FORMATION FLIGHT RELATIVE MOTION CONTROL USING SOLAR SAIL

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In paper the scheme of simultaneous relative motion and attitude control via solar radiation pressure is suggested. The control aim is to stabilize given closed relative orbits. The principle idea is to use special materials for solar sail that are able to change its optical properties. It is considered that solar sail is divided into a number of cells. Each of them can be absolutely black, i.e. it absorbs completely the solar radiation, or absolutely specular (white), i.e. it reflects all solar radiation. The necessary control force is developed by varying the average reflectivity of solar sail, and the control torque is achieved by the appropriate pattern of black and white cells

INTRODUCTION

Utilization of a group of satellites, for example formation flight, brings new possibilities in space missions. In addition, group of satellites is more reliable because even if one satellite fails, others can continue their operation.

The main problem of formation flying utilization is the deployment and maintenance of the particular group configuration. The simplest solution for this problem is to use thrusters that are installed onboard all or several satellites. On the other hand, thrusters require propellant, which can greatly affect the satellite lifetime or the payload mass. To overcome this problem environmental forces for formation flying motion control can be used [1]. This approach can be applied relatively easily by installing a special high area-to-mass ratio device such as a flat sail. There are two forces that can be used: aerodynamic drag [2–6] and solar radiation pressure (SRP) [7–11]. The principal idea here is to use a difference in environmental forces acting on each satellite in formation. This difference usually appears when a sail rotates but the effective size variation is also considered in literature [12].

In paper the case when both attitude and relative motion are controlled via solar sail with variable utilization. It is considered that sail is divided into cells which can either absorb all solar radiation or fully reflect it.

PROBLEM STATEMENT AND REFERENCE FRAMES

Deployment and maintenance of required relative orbit of two satellites is considered. It is assumed that each satellite has solar sail. The initial orbit of one satellite (leader) is circular. Second satellite (follower) is moving along the orbit which is close to the first one. Satellites move under the solar radiation pressure and J_2 perturbations.

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In paper the following reference frames are used:

- O_1XYZ is the inertial frame (IF) with the origin in the Earth centre of mass, O_1Z is perpendicular to the equator plane, O_1X is directed to the vernal equinox;
- $Oxyz$ is the orbital frame (OF), its origin in the leader satellite centre of mass, Oz directed along its radius vector, Oy is perpendicular to the orbit plane;
- $O\xi\eta\zeta$ is the body-fixed frame (BF), its axes are the principal axes of inertia (it is also assumed that $O\zeta$ is perpendicular to the sail plain);
- $Ox_sy_sz_s$ is the solar frame (SF), Oz_s is directed to the Sun, Oy_s is perpendicular to the ecliptic plane.

Transition between IF and OF is performed by the following matrix

$$\mathbf{S} = \begin{pmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3\end{pmatrix},\ \mathbf{e}_3 = \frac{\mathbf{r}}{r},\ \mathbf{e}_2 = \frac{\mathbf{r}\times\mathbf{v}}{|\mathbf{r}\times\mathbf{v}|},\ \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_3,$$

where $\mathbf{r}$ is the radius vector and $\mathbf{v}$ is the velocity of the leader satellite. Transition between IF and SF is determined by the

$$\mathbf{S}_{\text{sun}} = \begin{pmatrix}\mathbf{l}_1 & \mathbf{l}_2 & \mathbf{l}_3\end{pmatrix},\ \mathbf{l}_3 = \begin{pmatrix}\cos\lambda \\ \sin\lambda\cos\varepsilon \\ \sin\lambda\sin\varepsilon\end{pmatrix},\ \mathbf{l}_2 = \begin{pmatrix}0 \\ -\sin\varepsilon \\ \cos\varepsilon\end{pmatrix},\ \mathbf{l}_1 = \mathbf{l}_2 \times \mathbf{l}_3.$$

λ is the ecliptic longitude, ε is the obliquity of the ecliptic.

PROBLEM STATEMENT AND REFERENCE FRAMES

There are three types of motion equations that are used in this paper.

Orbital dynamics

Orbital dynamics is described by the following vector equation

$$\ddot{\mathbf{r}} = -\mu_E \frac{\mathbf{r}}{r^3} + \mathbf{g},$$

where μ_E is the Earth gravity constant and $\mathbf{g}$ is the result vector of the external disturbances. As it was mentioned earlier, only the effects of J_2 and SRP force are taken into account. The first has a form of

$$\mathbf{f}_{J_2} = \frac{3J_2\mu_E R_\oplus^2}{2r^4}\begin{pmatrix}3\sin^2 i \sin^2 u - 1 \\ -\sin^2 i \sin 2u \\ -\sin 2u \sin u\end{pmatrix}.$$

Here $J_2 = 1.082\times10^{-3}$, $R_\oplus$ is the mean Earth radius, i is the orbit inclination and u is the argument of latitude. SRP force on the elemental area can be written as follows

$$\begin{aligned} d\mathbf{F}_s = &-\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\times \\ &\times\left((1-\alpha)\mathbf{r}_s + 2\alpha\mu(\mathbf{r}_s,\mathbf{n})\mathbf{n} + \alpha(1-\mu)\left(\mathbf{r}_s + \frac{2}{3}\mathbf{n}\right)\right)dS, \end{aligned}$$

where $\Phi_0 = 1357\ W/m^2$ is the solar flux constant, $\mathbf{r}_s$ is the unit vector from the Sun to the satellite, $\mathbf{n}$ is the solar sail normal (SSN), α is the reflection coefficient, μ is the specularity coefficient. Further the case of $\mu = 1$ is considered, so

$$d\mathbf{F}_s = -\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\big((1-\alpha)\mathbf{r}_s + 2\alpha(\mathbf{r}_s,\mathbf{n})\mathbf{n}\big)dS .$$

Due to the α from point to point variation the total SRP force becomes

$$\mathbf{F}_s = -\frac{\Phi_0}{c}(\mathbf{r}_s,\mathbf{n})\Big(\Big(S - \int \alpha dS\Big)\mathbf{r}_s + 2(\mathbf{r}_s,\mathbf{n})\mathbf{n}\int \alpha dS\Big).$$

If denote $f = \dfrac{\int \alpha dS}{S}$ $(0 \le f \le 1)$ and $A = -\dfrac{\Phi_0 S}{c}$, then

$$\mathbf{F}_s = A(\mathbf{r}_s,\mathbf{n})\big((1-f)\mathbf{r}_s + 2f(\mathbf{r}_s,\mathbf{n})\mathbf{n}\big).$$

These equations are written for both satellites and are used in numerical simulation.

