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Effect of reaction wheels disbalances on the spacecraft stabilization accuracy

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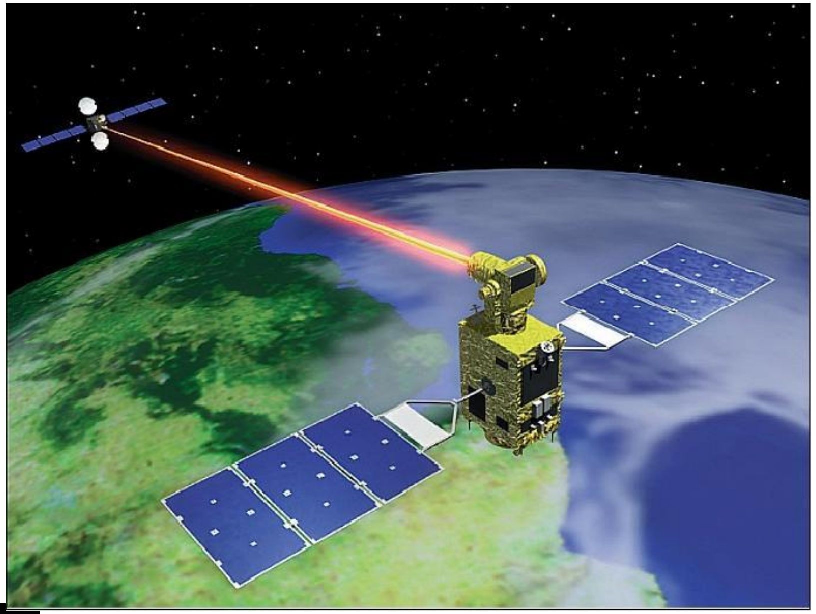
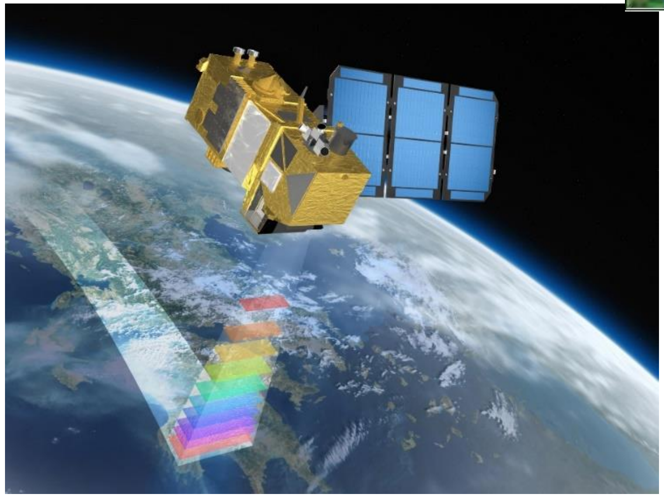
Introduction

Most types of mission require attitude control

- Solar panels pointing for battery recharging
- Communication antennas pointing
- Intersatellite communication
- Payload requirements





Introduction

Several types of actuators:

- Magnetorquers
- Reaction wheels/CMGs
- Thrusters

Gyroscopic attitude control

Pros:

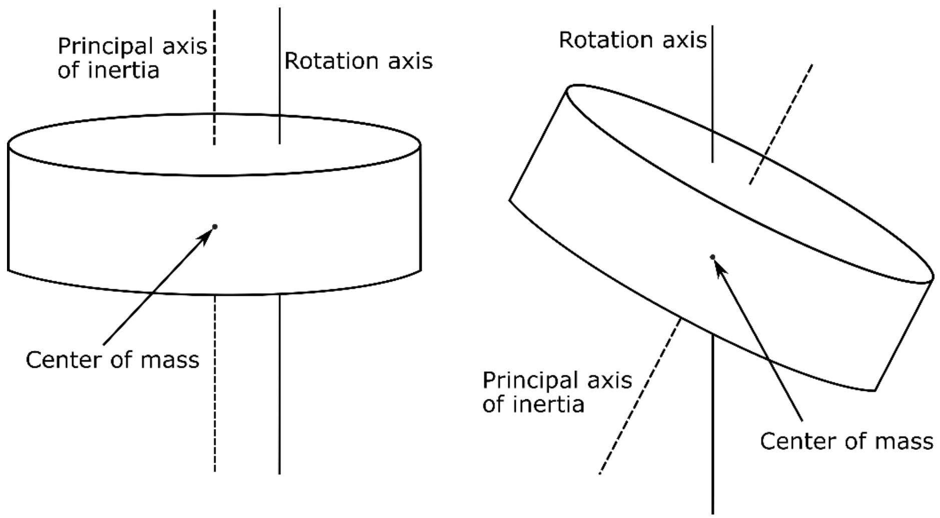
- Provide nice pointing accuracy
- Fast maneuvers

