
# Comparison of Relative Motion Control Algorithms for Point Capturing of Space Debris Object

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# Problem statement

## Assumptions:

- Active spacecraft is in the vicinity of space debris
- The relative rotational and translational motion is assumed to be known
- The active spacecraft is equipped:
  - with thrusters to control translational motion and reaction wheels to control attitude motion
  - with a capturing system (robotic arm, magnetic capturing system etc.)
- Passive space debris object is tumbling
- It is possible to catch the debris at one specific point on the debris surface

## Aims of the work:

- Develop different control algorithms for debris approaching
- Compare the performance of the algorithms

# Equations of relative motion

### The relative angular velocity:

$$\boldsymbol{\omega} = \boldsymbol{\omega}_S - \boldsymbol{\omega}_T$$

$$\frac{d^I \boldsymbol{\omega}}{dt} = \frac{d^I \boldsymbol{\omega}_S}{dt} - \frac{d^I \boldsymbol{\omega}_T}{dt}$$

$$\left(\frac{d^I \boldsymbol{\omega}}{dt}\right)^T = \mathbf{D}_S^T \left(\frac{d^I \boldsymbol{\omega}_S}{dt}\right)^S - \left(\frac{d^I \boldsymbol{\omega}_T}{dt}\right)^T$$

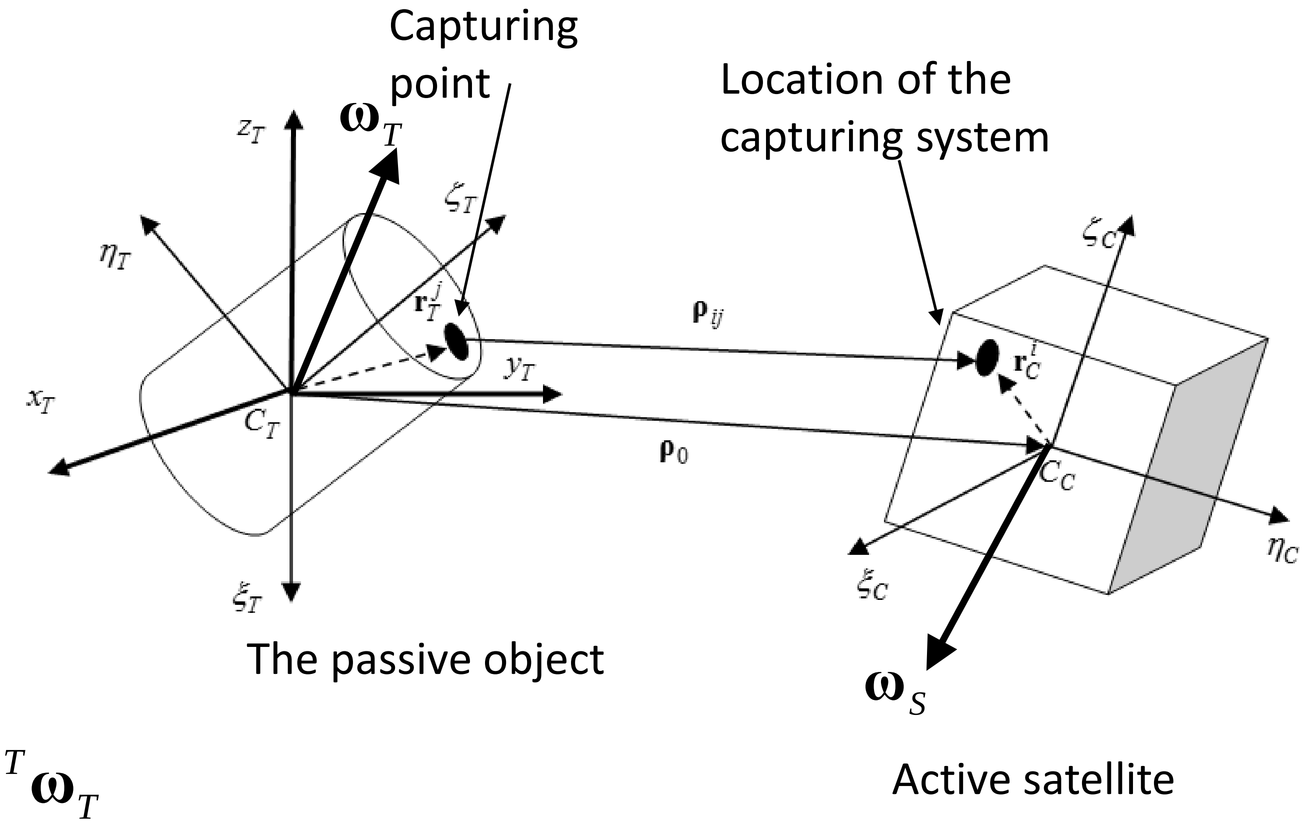
### Kinematical equations:

$$\dot{\mathbf{D}}_S^T = [\boldsymbol{\omega}]_\times \mathbf{D}_S^T$$

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q}\boldsymbol{\omega}^S$$

### Derivative of angular velocity:

$$\left(\frac{d^T \boldsymbol{\omega}}{dt}\right)^T = \left[(\boldsymbol{\omega})^T\right]_\times \mathbf{D}_S^T(\boldsymbol{\omega}_S) + \mathbf{D}_S^T \frac{d^T \boldsymbol{\omega}_S}{dt} - \frac{d^T \boldsymbol{\omega}_T}{dt}$$

