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Magnetically controllable attitude trajectory constructed using the particle swarm optimization method

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Introduction

Problem:

ensuring spacecraft 3-axis stabilization using magnetic attitude control system only

Restriction:

direction of magnetic torque, it cannot be applied along the geomagnetic induction vector

$M_{magn} = m \times B_{magn}$ is the control torque

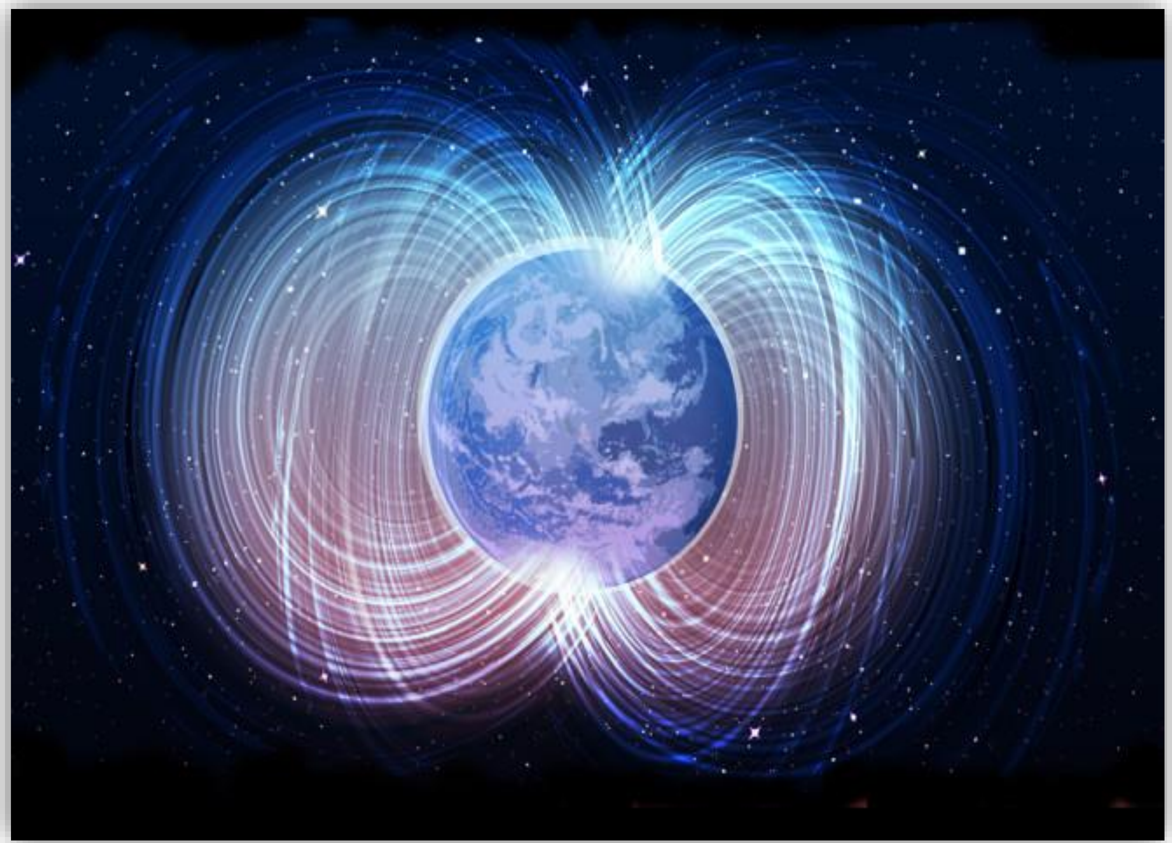
where

m is the satellite dipole moment

B_{magn} is the geomagnetic induction vector

Solution:

construction of an optimal magnetically controllable attitude trajectory using PSO algorithm



https://www.discovermagazine.com/environment/earths-magnetic-field-iprobably-i-isnt-reversing

Particle swarm optimization (PSO)

the non-gradient biologically inspired global optimization method

Optimization problem:

$$\min_{x \in \mathbb{U}} \Phi(x)$$

$\Phi(x)$ is the cost function

$\mathbb{U}$ is the search space

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Swarm characteristics:

P is the number of particles in the swarm

x_p is the particle position

v_p is the particle velocity

G is the number of generations

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Position and velocity change:

$$\mathbf{x}_p(i+1) = \mathbf{x}_p(i) + \mathbf{v}_p(i+1)$$

$$\mathbf{v}_p(i+1) = c_{in}\mathbf{v}_p(i) + c_{cog}\left[\mathbf{x}_{p,best}(i) - \mathbf{x}_p(i)\right] + c_{soc}\left[\mathbf{x}_{best}(i) - \mathbf{x}_p(i)\right]$$

i is the current generation number

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The inertial component
it is responsible for the search continuation in the same direction

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The cognitive component
the desire to return to its own better position found earlier

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The social component representing striving for a better position found in the particle vicinity

