

Attitude Control and Determination Accuracy Study of Small Satellite with Limited Set of Sensors

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Abstract

The problem of a small satellite Attitude Determination and Control System development is considered. The first task of the system is the attitude determination. Decent attitude estimation accuracy is required although only magnetometer and angular velocity sensor are available. Both sensors have high values of noise and changing bias. The extended Kalman Filter is utilized to estimate both the satellite attitude motion and sensors biases. The dynamical noises matrix should be estimated prior to the filter implementation. This is performed with the numerical simulations including disturbance source unaccounted for in the filter satellite motion model. The problem of active attitude control achieving both inertial and orbital frames stabilization is considered. Magnetorquers and reaction wheels are used to implement the Lyapunov feedback control. The results of the study for specific satellite parameters are presented in the paper.

Keywords: Attitude determination, attitude control, extended Kalman Filter, Lyapunov control

1. Introduction

Attitude Determination and Control System (ADCS) is one of the most important satellite subsystems. It must provide knowledge about the current state vector, construct the reference motion that satisfies mission requirements and, of course, it has to implement it. These problems became even more complicated when we talk about small satellites. Due to the energy, mass, volume and cost restrictions it is usually impossible to install large and precise sensors such as star trackers or laser gyros with low noise.

In this paper we suggest an approach to the ADCS algorithms synthesis. It includes Kalman Filtering for attitude determination, technique for reference motion construction (two different scenarios – inertial and orbital stabilization), and Lyapunov-based controller for ensuring the necessary reference angular motion.

2. Attitude motion determination

Different sensors might be utilized for the attitude determination: Sun sensors [1–3], magnetometers [4–7], angular velocity sensors [8,9] and even micro star tracker [10,11]. Here we consider a set of sensors that consists of magnetometer, sun sensor and angular rate sensor (ARS). They provide noised measurements. Magnetometer and ARS are biased, and this bias is time-dependent. Moreover, there are time intervals when satellite is located in the Earth shadow, thus sun sensor measurements are unavailable. In order to determine the current attitude and angular velocity two different Kalman Filters are suggested. First utilizes all three sensors, and the second one uses measurements from the magnetometer and ARS only. In addition to the state vector, both filters estimate the current bias of magnetometer and ARS.

Kalman Filter operates using the predictor-corrector scheme (Figure 1). Using estimations from the previous time step \mathbf{x}_{k-1}^+ and the satellite model of motion, it predicts the current state vector \mathbf{x}_k^- . After that, using new measurements it updates the state vector \mathbf{x}_k^+ .

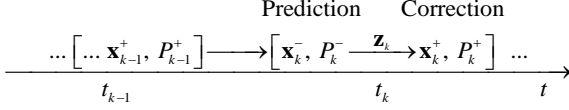


Fig 1. Kalman filter principle of operation

Let the system is described by the following equations

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}, t) + \mathbf{G}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{h}(\mathbf{x}, t) + \mathbf{v}(t)\end{aligned}$$

where \mathbf{x} is the state vector, \mathbf{z} is the measurements, \mathbf{G} is weighting matrix, \mathbf{w}, \mathbf{v} are the model and the measurements noises with corresponding covariance matrices \mathbf{D}, \mathbf{R} . We utilize extended Kalman Filter, so motion and measurements models must be linearized

$$\mathbf{H}_k = \left. \frac{\partial \mathbf{h}(\mathbf{x}, t)}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_k^-, t=t_k}, \quad \mathbf{F}_k = \left. \frac{\partial \mathbf{f}(\mathbf{x}, t)}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_k^-, t=t_k}$$