Angular dynamics

Angular dynamics is described in the BF by the Euler equations

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega} = \mathbf{M}_{\mathrm{control}} + \mathbf{M}_{grav}, \tag{1}$$

where $\mathbf{J}$ is the satellite inertia tensor, $\boldsymbol{\omega}$ is the angular velocity, $\mathbf{M}_{\mathrm{control}}$ is the control torque and $\mathbf{M}_{\mathrm{grav}} = 3\dfrac{\mu_E}{r^5}\mathbf{r}\times\mathbf{J}\mathbf{r}$ is the gravity gradient torque.

Attitude kinematics is defined by the quaternion $\Lambda = (\lambda_0, \boldsymbol{\lambda})$, $\lambda_0^2 + \boldsymbol{\lambda}^2 = 1$. Corresponding equations are the following

$$\begin{aligned} \dot{\lambda}_0 &= -0.5(\boldsymbol{\lambda}, \boldsymbol{\omega}), \\ \dot{\boldsymbol{\lambda}} &= 0.5(\lambda_0\boldsymbol{\omega} + \boldsymbol{\lambda}\times\boldsymbol{\omega}). \end{aligned}$$

These equations are used for the numerical simulation and control torque synthesis.

Relative motion dynamics

The control synthesis is based on the Hill-Clohessy-Wiltshire equations written in the curvilinear parameters $(a_0\theta_r, a_0\varphi_r, \rho)$ (see Figure 1). Since it is assumed that the leader satellite moves along circular orbit while the relative orbit is small with respect to the size of the orbit, these parameters more preferable that the regular one. Motion equations in the OF can be written as follows

$$\begin{aligned} &(a_0\ddot{\theta}_r) + 2\omega(a_0\dot{\varphi}_r) = 0, \\ &(a_0\ddot{\theta}_r) + \omega^2(a_0\theta_r) = 0, \\ &\ddot{\rho} - 2\omega\dot{\rho} - 3\omega^2(a_0\varphi_r) = 0. \end{aligned} \tag{2}$$

where ω is the orbital angular velocity of the leader satellite, $\rho = |\mathbf{r}_2| - |\mathbf{r}_1|$, $a_0 = |\mathbf{r}_1|$. Index “1” corresponds to the leader satellite and “2” to the follower.

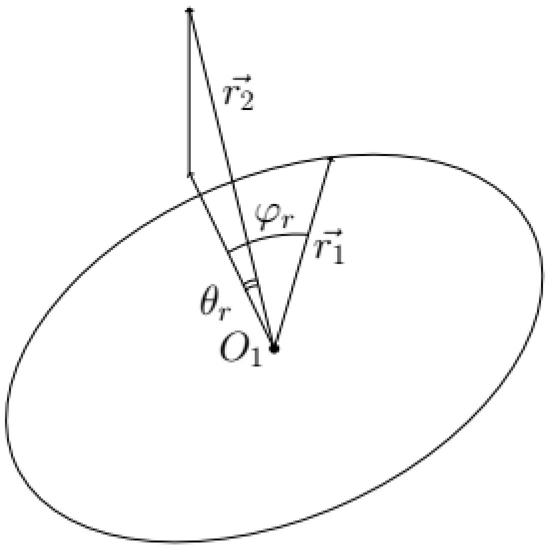

Figure 1 Curvilinear parameters

If control is taken into account Eq.(2) can be rewritten in the following form

$$\left(a_0\ddot{\theta}_r\right)+2\omega\left(a_0\dot{\varphi}_r\right)=u_x,$$

$$\left(a_0\ddot{\theta}_r\right)+\omega^2\left(a_0\theta_r\right)=u_y,$$

$$\ddot{\rho}-2\omega\dot{\rho}-3\omega^2\left(a_0\varphi_r\right)=u_z.$$

Here u_x, u_y, u_z are the components of control vector $\mathbf{u}=\dfrac{\mathbf{F}_{s,2}-\mathbf{F}_{s,1}}{m}$.

Solution of (2) is

$$a_0\theta_r=-3C_1\omega t+2C_2\cos\omega t-2C_3\sin\omega t+C_4;$$
$$a_0\varphi_r=C_5\cos\omega t+C_6\sin\omega t;$$
$$\rho=2C_1+C_2\sin\omega t+C_3\cos\omega t.$$

One can introduce new variables based on this solution.

$$a_0\theta_r=2B_2\cos\psi_1+B_3,$$
$$\rho=B_2\sin\psi_1+2B_1,$$
$$\left(a_0\dot{\theta}_r\right)=-2B_2\omega\sin\psi_1-3B_1\omega,$$
$$\dot{\rho}=B_2\omega\cos\psi_1,$$
$$a_0\varphi_r=B_4\cos\psi_2,$$
$$\left(a_0\dot{\varphi}_r\right)=-B_4\omega\sin\psi_2.$$