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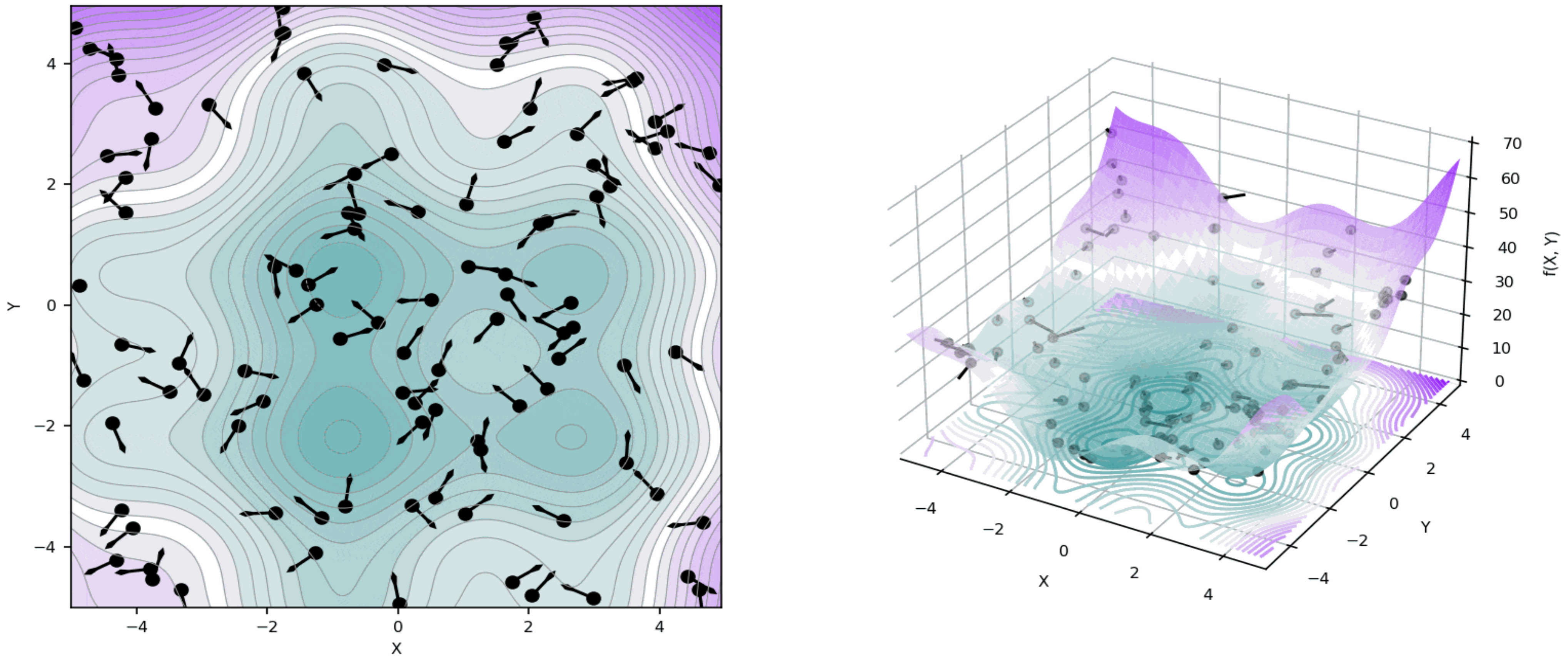
Search stop criteria:

1) the cost function derivative is small (<u>cost function stagnation</u>)
2) all particles are falling into some neighborhood of the best position (<u>swarm stagnation</u>)

Example

[1/100] $c_{in} = 0.8, \quad c_{cog} = 2.0, \quad c_{soc} = 2.0$

https://towardsdatascience.com/particle-swarm-optimization-visually-explained-46289eeb2e14



$$\Phi(x, y) = x^2 + (y + 1)^2 - 5\cos(1.5x + 1.5) - 3\cos(2x - 1.5) \rightarrow \min$$

$$\mathbb{U} = \{x, y \mid -5 \le x \le 5, -5 \le y \le 5\}$$

Motion model

Orbit parameters:

$h = 550\,\text{km}$ – attitude

$i = 57^\circ$ – inclination

$T_0 \approx 1.58\,\text{h}$ – orbital period

Satellite parameters:

$10 \times 20 \times 30$ cm – shape (parallelepiped)

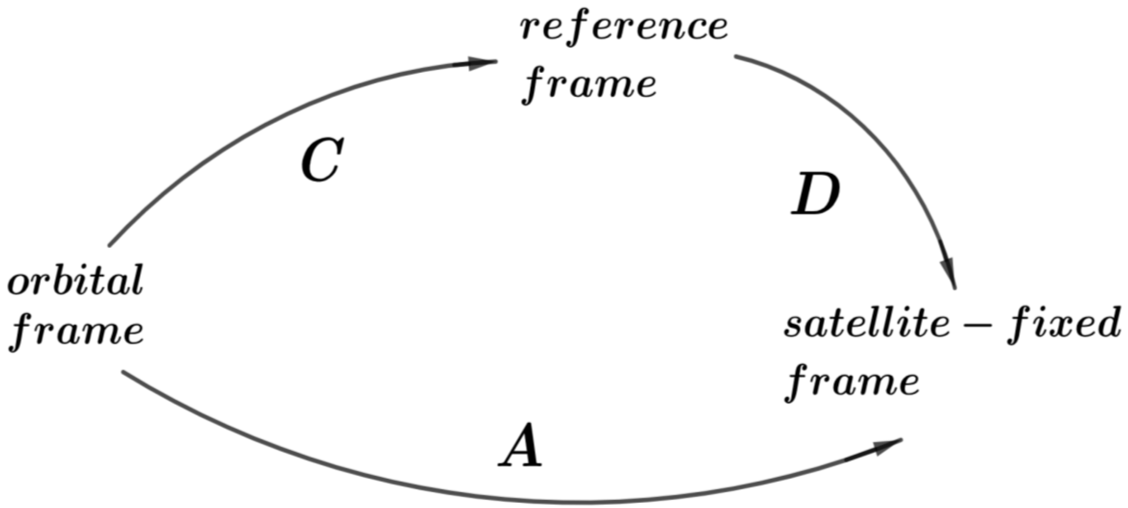
$c = (0,\ 1,\ 0)$ cm – center of mass displacement

$J = diag(0.15,\ 0.13,\ 0.11)\ \text{kg} \cdot \text{m}^2$ – inertia tensor

Equations of motion:

$$\begin{cases} \dot{\boldsymbol{\omega}}_{abs} = \mathbf{J}^{-1}(\mathbf{M}_{ctrl} + \mathbf{M}_{grav} + \mathbf{M}_{aero} + \\ \qquad + \mathbf{M}_{dist} - \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs}) \\ \dot{q}_0 = -0.5\,\mathbf{q}^T \boldsymbol{\omega}, \\ \dot{\mathbf{q}} = 0.5(q_0 \boldsymbol{\omega} + \mathbf{q} \times \boldsymbol{\omega}) \end{cases}$$

Reference frames:



Trajectory construction

Stage 1

search for a trajectory on which the projection of the control torque onto the Earth's geomagnetic induction vector is minimal

Attitude trajectory:

$$\alpha(t) = a_1 \sin\omega_0 t + a_2 \cos\omega_0 t + a_3 \sin 2\omega_0 t + a_4 \cos 2\omega_0 t,$$

$$\beta(t) = b_1 \sin\omega_0 t + b_2 \cos\omega_0 t + b_3 \sin 2\omega_0 t + b_4 \cos 2\omega_0 t,$$

$$\gamma(t) = g_1 \sin\omega_0 t + g_2 \cos\omega_0 t + g_3 \sin 2\omega_0 t + g_4 \cos 2\omega_0 t,$$