Predicted parameters are obtained using

$$\begin{aligned}\mathbf{x}_k^- &= \int_{t_{k-1}}^{t_k} \mathbf{f}(\mathbf{x}_{k-1}^+, t) dt; \\ \mathbf{P}_k^- &= \Phi_k \mathbf{P}_{k-1}^+ \Phi_k^T + \mathbf{Q}_k, \\ \mathbf{Q}_k &= \int_{t_{k-1}}^{t_k} \Phi_k \mathbf{G} \mathbf{D} \mathbf{G}^T \Phi_k^T dt = \Phi_k \mathbf{G} \mathbf{D} \mathbf{G}^T \Phi_k^T \Delta t\end{aligned}$$

where $\Phi_k = \mathbf{E} + \mathbf{F}_k \Delta t$, \mathbf{P} is Kalman filter estimation covariance matrix. Corrected parameters are defined by

$$\begin{aligned}\mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \\ \mathbf{x}_k^+ &= \mathbf{x}_k^- + \mathbf{K}_k [\mathbf{z}_k - \mathbf{h}(\mathbf{x}_k^-, t_k)] \\ \mathbf{P}_k^+ &= [\mathbf{E} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_k^-\end{aligned}$$

Satellite model of motion includes gravity gradient and reaction wheels control torques. Kinematics is described using quaternions

$$\begin{aligned}\mathbf{J}\dot{\boldsymbol{\omega}} &= \mathbf{M}_g - \boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega} + \mathbf{h}) - \dot{\mathbf{h}}, \\ \dot{\mathbf{Q}} &= \frac{1}{2} \mathbf{Q} \circ \boldsymbol{\omega} = \frac{1}{2} \begin{pmatrix} 0 & -\boldsymbol{\omega}^T \\ \boldsymbol{\omega} & [-\boldsymbol{\omega}]_x \end{pmatrix} \begin{pmatrix} q_0 \\ \mathbf{q} \end{pmatrix}.\end{aligned}$$

Here \mathbf{J} is the satellite tensor of inertia, $\boldsymbol{\omega}$ is its angular velocity, \mathbf{h} is the total angular momentum of reaction wheels, $\mathbf{Q} = (q_0, \mathbf{q})^T$ is the satellite attitude quaternion,

$$\mathbf{M}_g = \frac{3\mu}{R^5} \mathbf{R} \times \mathbf{J} \mathbf{R},$$

μ is the Earth gravitational parameter, \mathbf{R} is the satellite position, and skew symmetric of vector product is introduced

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_x \mathbf{b} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \mathbf{b}$$

In addition, sensor biases are included in the model of motion. It is supposed that they do not change over time, and their time variation is caused only by model noise. Hence, estimated state vector consists of twelve parameters: attitude quaternion vector part, angular velocity, magnetometer and ARS biases.

Model of motion must be linearized. We suppose that control torques, i.e. $\dot{\mathbf{h}}$, are constant between the time steps. After the mathematics, it can be represented by

$$\mathbf{F} = \begin{bmatrix} -[\boldsymbol{\omega}]_{x \ 3 \times 3} & \frac{1}{2} \mathbf{I}_{3 \times 3} & \mathbf{0}_{6 \times 3} \\ \left(\frac{6\mu}{R^5} \mathbf{J}^{-1} \mathbf{F}_g \right)_{3 \times 3} & (-\mathbf{J}^{-1} \mathbf{F}_x)_{3 \times 3} & \mathbf{0}_{6 \times 3} \\ \mathbf{0}_{3 \times 6} & \mathbf{0}_{3 \times 6} & \mathbf{0}_{6 \times 6} \end{bmatrix}_{12 \times 12}$$

where

$$\begin{aligned}\mathbf{F}_g &= [\mathbf{R}]_x \mathbf{J} [\mathbf{R}]_x - [\mathbf{J} \mathbf{R}]_x [\mathbf{R}]_x, \\ \mathbf{F}_x &= [\boldsymbol{\omega}]_x \mathbf{J} - [\mathbf{J} \boldsymbol{\omega}]_x.\end{aligned}$$

Expression for the linearized measurements matrix depends on the set of sensors used by Kalman Filter. For the full set of sensors it is

$$\mathbf{H} = \begin{bmatrix} 2[\mathbf{b}^s]_{x \ 3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ 2[\mathbf{s}^s]_{x \ 3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}_{9 \times 12}$$

and for the magnetometer and ARS it is

$$\mathbf{H} = \begin{bmatrix} 2[\mathbf{b}^s]_{x \ 3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}_{6 \times 12}$$

Here \mathbf{b} if the magnetic field direction, and \mathbf{s} is sun direction.