Cons:

- Saturation (can be solved by mangetorquers at LEO)
- Vibration

Why there is a problem?

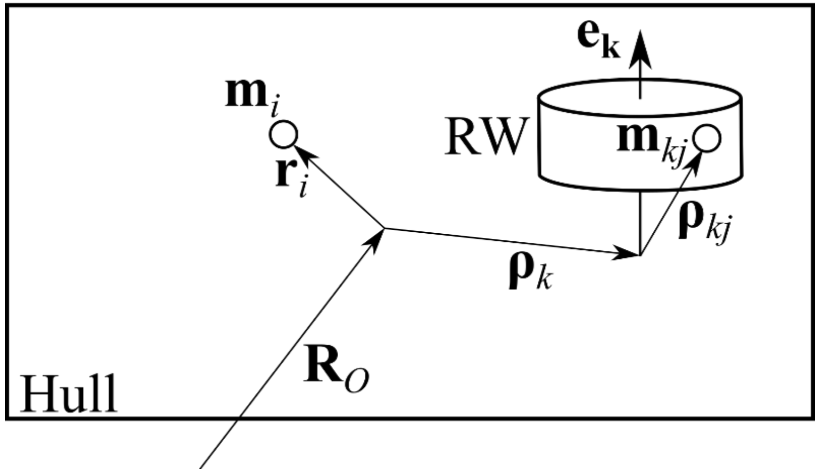
- Non-ideal balancing leads to vibration
- Two types of them: static and dynamic imbalances



What do we need?

- First of all – model of motion
- We use general equation of dynamics
- There are 6 + N degrees of freedom (N – amount of RWs)

$$\sum_i \left(m_i \ddot{\mathbf{R}}_i - \mathbf{F}_i \right)^T \delta \mathbf{R}_i + \sum_k \sum_j \left(m_{kj} \ddot{\mathbf{R}}_{kj} - \mathbf{F}_{kj} \right)^T \delta \mathbf{R}_{kj} = \sum_k M_k^{int} \delta \varphi_k$$



Model of motion

After the mathematics we got the following equations for dynamics

$$\mathbf{S}\begin{pmatrix}\dot{\mathbf{V}}_O\\ \dot{\boldsymbol{\omega}}\\ \dot{\Omega}_1\\ \vdots\\ \dot{\Omega}_n\end{pmatrix}=\mathbf{N}$$

$$\mathbf{S}=\begin{pmatrix} m\mathbf{E}_{3\times3} & -\left[\sum_k m_k\boldsymbol{\rho}_{kc}\right]_\times & -m_1\boldsymbol{\rho}_{1c}\times\mathbf{e}_1 & \dots & -m_n\boldsymbol{\rho}_{nc}\times\mathbf{e}_n\\ \left[\sum_k m_k\boldsymbol{\rho}_{kc}\right]_\times & \mathbf{J} & \left(\mathbf{I}_1-m_1[\boldsymbol{\rho}_1]_\times[\boldsymbol{\rho}_{1c}]_\times\right)\mathbf{e}_1 & \dots & \left(\mathbf{I}_n-m_n[\boldsymbol{\rho}_n]_\times[\boldsymbol{\rho}_{nc}]_\times\right)\mathbf{e}_n\\ -\left(m_1\boldsymbol{\rho}_{1c}\times\mathbf{e}_1\right)^T & \mathbf{e}_1^T\left(\mathbf{I}_1-m_1[\boldsymbol{\rho}_{1c}]_\times[\boldsymbol{\rho}_1]_\times\right) & I_1 & 0 & 0\\ \vdots & \vdots & 0 & \ddots & 0\\ -\left(m_n\boldsymbol{\rho}_{nc}\times\mathbf{e}_n\right)^T & \mathbf{e}_n^T\left(\mathbf{I}_n-m_n[\boldsymbol{\rho}_{nc}]_\times[\boldsymbol{\rho}_n]_\times\right) & 0 & 0 & I_n \end{pmatrix}$$

