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Lunar Frozen Orbits for Small Satellite Communication/Navigation Constellations

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Types and purposes of constellations

Classification of satellite constellations:

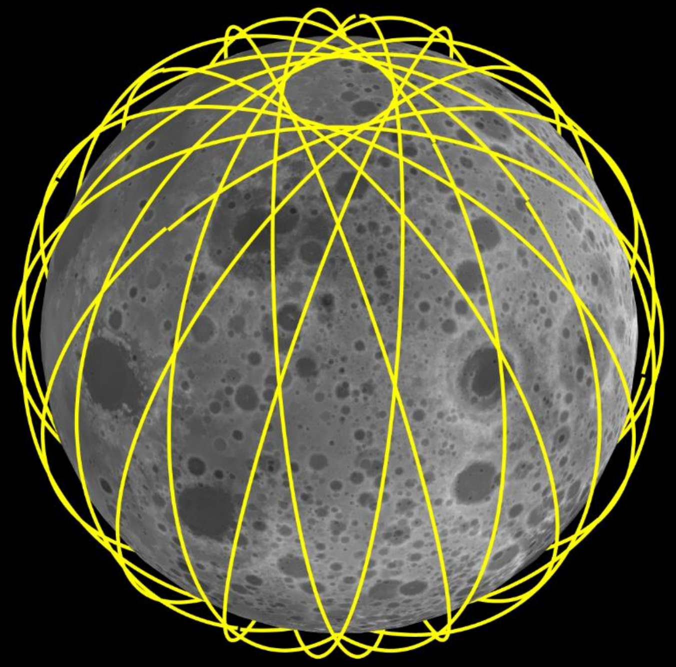
- Global or local coverage
- Symmetric or asymmetric configuration
- Types of orbits (circular/elliptical, identical or different inclinations and semimajor axes, etc.)

Two main applications of global coverage constellations:

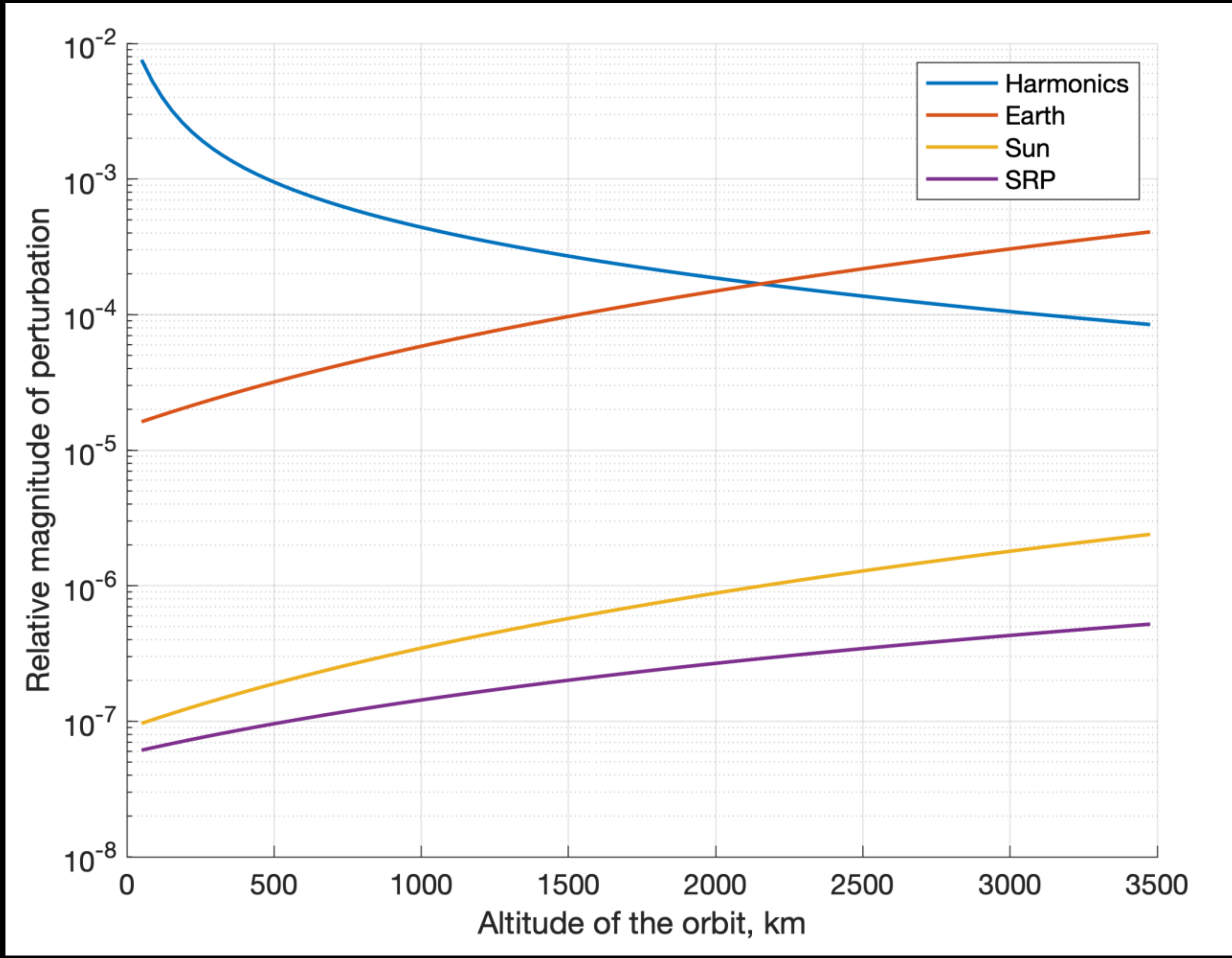
- Communication (at least 1-fold continuous coverage required)
- Global navigation (at least 4-fold continuous coverage required)