The equations that corresponds to these variables have form

$$
\begin{aligned}
\dot{B}_1 &= \frac{1}{\omega} u_x, \\
\dot{B}_3 &= -3B_1\omega - \frac{2}{\omega} u_z, \\
\dot{B}_2 &= \frac{1}{\omega}\left(u_z \cos\psi_1 - 2u_x \sin\psi_1\right), \\
\dot{\psi}_1 &= \omega - \frac{1}{B_2\omega}\left(u_z \sin\psi_1 + 2u_x \cos\psi_1\right), \\
\dot{B}_4 &= -\frac{1}{\omega} u_y \sin\psi_2, \\
\dot{\psi}_2 &= \omega - \frac{1}{\omega B_4} u_y \cos\psi_2.
\end{aligned}
\tag{3}
$$

It should be noted that B_1 corresponds to the drift velocity of the follower satellite along axis Ox of the OF, B_2 is the size of the in-plane ellipse, B_3 is the shift of this ellipse along Ox, finally, B_4 is the out-of-plane motion amplitude. The control purpose is to achieve the required B_i, $i = 1,2,3,4$ parameter i.e. to have required relative orbit.

CONTROL SYNTHESIS

The general control synthesis scheme is presented in Fig. 2.

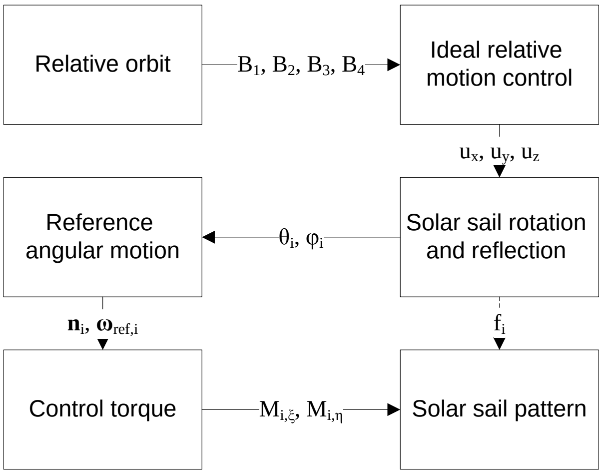

Figure 2. Control synthesis scheme

The control synthesis scheme is presented in Figure 2. First of all, the ideal control that provides required relative motion is found. Then corresponding integral reflection coefficient f_i and angles of SSN θ_i, φ_i are determined. Normal directions define the reference angular motion of each satellite. After that the control torque $\mathbf{M}_{i,\, control}$ (in $O\xi\eta$ plane) is calculated. Finally, $\mathbf{M}_{i,\, control}$ and f_i determine the solar sail reflection pattern. Further in this section each step is discussed.

Relative motion control

The purpose of the control is to deploy and maintain the required relative orbit. This orbit is defined by the B_i ($i = 1,2,3,4$). In paper the following relative orbit is considered

$B_1 = 0$, $B_2 = B_0$, $B_3 = 0$, $B_4 = 0$.

This means that the centre of the orbit is the origin of the OF and its shape is the ellipse with major and minor semi-axes $2B_0$ and B_0 respectively. The relative orbit stabilization is performed by two stages: firstly $B_1 = 0$ and $B_3 = 0$ are provided then $B_2 = B_0$ is achieved. The out-of-plane motion control is separated, so $B_4 = 0$ can be guaranteed independently.

On the first stage the following Lyapunov control function (LCF) is used

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^2 .$$

Its time derivative in accordance with (3) is

$$\dot{V} = B_1 \dot{B}_1 + B_3 \dot{B}_3 = \frac{1}{\omega} B_1 u_x + B_3 \left(3B_1\omega - \frac{2}{\omega} u_z \right).$$

So the control that ensures global asymptotic stability of $B_1 = 0$ and $B_3 = 0$ is the following (Barbashin-Krassovskii theorem)

$$\begin{aligned} u_x &= -k_1 B_1,\ k_1 > 0, \\ u_z &= \frac{1}{2}\left(3B_1\omega^2 + k_2\omega B_3\right),\ k_2 > 0. \end{aligned} \tag{4}$$

It should be noted that when B_1 is large the value of required u_z can exceed the accessible control effort (in this paper this value is $10^{-6}\,N$). So one should wait until B_1 becomes small which is possible since the first equation of (3) is independent.

On the second stage the following LCF is used

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}\left(B_2 - B_0\right)^2 + \frac{1}{2}B_3^2.$$

and its time derivative

$$\dot{V} = \frac{1}{\omega}\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right)u_x + \frac{1}{\omega}\left(\left(B_2 - B_0\right)\cos\psi_1 - 2B_3\right)u_z - 3B_1B_3\omega.$$

Then the control is

$$\begin{aligned} u_x &= -k_x\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right),\ k_x > 0 \\ u_z &= -k_z\left(\left(B_2 - B_0\right)\cos\psi_1 - 2B_3\right),\ k_z > 0 \end{aligned} \tag{5}$$

Which leads to

$$\dot{V} = -\frac{1}{\omega}\left(B_1 - 2\left(B_2 - B_0\right)\sin\psi_1\right)^2 - \frac{1}{\omega}\left(\left(B_2 - B_0\right)\cos\psi_1 - 2B_3\right)^2 - 3B_1B_3\omega.$$

The last term doesn't depend on control and its sign is undefined. From the other side its value after the first stage will be smaller than the sum of the first and second terms. This is why the first stage is required.

Practically, in both cases (4) and (5) the following scheme was used

$$u = \begin{cases} -u_{\max}\, sign(a),\ a > 1 \\ -u_{\max}\, a,\ a \le 1 \end{cases}$$

Additionally, as B_1 determine the drift velocity it could be used to control the convergence speed of B_3 to zero.

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The out-of-plane motion control has a form

$$u_y = -k_y B_4 \cos\psi_2, \ k_y > 0.$$

Control u_x, u_y, u_z is an ideal one. It should be implemented through the solar sails rotation and integral reflectivity coefficients f_i.

