}}$ is the mean square deviation of disturbances acting on the satellite body, $\sigma_{\ddot{q}}$ is mean square deviation acting on flexible elements.

Note that when conducting a numerical study, the number of estimated vibration modes is inevitably limited. For objective reasons the dynamics of the system taking into account a large number of modes cannot be implemented in on-board satellite computer. Therefore only senior modes are considered that have the greatest influence on the motion of the satellite body and have, as a rule, a low natural damping decrement. The influence of higher-frequency modes on the motion of a spacecraft with a flexible elements can be considered as a perturbation, but this approach will increase the errors in the estimates of the state vector. On the other hand, a decrease in the dimension of the estimated state vector will reduce the computational cost for the satellite on-board computer. Moreover, it turns out that the addition of extra measurements for the flexible satellite state vector estimation, for example, measurements from camera abserving the antenna, does not necessarily have a positive effect on the overall determination accuracy. This is due to the fact that flexible motion observations are correspond to the complete model with an “infinite” number of flexible modes. In the determination algorithm, however, only a few low frequency modes are taken into account. As a result, the algorithm attempts to attribute the observed flexible elements deviations to the modes in state vector only, which increases the error.

5. Numerical study

Joint modeling of control algorithms and state determination is carried out for the LQR-based control algorithm. For reduced LQR, measurements of the star sensor and angular velocity sensor are used directly. The conditions for modeling are set as follows:

$$\boldsymbol{\omega} = \begin{pmatrix} 0.02 & 0.01 & 0.03 \end{pmatrix}\ \text{deg/s},$$

$$\boldsymbol{\Lambda} = \begin{pmatrix} 0.5 & 0.5 & 0.5 & -0.5 \end{pmatrix}$$

$$q_a = \mathbf{e}_{n_a\text{x}1}\cdot 0.1,$$

$$q_p = \mathbf{e}_{n_p\text{x}1}\cdot 0.1,$$

where n_a and n_p are number of flexible modes in antenna and panel. The numerical simulation is carried out with 0.125 s time step.

5.1. LQR-based control algorithm

As it was mentioned earlier, the control law (4) requires the determination of both the attitude motion of satellite body and the flexible motion of the vibration modes. To solve this problem, an extended Kalman filter is used. In this case, a closed-loop control loop is simulated in the case when the number of modes in the system model is equal to $n_a = 7$, $n_p = 2$, whereas when evaluated in the Kalman filter, it is implied $n_a = 5$, $n_p = 1$. That is, the case of partial knowledge of the vibration modes is considered, and for a panel that is more rigid, only one main vibration mode is taken into account. The simulation results are shown in Fig. 2-5.

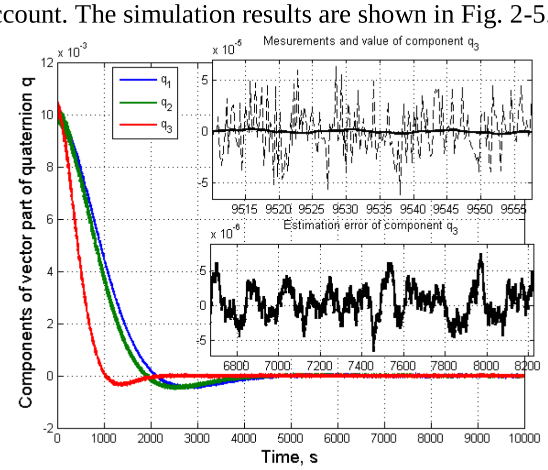

Fig. 2. Attitude quaternion of the satellite body, star tracker measurements (upper right) and estimation of EKF (lower right)

Fig. 2 shows three components of the vector part of the attitude quaternion of satellite body. In the upper right part for the component q_3 the accuracy of the final stabilization in more detail is presented (about 10^{-5} in the vector part of the quaternion, i.e. 4 arc seconds) and the accuracy of the readings of the star sensor. In the lower right part of the figure an estimation error of quaternion using the Kalman filter is shown. It can be seen that its use can reduce the error by about ten times. However, the readings of the star sensor are accurate enough to be used directly without processing, which is considered when application of the reduced LQR. The stabilization accuracy achieves $3{\cdot}10^{-5}$ s^{-1} in angular velocity. In Fig.3 also presented angular velocity sensor measurements in right upper part and EKF estimations in lower right part.

