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Formation Flying Control of the Relative Trajectory Shape and Size Using Lorenz Forces

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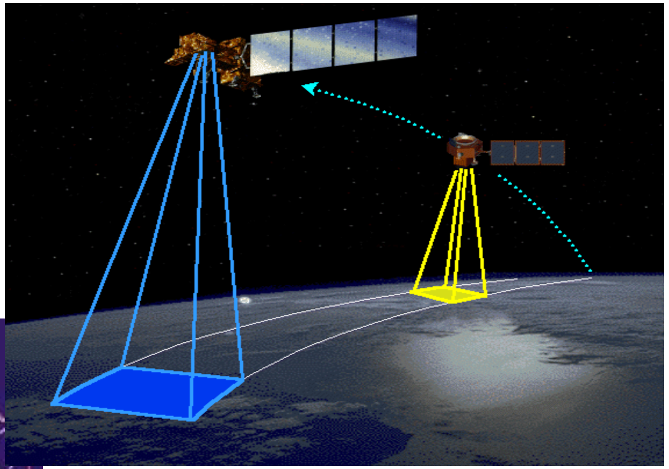
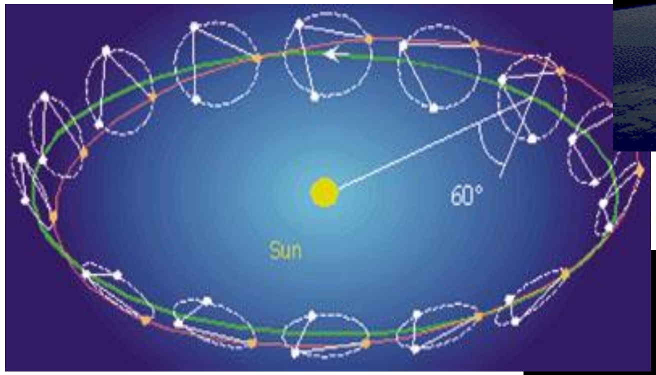
Satellite formation flying applications

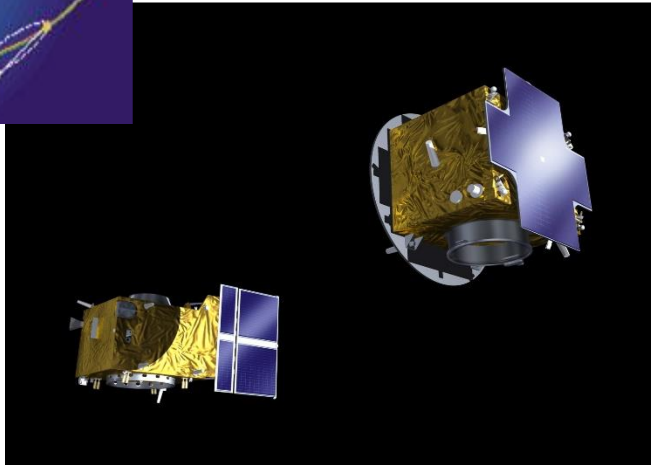
- Earth remote sensing
- Gravitational waves detector
- Solar coronagraphy
- Magnetic field measurements
- Measurements of Earth's gravity field



Landsat 7 & EO-1 Credit: NASA



LISA Credit: ESA



Proba-3 Credit: ESA Portal

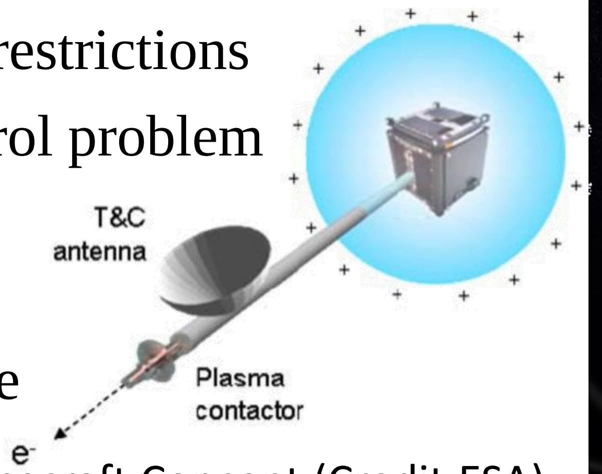
Formation Flying Using Lorenz Force

- The Lorentz force is experienced by any charged spacecraft in LEO

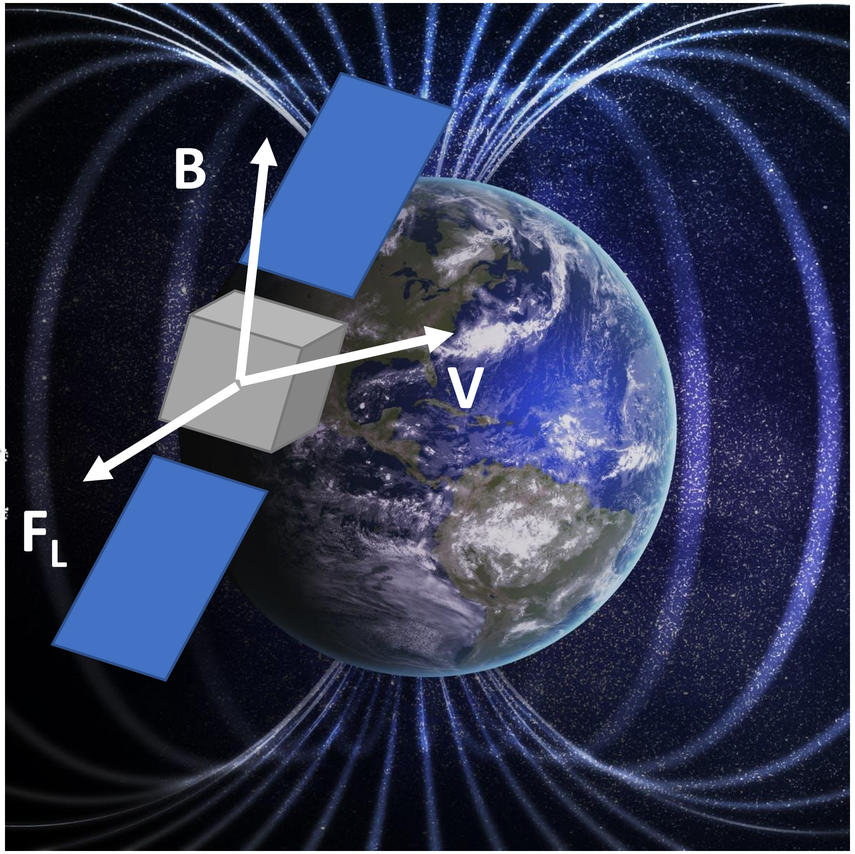
$$\mathbf{F}_L = q\left(\left(\mathbf{V} - \boldsymbol{\omega}_E \times \mathbf{R}\right) \times \mathbf{B}\right)$$