Relative motion control implementation

Let θ be the angle between the SSN and Sun direction, φ is the angle between normal projection to the plane $Ox_s y_s$ of the SF and axis Ox_s. Then in the SF the SSN is as follows

$$\mathbf{n} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}.$$

Relative motion control force $\mathbf{u}$ in the SF

$$\begin{aligned} u_{x_s} &= 2Af_2\cos^2\theta_2\sin\theta_2\cos\varphi_2 - 2Af_1\cos^2\theta_1\sin\theta_1\cos\varphi_1, \\ u_{y_s} &= 2Af_2\cos^2\theta_2\sin\theta_2\sin\varphi_2 - 2Af_1\cos^2\theta_1\sin\theta_1\sin\varphi_1, \\ u_{z_s} &= A(1-f_2)\cos\theta_2 - A(1-f_1)\cos\theta_1 + Af_2\cos^3\theta_2 - 2Af_1\cos^3\theta_1. \end{aligned} \tag{6}$$

These equations are the non-linear equations w.r.t. f_i, φ_i θ_i. Moreover, here are only three equations for six unknown variables. This freedom is used for the maximization of the possible values u. First of all, as the SRP force is decreasing when θ_i tends to 90 degrees, it is reasonable consider the case of small θ_i, so (6) transforms to

$$\begin{aligned} u_{x_s} &= 2Af_2\theta_2\cos\varphi_2 - 2Af_1\theta_1\cos\varphi_1, \\ u_{y_s} &= 2Af_2\theta_2\sin\varphi_2 - 2Af_1\theta_1\sin\varphi_1, \\ u_{z_s} &= Af_2 - Af_1. \end{aligned} \tag{7}$$

From the last equation one can see, that only f_i define u_{z_s}. It should be noted that the maximum torque will be when $f = 0.5$ while when $f = 0$ and $f = 1$ the torque is zero. So it is reasonable to demand f_i to be as close to 0.5 as possible, so

$$(f_1 - 0.5)^2 + (f_2 - 0.5)^2 \to \min$$

with the constraint

$$f_2 - f_1 = \frac{u_{z_s}}{A},$$
$$0 < f_{\min} \le f_i \le f_{\max} < 1, \ i = 1,2.$$

The solution in the inner domain is as follows

$$\begin{aligned} f_1 &= 0.5 - \frac{u_{z_s}}{2A}, \\ f_2 &= 0.5 + \frac{u_{z_s}}{2A}. \end{aligned} \tag{8}$$

It exists when

$$2f_{\min} - 1 \le \frac{u_{z_s}}{A} \le 2f_{\max} - 1. \tag{9}$$

In the further discussion it is supposed that $f_{\min} = 0.25$ and $f_{\max} = 0.75$. So (9) rewrites as

$-0.5 \le \dfrac{u_{z_s}}{A} \le 0.5$.

When f_i is known one can find φ_i from the maximization (with θ_i fixed) of

$L = \left(f_2\theta_2 \cos\varphi_2 - f_1\theta_1 \cos\varphi_1 \right)^2 + \left(f_2\theta_2 \sin\varphi_2 - f_1\theta_1 \sin\varphi_1 \right)^2$.

It means that the result values of φ_i give the largest domain of the possible control force. This problem has two groups of solutions

$$\varphi_1 = \varphi_2,\ \theta_1\theta_2 < 0,$$

$$\varphi_1 = \varphi_2 + \pi,\ \theta_1\theta_2 > 0.$$

It should be noted that relative attitude of two satellites is the same for both solutions. So further the case $\varphi_1 = \varphi_2 = \varphi$ is considered. The first and the second equations of (7) one can rewrite as follows

$$\left(f_2\theta_2 - f_1\theta_1 \right)\cos\varphi = \frac{u_{x_s}}{2A},$$
$$\left(f_2\theta_2 - f_1\theta_1 \right)\sin\varphi = \frac{u_{y_s}}{2A}.$$

Then

$$tg\varphi = \frac{u_{y_s}}{u_{x_s}},$$

$$\left| f_2\theta_2 - f_1\theta_1 \right| = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}.$$

If $u_{x_s} \cos\varphi > 0$

$$f_2\theta_2 - f_1\theta_1 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}. \tag{10}$$

To find θ_i one can solve the following optimization problem

$$L = \theta_1^2 + \theta_2^2$$

with constraints (10) and $-\theta_{\max} \le \theta_i \le \theta_{\max}$. The solution of this problem gives the minimum possible values of θ_i that fulfill constraints. This keeps θ_i in the vicinity of zero.

The solution in the inner domain is as follows

$$\theta_1 = -\frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_1}{f_1^2 + f_2^2},$$
$$\theta_2 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_2}{f_1^2 + f_2^2}.$$

Thus, once $\mathbf{u}$ is known the attitude of SSN can be found.

Attitude control

The expression for the SRP torque for the square sail and given sun direction in the BF $\mathbf{r}_s = (\sin\theta\cos\beta \quad \sin\theta\sin\beta \quad -\cos\theta)^T$ is as follows

$$\begin{pmatrix} M_\xi \\ M_\eta \\ M_\zeta \end{pmatrix} = \frac{\Phi_0}{c}\cos\theta \begin{pmatrix} -P\cos\theta \\ -Q\cos\theta \\ \mathrm{Q}\sin\theta\sin\beta + P\sin\theta\cos\beta \end{pmatrix},$$

Where

$$P = -\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \eta\alpha d\xi d\eta\ ,\ Q = -\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \xi\alpha d\xi d\eta\ .$$

One can notice that there are only two components that can be defined independently by the optical properties variation and SSN attitude. So there can be only two independent control torques that can be produced. Taking this into account the following LCF is considered

$$V = \frac{1}{2}\left(J_\xi\omega_{\mathrm{rel},1}^2 + J_\eta\omega_{\mathrm{rel},2}^2\right) + k_{\mathrm{a}}\left(1 - \left((0 \quad 0 \quad 1)^{\mathrm{T}}, \mathbf{Bn}\right)\right). \tag{11}$$