Earth and Moon constellations: what's the difference?

- Far more expensive to deploy a GPS-like or Iridium-like constellation
- Cheap small satellites could help, but much lower orbits are needed due to antenna power constraints
- Dynamical environment is much more complicated despite the Moon has no atmosphere



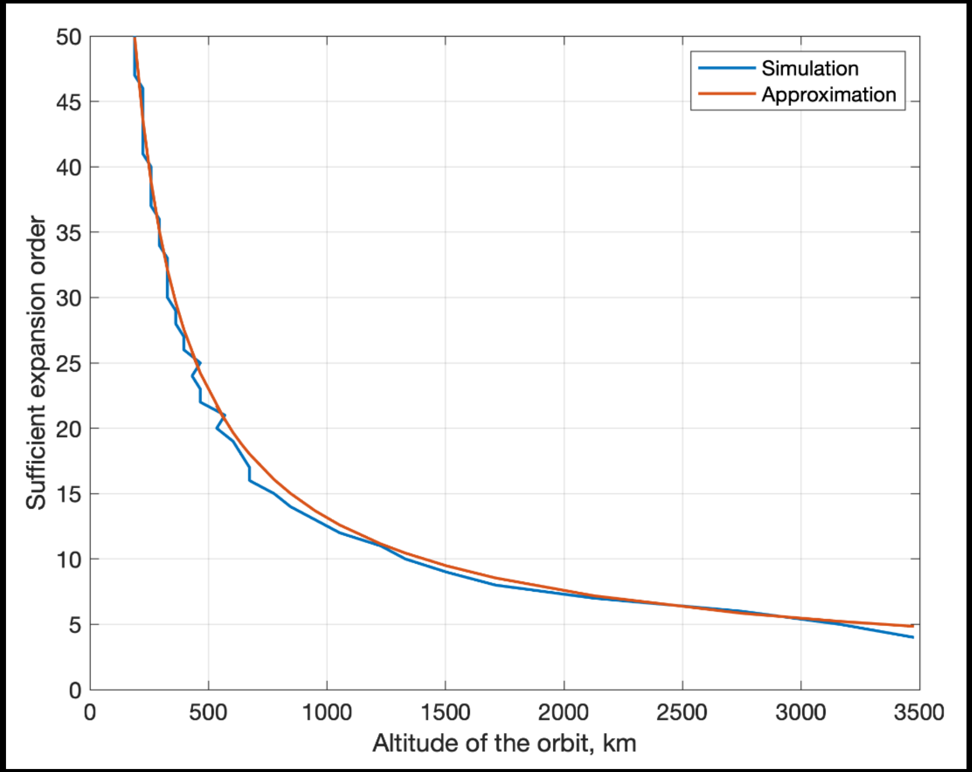
What perturbations to be included?