- A charging device is required onboard
- No fuel consumption
- No full controllability due to direction restrictions
- Lorenz-force is applied to various control problem

- A Lypunov-based control is proposed in this work to obtain a required relative trajectory



LAO Spacecraft Concept (Credit:ESA)



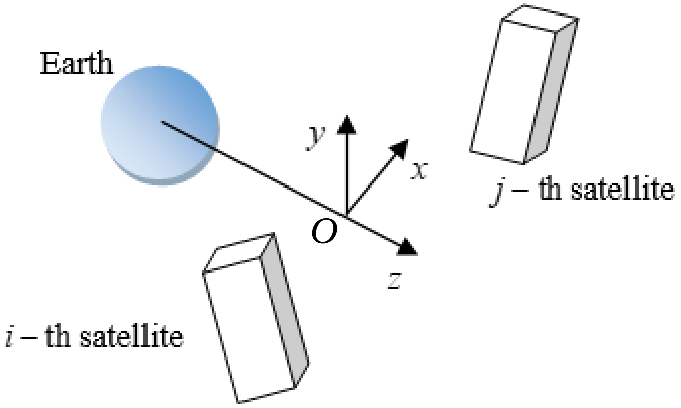
Lorenz Force direction

Saaj, C.M.; Lappas, V.; Richie, D.; Peck, M.; Streetman, B.; Schaub, H.; Izzo, D. Electrostatic forces for satellite swarm navigation and reconfiguration. ESA Report, 2006

Motion equations

Hill-Clohessy-Wiltshire equations:

$$\begin{cases} \ddot{x}_{ij} + 2\omega \dot{z}_{ij} = 0, \\ \ddot{y}_{ij} + \omega^2 y_{ij} = 0, \\ \ddot{z}_{ij} - 2\omega \dot{x}_{ij} - 3\omega^2 z_{ij} = 0, \end{cases} \Rightarrow \begin{cases} x_{ij} = 2B_2^{ij} \cos\left(\omega t + \psi_{ij}\right) + B_3^{ij}, \\ y_{ij} = B_4^{ij} \cos\left(\omega t + \varphi_{ij}\right), \\ z_{ij} = 2B_1^{ij} + B_2^{ij} \sin\left(\omega t + \psi_{ij}\right), \end{cases}$$



Reference frame

Controlled motion equations

$$\dot{B}_1^{ij} = \frac{1}{\omega} u_{ij}^x,$$

$$\dot{B}_2^{ij} = \frac{1}{\omega}\left(u_{ij}^z \cos\psi_{ij} - 2u_{ij}^x \sin\psi_{ij}\right),$$

$$\dot{B}_3^{ij} = -3\omega B_1^{ij} - \frac{2}{\omega} u_{ij}^z,$$

$$\dot{B}_4^{ij} = -\frac{1}{\omega} u_{ij}^y \sin\varphi_{ij},$$

$$\dot{\psi}_{ij} = -\frac{1}{\omega A}\left(u_{ij}^z \sin\psi_{ij} + 2u_{ij}^x \cos\psi_{ij}\right),$$

$$\dot{\varphi}_{ij} = -\frac{1}{\omega B} u_{ij}^y \sin\varphi_{ij}.$$

Controlled parameters:

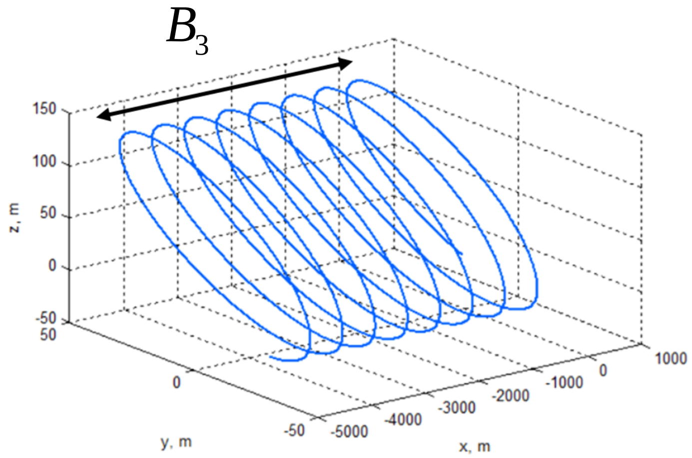
Relative drift

In-plane amplitude

Relative shift

Out-of-plane amplitude

Not controlled parameters:
Phase angles



Relative Shift

Lyapunov-based control

Two stages of relative motion control:

1. Relative drift and relative shift control

Lyapunov function candidate:

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}\Delta B_3^2$$

Its derivative

$$\dot{V} = B_1\dot{B}_1 + \Delta B_3 \Delta\dot{B}_3 = \\ = \frac{1}{\omega}B_1 u_x + \Delta B_3\left(-3B_1\omega - \frac{2}{\omega}u_z\right).$$