Here J_ξ, J_η are the in-plane moments of inertia, $\omega_{rel,1}$, $\omega_{rel,1}$ are the corresponding relative angular velocity components ($\boldsymbol{\omega}_{rel} = \boldsymbol{\omega} - \boldsymbol{\omega}_{ref}$), $\boldsymbol{\omega}_{ref} = \mathbf{n}\times\dot{\mathbf{n}}$, $\mathbf{n}$ is the required SSN attitude in the IF, $k_a > 0$ and $\mathbf{B}$ is the transition matrix between the IF and the BF. The goal of the control is to guarantee asymptotic stability of the motion when the axes $O\zeta$ of the BF and $\mathbf{n}$ coincide.

Derivative of (11) is

$$\dot{V} = J_\xi\omega_{\mathrm{rel},1}\dot{\omega}_{\mathrm{rel},1} + J_\eta\omega_{\mathrm{rel},2}\dot{\omega}_{\mathrm{rel},2} - k_{\mathrm{a}}\left((0 \quad 0 \quad 1)^{\mathrm{T}}, \frac{d}{dt}(\mathbf{Bn})\right). \tag{12}$$

As there is no need to control the third component of the angular velocity it is reasonable to take relative angular velocity vector as $\boldsymbol{\omega}_{rel} = (\omega_{rel,1} \quad \omega_{rel,2} \quad 0)^T$. In this case

$$\frac{d}{dt}(\mathbf{Bn}) = -\boldsymbol{\omega}_{rel}\times\mathbf{Bn}\ .$$

And (12) one can rewrite as follows

$$\dot{V} = \boldsymbol{\omega}_{rel}^T\left(\mathbf{J}\,\dot{\boldsymbol{\omega}}_{\mathrm{rel}} + k_{\mathrm{a}}\mathbf{Bn}\times(0 \quad 0 \quad 1)^{\mathrm{T}}\right).$$

To guarantee $\dot{V} < 0$ it is sufficient if

$$\mathbf{J}\,\dot{\boldsymbol{\omega}}_{\mathrm{rel}} + k_{\mathrm{a}}\mathbf{Bn}\times(0 \quad 0 \quad 1)^{\mathrm{T}} = -k_\omega\boldsymbol{\omega}_{\mathrm{rel}}$$

and the control torque

$$\mathbf{M}_{\mathrm{control}} = -k_\omega\boldsymbol{\omega}_{\mathrm{rel}} - \mathbf{M}_{\mathrm{ext}} + \boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega} - \mathbf{J}\boldsymbol{\omega}\times\mathbf{B}\boldsymbol{\omega}_{\mathrm{ref}} + \mathbf{JB}\dot{\boldsymbol{\omega}}_{\mathrm{ref}} - k_{\mathrm{a}}\mathbf{Bn}\times(0 \quad 0 \quad 1)^{\mathrm{T}}. \tag{13}$$

Here only two first components of $\mathbf{M}_{\mathrm{control}}$ are taken. The last component is defined by the other ones.

Solar sail pattern

The final step of the control synthesis is the determination of the solar sail pattern for each satellite. This pattern must give the required integral reflection coefficients and control torques components M_ξ and M_η. In general the reachability domain is the inner area of rhombus $LMNK$ which is presented in Figure 3.

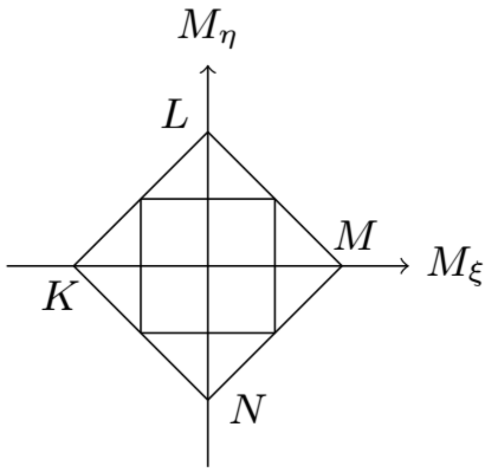

Figure 3. The torque reachability domain

One can see, that in the domain between rhombus and square the component of control torque can't be chosen independently. To overcome this the control torque components are limited by the square.

Each solar sail is considered to be divided into $n \times n$ cells. Each cell can either reflect all radiation or absorb it. The problem is to find the fully reflective cells that give the required torque components and integral reflectivity coefficient. There are various approaches. Here presented on of them which requires neither complex calculations nor large amount of memory.

Let initially each cell have the reflectivity coefficient $\alpha = 0$. The required torque will be produced by the rectangle with $\alpha = 1$. Its center is located at the edge of the square, which is described in the following way: its center coincides with sail center, and the edge length equals to a half of sail's edge length (Figure.4). The SRP torque which is produced by this rectangle is as follows

$$\mathbf{M_s} = \mathbf{r} \times \mathbf{F_s} = \begin{pmatrix} \xi \\ \eta \\ 0 \end{pmatrix} \times \begin{pmatrix} F_{s,\xi} \\ F_{s,\eta} \\ F_{s,\zeta} \end{pmatrix} = \begin{pmatrix} \eta F_{s,\zeta} \\ -\xi F_{s,\zeta} \\ \xi F_\eta - \eta F_\xi \end{pmatrix} = \begin{pmatrix} M_\xi \\ M_\eta \\ M_\zeta \end{pmatrix},$$

where $\mathbf{r} = (\xi \quad \eta \quad 0)^{\mathrm{T}}$ is the center position vector. So