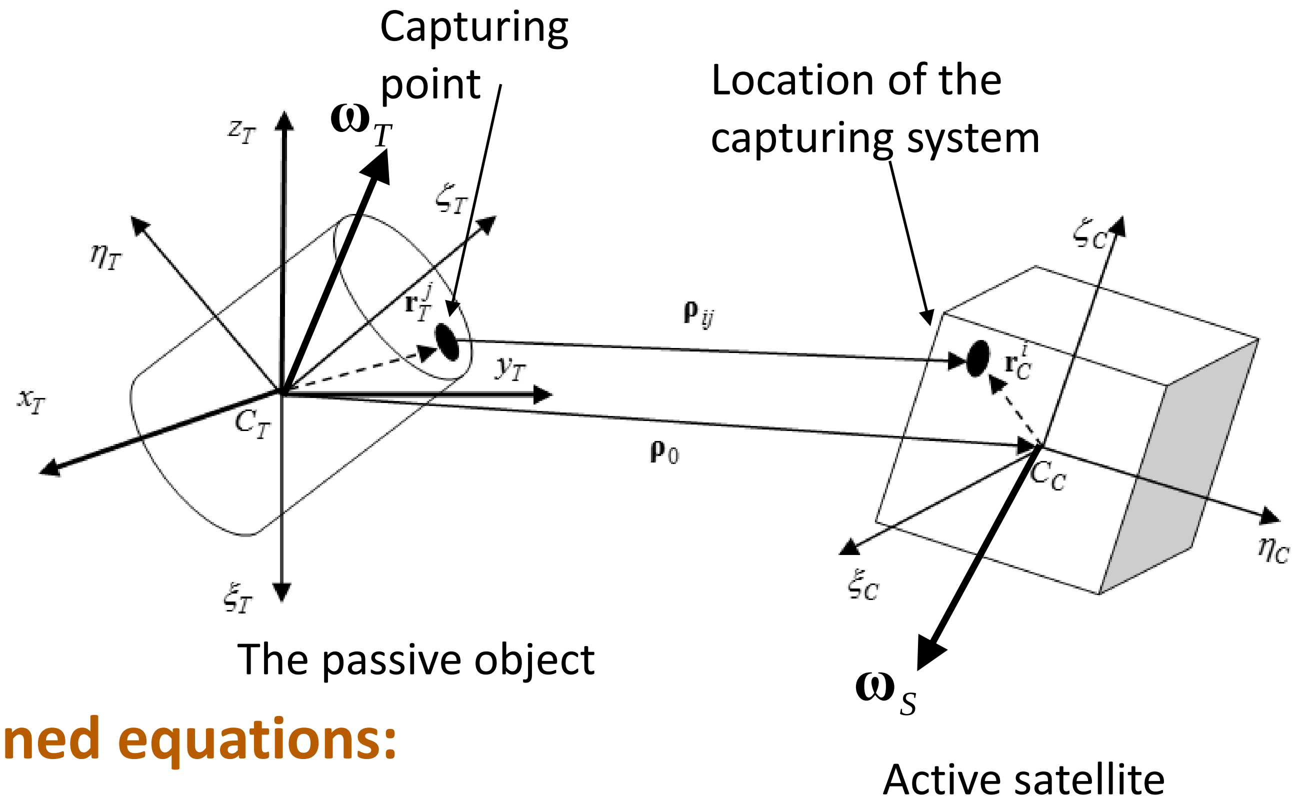


# Dynamical equations of relative rotational motion

## Dynamical Euler equations:

$$\left.\frac{d\mathbf{H}_S}{dt}\right|_I = \left.\frac{d\mathbf{H}_S}{dt}\right|_S + \boldsymbol{\omega}_S \times \mathbf{H}_S = \mathbf{N}_S + \mathbf{T}_S$$

$$\left.\frac{d\mathbf{H}_T}{dt}\right|_I = \left.\frac{d\mathbf{H}_T}{dt}\right|_T + \boldsymbol{\omega}_T \times \mathbf{H}_T = \mathbf{N}_T$$




## After substituting derivative of angular velocity into the above mentioned equations:

$$\mathbf{I}_T\dot{\boldsymbol{\omega}}^T = \mathbf{I}_T\mathbf{D}(q)\mathbf{I}_S^{-1}[-\mathbf{D}(\mathbf{q})^{-1}\left(\boldsymbol{\omega}^T + \boldsymbol{\omega}_T^T\right) \times \mathbf{I}_S\mathbf{D}(\mathbf{q})^{-1}\left(\boldsymbol{\omega}^T + \boldsymbol{\omega}_T^T\right) - \ldots$$

$$-\mathbf{D}(\mathbf{q})^{-1}\left(\boldsymbol{\omega}^T + \boldsymbol{\omega}_T^T\right)\mathbf{h}_{WC} - \dot{\mathbf{h}}_{WC} + \mathbf{T}_C + \mathbf{N}_C] - \mathbf{I}_T\boldsymbol{\omega}_T^T \times \boldsymbol{\omega}^T + [\boldsymbol{\omega}_T^T \times \mathbf{I}_T\boldsymbol{\omega}_T^T]$$

# Relative translational motion

## Equations of relative translational motion:

$$\ddot{x}_{ij} - 2\omega_{OT}\dot{y}_{ij} - \dot{\omega}_{OT} y_{ij} - 3\omega_{OT}{}^{2} x_{ij} = a_x + d_x$$

$$\ddot{y}_{ij} + 2\omega_{OT}\dot{x}_{ij} + \dot{\omega}_{OT} x_{ij} = a_y + d_y$$

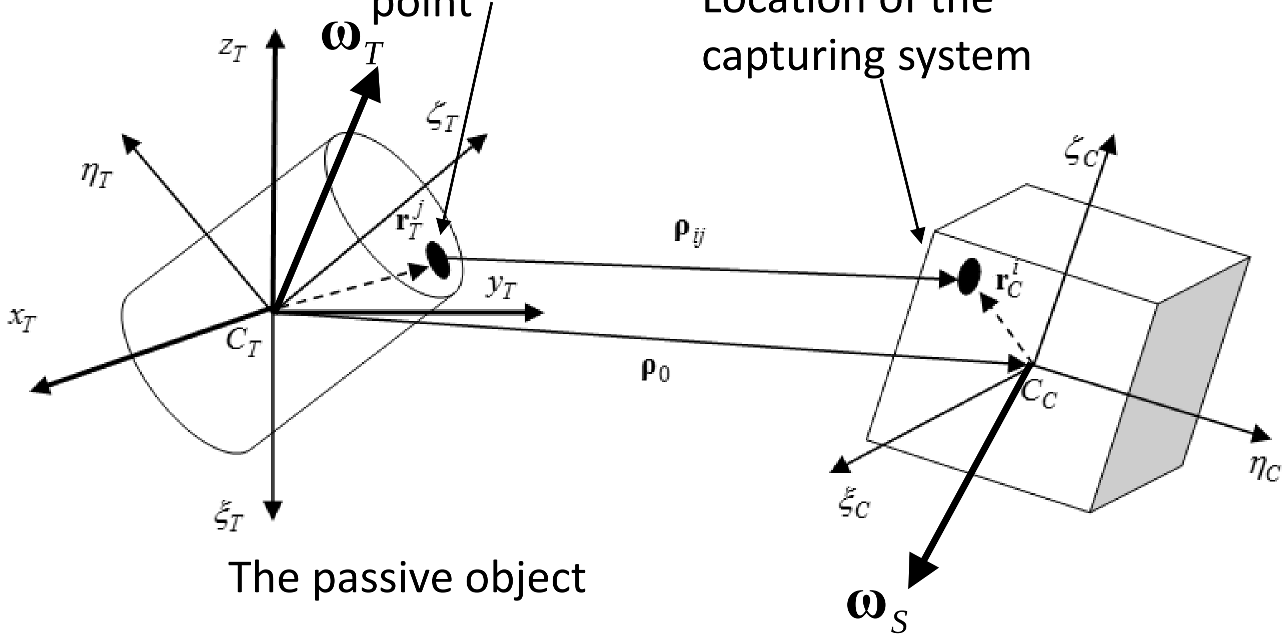
$$\ddot{z}_{ij} + \omega_{OT}^{2} z_{ij} = a_z + d_z$$

## Relative position of the points:

$$\boldsymbol{\rho}_{ij} = \boldsymbol{\rho}_0 + \mathbf{r}_S^i - \mathbf{r}_T^j$$

$$\dot{\boldsymbol{\rho}}_{ij} = \dot{\boldsymbol{\rho}}_0 + \boldsymbol{\omega} \times \mathbf{r}_S^i$$

$$\ddot{\boldsymbol{\rho}}_{ij} = \ddot{\boldsymbol{\rho}}_0 + \dot{\boldsymbol{\omega}} \times \mathbf{r}_S^i + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_S^i)$$



# Desired relative position for capturing

- **Relative directions**

$$\mathbf{e}^T = \mathbf{D}_S^T \mathbf{e}_S^S + \mathbf{e}_T^T, \quad \mathbf{e}_S = \frac{\mathbf{r}_S^i}{\left|\mathbf{r}_S^i\right|}, \ \mathbf{e}_T = \frac{\mathbf{r}_T^j}{\left|\mathbf{r}_T^j\right|}$$