The resulting control

$$u_x = -k_a B_1,\ k_a > 0,$$
$$u_z = \frac{1}{2}\left(-3B_1\omega^2 + k_b\omega\Delta B_3\right),\ k_b > 0.$$

2. In-plane and out-of-plane amplitudes control

Lyapunov function candidate:

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}\Delta B_2^2 + \frac{1}{2}\Delta B_3^2 + \frac{1}{2}\Delta B_4^2$$

Its derivative

$$\dot{V} = -\frac{1}{\omega}\left(B_1 - 2\Delta B_2 \sin\psi\right)u_x + \\ + \frac{1}{\omega}\left(\Delta B_2 \cos\psi - 2\Delta B_3\right)u_z - \\ - 3B_1\Delta B_3\omega - \frac{1}{\omega}\sin\varphi\Delta B_4 u_y$$

The resulting control

$$u_x = k_x\left(B_1 - 2\Delta B_2 \sin\psi\right),\ k_x > 0,$$
$$u_y = k_y\Delta B_4 \sin\varphi,\ k_y > 0,$$
$$u_z = -k_z\left(\Delta B_2\cos\psi - 2\Delta B_3\right),\ k_z > 0.$$

Control implementation using Lorenz force

- Lyapunov-based control cannot be implemented using Lorenz-force as it is
- At each point in the orbit the Lorenz-force direction is defined
- The only control value is the scalar value of satellite charge q
- The virtual charges required for each component implementation:

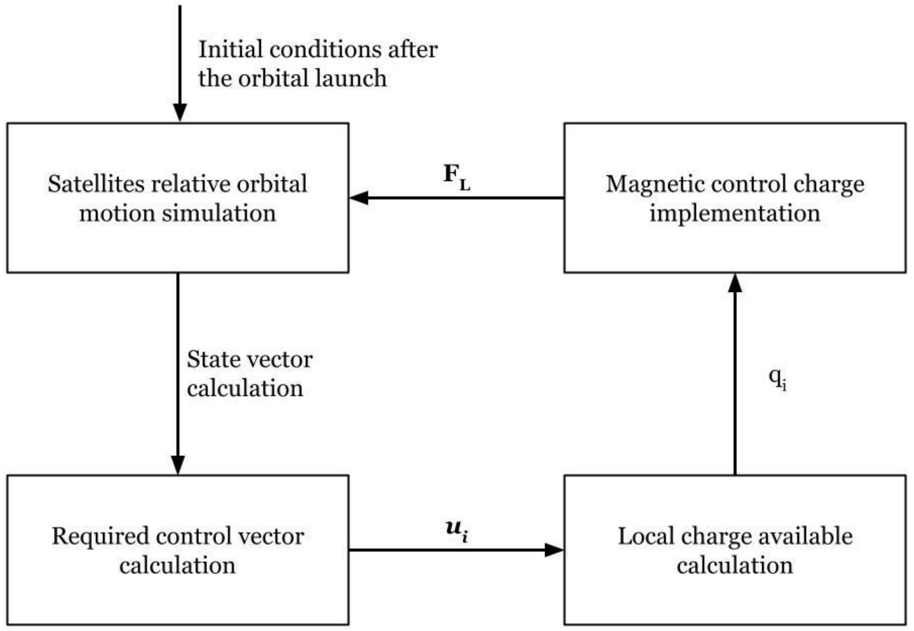
$$q_x = \frac{u_x}{L_x},\ q_y = \frac{u_y}{L_y},\ q_z = \frac{u_z}{L_z}$$

- Calculate the single charge value for Lorentz force that is close to calculated control vector

$$q_{mean} = \sqrt{\frac{q_x^2 + q_y^2 + q_z^2}{3}} sign\left(q_x + q_y + q_z\right)$$

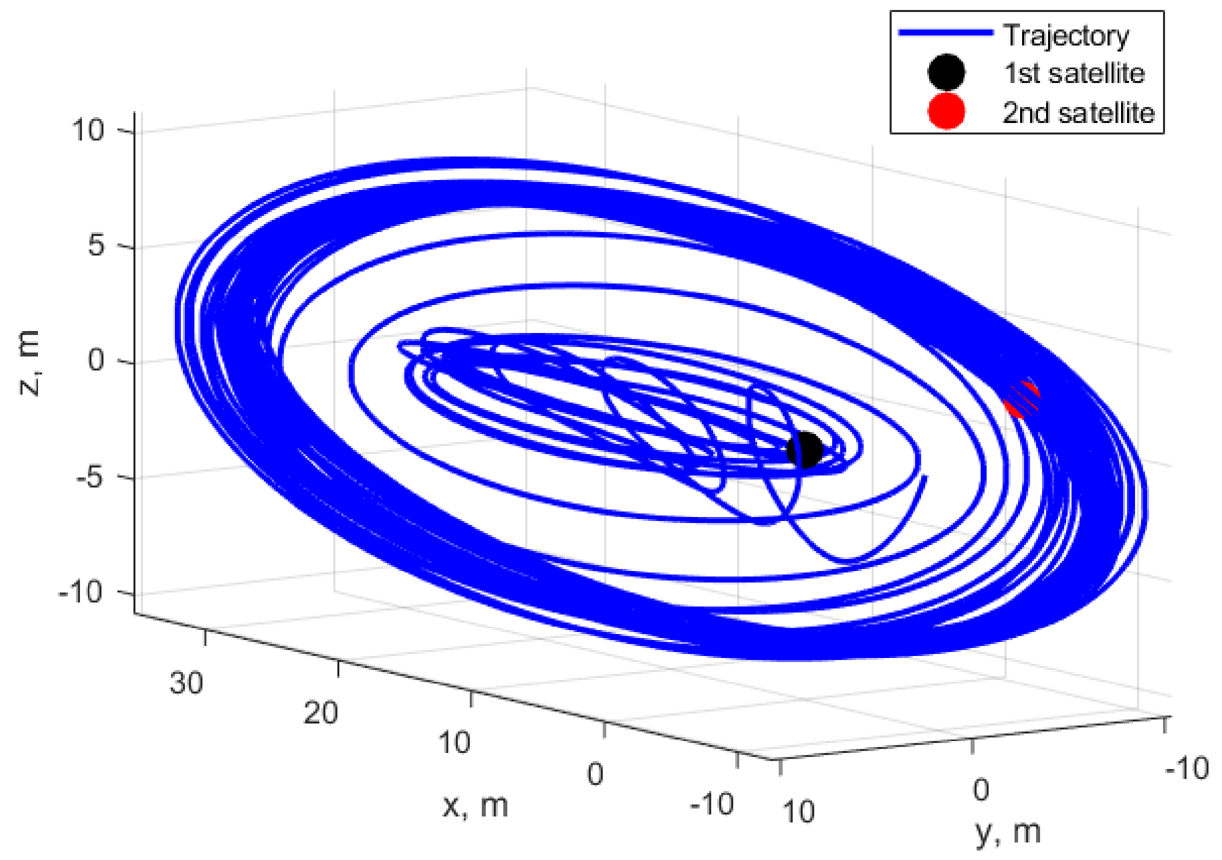
- The limitation to the maximum possible charge is applied