$$\frac{\xi}{\eta} = -\frac{M_\eta}{M_\xi},$$

$$\xi^2 + \eta^2 = \left(\frac{a}{4}\right)^2,$$

$$\operatorname{sign}(\eta) = -\operatorname{sign}(M_\xi).$$

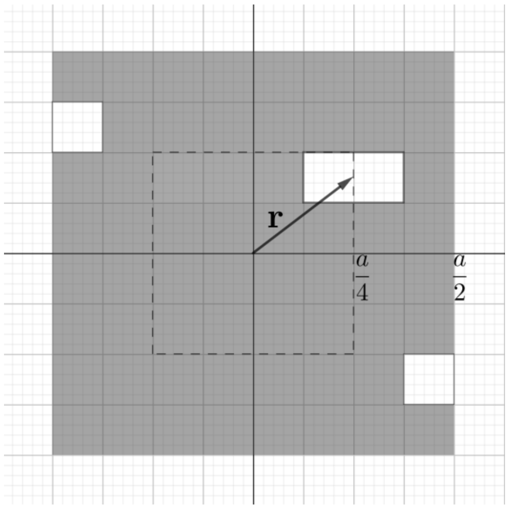

Figure 4. Solar sail pattern

The area of the rectangle is determined by third component of SRP force

$$F_{s,\zeta} = -\frac{\Phi_0}{c}\cos^2\theta S_{\mathrm{r}}\ .$$

$$M_\xi^2 + M_\eta^2 = \left(\xi^2+\eta^2\right)F_{s,\zeta}^2,$$
$$\operatorname{sign}\left(F_{s,\zeta}\right) = -1.$$

The rectangle with given center and area is not unique but it should fit cell pattern on the sail surface. First of all it should be noted that

$$S_r \leqslant \left(\frac{a}{2}\right)^2$$

since the control parameters in (13) are chosen to guarantee control torque to be in the square in Figure 3. Next the following algorithm is used

1. When $\left(\frac{a}{n}\right)^2 \le S_r$ (which means that area corresponds to at least one cell) the sides of the rectangle are taken to fulfill

$$\left(\frac{S_r n^2}{a^2} - AB\right)\xrightarrow[A,B\in\mathbb{N}]{} 0.$$

2. In case $\left(\frac{a}{n}\right)^2 \ge S_r$. The area is taken as $S_r = \left(\frac{a}{n}\right)^2$, but the center of rectangle is moving to the point $\begin{pmatrix}\frac{\xi}{k} & \frac{\eta}{k} & 0\end{pmatrix}^{\mathrm{T}}$. Here

$$k = \frac{\sqrt{M_\xi^2+M_\eta^2}}{\sqrt{\xi^2+\eta^2}\left|F_\zeta\right|}.$$

When $k \ge n$ (it means that distance from the center of sail to the center of the rectangle is less than the half-size of the cell). In this case $S_{\text{пр}} = 0$ and no torque is produced.

Once the cell pattern for torque implementation is determined the f should be realized. The number of cells with $\alpha = 1$ is determined from

$N_{\mathrm{req}} = f \cdot n^2$.

But there are already N cells with $\alpha = 1$ which realize the control torque. In general, here several options:

1. $N = N_{req}$. Which means that there is no need in cell pattern modifications and required f is achieved.

2. $N < N_{req}$. In this case the symmetric pairs with zero torque are added. If $N - N_{req}$ is an odd number then one cell is left (this be an error in force realization).

3. $N > N_{req}$. Torque producing cells give pattern with $f \le 0.25$ since no more than quarter of sail has $\alpha = 1$. On the other hand in (9) minimum force producing $f_{\min} = 0.25$. So this situation is avoided.

Thus, the control synthesis is finished. The presented scheme allows determining the solar sail pattern which give the required control force and torque with given constraints.

NUMERICAL EXAMPLE

Numerical simulation is carried out with the following parameters

Leader orbit radius $R_{orb} = 9000\ \mathrm{km}$;

Initial relative orbit: $\begin{array}{l} \mathbf{r}_{\mathrm{rel}} = \begin{pmatrix} 200 & 100 & 50 \end{pmatrix} \mathrm{m} \\ \mathbf{v}_{\mathrm{rel}} = \begin{pmatrix} 0.05 & 0.5 & 1 \end{pmatrix} \mathrm{m/s} \end{array}$;

Mass of each satellite: $m = 10\ \mathrm{kg}$;

Solar sail size: $5 \times 5\ \mathrm{m}$;

Inertia tensor: $\mathbf{J} = \mathrm{diag}\begin{pmatrix} 2.1 & 2.1 & 3.8 \end{pmatrix} \mathrm{kg} \cdot \mathrm{m}^2$;

Initial angular velocity: $\begin{array}{l} \boldsymbol{\omega}_1 = \begin{pmatrix} 0.005 & 0.003 & 0.001 \end{pmatrix} \mathrm{s}^{-1} \\ \boldsymbol{\omega}_2 = \begin{pmatrix} 0.001 & 0.003 & 0.002 \end{pmatrix} \mathrm{s}^{-1} \end{array}$;

Attitude of the SSN: $\begin{array}{l} \theta_1 = 10^\circ \\ \theta_2 = 10^\circ \end{array}$;

Control law parameters: $\begin{array}{l} k_1 = k_3 = k_4 = 20,\ k_2 = 10^{-6}\ \mathrm{s}^{-1},\ k_y = 10\ \mathrm{s}^{-1} \\ k_\omega = 0.02\ \mathrm{N} \cdot \mathrm{m} \cdot \mathrm{s},\ k_a = 10^{-4}\ \mathrm{N} \cdot \mathrm{m} \end{array}$;

Maximum allowed control force: $u_{\max} = 10^{-6}\ \mathrm{N}$;

Maximum allowed control torque: $M = 3 \cdot 10^{-5}\ \mathrm{N} \cdot \mathrm{m}$.