Kalman filter depends on several parameters that must be determined [12]. First of all, it is the model noise, i.e. covariance matrix \mathbf{D} . We suppose that it is diagonal, and there are three parameters to be determined: noise of the dynamics, i.e. unknown external torques that affect the satellite, noise of the magnetometer bias and noise of the ARS bias. They are determined numerically using computer simulation of the satellite motion in the complicated model of motion and minimizing the estimation errors (see Simulation section).

3. Reference motion construction

The following coordinate systems are used in the paper:

- $O_a Y_1 Y_2 Y_3$ – Inertial Frame (IF): its origin O_a is located in the Earth center of mass, $O_a Y_1$ is directed to the Vernal equinox of the J2000 epoch, $O_a Y_3$ is orthogonal to the ecliptic plane;
- $O x_1 x_2 x_3$ – Body-Fixed Frame (BF): its origin O is located in the spacecraft center of mass, and the axes are its principal axes of inertia.
- $O y_1 y_2 y_3$ – Orbital Frame (OF): its origin O is located in the spacecraft center of mass, $O y_1$ is aligned with satellite radius vector, $O y_3$ is aligned with orbital angular momentum.
- $O z_1 z_2 z_3$ – Reference Frame (RF): its origin O is located in the spacecraft center of mass, and axes depend on the mission scenario.

In this paper we consider two different scenarios: inertial and orbital stabilization. However, suggested technique can be applied in other cases, when required angular motion is more complicated.

For the reference motion construction the direction cosine matrix kinematics is used:

$$\dot{\mathbf{B}} = -[\boldsymbol{\omega}_0]_{\times} \mathbf{B},$$

where $\mathbf{B} = (\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3)^T$ is the reference attitude, \mathbf{e}_k are the basis vectors of the Reference Frame, $\boldsymbol{\omega}_0$ is the reference angular velocity. In the case of inertial stabilization (e.g. for the accumulator recharging when satellite's solar panels are directed to the sun) \mathbf{B} is constant, thus $\boldsymbol{\omega}_0 \equiv 0$.

Consider the case of orbital stabilization in more details. Here the satellite attitude might be represented by two sequential rotations, i.e.

$$\mathbf{B} = \mathbf{M}\mathbf{N},$$

where \mathbf{N} describes rotation from Inertial Frame to Orbital Frame, and \mathbf{M} is rotation from Orbital Frame to the Reference Frame. \mathbf{M} is supposed to be constant, and \mathbf{N} is described by

$$\mathbf{N} = (\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3)^T,$$

$$\mathbf{e}_1 = \frac{\mathbf{R}}{R}, \quad \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1, \quad \mathbf{e}_3 = \frac{\mathbf{R} \times \mathbf{V}}{|\mathbf{R} \times \mathbf{V}|}.$$

Here \mathbf{R} is satellite position, \mathbf{V} is its velocity. Their time derivatives (in a simple orbital motion model where only central gravitation field affects the satellite) are

$$\dot{\mathbf{e}}_1 = \frac{\mathbf{V} - \mathbf{e}_1(\mathbf{e}_1, \mathbf{V})}{R}, \quad \dot{\mathbf{e}}_3 = 0, \quad \dot{\mathbf{e}}_2 = -\mathbf{e}_2 \frac{(\mathbf{e}_1, \mathbf{V})}{R|\mathbf{R} \times \mathbf{V}|}.$$

Then orbital angular velocity is defined by

$$[\boldsymbol{\omega}_{orb}]_{\times} = -\dot{\mathbf{N}}\mathbf{N}^T.$$

Since \mathbf{M} is constant, reference angular velocity is

$$\boldsymbol{\omega}_0 = \mathbf{M}\boldsymbol{\omega}_{orb}.$$

If satellite moves along the circular orbit, then $\dot{\boldsymbol{\omega}}_{orb} = 0$ and $\dot{\boldsymbol{\omega}}_0 = 0$. Otherwise, we can use the same kinematics for matrices and define angular accelerations, or find them numerically.