$$q_{mean} = \begin{cases} q_{\max} sign(q_{mean}), & \text{if } |q_{mean}| \geq q_{\max}, \\ q_{mean}, & \text{if } |q_{mean}| < q_{\max}. \end{cases}$$



Motion Simulation Control Scheme

Numerical simulation



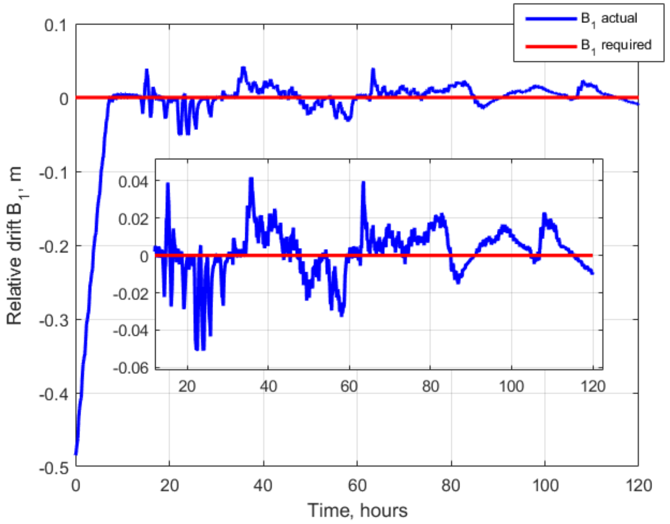
Relative trajectory of one active satellite

Initial conditions	
Initial relative drift, C_1	rand([-0.5;0.5]) m
Initial relative position constants $C_2 - C_6$	rand([5;5]) m
Satellite parameters	
Mass of the satellites, m	1 kg
Maximum charge, q_{max}	10 μC
Orbital parameters	
Orbit altitude, h	500 km
Orbit inclination, i	51.7°
Algorithms parameters	
Control gains k_a , k_b	10^{-6}, 10^{-4}
Control gains k_x, k_y, k_z	10^{-6}, 10^{-8}, 10^{-7}
Maximal charge change rate, dq / dt	10^{-7} C/s
Required relative orbit parameters $B_1 - B_4$	[0, 10, 10, 10] m
Second stage algorithm threshold for B_1 and B_3	0.05 m, 2.5 m

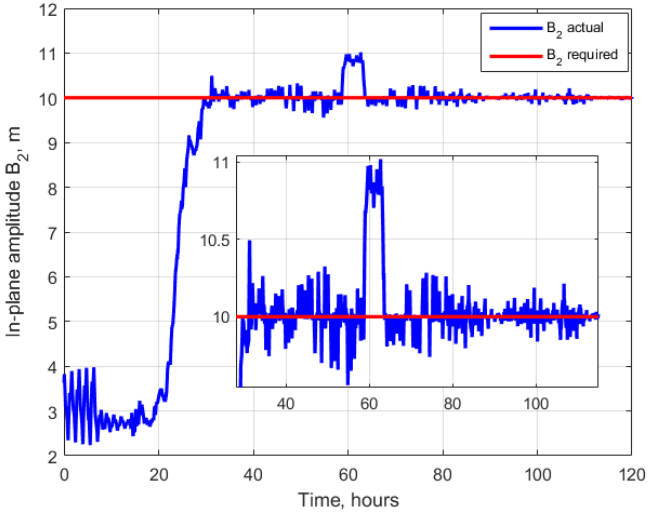
Numerical simulation results

- The first stage of the algorithm took about 15 hours, when the relative shift and relative drift reached the required values
- The second stage was also about 15 hours, during this period the trajectory amplitudes get to the vicinity of the required values
- Starting from 30 hours from the simulation beginning the trajectory can be considered as converged to the trajectory with desired shape and size