- **For capturing must be satisfied:**

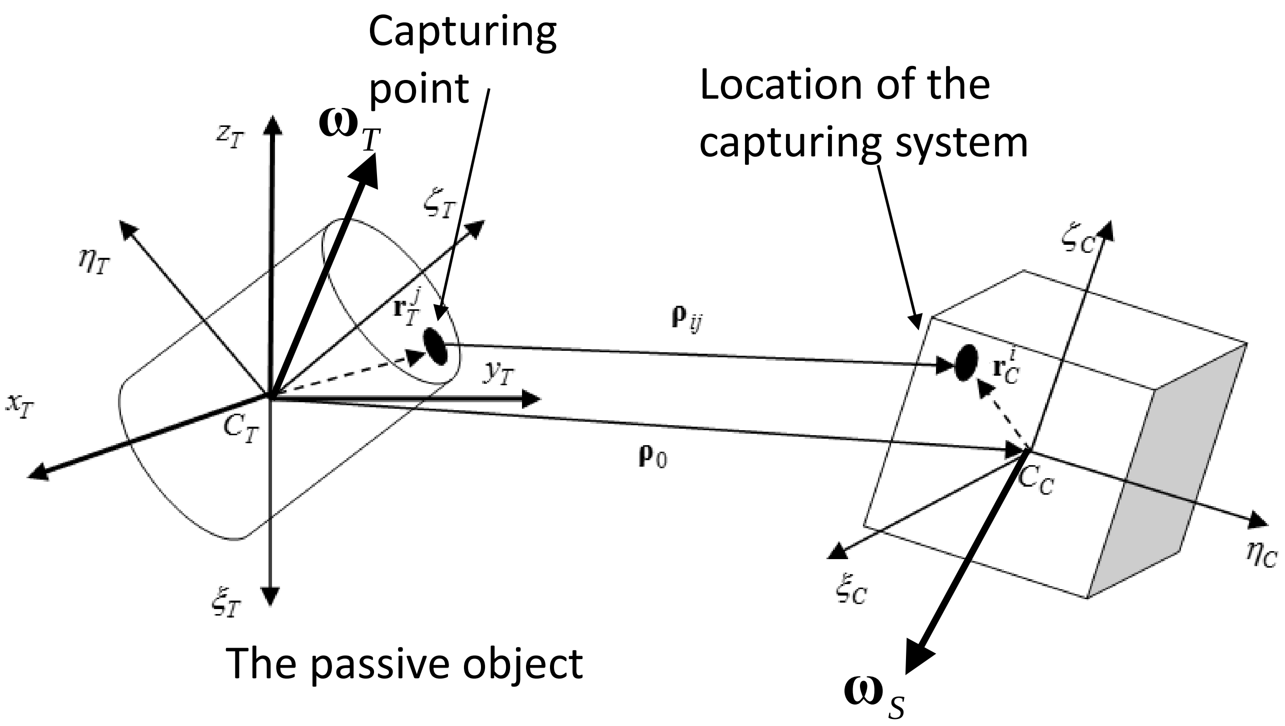
$$\mathbf{e} = 0 \ \text{и} \ \dot{\mathbf{e}} = 0$$

- **Desired position of center of mass:**

$$\boldsymbol{\rho}_d^T = K \cdot \mathbf{e}_T^T, \quad K > \left|\mathbf{r}_C^i\right| + \left|\mathbf{r}_T^j\right|$$

- **Dynamical equation:**

$$\left(\frac{d^2\mathbf{e}}{dt^2}\right)^T = \left(\dot{\boldsymbol{\omega}}^\times + \boldsymbol{\omega}^\times \boldsymbol{\omega}^\times\right)\left(\mathbf{e}^T - \mathbf{e}_T^T\right)$$




$$\ddot{\boldsymbol{\rho}}_d^T = \dot{\boldsymbol{\omega}}_T \times \ddot{\boldsymbol{\rho}}_d^T + \boldsymbol{\omega}_T \times (\boldsymbol{\omega}_T \times \ddot{\boldsymbol{\rho}}_d^T)$$

# SDRE-based control algorithm

- Nonlinear dynamical system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x},\mathbf{u})$$

- After linearization

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\big|_{\mathbf{x}(t_k)}\mathbf{x} + \mathbf{B}(\mathbf{x})\big|_{\mathbf{x}(t_k)}\mathbf{u}$$

- Riccati equation

$$\mathbf{P}(\mathbf{x})\mathbf{A}(\mathbf{x}) + \mathbf{A}^T(\mathbf{x})\mathbf{P}(\mathbf{x}) - \\ - \mathbf{P}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}\mathbf{B}^T(\mathbf{x})\mathbf{P}(\mathbf{x}) + \mathbf{Q} = 0$$

- Minimization of the functional

$$J = \frac{1}{2}\int_0^{t_f}\left[\mathbf{x}(t)^T\mathbf{Q}\,\mathbf{x}(t) + \mathbf{u}(t)^T\mathbf{R}\,\mathbf{u}(t)\right]dt$$

- Control law

$$\mathbf{u}(\mathbf{x}) = -\mathbf{R}^{-1}\left(\mathbf{B}^T(\mathbf{x})\mathbf{P}(\mathbf{x})\right)\mathbf{x}$$

# State-dependent coefficients for dynamical system

- **For relative angular motion**

$$\begin{bmatrix} \dot{\mathbf{e}} \\ \ddot{\mathbf{e}} \end{bmatrix}_{6\times1} = \begin{bmatrix} 0 & \mathbf{E} \\ \left((\dot{\boldsymbol{\omega}})^{\times} + \boldsymbol{\omega}^{\times}\boldsymbol{\omega}^{\times}\right) & 0 \end{bmatrix}_{6\times6} \begin{bmatrix} \mathbf{e} \\ \dot{\mathbf{e}} \end{bmatrix}_{6\times1} + \begin{bmatrix} 0 \\ (\mathbf{e}^{T} - \mathbf{e}_{T}^{T})^{\times} \mathbf{D}\mathbf{I}_{S}^{-1} \end{bmatrix}_{6\times3} \left[\mathbf{T}_{C}\right]_{3\times1}$$

- **For relative translational motion**

$$\Delta\boldsymbol{\rho} = \boldsymbol{\rho}_0 - \boldsymbol{\rho}_d$$

$$\begin{bmatrix} \Delta\dot{\boldsymbol{\rho}} \\ \Delta\ddot{\boldsymbol{\rho}} \end{bmatrix}_{6\times1} = \begin{bmatrix} 0 & \mathbf{E} \\ \mathbf{N} & \mathbf{M} \end{bmatrix}_{6\times6} \begin{bmatrix} \Delta\boldsymbol{\rho} \\ \Delta\dot{\boldsymbol{\rho}} \end{bmatrix}_{6\times1} + \begin{bmatrix} 0 \\ \mathbf{E} \end{bmatrix}_{6\times3} \left[\mathbf{a}\right]_{3\times1} - \dot{\boldsymbol{\omega}}_T \times \ddot{\boldsymbol{\rho}}_d^{T} + \boldsymbol{\omega}_T \times (\boldsymbol{\omega}_T \times \ddot{\boldsymbol{\rho}}_d^{T})$$

# Attractive and repulsive virtual potentials control approach

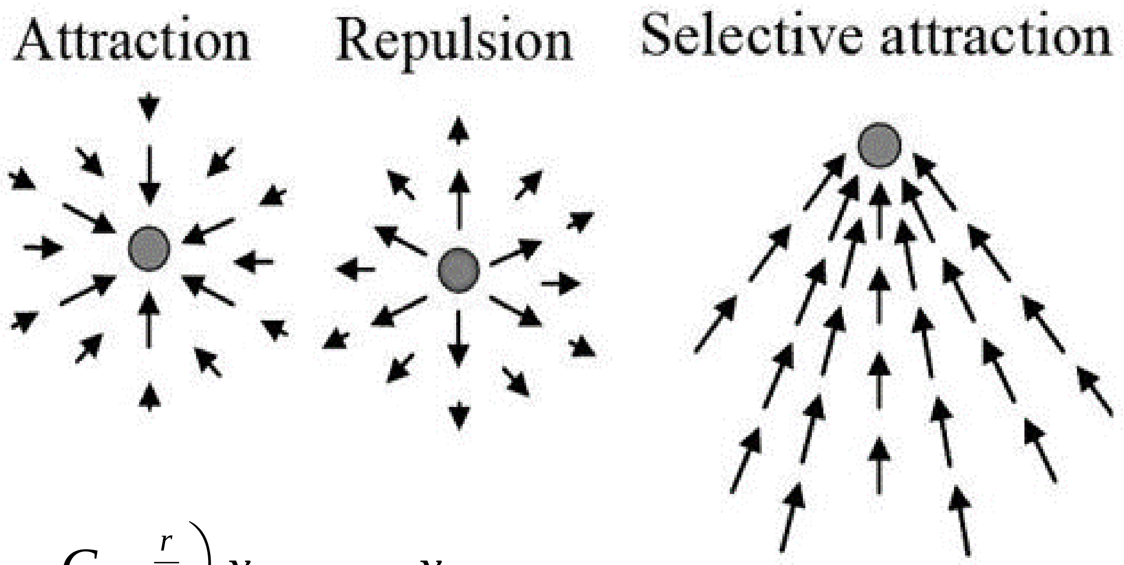
- **Attractive and repulsive potential functions**