ω_0 – orbital angular velocity

Cost function:

$$\Phi_1 = \frac{dt}{T_0}\sqrt{\sum_{t_0=0}^{T_0}\left(\frac{\mathbf{M}_{ctrl}(t)}{\left|\mathbf{M}_{ctrl}(t)\right|}, \frac{\mathbf{B}_{magn}(t)}{\left|\mathbf{B}_{magn}(t)\right|}\right)^2} \rightarrow \min$$

$$\mathbb{U} = \left\{a_k, b_k, g_k \in \left(-3.5\cdot 10^{-2}, 3.5\cdot 10^{-2}\right) \text{ rad}, \ k = \overline{1,4}\right\}$$

Required control torque:

$$\mathbf{M}_{ctrl}(\alpha,\beta,\gamma) = \mathbf{J}\dot{\boldsymbol{\omega}}_{abs}(\alpha,\beta,\gamma) + \boldsymbol{\omega}_{abs}(\alpha,\beta,\gamma)\times \mathbf{J}\boldsymbol{\omega}_{abs}(\alpha,\beta,\gamma) - \mathbf{M}_{grav}(\alpha,\beta,\gamma) - \mathbf{M}_{aero}(\alpha,\beta,\gamma)$$

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Attitude trajectory parameters finding by PSO:

$$a_1 = 1.016 \cdot 10^{-2} \text{ rad}, \quad b_1 = -4.028 \cdot 10^{-3} \text{ rad}, \quad g_1 = 8.067 \cdot 10^{-3} \text{ rad},$$
$$a_2 = 2.545 \cdot 10^{-2} \text{ rad}, \quad b_2 = -9.717 \cdot 10^{-3} \text{ rad}, \quad g_2 = -4.127 \cdot 10^{-3} \text{ rad},$$
$$a_3 = 1.449 \cdot 10^{-3} \text{ rad}, \quad b_3 = -4.841 \cdot 10^{-5} \text{ rad}, \quad g_3 = -2.433 \cdot 10^{-2} \text{ rad},$$
$$a_4 = 1.124 \cdot 10^{-3} \text{ rad}, \quad b_4 = 2.231 \cdot 10^{-4} \text{ rad}, \quad g_4 = -4.637 \cdot 10^{-4} \text{ rad}.$$

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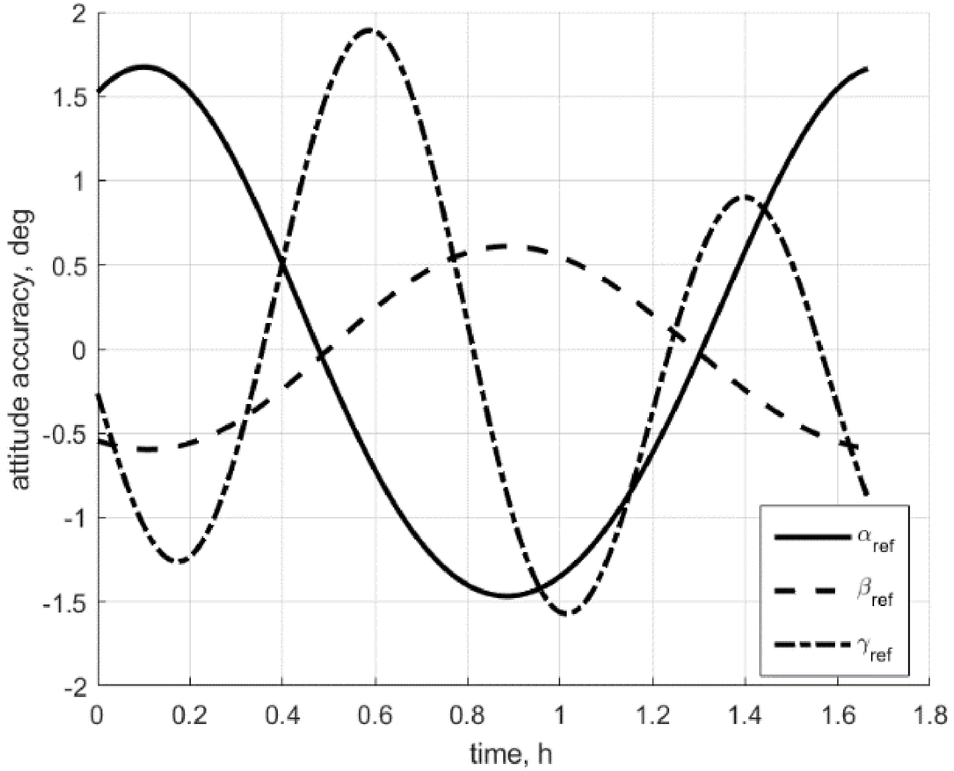
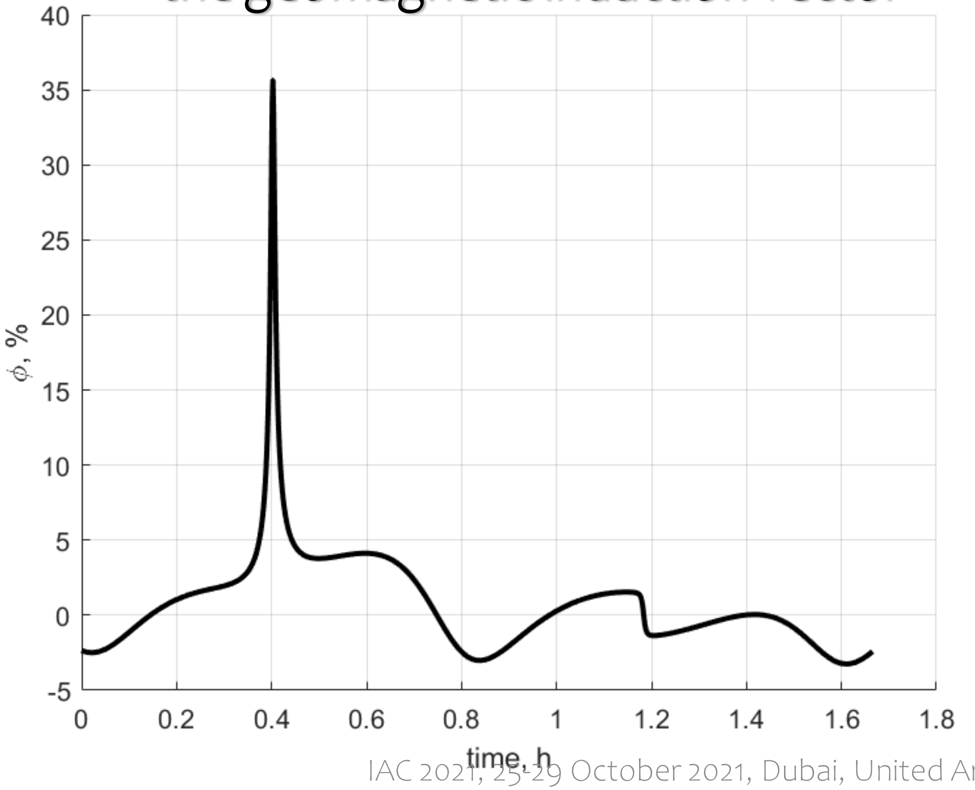
Reference trajectory



