International Workshop on Satellite Constellation & Formation Flying
16-19 July 2019, Glasgow, Scotland

FORMATION FLIGHT RELATIVE MOTION CONTROL USING SOLAR SAIL

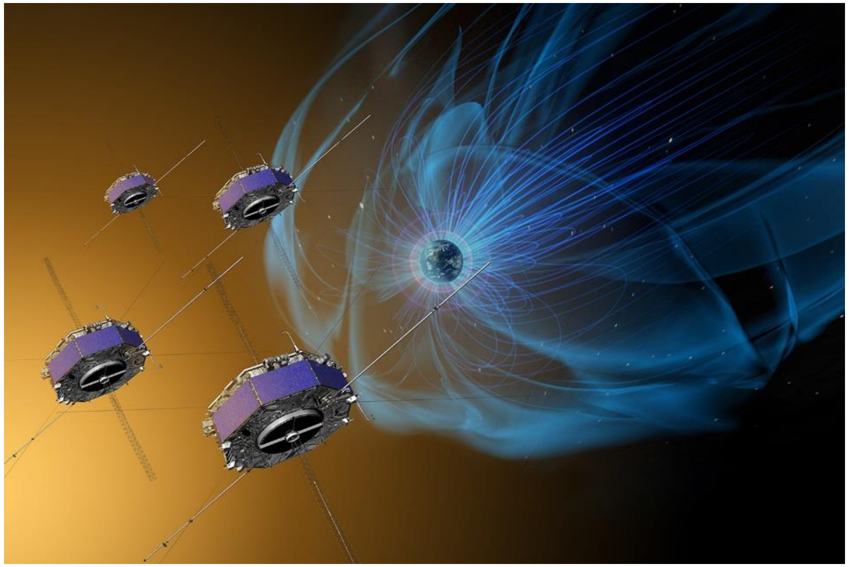
Yaroslav Mashtakov
Mikhail Ovchinnikov
Tatiana Petrova
Stepan Tkachev

Keldysh Institute of Applied Mathematics of RAS

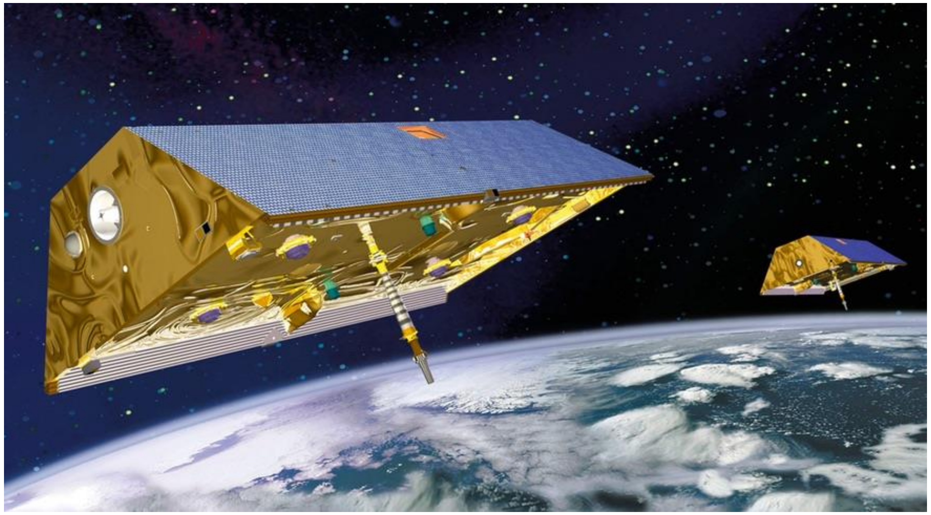
Introduction

Formation flying advantages:

- Spatial measurements
- Increased reliability
- Decreased cost



Magnetospheric MultiScale



Gravity Recovery and Climate Experiment

Swarm

Introduction

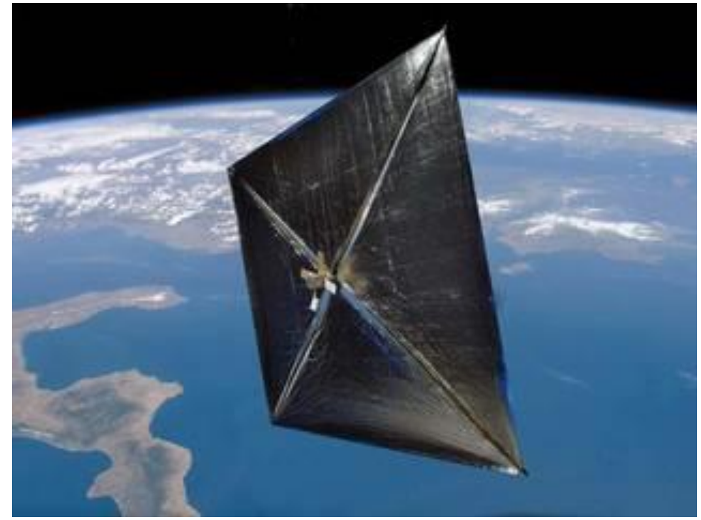
We need active relative motion control. How?