$$V = V_a + V_r = -C_a e^{\frac{-r}{la}} + C_r e^{\frac{-r}{lr}}$$

- **The force produced by the potential function**

$$-\nabla V = F$$

- **Dynamical equations**




$$\ddot{x} = n^2 x + 2n\dot{y} + \left( \frac{-C_a}{l_a} e^{\frac{-r}{l_a}} + \frac{C_r}{l_r} e^{\frac{r}{l_r}} \right) \frac{x}{r} + f_r e_r \frac{x}{|\mathbf{r}|} - f_r \left( V_{\tau x} - V_{\tau xd} \right),$$

$$\ddot{y} = -2n\dot{x} + \left( \frac{-C_a}{l_a} e^{\frac{-r}{l_a}} + \frac{C_r}{l_r} e^{\frac{r}{l_r}} \right) \frac{y}{r} + f_r e_r \frac{y}{|\mathbf{r}|} - f_r \left( V_{\tau y} - V_{\tau yd} \right),$$

$$\ddot{z} = -n^2 z + \left( \frac{-C_a}{l_a} e^{\frac{-r}{l_a}} + \frac{C_r}{l_r} e^{\frac{r}{l_r}} \right) \frac{z}{r} + f_r e_r \frac{z}{|\mathbf{r}|} - f_r \left( V_{\tau z} - V_{\tau xz} \right),$$

# Numerical simulations examples

- **Initial conditions and system parameters**

$h = 700$ км; $e = 0.03$; $i = 70^\circ$

$$\mathbf{I}_T = diag(1, 1.4, 1)\ \mathrm{kg\cdot m^2}$$

$$\mathbf{I}_S = diag(2, 4, 4)\ \mathrm{kg\cdot m^2}$$

$$m = 45\ \mathrm{kg}$$

$$\mathbf{q}_{T0} = [0,\ 0,\ 1,\ 0]^T$$

$$\mathbf{q}_{S0} = [0.59,\ 0.2,\ 0.6,\ 0.5]^T$$

$$\boldsymbol{\omega}_T^T = [0.2,\ 0.4,\ 0]^T\ \mathrm{rad/s}$$

$$\boldsymbol{\rho}_0 = r_0 = [x_0,\ y_0,\ z_0]^T = [5,\ 5,\ 5]^T\ \mathrm{m}$$

$$\dot{\boldsymbol{\rho}}_0 = \dot{r}_0 = [1,\ 2,\ 3]^T\ \mathrm{m/s}$$

$$\boldsymbol{\rho}_{i1} = [1,\ 1,\ 1]^T\ \mathrm{m}$$

$$\boldsymbol{\rho}_{jo} = [1,\ 1,\ 1]^T\ \mathrm{m}$$

- **Control parameters**

- **For virtual potentials**

Attraction and repulsion

$$l_a, = 5,\ l_r = 2.755$$

$$C_a = 5,\ C_r = 4.5$$

$$f_r = 1,$$

Selective attraction

$$l_a, = 5,\ l_r = 2.755$$

$$C_a = 12,\ C_r = 1$$

$$f_r = 0$$

- **For SDRE**

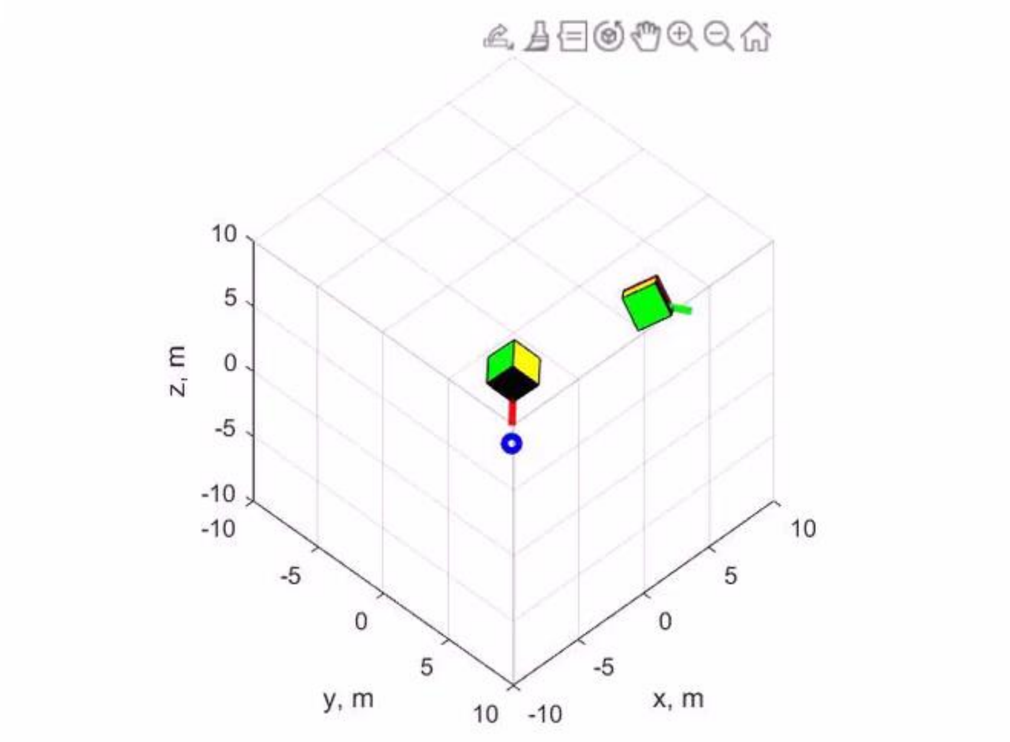
$$\mathbf{Q}_{attitude} = \mathbf{I}_6,$$

$$\mathbf{R}_{attitude} = 10^{-3}\mathbf{I}_3,$$

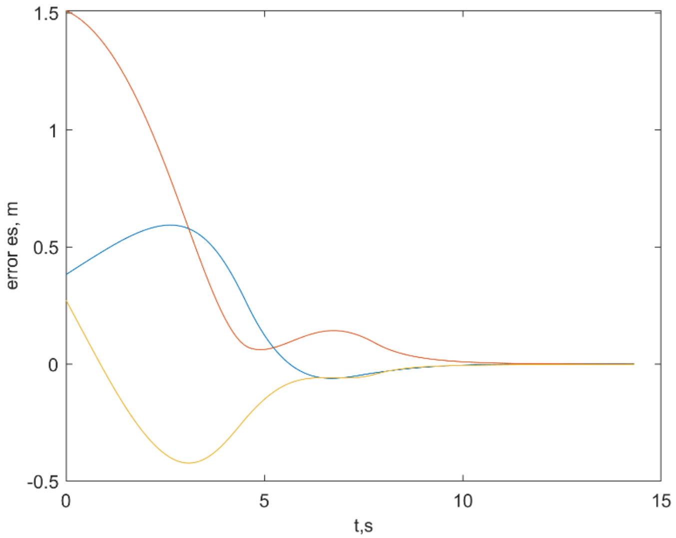
$$\mathbf{Q}_{translation} = 1\mathbf{I}_6,$$

$$\mathbf{R}_{translation} = 10\mathbf{I}_3.$$

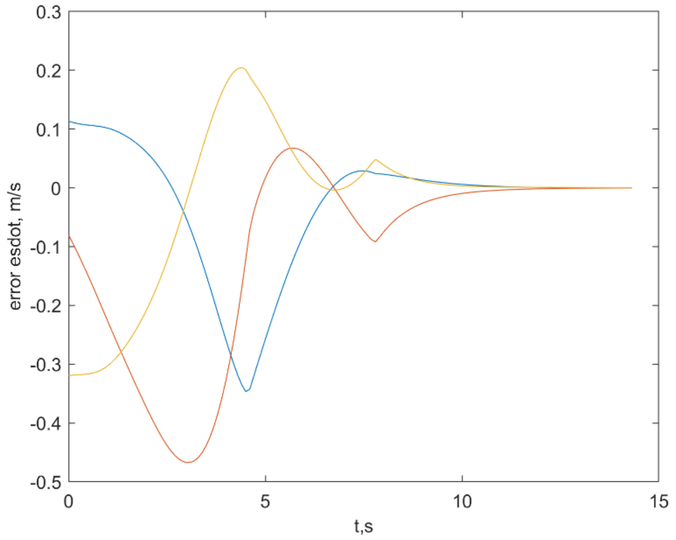
# Relative trajectory using SDRE-based control