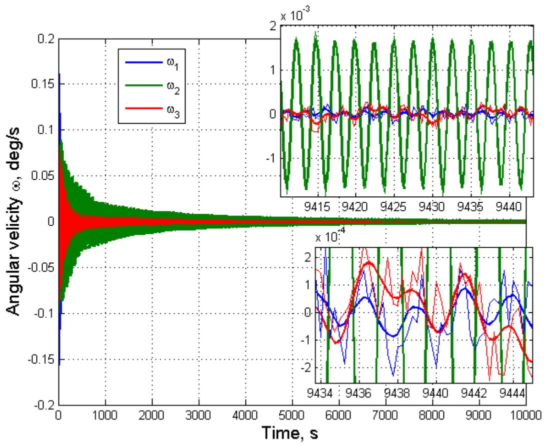

Fig. 3. Angular velocity of the satellite body, angular velocity sensor measurements (upper right) and estimation of EKF (lower right)

Fig. 4 and 5 show that the full damping of the natural vibrations of the antenna and panel takes a long time, but in general the attitude control system copes with the problem. It is interesting to note that the determination of the deviations of the panel and antenna, depicted for example in detail in Fig. 4 and 5 in upper right part for one of the modes, has a high accuracy.

An important result of the algorithm implementation is the control torque limitation. The control algorithm is turned on after reaching a predetermined error value of Kalman filter estimations only, and the control coefficients are selected using an iterative procedure depending on the initial conditions. In this case, only the matrix **R** changes: when the allowable control is exceeded, its diagonal elements increase by 10 times. Due to this procedure, a restriction on the value of the control of reation wheels of 0.4 N·m is achieved.

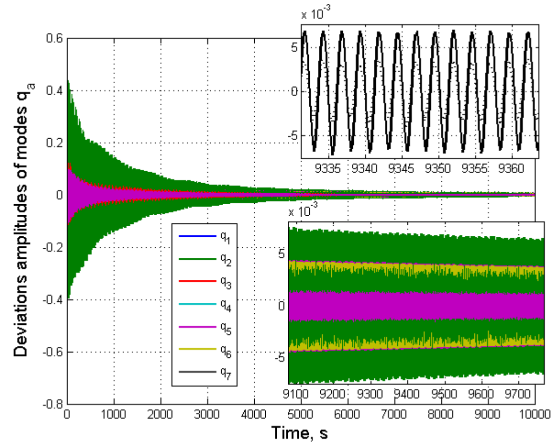

Fig. 4. Deviations of antenna modes and estimation of EKF for one of the mode (upper right)

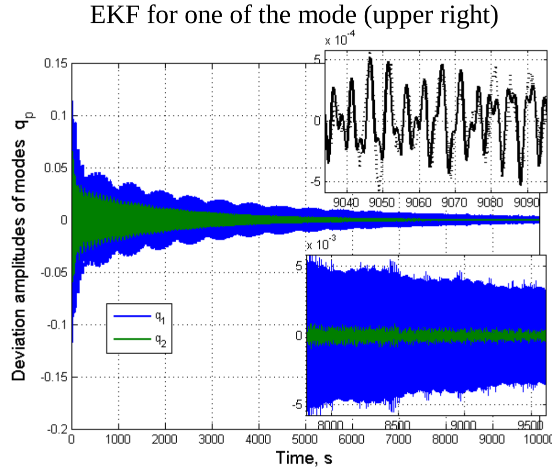

Fig. 5. Deviations of panel modes and estimation of EKF for one of the mode (upper right)

5.2. Reduced LQR-based control algorithm

According to the results in Fig. 2 the measurements of the star sensor and angular velocity sensor is derectly used for reduced LQR. The simulation results are presented in Fig. 6-8 for an satellite having 7 modes of antenna vibrations and 2 modes of panel vibrations (the satellite has information about 5 and 1 mode only, respectively).