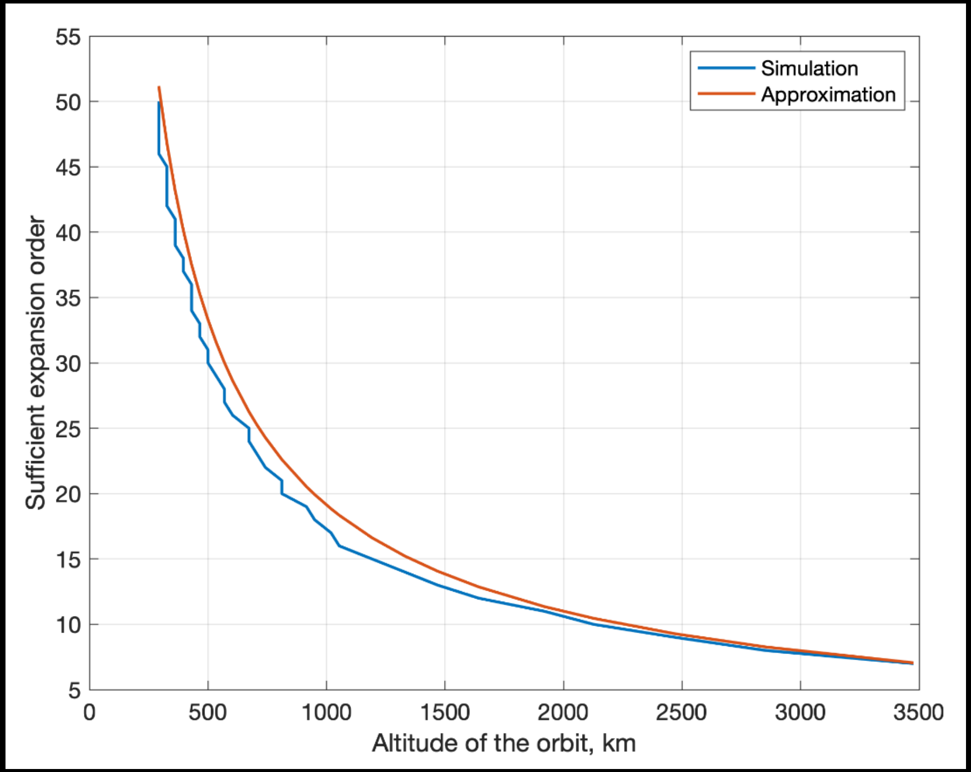


For low and medium lunar orbits ($h < 2R_☾ \approx 3500$ km), the following perturbations dictate the orbital evolution:

- Nonsphericity of lunar gravitational potential
- Third-body gravitational perturbations (Earth, Sun)
- Solar radiation pressure