4. Attitude control algorithms

In order to control the attitude Lyapunov-based controller is implemented. It is based on Barbashin-Krasovskii-LaSalle principle [13,14] and provides asymptotic stability of the reference motion [15–18]. Let the satellite motion is described by

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega}) = \mathbf{M}_g + \mathbf{M}_{ctrl},$$

$$\dot{\mathbf{Q}} = \frac{1}{2} \mathbf{Q} \circ \boldsymbol{\omega}.$$

Reference motion is described by the reference quaternion \mathbf{U} and angular velocity $\boldsymbol{\omega}_0$, $\mathbf{S} = (s_0, \mathbf{s})^T$ is the quaternion from Reference Frame to the Body Frame (its origin in the satellite center of mass and axes are the satellite's principal axes of inertia), \mathbf{A} is the direction cosine matrix that corresponds to the quaternion \mathbf{S} , and $\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs} - \mathbf{A}\boldsymbol{\omega}_0$ is the relative angular velocity. Consider the Lyapunov-candidate function

$$V = \frac{1}{2} (\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}) + k_q (1 - s_0).$$

where $k_q = const > 0$. Its time derivative is

$$\dot{V} = (\boldsymbol{\omega}_{rel}, \mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{J}(\boldsymbol{\omega}_{rel} \times \mathbf{A}\boldsymbol{\omega}_0) - \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_0 + k_q \dot{\mathbf{s}}).$$

Hence, the control that ensures asymptotic stability of the reference motion is

$$\mathbf{M}_{ctrl} = -\mathbf{M}_{ext} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{J}(\boldsymbol{\omega}_{rel} \times \mathbf{A}\boldsymbol{\omega}_0) + \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_0 - k_q \boldsymbol{\omega}_{rel} - k_s \mathbf{s}.$$

There are reaction wheels installed on the satellite, so in order to implement this control torque we have to choose $\dot{\mathbf{h}}$ as follows:

$$\dot{\mathbf{h}} = -\boldsymbol{\omega} \times \mathbf{h} - \mathbf{M}_{ctrl}.$$

It allows us to include reaction wheels into the simulation and estimate the required torque and angular momentum.

5. Results of numerical experiments

In order to test suggested techniques the computer simulation is carried out:

- Satellite utilizes IGRF model for magnetic field, and external field additionally perturbed by periodic function with amplitude of 75 nT
- J2 perturbation for orbital motion
- The magnetometer noise is $\sigma = 50$ nT, bias is periodic function with a period of one revolution, its amplitude is 500 nT
- Sun sensor noise is $\sigma = 0.1$ deg
- The ARS noise is $\sigma = 0.005$ deg/s, bias is periodic function with a period of one revolution, its amplitude is 0.05 deg/s
- Errors in reaction wheels installation setup are included in the simulation
- Errors in tensor of inertia knowledge are included in the simulation

For different sets of filter parameters (noise of the dynamical model σ_d , noise of the magnetometer bias model σ_{db} , noise of the ARS bias model σ_{dw}) Monte Carlo simulation is carried out. After that, we get an expected accuracy of attitude and angular velocity errors (see Fig. 2), and choose the best set of parameters.