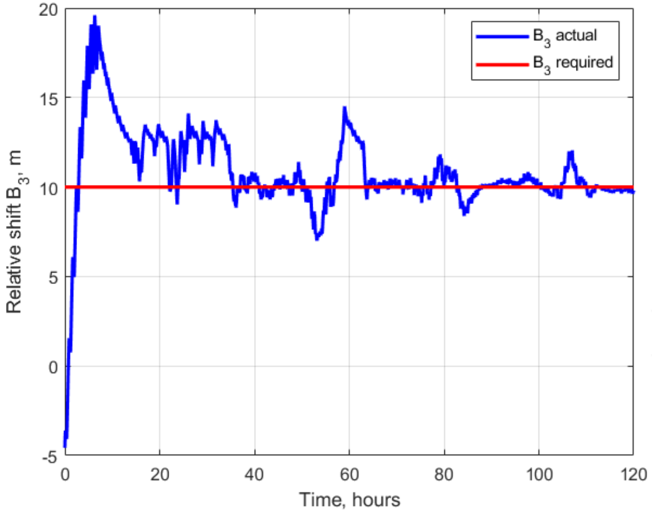
Relative drift



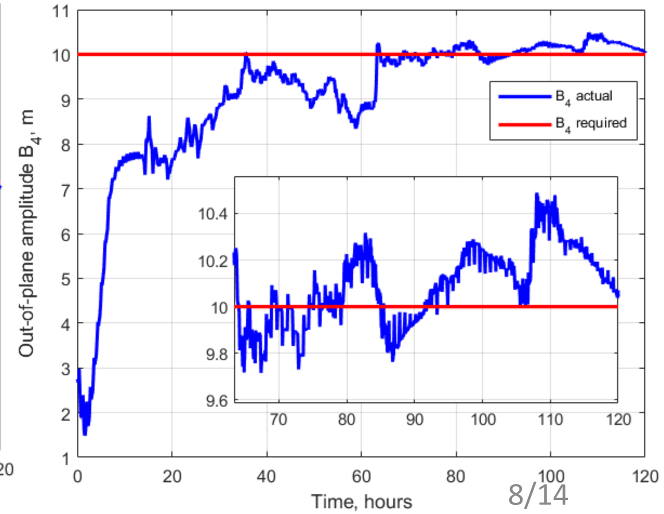
In-plane amplitude



Relative shift



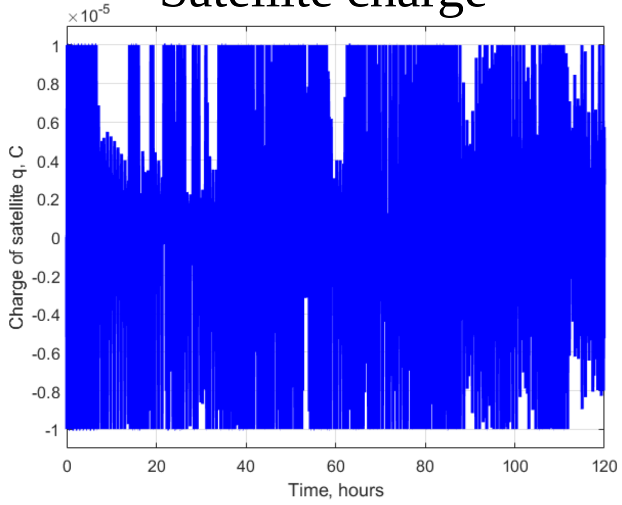
Out-of-plane amplitude



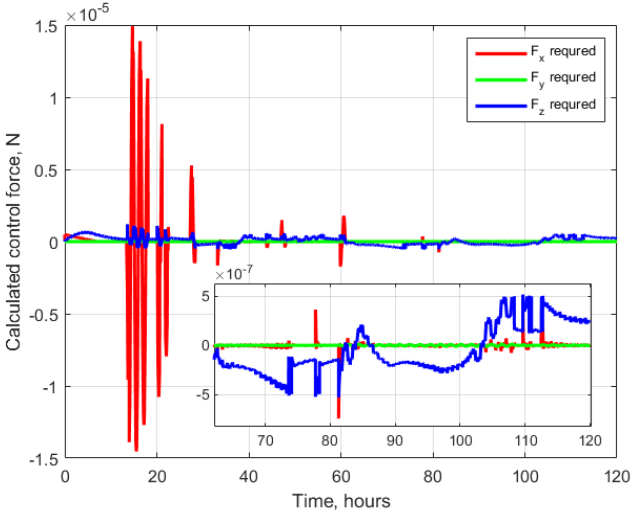
Numerical simulation results

- The value of the charge is limited by 10 μC
- The speed of charging is also limited
- The sudden high values for the required control along x direction are caused by transition to the second stage of the algorithm
- The implemented control through Lorenz force differs significantly from the calculated control
- A sinusoidal behavior of the peaks of Lorenz forces with period of about 24 hours can be explained by the rotational motion of the tilted Earth magnetic dipole that cause the slight change in the possible direction of the Lorenz force

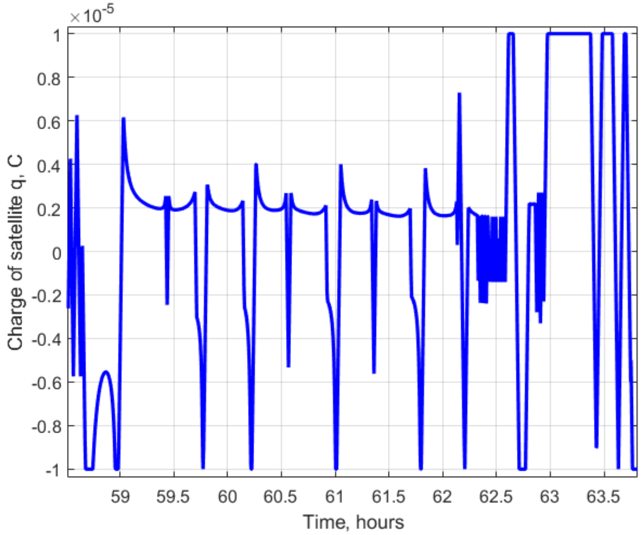
Satellite charge



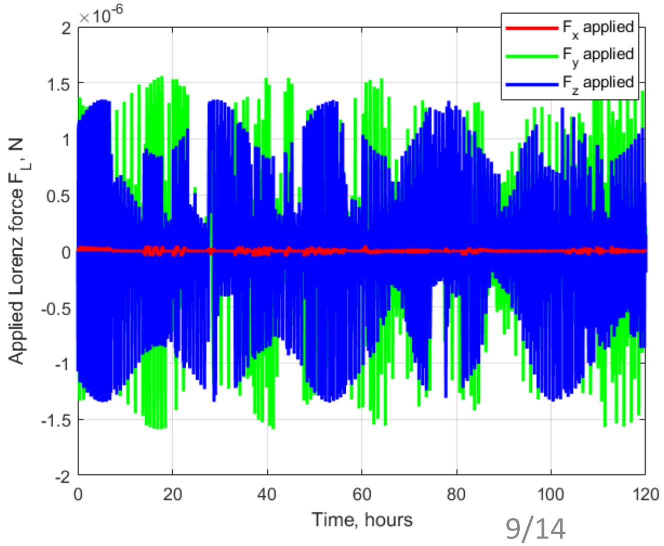
Calculated control



Satellite charge (zoomed)