Control torque projection onto the geomagnetic induction vector



Trajectory construction

Stage 2

construct a magnetic control that provides convergence

Required control torque (using Lyapunov function):

$$M_{ctrl} = -k_\omega \omega_{rel} - k_a S + \omega_{abs} \times J\omega_{abs} + \\ + J\dot{A}(\omega_0 + \omega_{ref}) + JA\dot{\omega}_{ref} - M_{grav} - M_{aero}$$

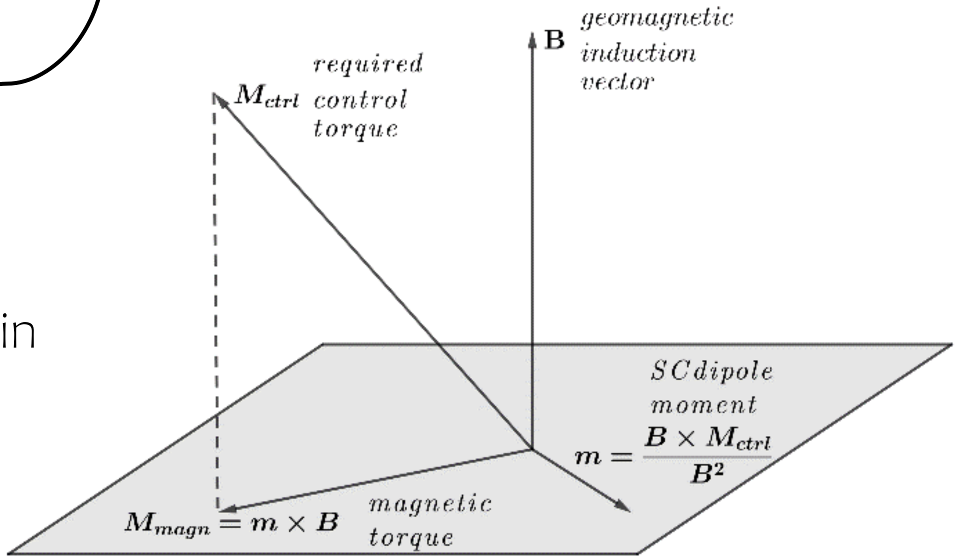
Receiving magnetic control torque:

$$M_{magn} = m \times B_{magn} = \frac{B_{magn} \times M_{ctrl} \times B_{magn}}{B_{magn}^{\;2}}$$

Cost function:

$$\Phi_2 = \left(\sum_{t_0=0}^{T_0} \left(\left(\omega_{rel}^T B_{magn} \right) \cdot \left(M_{ctrl}^T B_{magn} \right) \right)^2 + \sum_{t_0=0}^{T_0} \left(\left(S^T B_{magn} \right) \cdot \left(M_{ctrl}^T B_{magn} \right) \right)^2 \right) \to \min$$

$$\mathbb{U} = \left\{ k_\omega \in \left(5 \cdot 10^{-5}, 10^{-2}\right) \text{ Nms},\ k_a \in \left(10^{-8}, 5 \cdot 10^{-5}\right) \text{ Nm} \right\}$$



Trajectory construction

Stage 2

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Control gains finding by PSO:

$$k_\omega = 1.119 \cdot 10^{-4} \text{ Nms}, \; k_a = 6.578 \cdot 10^{-8} \text{ Nm}$$

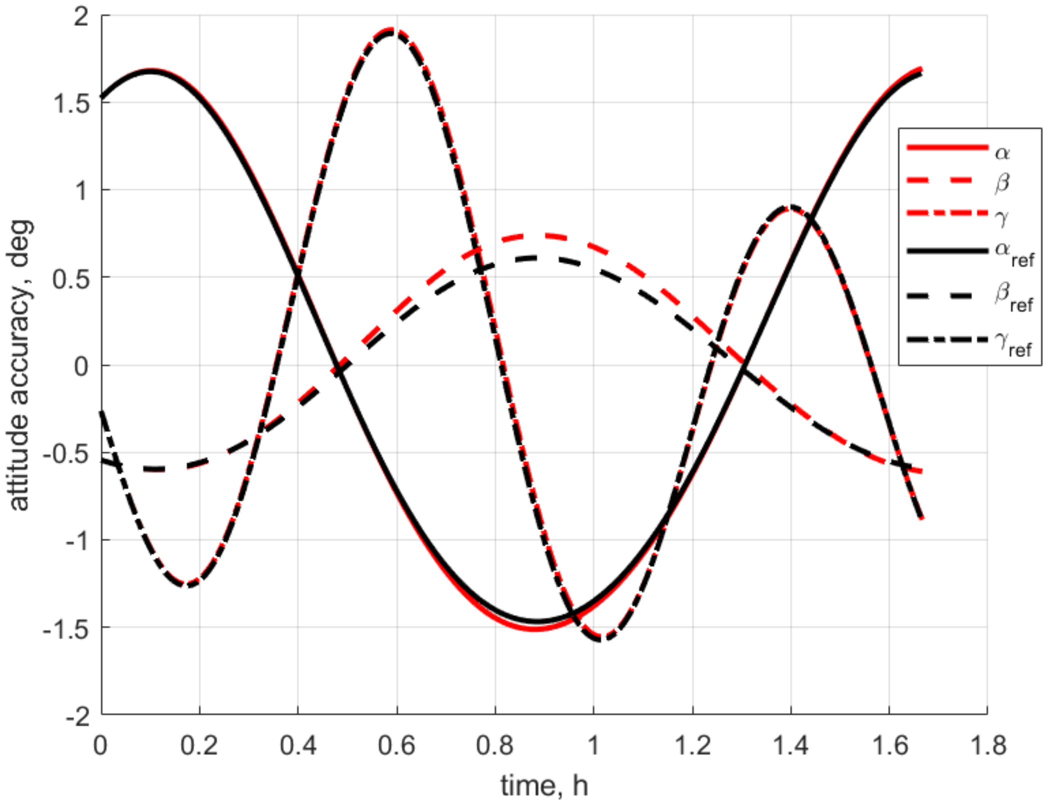
* To prove the **asymptotic stability** of the resulting motion, we linearize the equations of motion and use the **Floquet theory**. The norm of eigenvalues of the obtained monodromy matrix (**characteristic multipliers**, ρ) lie inside the unit circle: $|\rho| < 1$