Practical approach to choose a degree of the spherical harmonics model

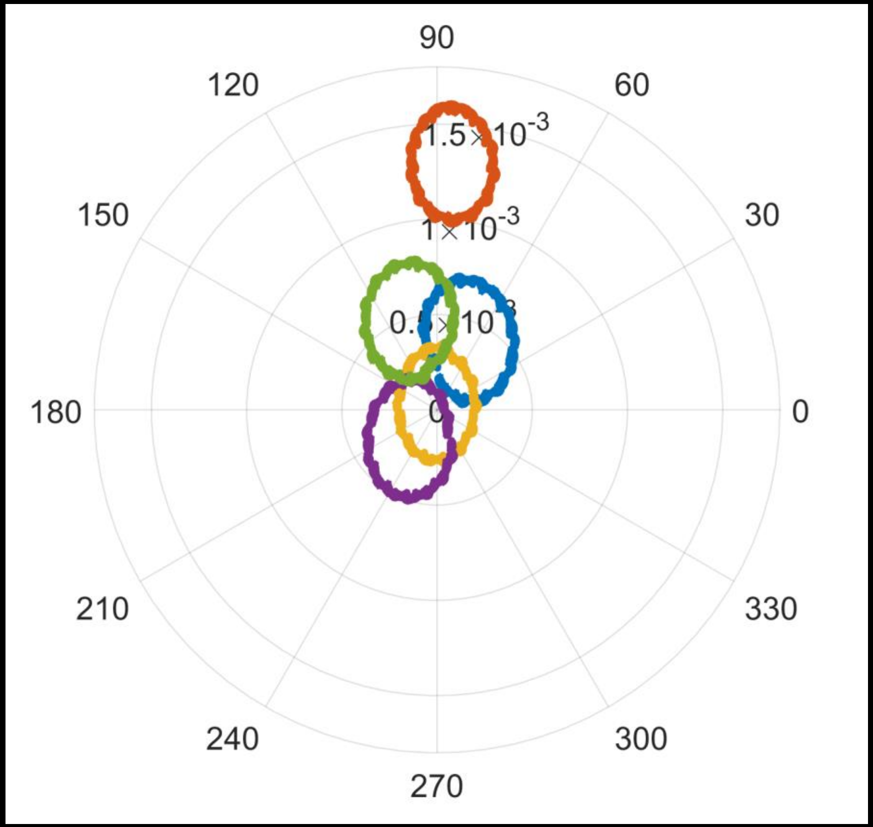




$\varepsilon = 10^{-6}$ $\quad N = \left(\dfrac{25}{h[10^3\ \mathrm{km}]}\right)^{0.8}$

$\varepsilon = 10^{-7}$ $\quad N = \left(\dfrac{40}{h[10^3\ \mathrm{km}]}\right)^{0.8}$

Lunar frozen orbits: unified definition



One-year evolution of the eccentricity vector for 5 spacecraft with uniformly distributed (in terms of RAAN) orbital planes $i = 40°$

A frozen orbit is such an orbit that the eccentricity vector components $e_x = e\cos\omega,\ \ e_y = e\sin\omega$ are nearly constant (relatively small variations are only allowed in the e_x-e_y plane)

Special case of perturbations: J2+J3 harmonics of gravitational potential

$\cos^2 i = 1/5$, usually $\omega = 90°$ or $270°$

Special case of perturbations: gravity due to a third body in a circular orbit

$\cos^2 i = 3/5 \cdot (1 - e^2)$, $\omega = 90°$ or $270°$

Frozen orbit design: Bayesian optimization approach

To generate a (quasi-)frozen orbit with given inclination and RAAN values, the following optimization problem is posed:

minimize $\quad J = \left(e_f - e_0\right)^2 + \left(\cos\omega_f - \cos\omega_0\right)^2 + \left(\sin\omega_f - \sin\omega_0\right)^2$

over $\; a_0, e_0, \omega_0 \qquad$ s.t. $\; a_0 - R_{☾} = h_{\text{ref}} \pm 10\%\;$ and $\; e_0 \in [0, e_{\max}]$

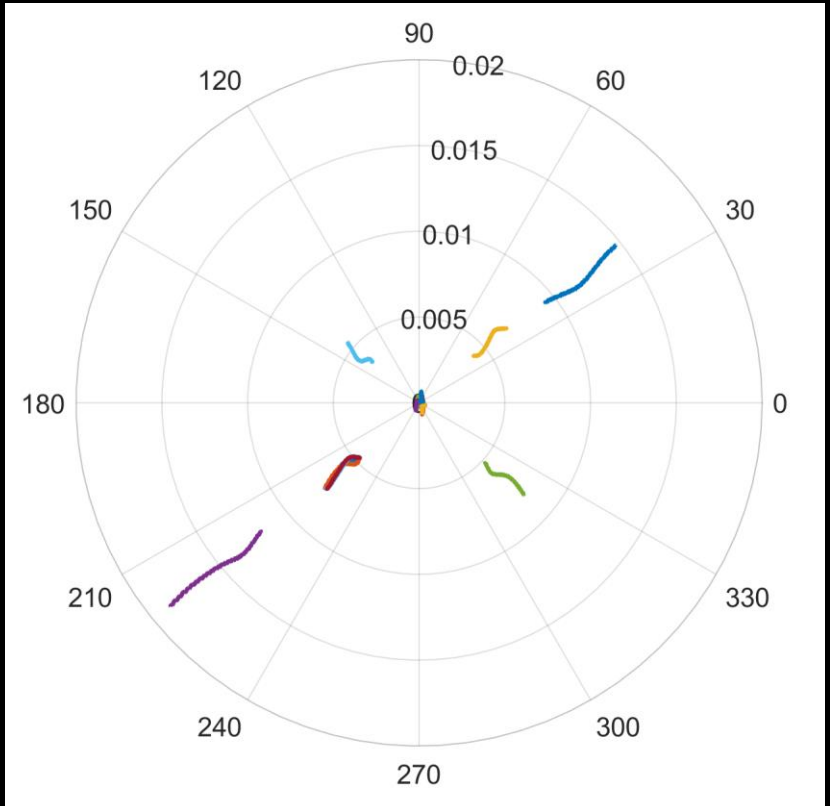
The spacecraft trajectory is propagated for $t_f - t_0 = 365$ days by MATLAB's ode113 solver.