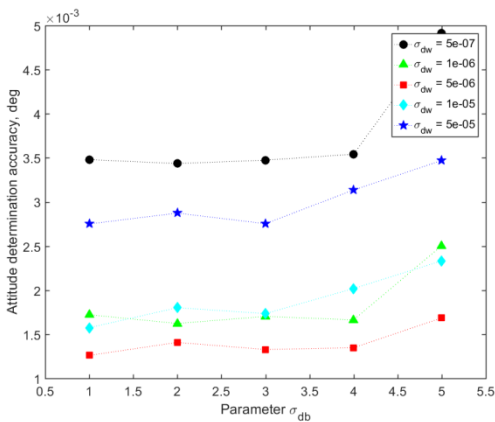
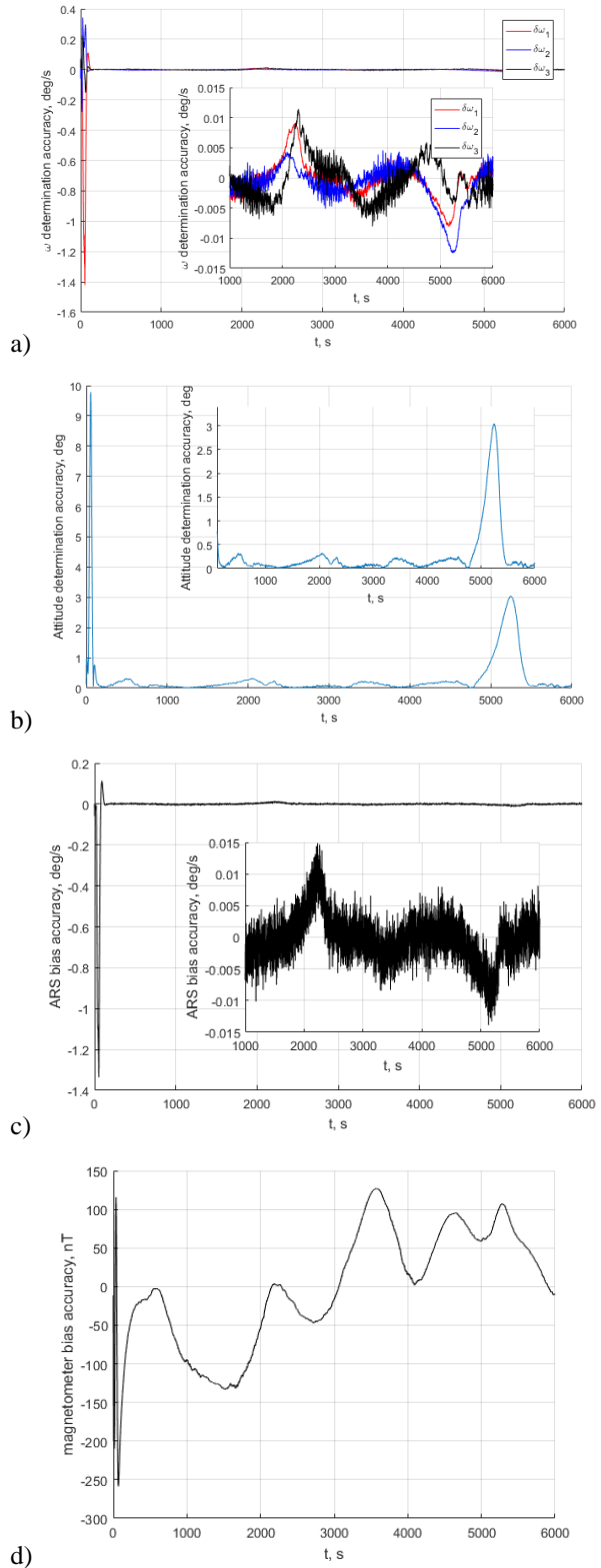


Fig 2. Example of the Monte Carlo simulation

Results of the simulation with the best set of parameters are presented in Fig. 3-5. It should be noted that ADCS based on sun sensor, magnetometer and ARS demonstrates better performance (accuracy around 0.5 degrees), but depends on the angle between the sun direction and magnetic field.