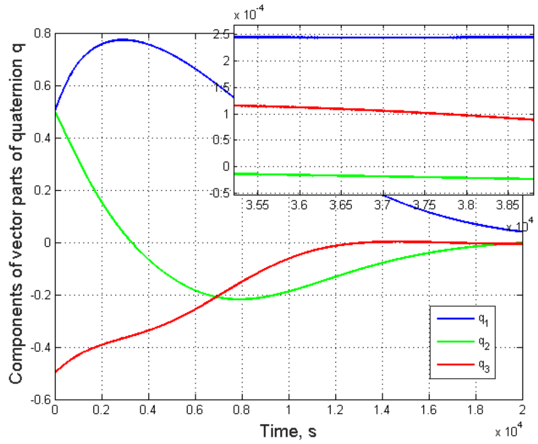

Fig. 6. Attitude quaternion of the satellite body

Comparison of Fig. 2 and Fig. 6 shows that in comparison with to the conventional LQR stabilization time is significantly increased. The accuracy of the final stabilization is also high and is $2.5 \cdot 10^{-4}$ for one of the components. Note that stabilization accuracy could be improved. The error is caused primarily by a constant disturbing torque, in this case acting along one axis (it is along this axis that the attitude error is greater). Moreover, in the simulation, the value of the elements of the weight matrix changed, reflecting the penalty for the attitude error. At the beginning, their value is reduced to provide a limit on the magnitude of the control torque, then, after ensuring stabilization of the satellite and flexible modes, the contribution of the attitude error to the functional increases to achieve the desired attitude with high accuracy.

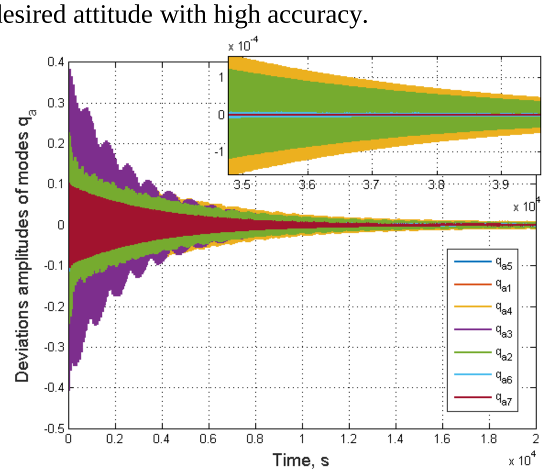

Fig. 7. Deviations of antenna modes

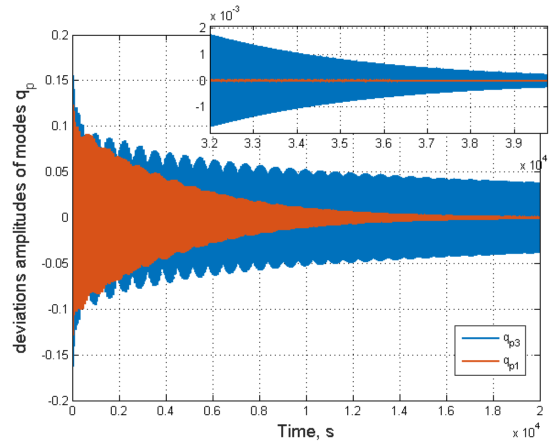

Fig. 8. Deviations of antenna modes

Comparison Fig. 4 and 7 shows that damping the oscillations of the antenna is slower when using reduced LQR. The same applies to Fig. 5 and 8, showing damping panel vibrations. In general, the stabilization problem is solved by reduced LQR, but its performance is worse. An important advantage of this approach is a significant reduction in computational complexity compared to the conventional LQR, since, firstly, the dimension of the phase vector is smaller and, as a result, the matrix in the Riccati equation has a size of 6x6 (instead of 18x18), and, secondly, there is no need to use estimation algorithm, which significantly reduces computational costs. Comparative calculations show that the function that implements the extended Kalman filter works for about 300 seconds when modeling on an interval of 2000 seconds, while LQR works for about 30 seconds. In the case of reduced LQR, the control function works for about 20 seconds, and measurements are submitted directly. Thus, LQR with a extended Kalman filter requires 16.5 times more computational time than reduced LQR.

Conclusions

The paper considers algorithms for the state vector determination and the attitude motion control of a satellite with two flexible elements – a rotating solar panel and a large-sized antenna. It is shown that stabilization of the satellite with damping of its own vibrations of flexible structures using sensors and actuators installed only on the satellite body is possible. In this case, a zero decrement of damping of the natural vibrations of the structure is assumed. The control algorithm that does not require state estimation and does not excite vibrations is proposed. The latter, providing less accuracy and more attitude stabilization time, significantly reduces the computational cost for the on-board computer.

Acknowledgements

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