$$\mathbf{N}=\begin{pmatrix} \mathbf{F}_s+\sum_k\mathbf{F}_k-\sum_k m_k\left(\boldsymbol{\omega}\times\boldsymbol{\Omega}_k\right)\times\boldsymbol{\rho}_{kc}-\sum_k m_k\left(\boldsymbol{\omega}+\boldsymbol{\Omega}_k\right)\times\left(\boldsymbol{\omega}+\boldsymbol{\Omega}_k\right)\times\boldsymbol{\rho}_{kc}\\ \mathbf{N}_\omega\\ M_1^{int}+\mathbf{e}_1^T\mathbf{M}_1-\mathbf{e}_1^T\left(m_1\boldsymbol{\rho}_{1c}\times\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{\rho}_1+\mathbf{I}_1\left(\boldsymbol{\omega}\times\boldsymbol{\Omega}_1\right)+\boldsymbol{\omega}\times\mathbf{I}_1\left(\boldsymbol{\omega}+\boldsymbol{\Omega}_1\right)\right)\\ \vdots\\ M_n^{int}+\mathbf{e}_n^T\mathbf{M}_n-\mathbf{e}_n^T\left(m_n\boldsymbol{\rho}_{nc}\times\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{\rho}_n+\mathbf{I}_n\left(\boldsymbol{\omega}\times\boldsymbol{\Omega}_n\right)+\boldsymbol{\omega}\times\mathbf{I}_n\left(\boldsymbol{\omega}+\boldsymbol{\Omega}_n\right)\right) \end{pmatrix}$$

These equations are verified using momentum, angular momentum and energy conservation laws

What are the problems?

- It requires very small time steps
- Calculation burden might be unacceptable for Monte-Carlo simulation
- We need some estimation of the obtained accuracy, at least in the most common modes of attitude motion
- We consider inertial stabilization and standard Lyapunov-based controller

$$\mathbf{M}_{ctrl} = -k_{\omega}\boldsymbol{\omega} - k_q \mathbf{q} + \boldsymbol{\omega} \times \tilde{\mathbf{J}}\boldsymbol{\omega}$$

Attitude and stabilization accuracy

- Imbalances act as a small disturbances
- Undisturbed motion is asymptotically stable
- Motion is in the vicinity of required one

$$\mathbf{Q}_{ref} = (1,\ 0,\ 0,\ 0), \quad \boldsymbol{\omega}_{ref} = (0,\ 0,\ 0)$$

- Two different cases: purely static and purely dynamic imbalance

Equations of motion analysis

- Main idea – looking for solution as a power series

$$\boldsymbol{\omega} = \boldsymbol{\omega}^0 + \varepsilon \boldsymbol{\omega}^1 + \ldots$$

$$\boldsymbol{\Omega}_k = \boldsymbol{\Omega}_k^0 + \varepsilon \boldsymbol{\Omega}_k^1 + \ldots = \mathbf{e}_k \left(\Omega_k^0 + \varepsilon \Omega_k^1 + \ldots \right)$$

$$\mathbf{q} \approx \frac{1}{2} \left(\boldsymbol{\varphi}^0 + \varepsilon \boldsymbol{\varphi}^1 + \ldots \right).$$

- Epsilon here is the ratio between imbalance value and RW moment of inertia

Dynamic imbalance

- Zero approximation converges to zero
- Equations of first approximation give

$$\left(\tilde{\mathbf{J}} - \sum_k I_k^0 \mathbf{e}_k \mathbf{e}_k^T\right)\ddot{\boldsymbol{\varphi}}^1 + k_\omega \dot{\boldsymbol{\varphi}}^1 + \frac{k_q}{2}\boldsymbol{\varphi}^1 = \sum_k \boldsymbol{\Omega}_k^0 \times \mathbf{I}_k^1 \boldsymbol{\Omega}_k^0, \quad \boldsymbol{\Omega}_k^0 = const$$