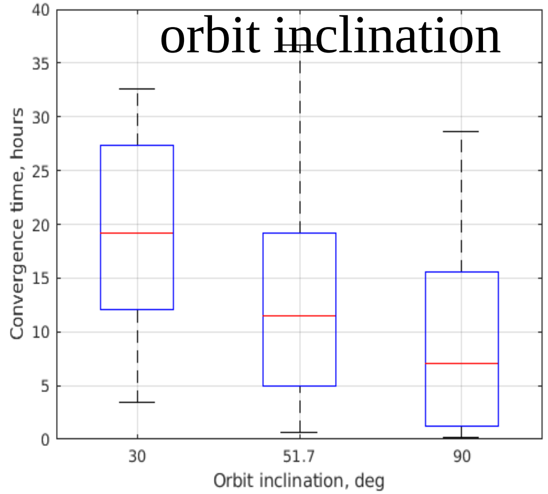
Implemented Lorenz-force



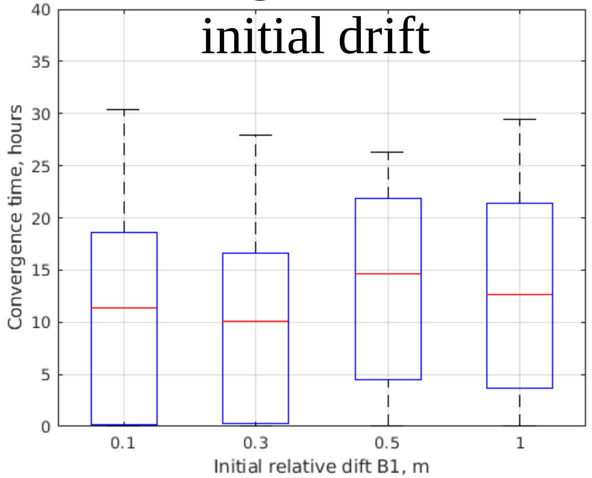
Monte-Carlo study results

- Convergence time reduces significantly with larger values of the maximum charge
- The closer the orbit to the polar – the less the convergence time and the less the errors in trajectory after the convergence
- The initial relative drift value for the considered range almost does not affect the convergence time