The Bayesian optimizer (MATLAB's bayesopt) is used.

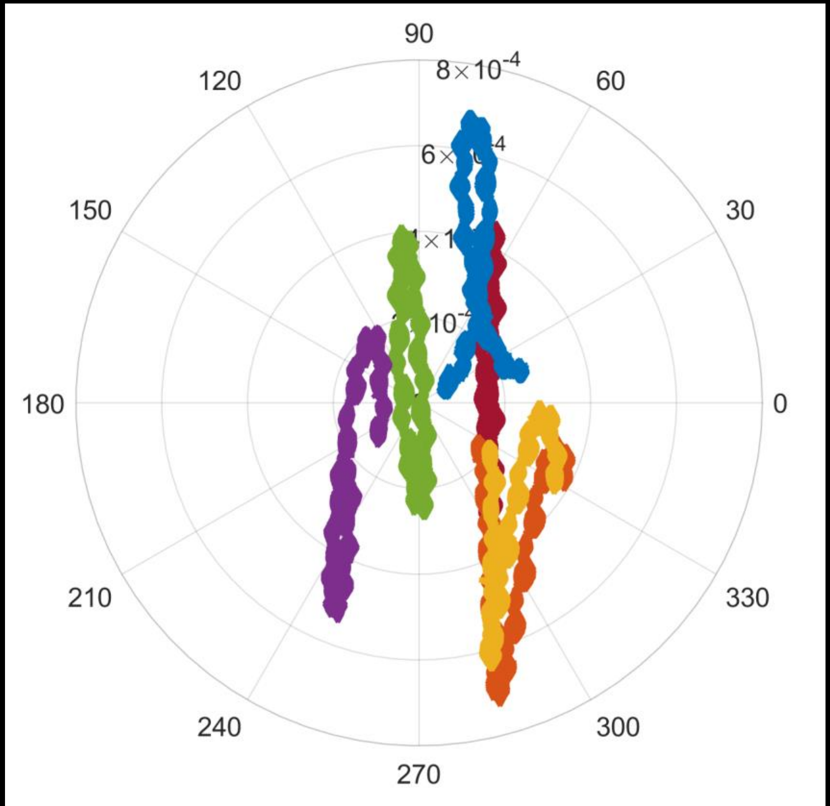
Algorithm parameters: MaxObjectiveEvaluations = 300, stopping criteria $J < \varepsilon = 10^{-4}$

Practically stable (quasi-frozen) orbits

One-year evolution of the eccentricity vector for 15 spacecraft with uniformly distributed (in terms of RAAN) orbital planes $i = 80°$



One-year evolution of the eccentricity vector for the most stable (almost perfectly frozen) 6 orbits out of 15 orbits from the left figure



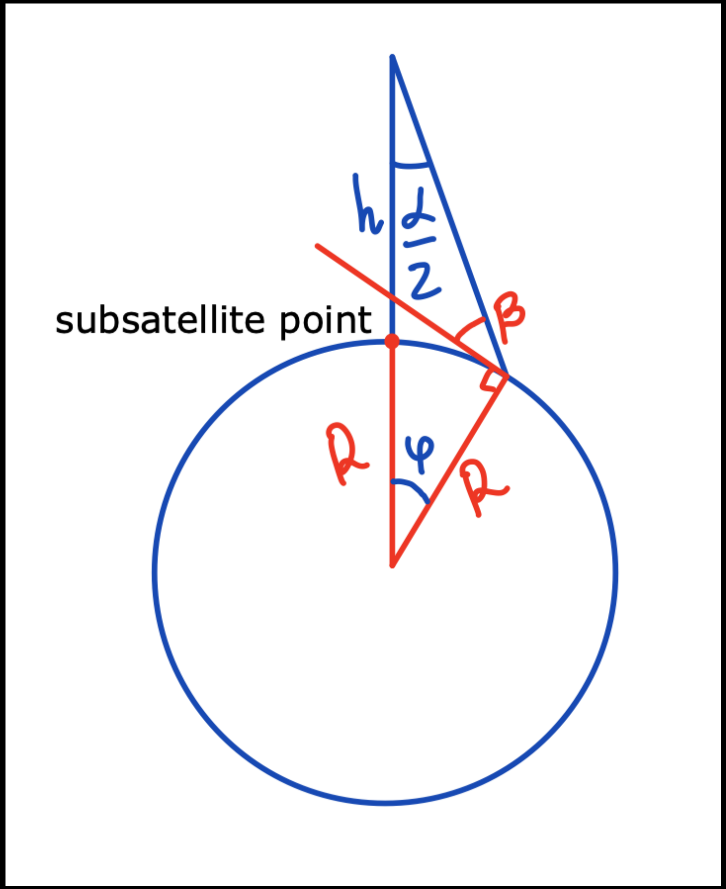
Coverage geometry: beam width, footprint size, elevation angle