Relative trajectory



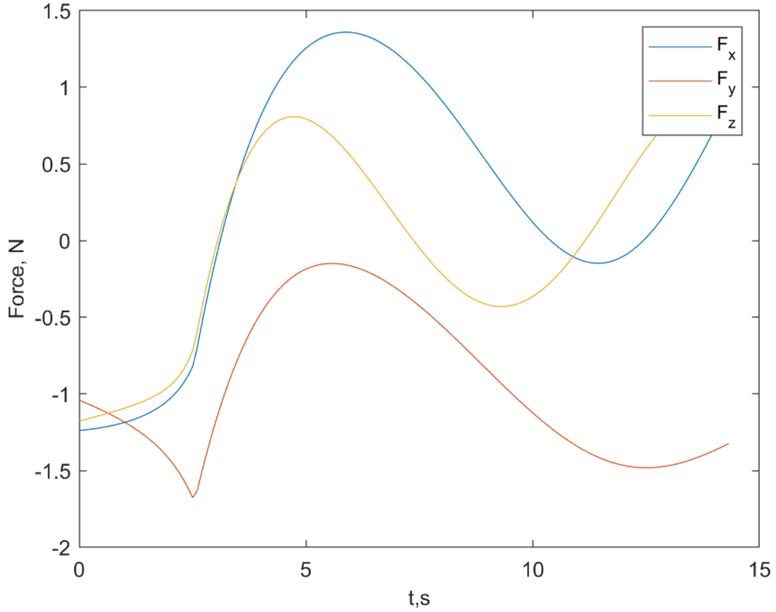

Errors of capturing vector



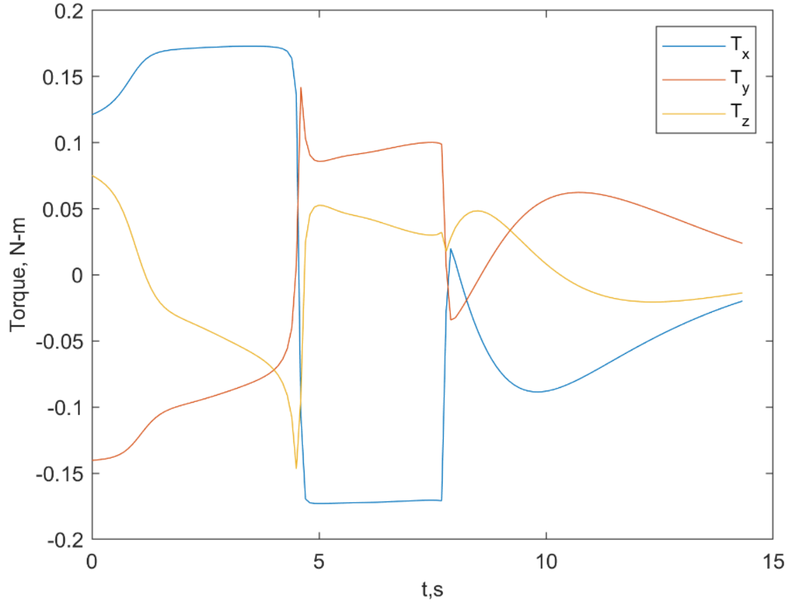

Errors of capturing vector velocity

# Control force and torques: SDRE-based control

- The time of approaching is 14s
- The required control values are high
- No collision avoidance