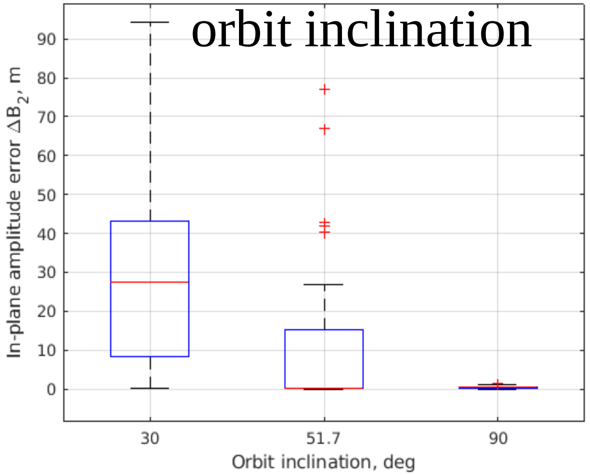
Convergence time vs orbit inclination



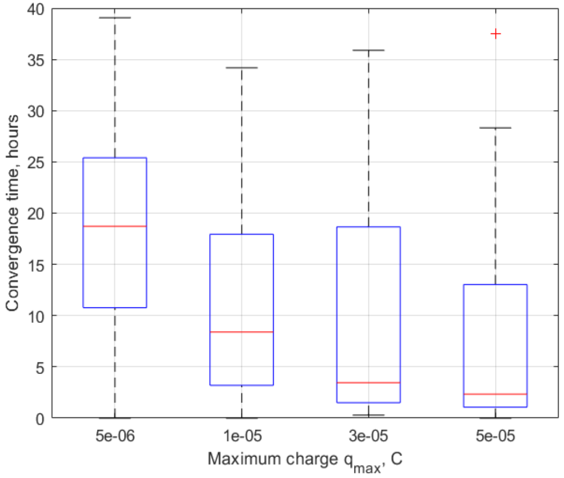
Convergence time vs initial drift



In-plane accuracy vs orbit inclination

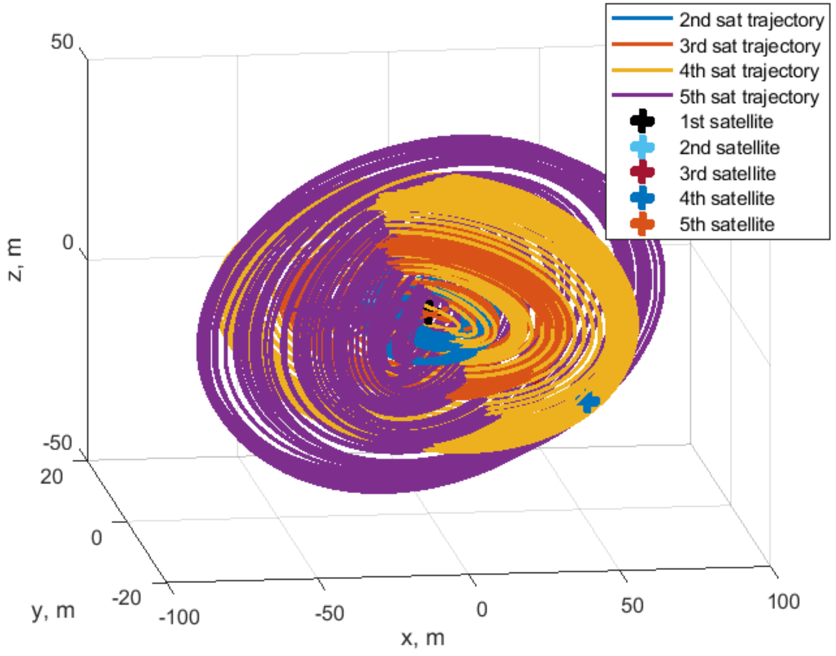


Convergence time vs q_{max}

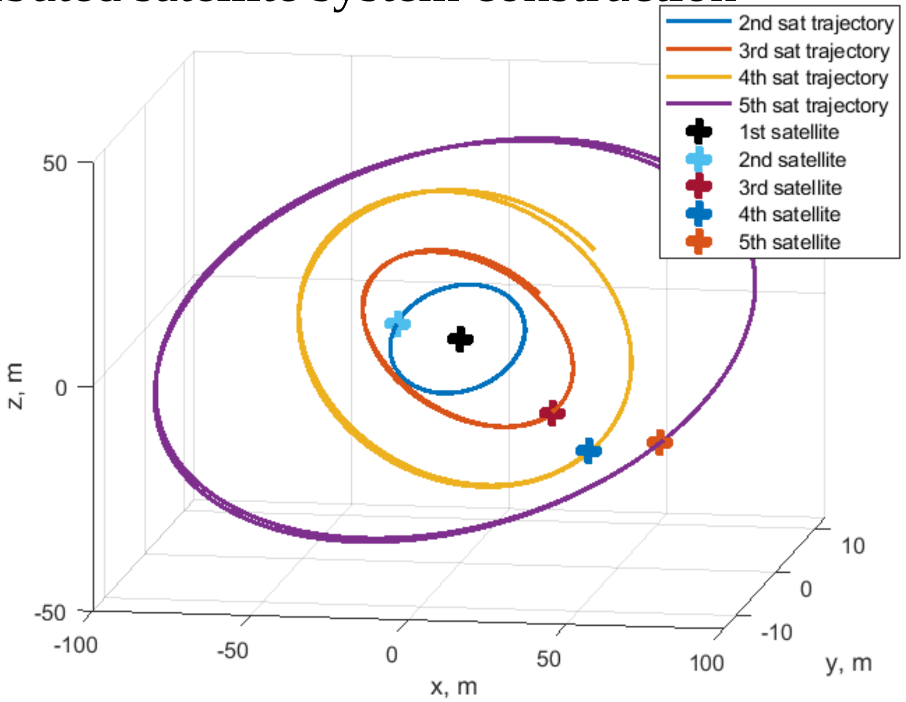


Nested ellipses example

- Consider a formation flying consisting of 5 satellites
- The required drift and shift is zero for all the satellites, the out-of-plane amplitude is 10 m and the in-plane amplitudes differs in 10m
- The proposed control leads to distributed satellite system construction