Thrusters
- ✓ Simple
- ✓ Reliable
- × Consume propellant

Atmospheric drag
- ✓ Does not consume fuel
- × Limited control
- ❑ Available only at LEO

Solar radiation pressure
- ✓ Does not consume fuel
- × Limited control
- ❑ Available at high orbits (>500-600 km)

Introduction



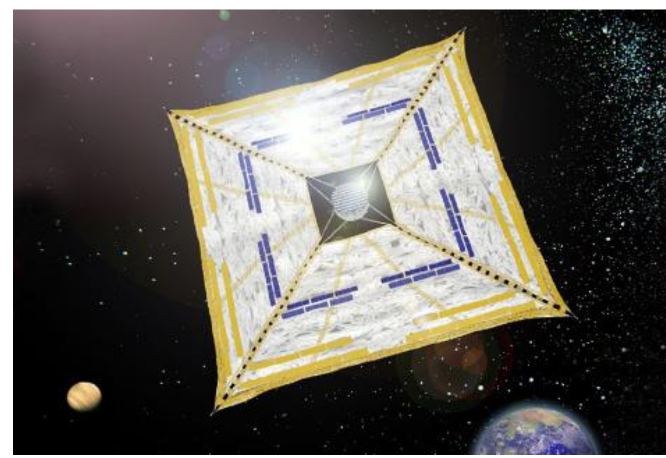
We will use SRP. How?

Conventional solar sail

- Change of the attitude → change of the force
- Additional attitude control means are necessary

Sail with variable reflective properties

- We can change reflectivity to change the force
- Does not require significant attitude changes



Smart glass

IKAROS mission

Problem statement

What we know?

- Two identical satellites at near-circular orbit with a solar sail
- Sail is divided into squares: each one can either reflect or absorb all light
- Model of motion: central geopotential + J2 + SRP for orbital motion, GG and SRP torque for attitude

What we want?

- Ensure the necessary relative motion

Relative motion description

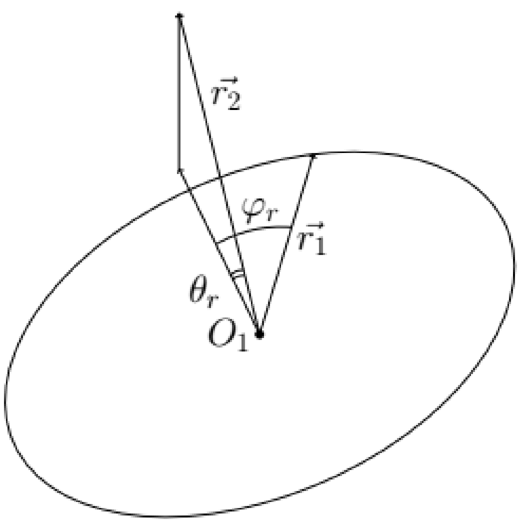
Relative curvilinear dynamics

(w/o disturbances, circular chief orbit)

$$\frac{d^2}{dt^2}\rho - 2n\frac{d}{dt}(a_0\varphi) - 3n^2\rho = 0$$

$$\frac{d^2}{dt^2}(a_0\varphi) + 2n\frac{d}{dt}\rho = 0$$

$$\frac{d^2}{dt^2}(a_0\theta) + n^2(a_0\theta) = 0$$



Its solution (similar to HCW)

$$\rho = \boxed{A}\cos(nt + \boxed{\gamma}) + 2\boxed{C},$$

$$a_0\varphi = -2A\sin(nt+\gamma) - 3Cnt + \boxed{G}$$

$$a_0\theta = \boxed{B}\sin(nt + \boxed{\delta})$$

$\boxed{}$ – first integrals of motion

We will use them (partially) as new variables

Osculating coordinates

New variables:

$$\rho = A\sin\psi + 2C,$$
$$\frac{d}{dt}\rho = An\cos\psi,$$
$$a_0\theta = 2A\cos\psi + D,$$
$$\frac{d}{dt}(a_0\theta) = -2An\sin\psi - 3C\omega,$$
$$a_0\varphi = B\sin\lambda,$$
$$\frac{d}{dt}(a_0\varphi_0) = Bn\cos\lambda.$$