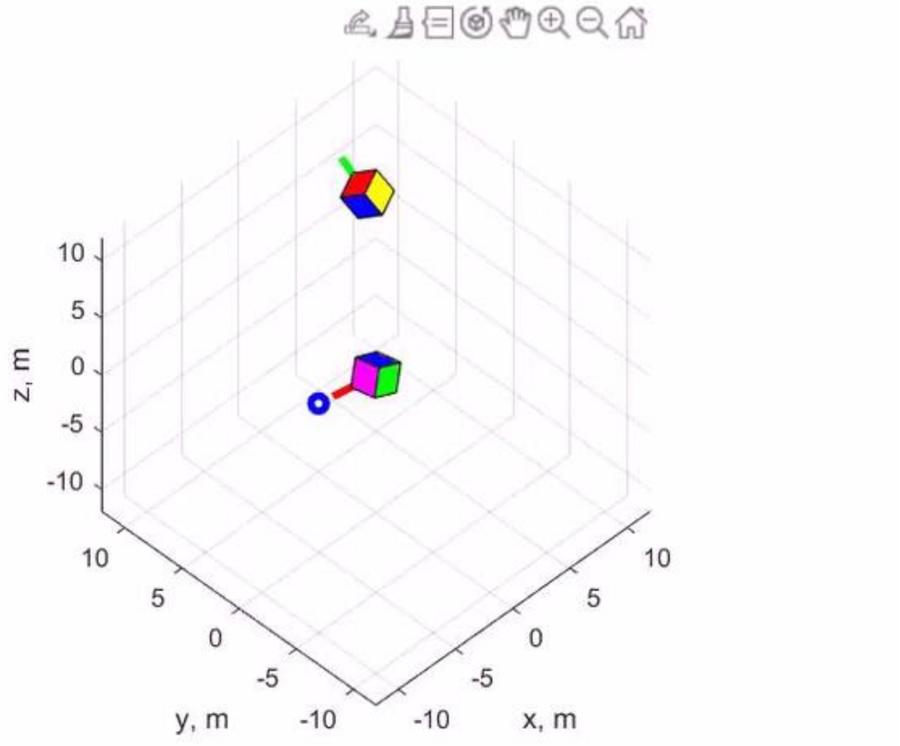



Control force for translational motion

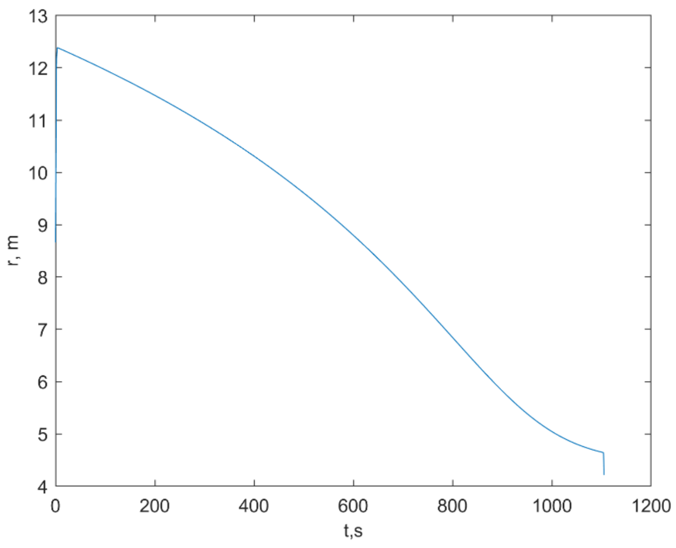



Control Torque for attitude motion

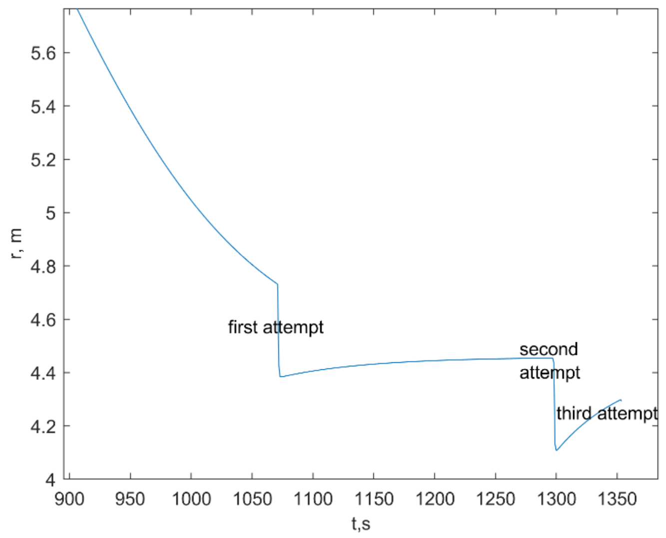
# Trajectory and relative distance: virtual potentials




Relative trajectory



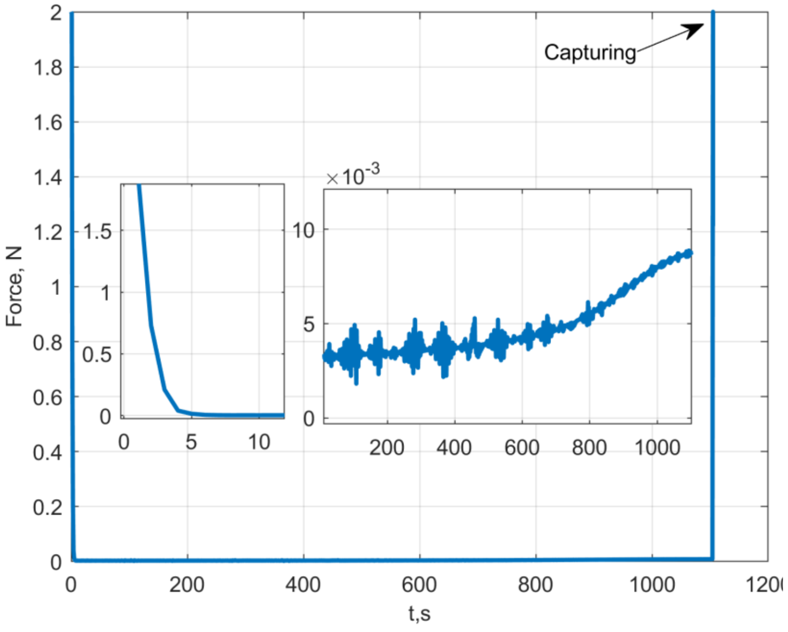

Relative distance, single attempt



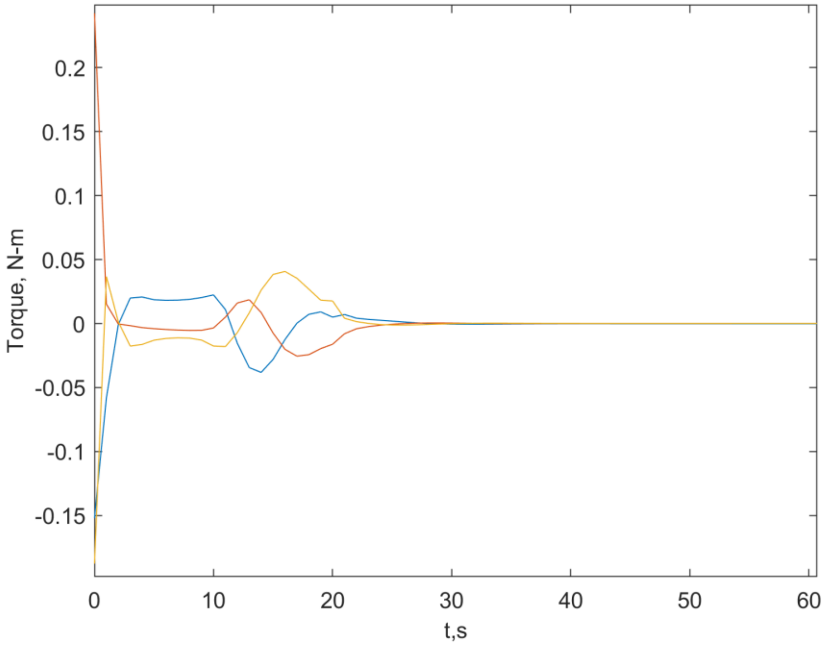

Relative distance, two attempts

# Control force and torque: virtual potentials

- **The time of approaching is 1100s**
- **Low control values**
- **Collision avoidance included**