The results of numerical modeling are presented in figures 5-12. Figures 5-8 shows the evolution of the parameters B_i which determine the relative orbit. In figure 9 the relative orbit is shown. Figure 10 contains the integral reflectivity coefficient of the first satellite while figure 11 presents its angle between SSN and sun direction. Finally, Figure 12 shows one of the components of the control torque.

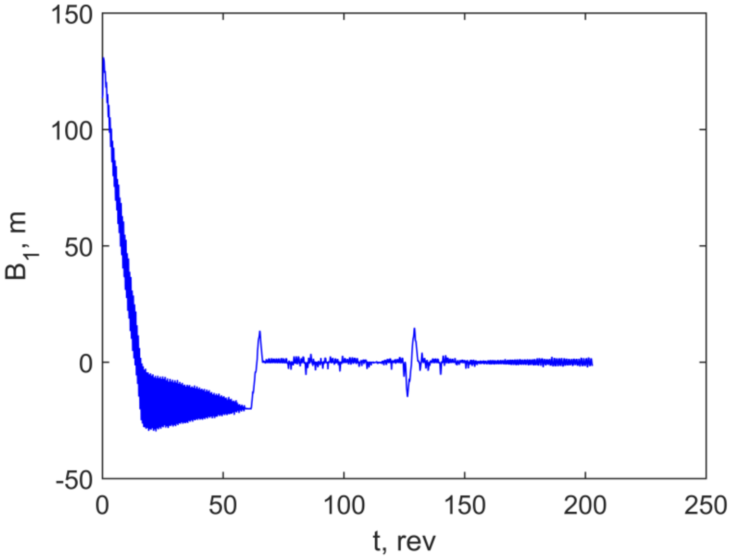

Figure 5. Parameter B1 evolution

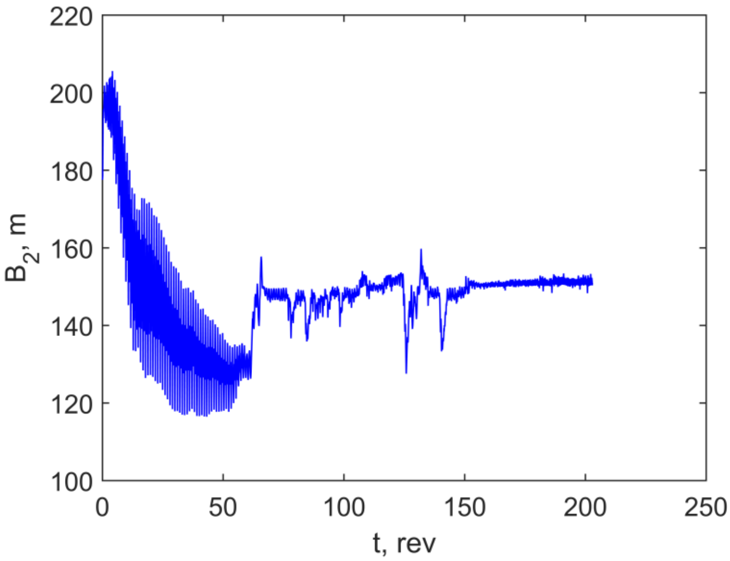

Figure 6. Parameter B2 evolution

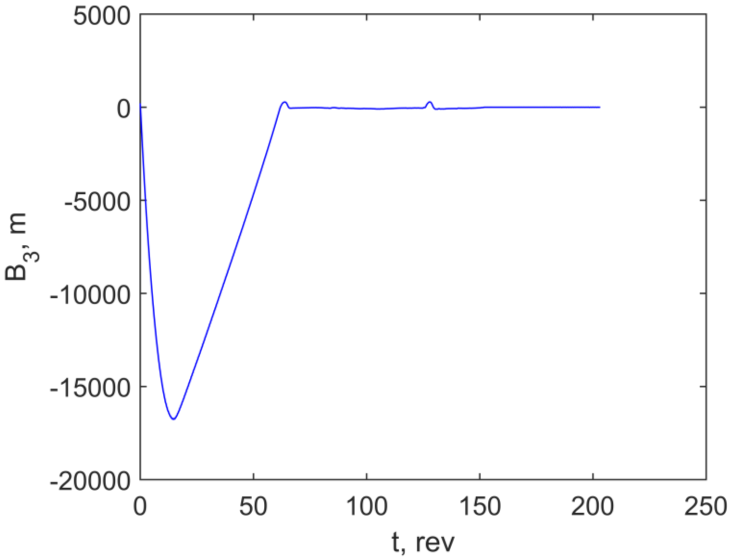

Figure 7. Parameter B3 evolution

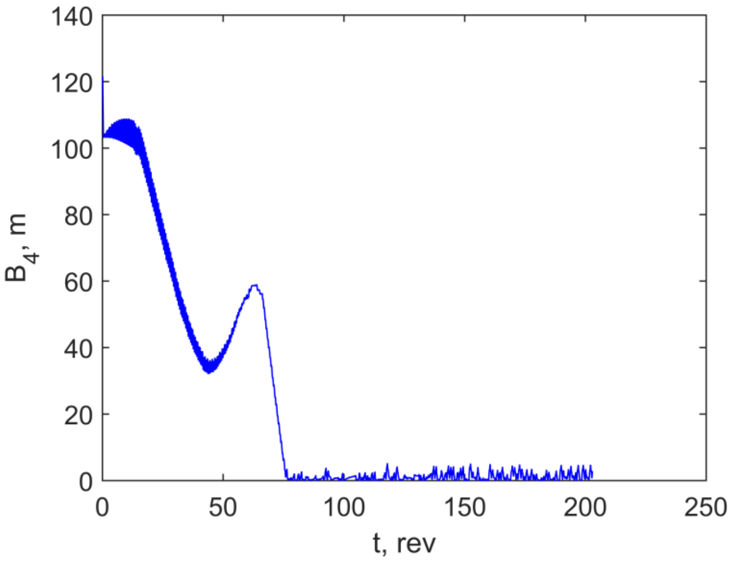

Figure 8. Parameter B4 evolution

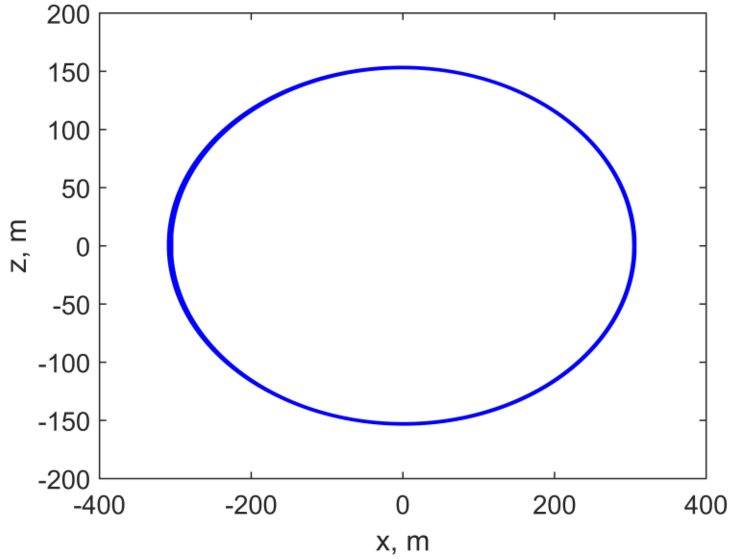

Figure 9. Relative orbit

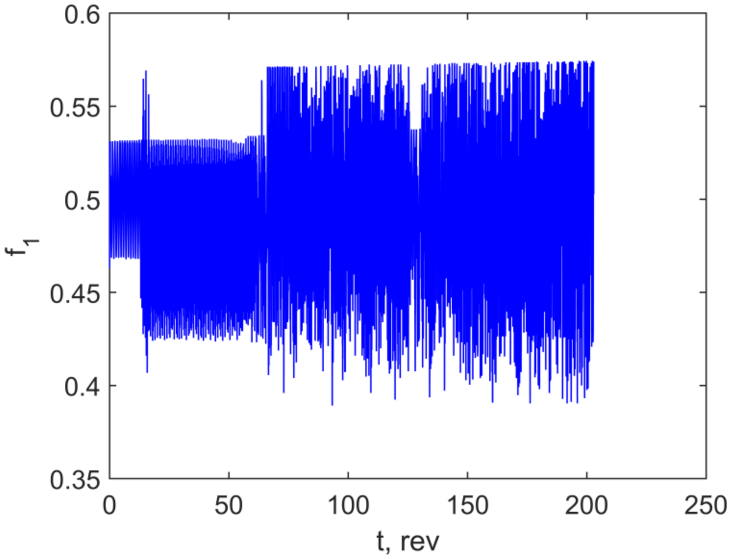

Figure 10. Integral reflectivity coefficient

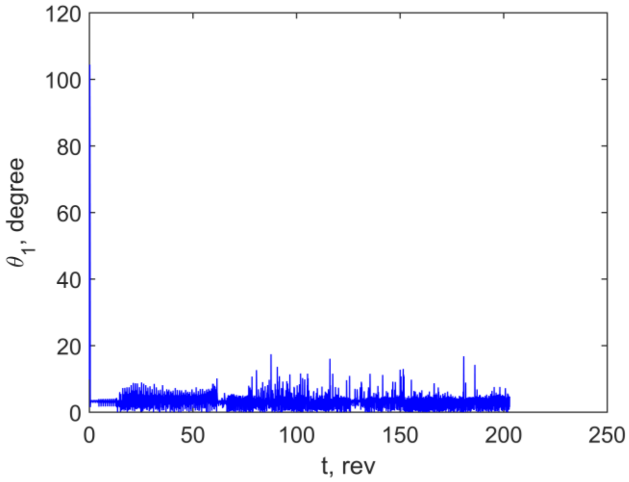

Figure 11. The angle between SSN and sun direction

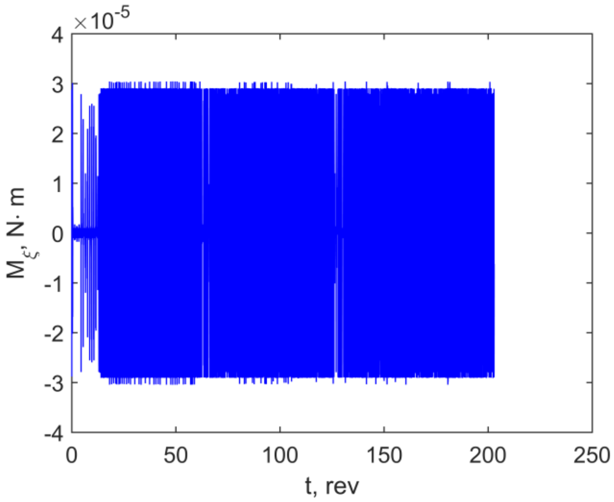

Figure 12. One of the control torque components

From Figures 5-9 one can see that control successfully solve the task and formation reach the required relative orbit. It also can be seen in Figure 5 that $B_1 \approx -20\ m$ during first ~ 50 revolutions. This allows reaching required B_3 value much faster. The Figures 10 and 11 show that the f and θ are bounded in the desired regions except the initial region for θ. This value depends on initial conditions and one can see that control together with the gravity gradient torque bring the satellite to the required attitude. Finally, from Figure 12 one can see that torque is also bounded in the desired region.

CONCLUSION

In paper the scheme of the two satellites formation flying control using the solar sail is proposed. It was shown that it is possible to control relative motion and corresponding attitude control using solar sail only.

ACKNOWLEDGMENTS

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