Trajectory construction

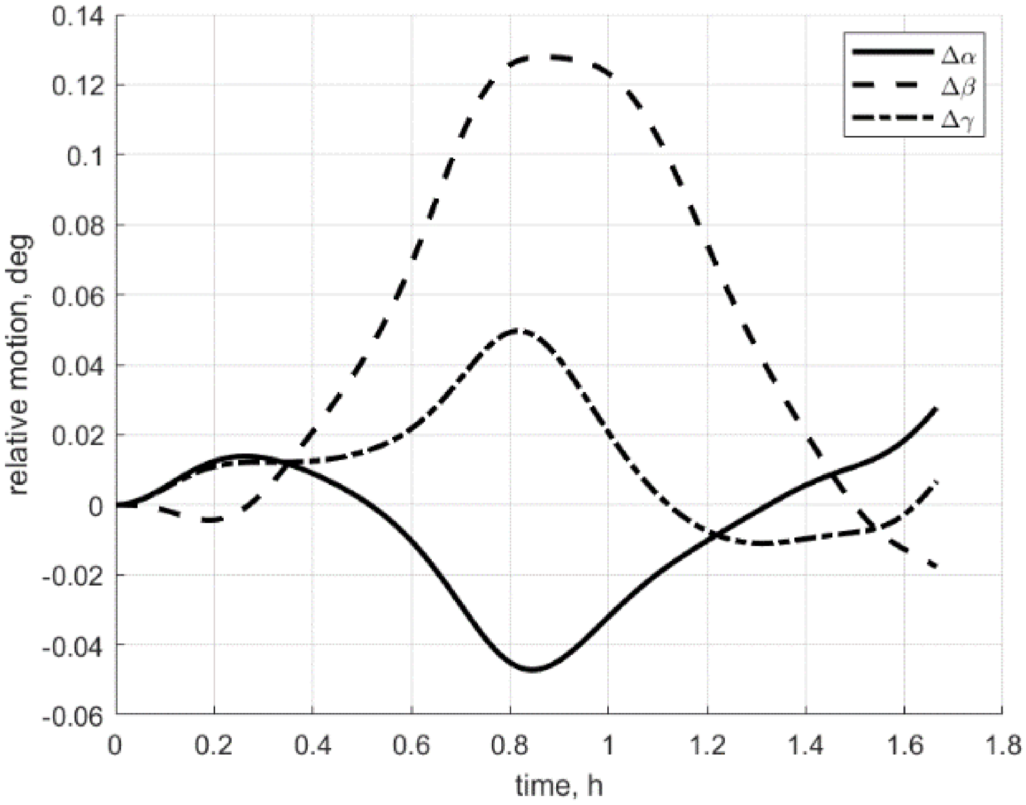
Stage 2

construct a magnetic control that provides convergence

Reference and real trajectory



Reference trajectory deviation



Trajectory construction

Stage 2

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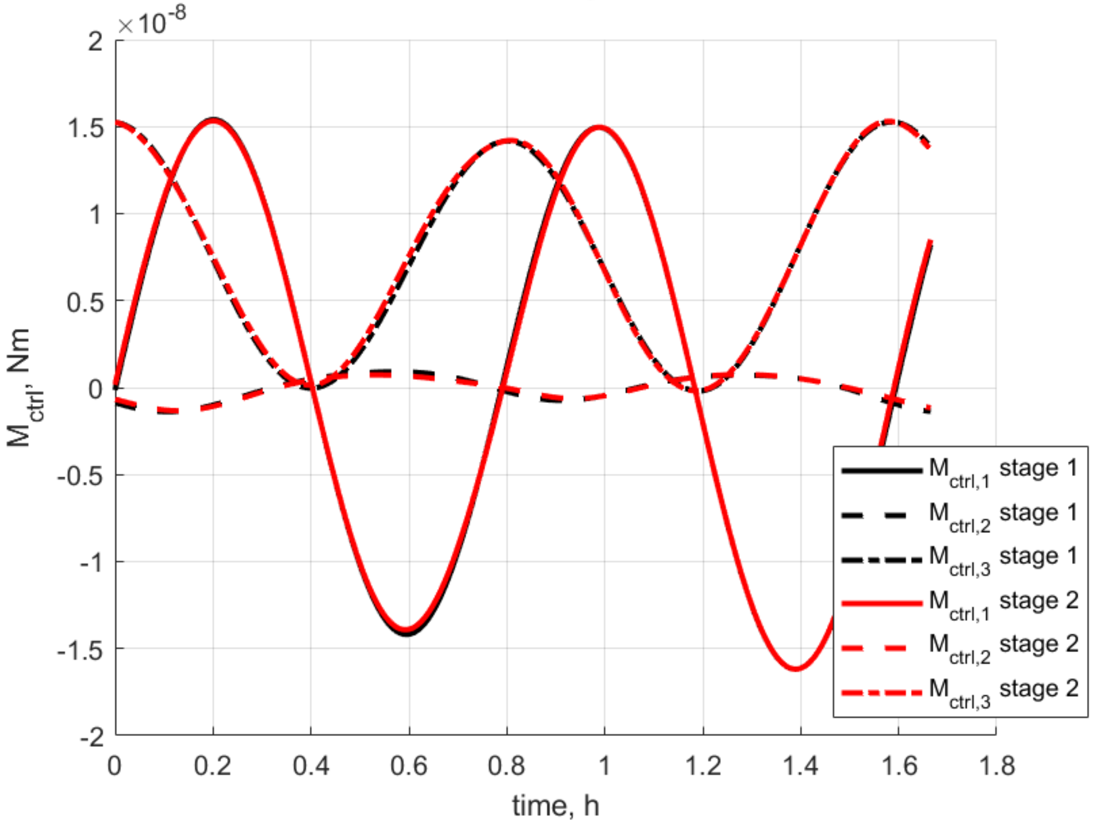
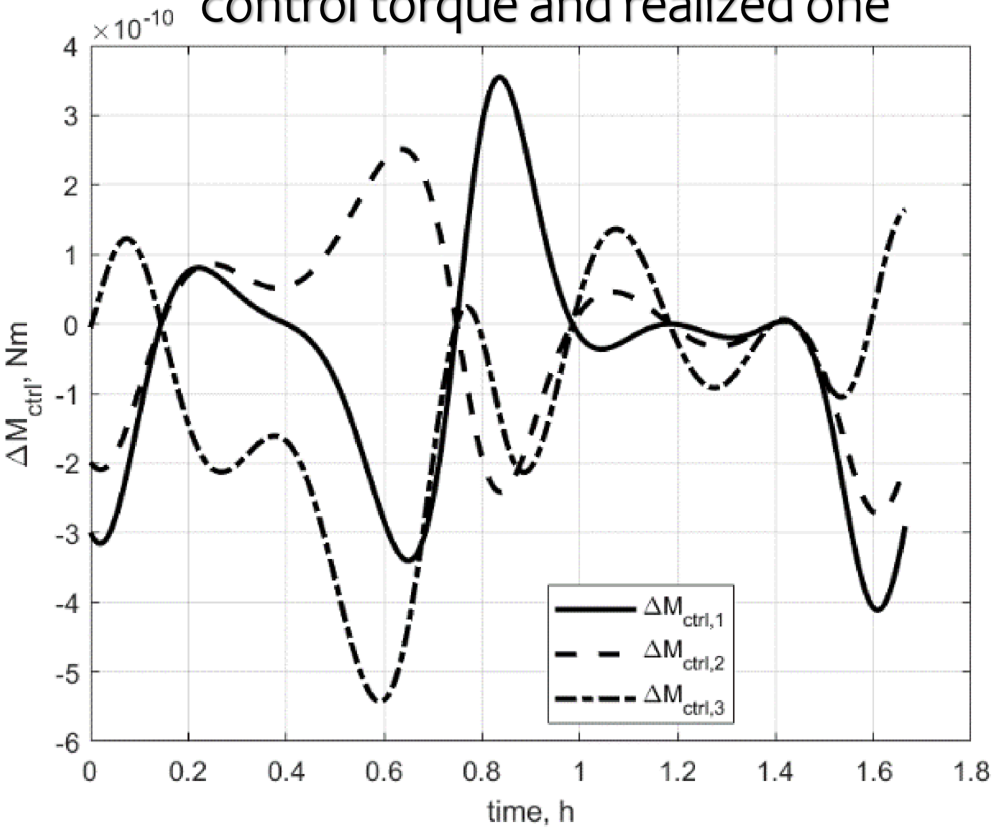
Required and real (magnetic) control torque