The antenna beam width α defines the minimum elevation angle β:

$$\sin\frac{\alpha}{2} = \frac{R}{R+h}\cos\beta$$

For an omnidirectional/hemispherical antenna, the footprint half-size φ is obtained from the constraint $\beta \geq 5°$:

$$\varphi = 85° - \sin^{-1}\left(\frac{R}{R+h}\cos 5°\right)$$

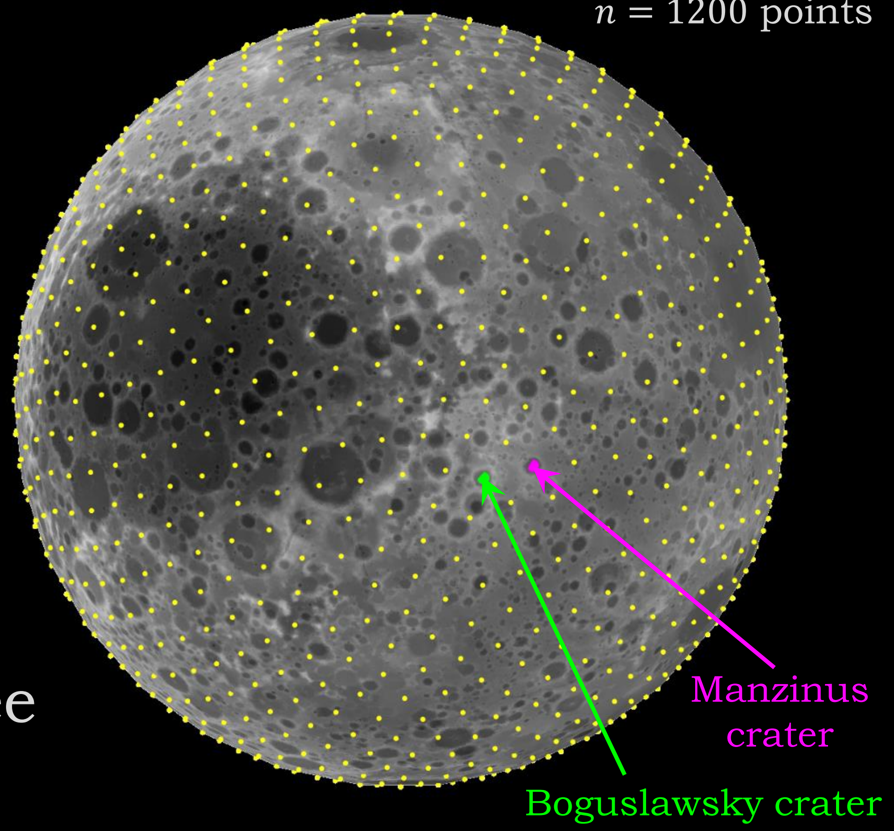


Quasi-uniform surface grid for global coverage analysis

To generate a (quasi-)uniform surface grid of n points for global coverage analysis, we use the interior-point optimization method (MATLAB's fmincon) to find a minimum of the functional

$$J = \sum_{1 \le i < j \le n} \log \frac{1}{\|\boldsymbol{r}_i - \boldsymbol{r}_j\|}$$

which choice was inspired by the paper of Lee and Mortari*



*Lee, S. & Mortari, D. Quasi-equal area subdivision algorithm for uniform points on a sphere with application to any geographical data distribution // Computers & Geosciences, 2017, Vol. 103, pp. 142–151