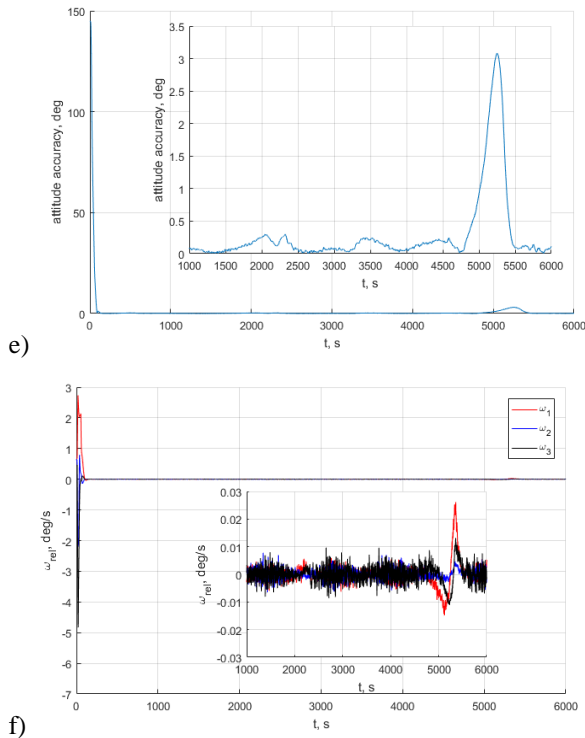


Fig. 3. Kalman filter based on magnetometer, sun sensor and ARS: a) angular velocity determination accuracy, b) attitude determination accuracy, c) ARS bias determination accuracy, d) magnetometer bias determination accuracy, e) provided attitude accuracy, f) provided angular velocity accuracy]

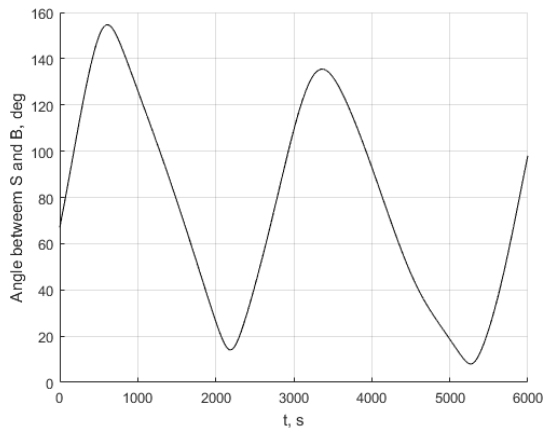
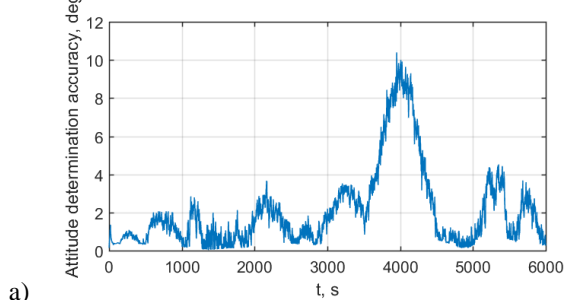
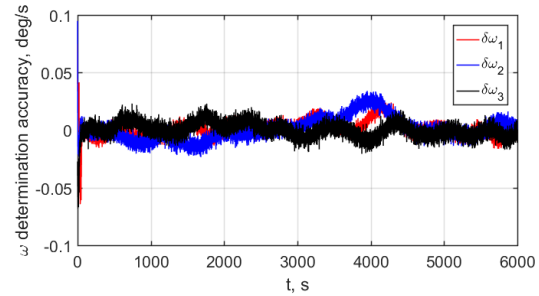


Fig. 4. Angle between local magnetic field and sun direction



a)



b)

Fig. 5. Kalman Filter based on magnetometer and ARS: a) attitude determination accuracy b) angular velocity determination accuracy

6. Conclusions

In the paper we suggested an approach to the algorithm synthesis for the small satellite ADCS. Two different Filters were suggested: based on ARS, magnetometer and sun sensor, and based on ARS and magnetometer. Computer simulation showed that first one can provide an accuracy of better than 0.5 degrees (in good conditions when angle between sun direction and magnetic field is sufficient), and the second one can provide accuracy of around 2-5 degrees.

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