The difference between the required control torque and realized one



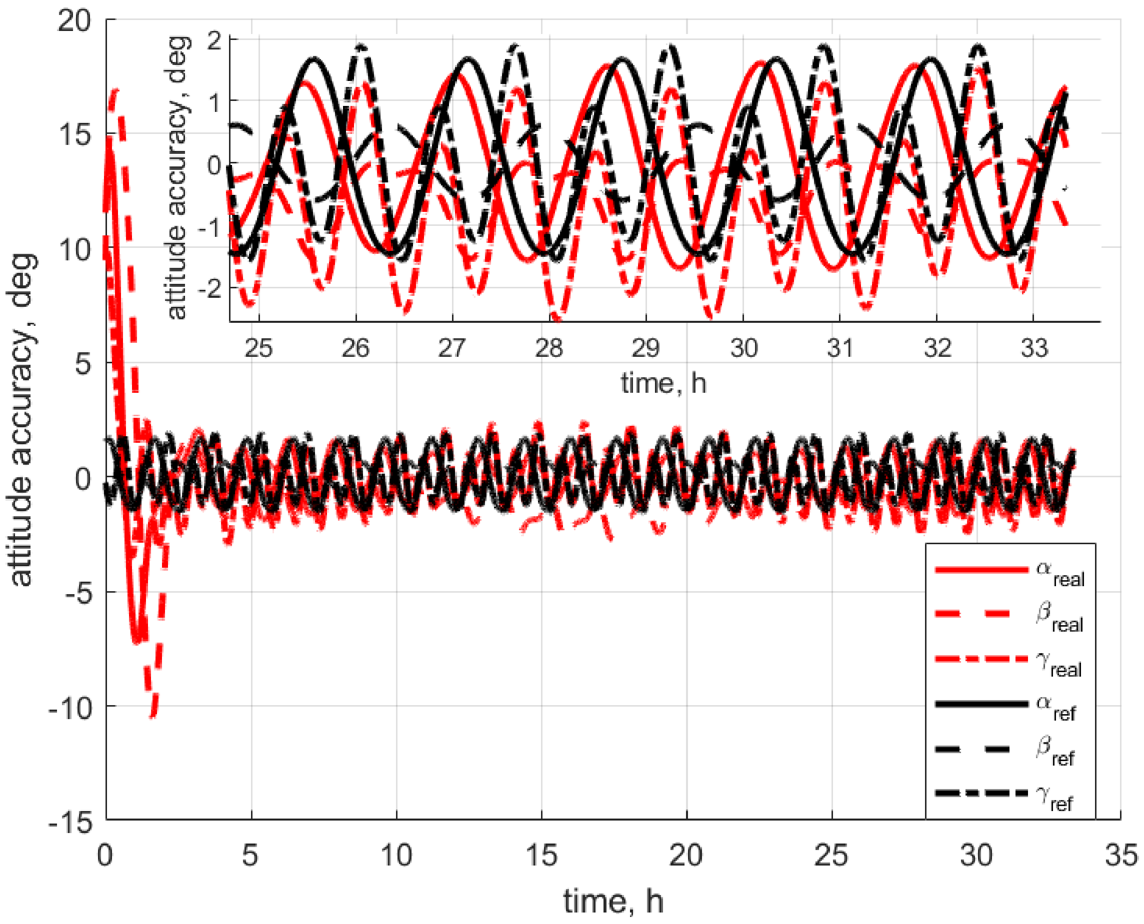
Numerical example

Parameters for numerical simulation

Simulation time	$T = 20T_0 \approx 33\,\text{h}$
SC initial angular velocity	$\omega = [1,2,3]\cdot 10^{-4}$ rad/s
SC initial orientation	$\alpha = 11.5^\circ \cdot \frac{\pi}{180^\circ} \approx 0.2\,\text{rad}$ $\beta = 9.5^\circ \cdot \frac{\pi}{180^\circ} \approx 