- Right part depends only on time
- Final estimation

$$\Delta\boldsymbol{\omega} \approx \varepsilon \sum_k \Omega_k^0 \left(-\mathbf{u}_k \sin\left(\alpha_k^0 + \Omega_k^0 t\right) + \mathbf{v}_k \cos\left(\alpha_k^0 + \Omega_k^0 t\right)\right)$$

- And in simplified form

$$\Delta\omega_{ik} \sim \Omega_k^0 d$$

Static imbalance

- Zero order approximation is the same
- First order approximation is

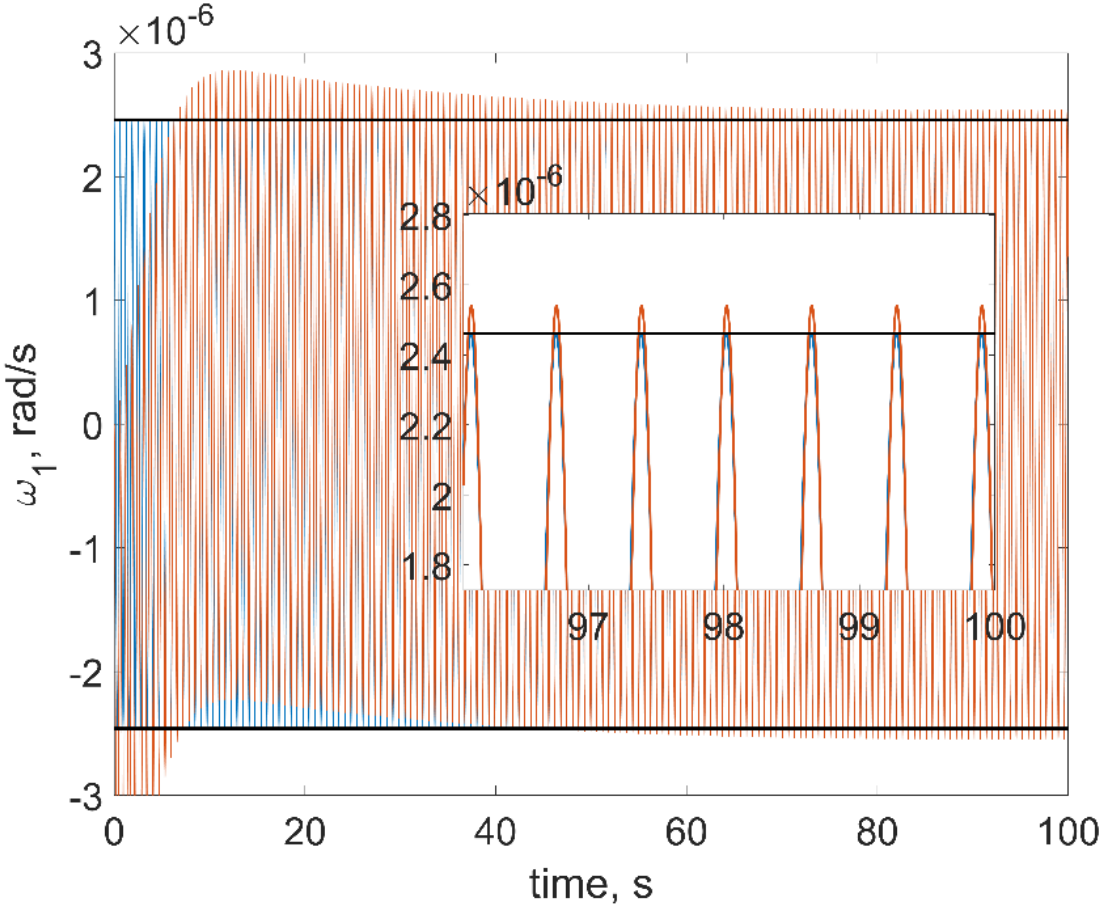
$$\left(\tilde{\mathbf{J}} - \sum_k \tilde{I}_k \mathbf{e}_k \mathbf{e}_k^T\right)\dot{\boldsymbol{\omega}}^1 + k_\omega \boldsymbol{\omega}^1 + \frac{k_q}{2}\boldsymbol{\varphi}^1 = -\sum_k m_k \boldsymbol{\rho}_k \times \boldsymbol{\Omega}_k^0 \times \boldsymbol{\Omega}_k^0 \times \delta\boldsymbol{\rho}_{kc}$$