After some mathematics system is described by

$$\dot{A} = \frac{1}{n}\Big((u_x + g_x)\cos\psi - 2(u_y + g_y)\sin\psi\Big),$$
$$\dot{B} = \frac{1}{n}(u_z + g_z)\cos\lambda,$$
$$\dot{C} = \frac{1}{n}(u_y + g_y),$$
$$\dot{D} = -3Cn - \frac{2}{n}(u_x + g_x),$$
$$\dot{\lambda} = n - \frac{1}{nB}(u_z + g_z)\sin\lambda.$$
$$\dot{\psi} = n - \frac{1}{An}\Big((u_x + g_x)\sin\psi + 2(u_y + g_y)\cos\psi\Big).$$

$\mathbf{u}, \mathbf{g}$ are control and disturbances (J2, SRP, nonlinearities etc.)

What we have to do?

Reference motion: ellipse that lies in orbital plane with a given semiaxis and zero shift

$$A = A_0, \quad B = 0, \quad C = 0, \quad D = 0$$

In-plane amplitude

out-of-plane amplitude

drift

shift

How?

Ideal controller → Attitude and integral reflectivity calculation → Necessary torque calculation → Sail pattern

Lyapunov approach

In-plane motion stabilization

$$V = \frac{1}{2}C^2 + \frac{1}{2}D^2$$

$$\dot{V} = C\dot{C} + D\dot{D} = \frac{1}{n}Cu_y + D\left(3Cn - \frac{2}{n}u_x\right)$$

$$u_y = -k_1 C,\ k_1 > 0$$

$$u_x = \frac{1}{2}\left(3Cn^2 + k_2 nD\right),\ k_2 > 0$$

In-plane amplitude

$$V = \frac{1}{2}C^2 + \frac{1}{2}D^2 + \frac{1}{2}\left(A - A_0\right)^2$$

$$\dot{V} = \frac{1}{n}\left(C - 2\left(A - B_0\right)\sin\psi\right)u_y +$$

$$+\frac{1}{n}\left(-2D + \left(A - A_0\right)\cos\psi\right)u_x - 3CD\omega$$

$u_{\max} > \left|3CD\omega^2\right|$ stability condition

$$u_y = -k_3\left(C - 2\left(A - A_0\right)\sin\psi\right),\ k_3 > 0$$

$$u_x = -k_4\left(-2D + \left(A_2 - A_0\right)\cos\psi\right),\ k_4 > 0$$

Out-of-plane stabilization

$$u_z = -k_z B\cos\lambda,\ k_z > 0$$

Sail attitude and reflection

Solar sail normal

$$\mathbf{n} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

Control for small θ_i

$$u_{x_s} = 2Af_2\theta_2\cos\varphi_2 - 2Af_1\theta_1\cos\varphi_1,$$
$$u_{y_s} = 2Af_2\theta_2\sin\varphi_2 - 2Af_1\theta_1\sin\varphi_1,$$
$$u_{z_s} = Af_2 - Af_1.$$

---

f=0 and f=1 zero torque
f=0.5 maximum torque

$$\left(f_1 - 0.5\right)^2 + \left(f_2 - 0.5\right)^2 \to \min$$
$$f_2 - f_1 = \frac{u_{z_s}}{A},$$
$$0 < f_{\min} \le f_i \le f_{\max} < 1,\ i = 1, 2.$$

$$f_1 = 0.5 - \frac{u_{z_s}}{2A} \qquad f_{\min} = 0.25,\ f_{\max} = 0.75$$
$$f_2 = 0.5 + \frac{u_{z_s}}{2A} \qquad -0.5 \le \frac{u_{z_s}}{A} \le 0.5$$

Maximum u_x and u_y

$$L = \left(f_2\theta_2\cos\varphi_2 - f_1\theta_1\cos\varphi_1\right)^2 + \\ + \left(f_2\theta_2\sin\varphi_2 - f_1\theta_1\sin\varphi_1\right)^2.$$

$$\varphi_1 = \varphi_2,\ \theta_1\theta_2 < 0$$
$$\varphi_1 = \varphi_2 + \pi,\ \theta_1\theta_2 > 0$$

Minimum θ_i

$$L = \theta_1^2 + \theta_2^2,$$
$$f_2\theta_2 - f_1\theta_1 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}$$

$$\theta_1 = -\frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_1}{f_1^2 + f_2^2},$$
$$\theta_2 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\frac{f_2}{f_1^2 + f_2^2}.$$