0.165\,\text{rad}$ $\gamma = 9.7^\circ \cdot \frac{\pi}{180^\circ} \approx 0.17\,\text{rad}$
Magnetic field model	inclined dipole
Inaccuracy of knowledge of the density of the atmosphere	20%
External random disturbances	$\mathbf{M}_{dist} \sim 10^{-9}\ Nm$

Numerical example

Reference and real trajectory



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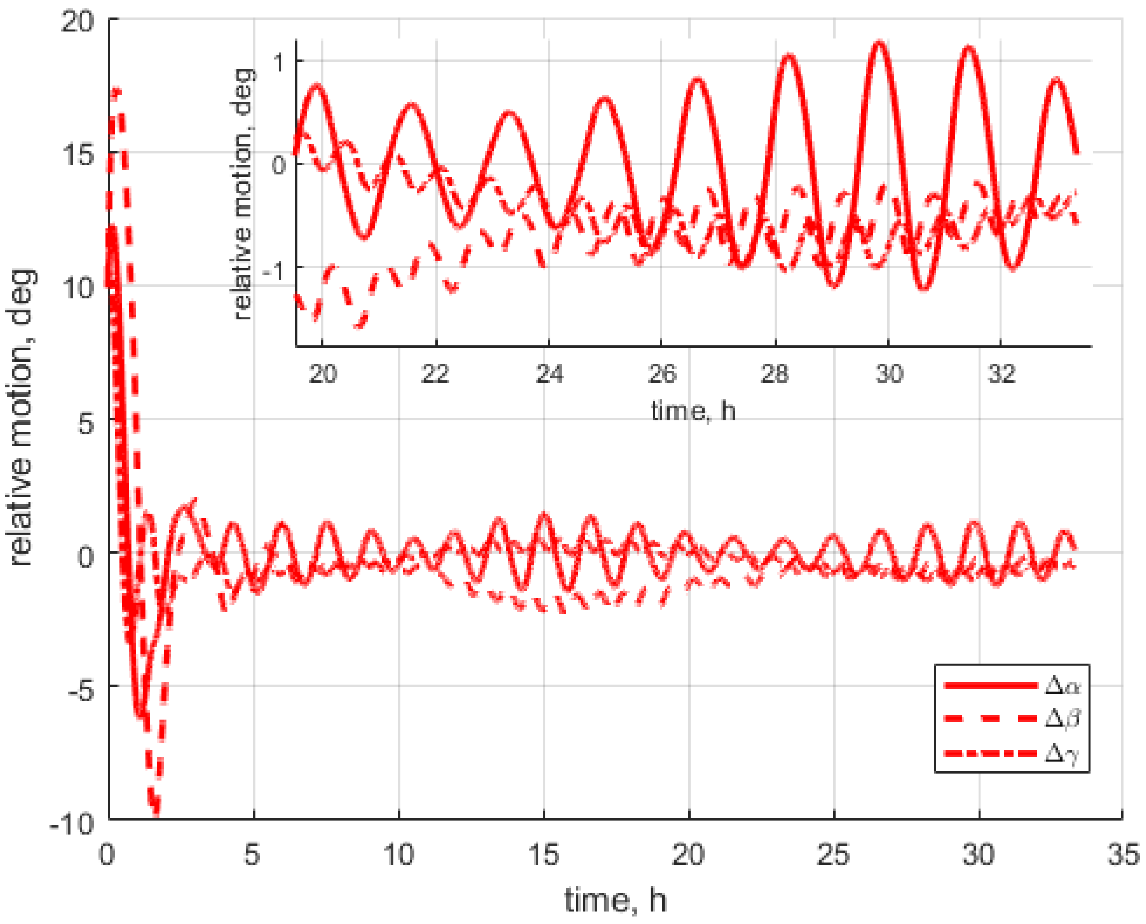
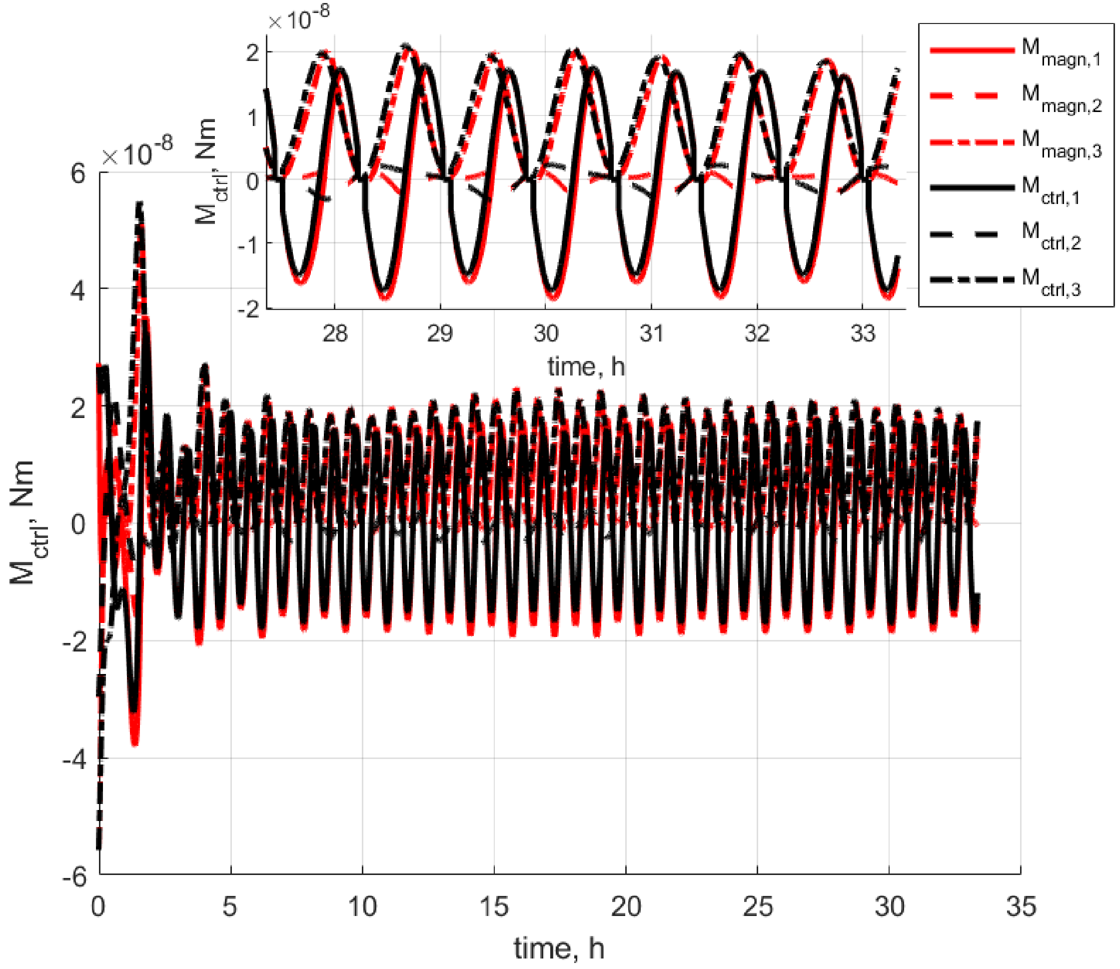