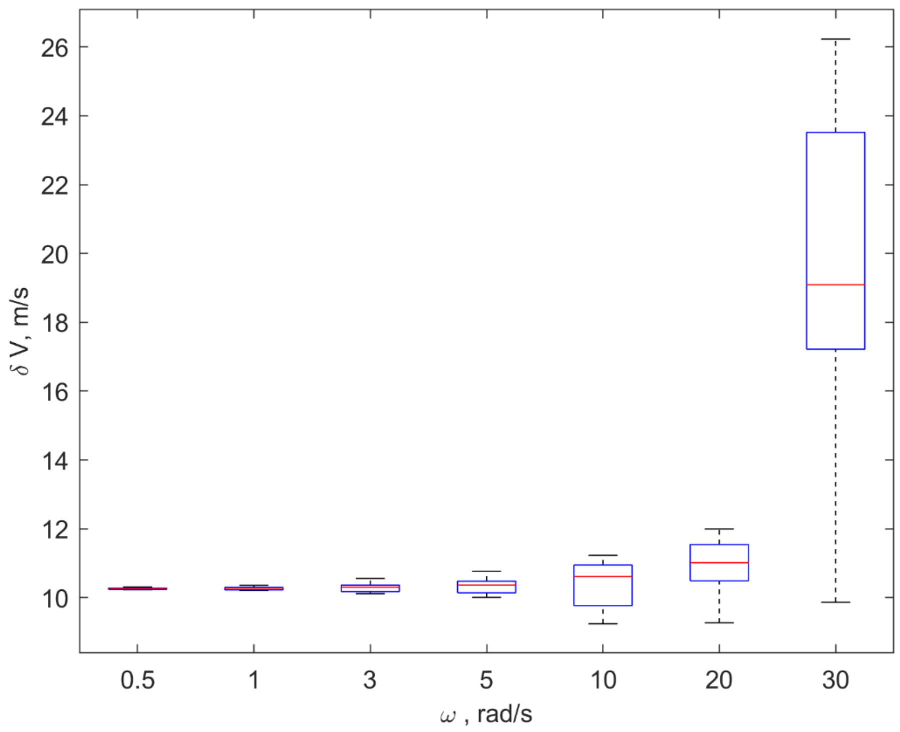



Control force for translational motion

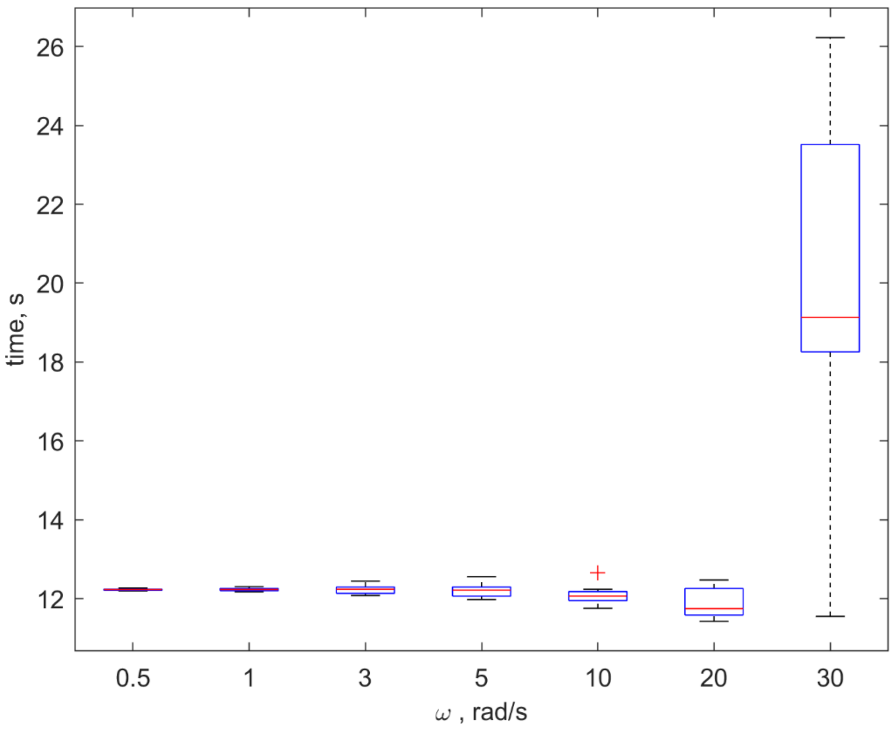



Control torque for rotational motion

# Monte-Carlo study: tumbling debris angular velocity SDRE-based control

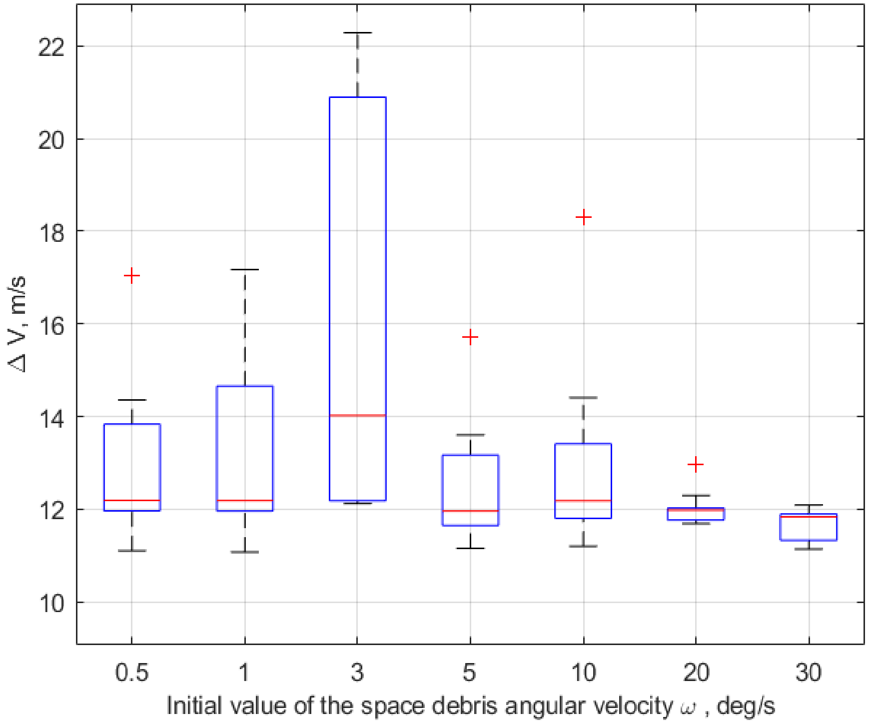



Required Delta-V vs the initial angular velocity of the debris

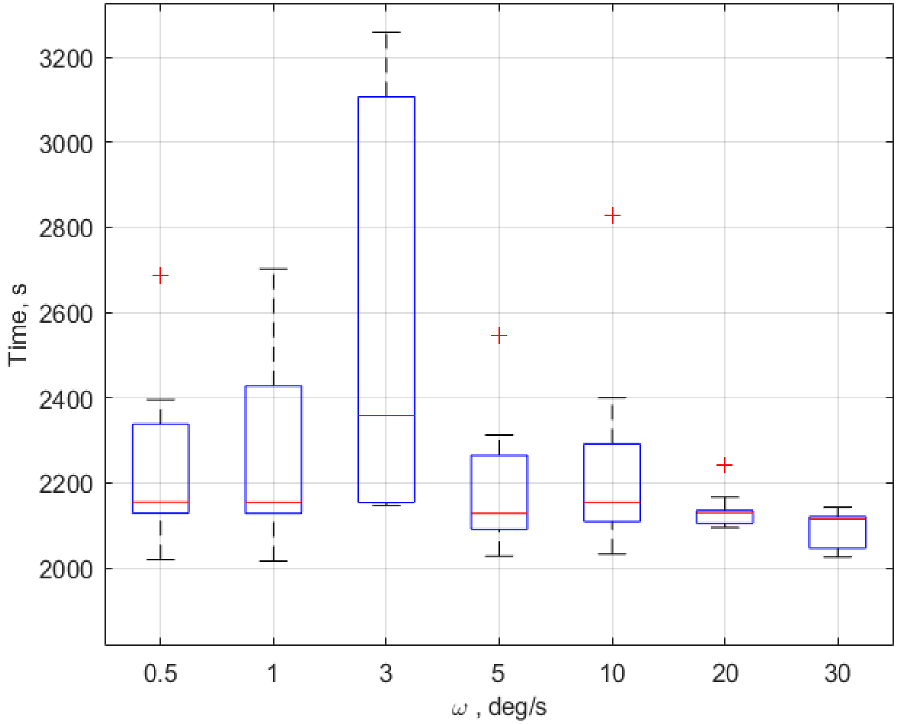



Approaching time vs the initial angular velocity of the debris

# Monte-Carlo study: tumbling debris angular velocity
# Virtual potentials control



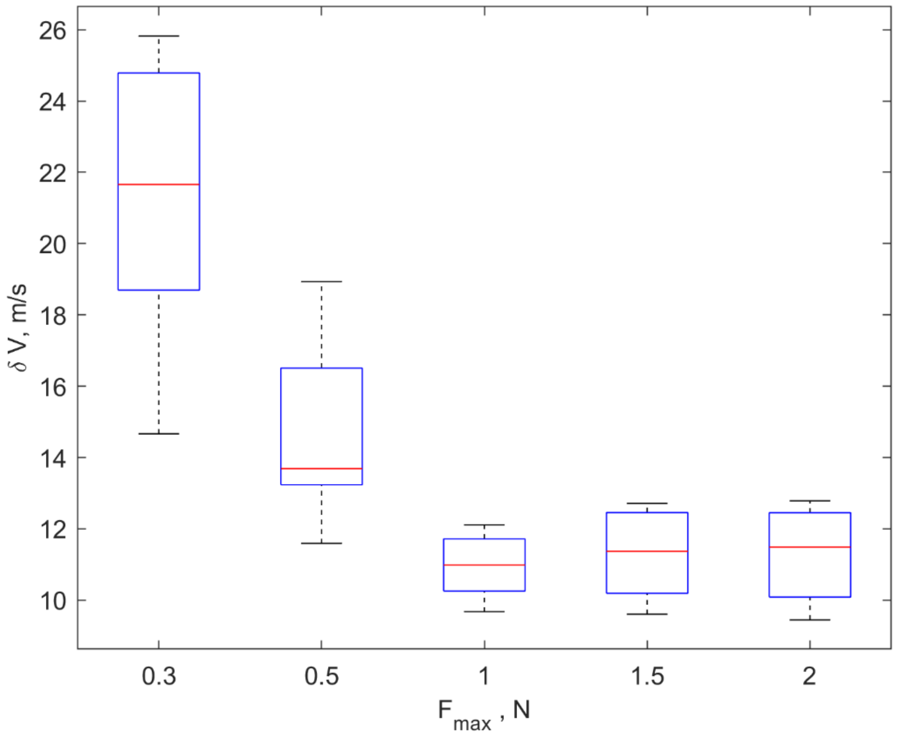

Required Delta-V vs the initial angular velocity of the debris



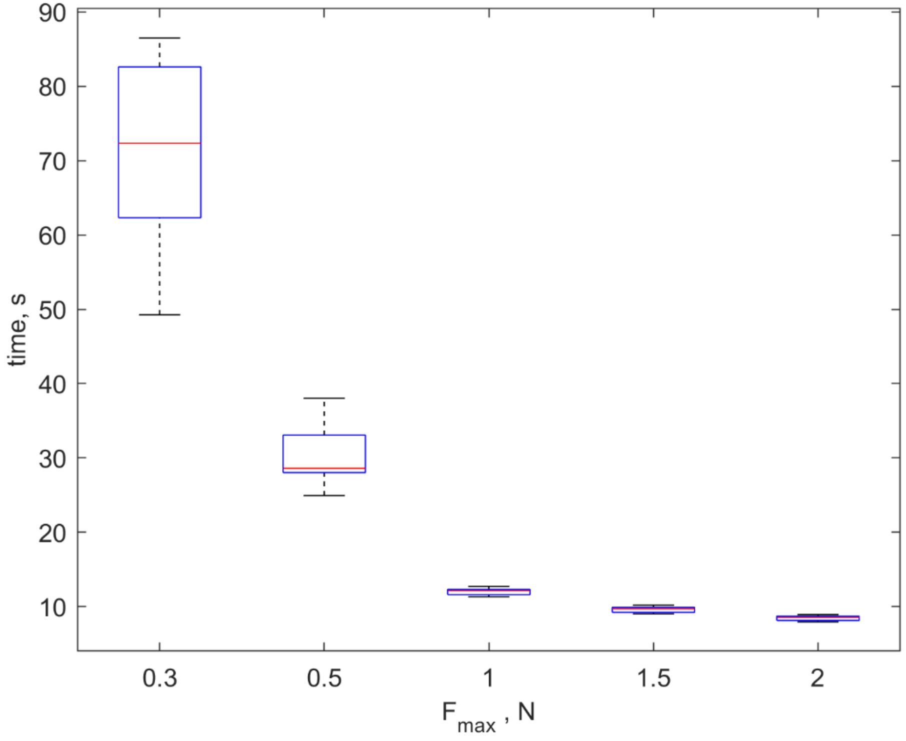

Capturing time vs the initial angular velocity of the debris