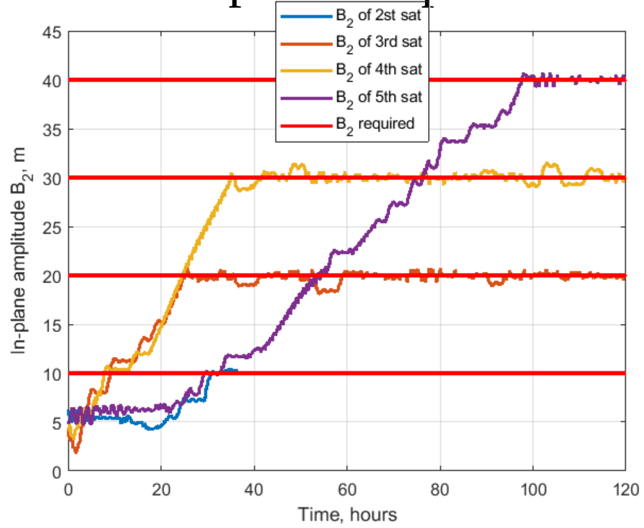


Relative trajectory for the whole simulation time

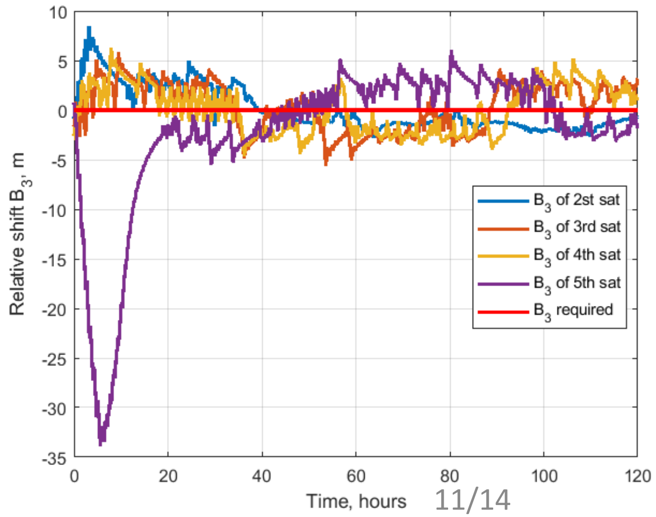


Relative trajectory for the last few hours

In-plane amplitude



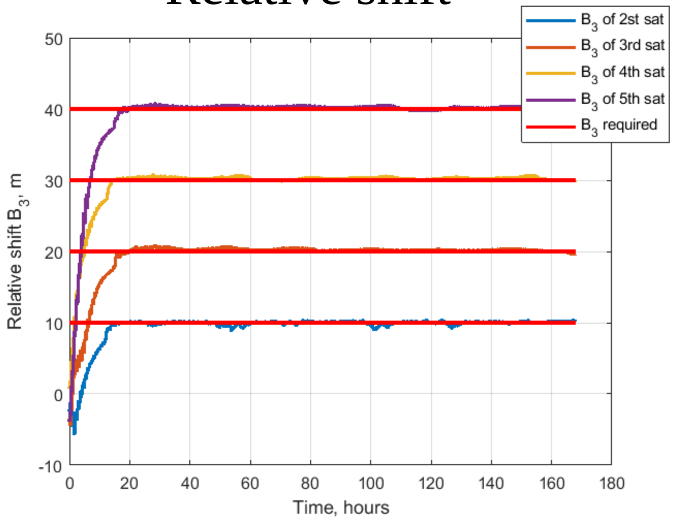
Relative shift



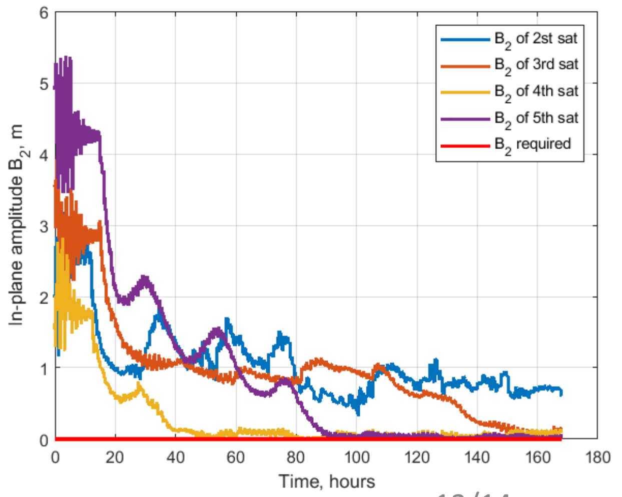
Train formation example

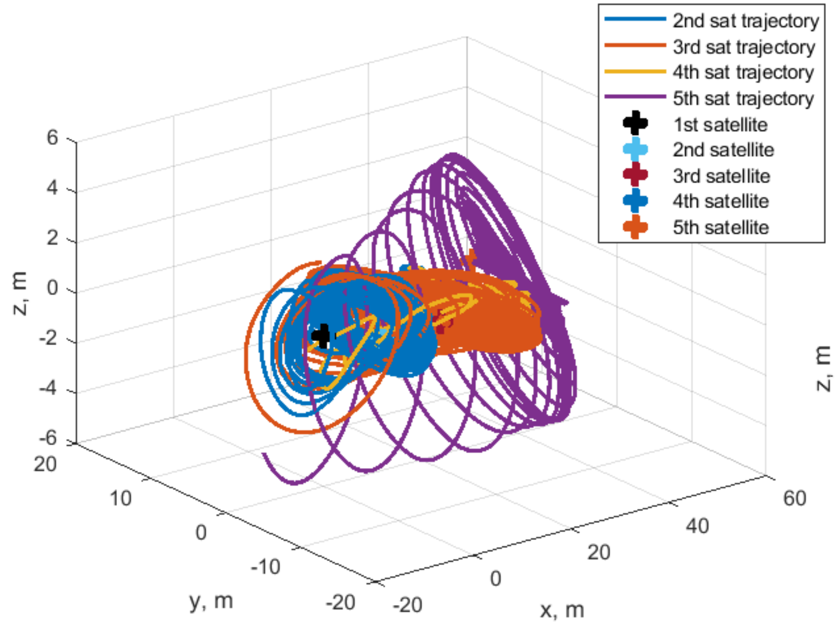
- Consider a formation flying consisting of 5 satellites with same initial conditions
- The required drift, the out-of-plane and in-plane amplitudes are zero for all
- A train formation flying is obtained

Relative shift

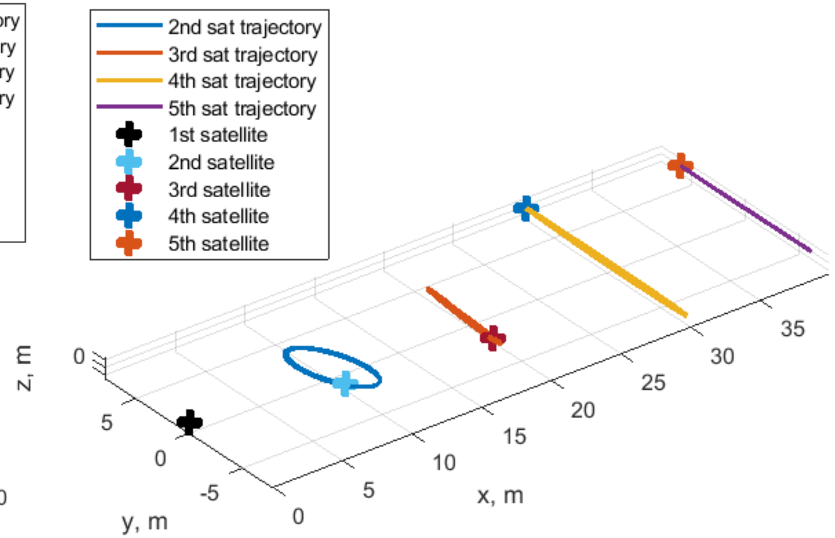


In-plane amplitude





Relative trajectory for the whole simulation time



Relative trajectory for the last few hours

Conclusions

- A control algorithm based on the Lorenz-force application is developed
- The problems of the construction and maintenance of the small satellites formation flying relative motion are addressed
- The proposed Lyapunov-based control is aimed to achieve the required relative drift, relative shift and in-plane and out-of-plane amplitudes
- The results of the application of the proposed algorithm is demonstrated in three cases: the case of one controlled satellite in two satellite formation flying, the two examples of multiple satellites formation flying in nested ellipses and train configurations

Thank you for attention!

Our web-site:
http://keldysh.ru/microsatellites/eng/