Numerical example

Required and real (magnetic) control torque



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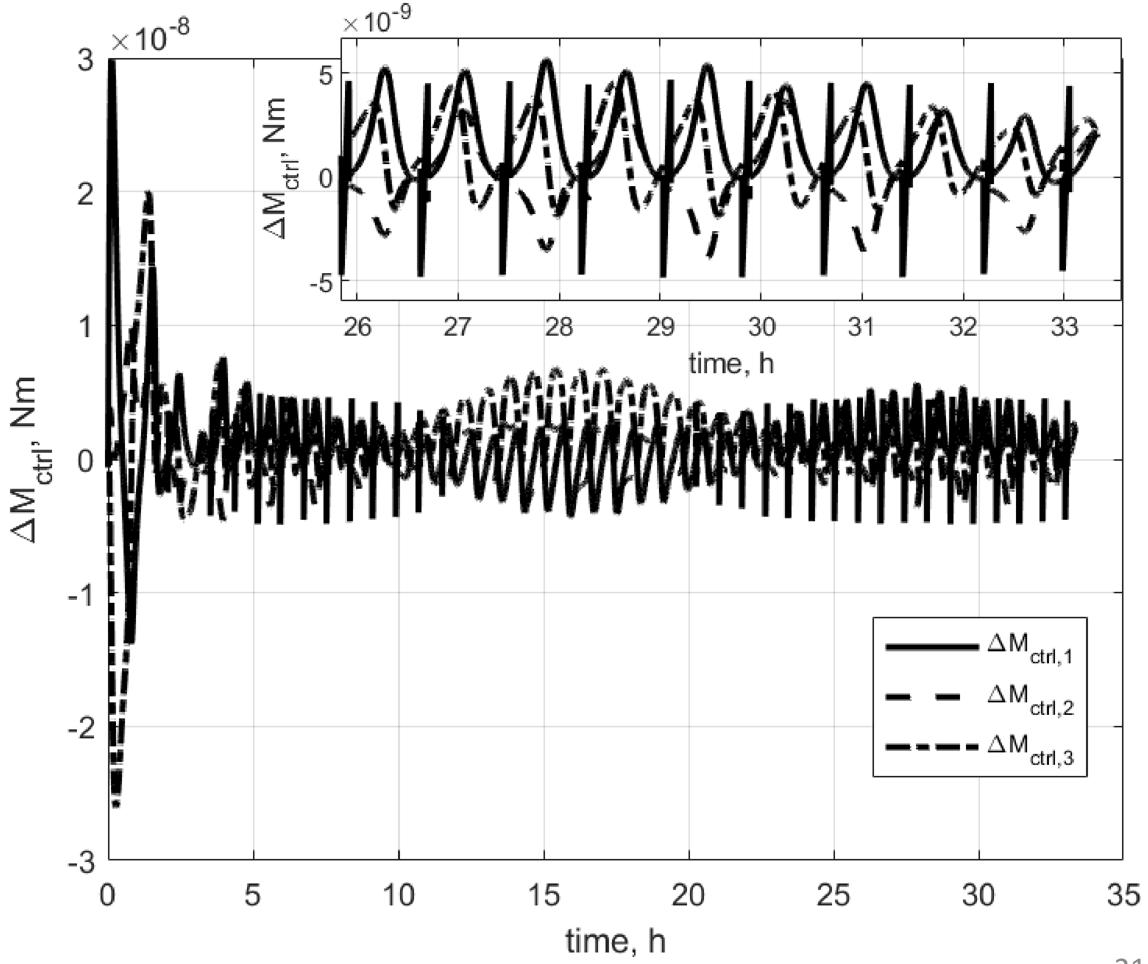

Conclusion

- a **method for constructing an attitude trajectory** is proposed
- the **particle swarm optimization** (PSO) method is applied to find the

  optimal trajectory coefficients and optimal control gains
- the magnetic attitude control system fully maintains the trajectory
- **numerical example** is given
- the orientation **accuracy** is about **2 degrees**

Thank you for listening!