- Again, right part depends only on time
- Solution is

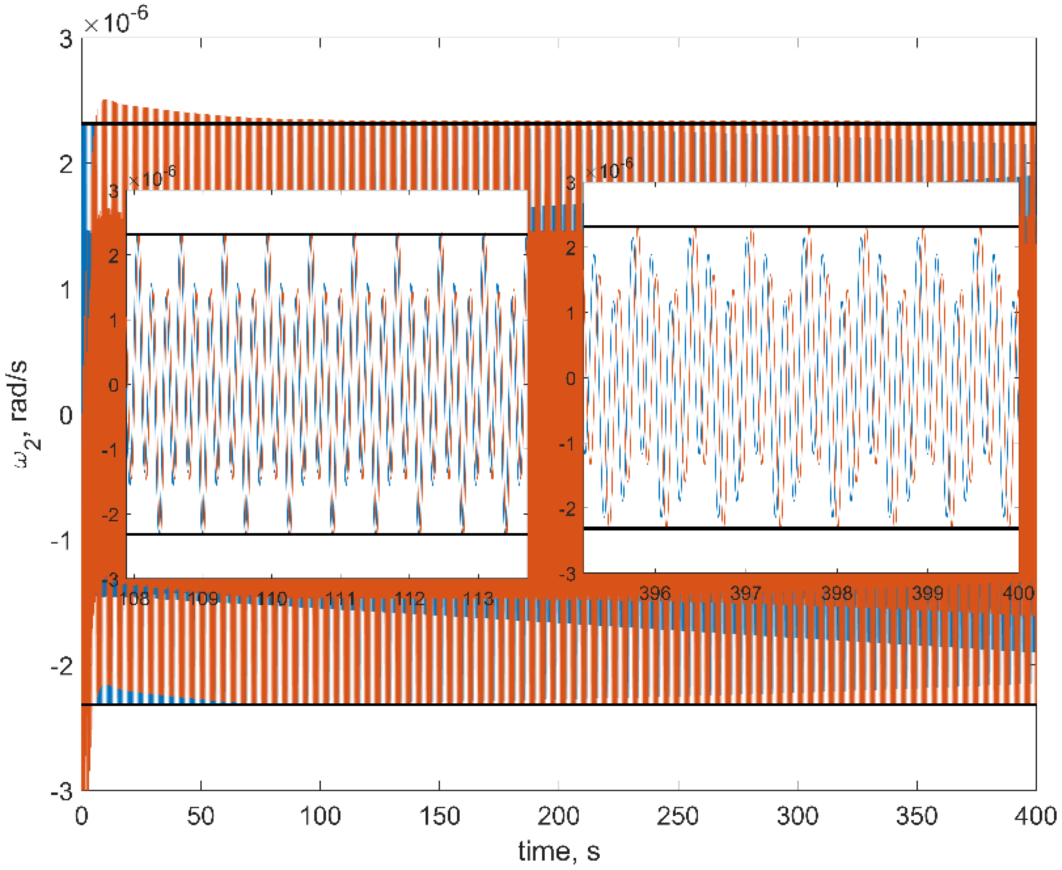
$$\Delta\boldsymbol{\omega} \approx \sum_k \rho_k \delta\rho_{kc} \Omega_k^0 \left(-\mathbf{u}_k \sin\left(\alpha_k^0 + \Omega_k^0 t\right) + \mathbf{v}_k \cos\left(\alpha_k^0 + \Omega_k^0 t\right)\right)$$

- Depends on position of RWs: the farther they are from center of mass – the greater stabilization error

Simulation results



Static Imbalance



Dynamic Imbalance

Conclusion

- We presented a fully coupled model of motion for the satellite with imbalanced RWs
- It is verified using conservation laws for free motion
- Two cases of imbalances are considered for inertial stabilization
- Analytical estimations and numerical simulation are in a good accordance (several percent difference)

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