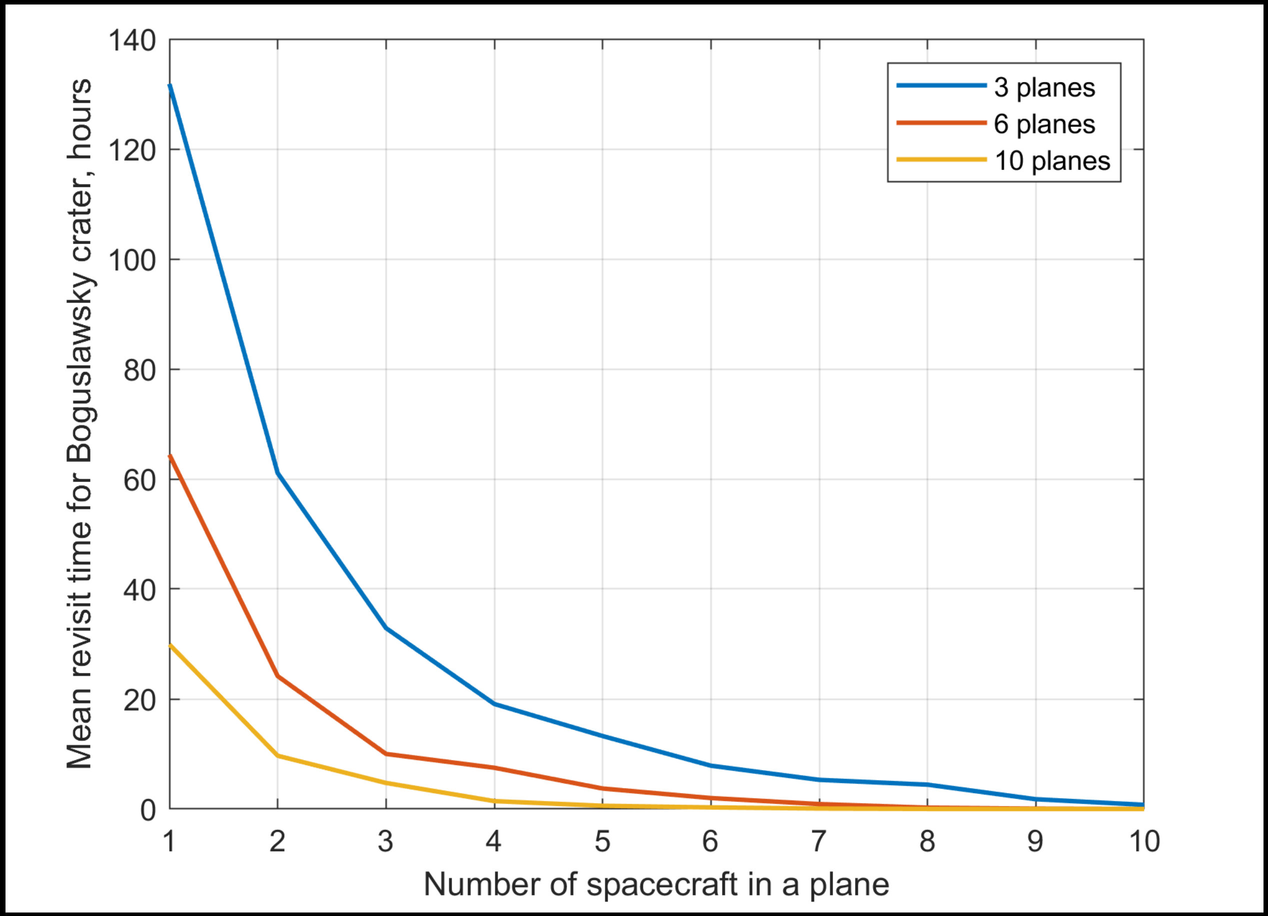
Global coverage constellation in near-polar quasi-frozen orbits

For a LLO constellation ($h_{\text{ref}} = 132$ km, $i = 80°$), the minimum number of orbital planes is