# Monte-Carlo study: maximum value of thrust

## SDRE-based control

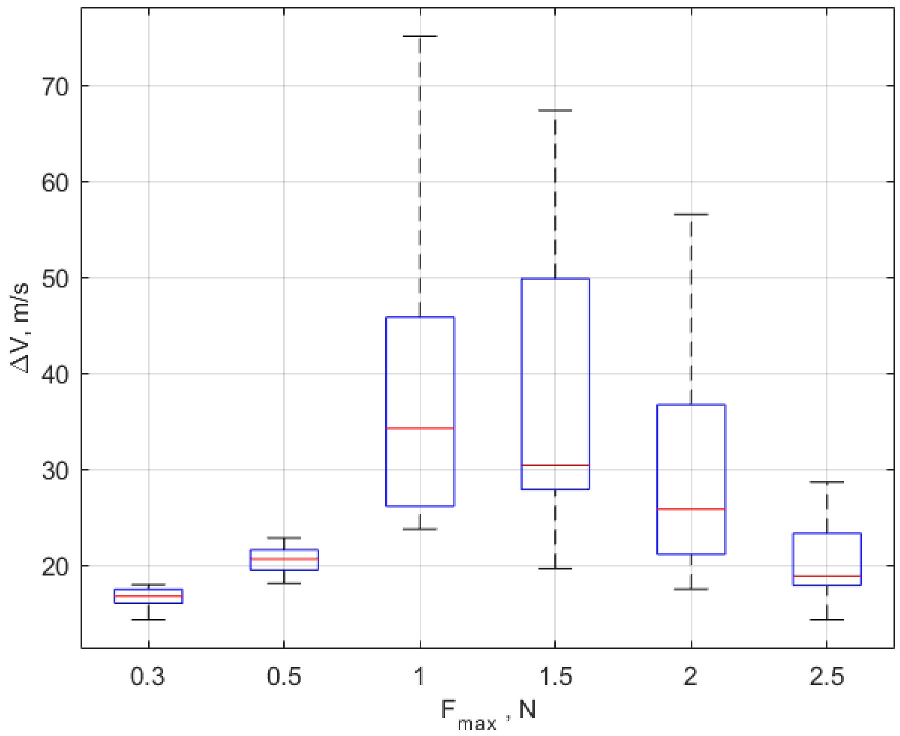



Delta-V vs the thrust value constraint

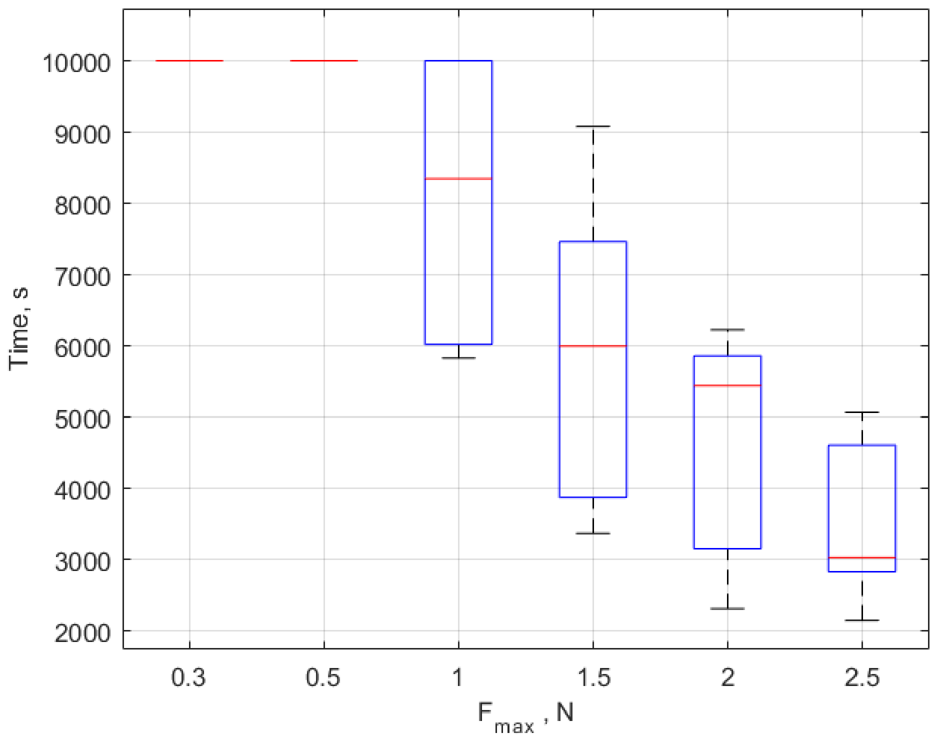



Approaching time vs the thrust value constraint

# Monte-Carlo Study: maximum value of thrust virtual potentials control




Rendezvous Delta-V time vs the thrust value constraint




Approaching time vs the thrust value constraint

# Conclusions

- Two types of control algorithms are developed: SDRE-based and virtual potentials approaches
- The algorithms performance in terms of required Delta-V and approaching time is compared
- The tumbling debris angular velocity greatly affects the cost of SDRE-control
- The virtual potential control takes long time to approach the debris

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# Thanks for your attention!