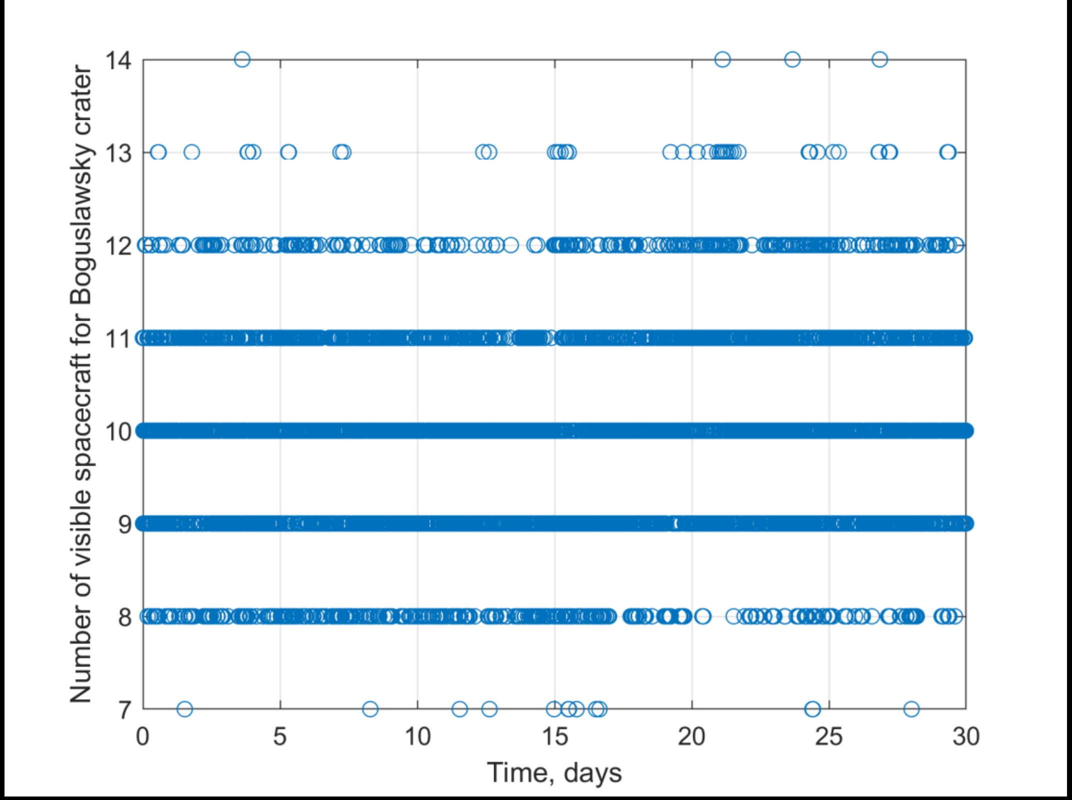
$$N_p = \left\lceil \frac{180}{2\varphi} \right\rceil = 6$$

with minimum number of satellites in each plane

$$N_s = \left\lceil \frac{360}{2\varphi} \right\rceil = 11$$



Navigation constellation in near-polar quasi-frozen orbits



Number of satellites visible from Boguslawsky crater among 180 satellites of near-polar LLO constellation (15 planes, 12 satellites in each)

>300 satellites in a LLO constellation required to achieve an N-fold ($N \geq 4$) continuous lunar coverage

High collision risk: 540 m minimum intersatellite distance if 10 satellites are uniformly distributed in each of 30 orbital planes

However, for navigation in lunar polar regions, significantly lower number of satellites – as well as orbital planes – are enough for a reliable fix

Conclusions

- (Quasi-)frozen low and medium lunar orbits are the only viable option for placing a constellation of small satellites with no or limited resources for long-term station-keeping
- Near-polar frozen LLOs are suitable for almost continuous global coverage with a constellation of <100 CubeSat-type satellites with a low-power omnidirectional/hemispherical antenna (could be augmented with several s/c in MLOs)
- For navigation purposes in lunar polar regions, it is enough to place <150 satellites in near-polar frozen LLOs
- More details on the frozen orbit design are put in the paper

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Backup slides

Gravitational potential expansion

Central gravitational field potential: $U = \dfrac{GM}{r}$

Gravitational potential of an arbitrary celestial body:

$$U = \frac{GM}{r} - \frac{GM}{r}\left(\sum_{n=2}^{\infty} J_n \left(\frac{R_{eq}}{r}\right)^n P_n(\sin\phi) + \sum_{n=2}^{\infty}\sum_{k=1}^{n}\left(\frac{R_{eq}}{r}\right)^n P_n^{(k)}(\sin\phi)(C_{nk}\cos k\lambda + S_{nk}\sin k\lambda)\right)$$

Latitude

Longitude

Zonal harmonics

Tesseral harmonics

Effects due to second zonal harmonic J_2

Orbital elements are no longer constants: the orbital semi-major axis, inclination, and eccentricity start oscillating around their mean values with a (relative) amplitude of the order of J_2.

Out-of-plane precession:

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{R_{eq}}{p}\right)^2 n \cos i$$

$$n = \sqrt{\frac{\mu}{a^3}}$$ – mean motion

In-plane precession:

$$\dot{\omega} = \frac{3}{4} J_2 \left(\frac{R_{eq}}{p}\right)^2 n \left(5\cos^2 i - 1\right)$$