Attitude motion control

SRP torque

$$\begin{pmatrix} M_\xi \\ M_\eta \\ M_\zeta \end{pmatrix} = \frac{\Phi_0}{c}\cos\theta \begin{pmatrix} -P\cos\theta \\ -Q\cos\theta \\ Q\sin\theta\sin\beta + P\sin\theta\cos\beta \end{pmatrix}$$

$$P = -\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \eta\alpha\, d\xi\, d\eta \qquad Q = -\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \xi\alpha\, d\xi\, d\eta$$

Lyapunov control function

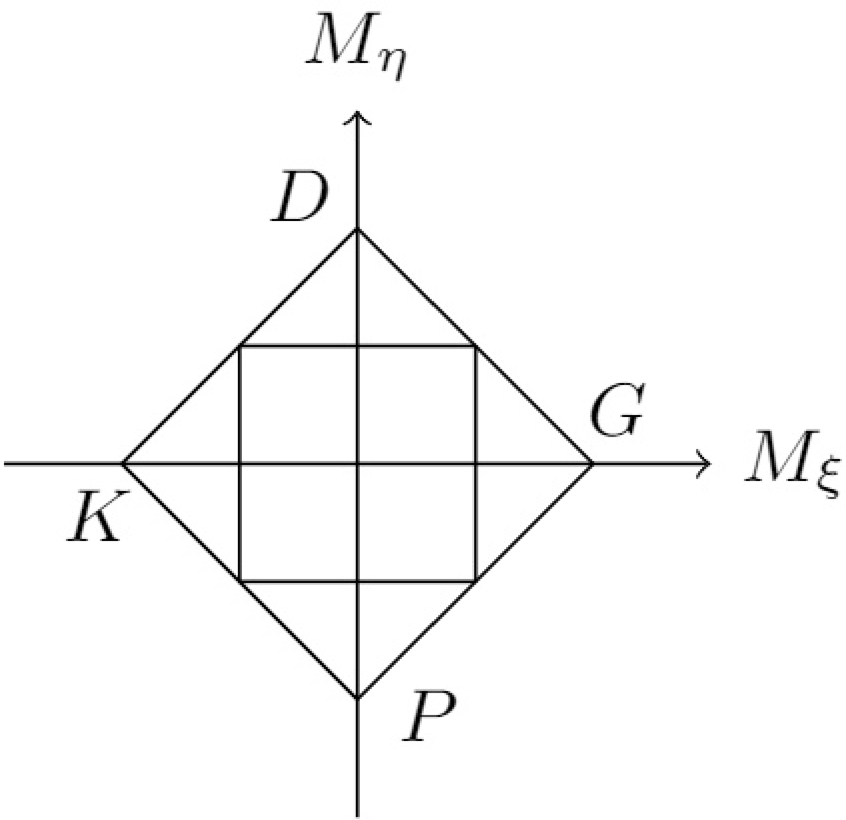
$$V = \frac{1}{2}\left(J_\xi\omega_{\mathrm{rel},1}^2 + J_\eta\omega_{\mathrm{rel},2}^2\right) + k_{\mathrm{a}}\left(1 - \left(\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}, \mathbf{Bn}\right)\right)$$

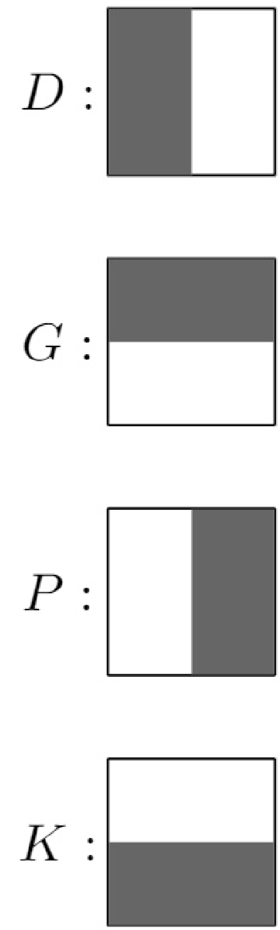
$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega} - \boldsymbol{\omega}_{ref},\ \boldsymbol{\omega}_{ref} = \mathbf{n}\times\dot{\mathbf{n}}$$

$$\dot{V} = \boldsymbol{\omega}_{rel}^T\left(\mathbf{J}\,\dot{\boldsymbol{\omega}}_{\mathrm{rel}} + k_{\mathrm{a}}\mathbf{Bn}\times\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}\right)$$

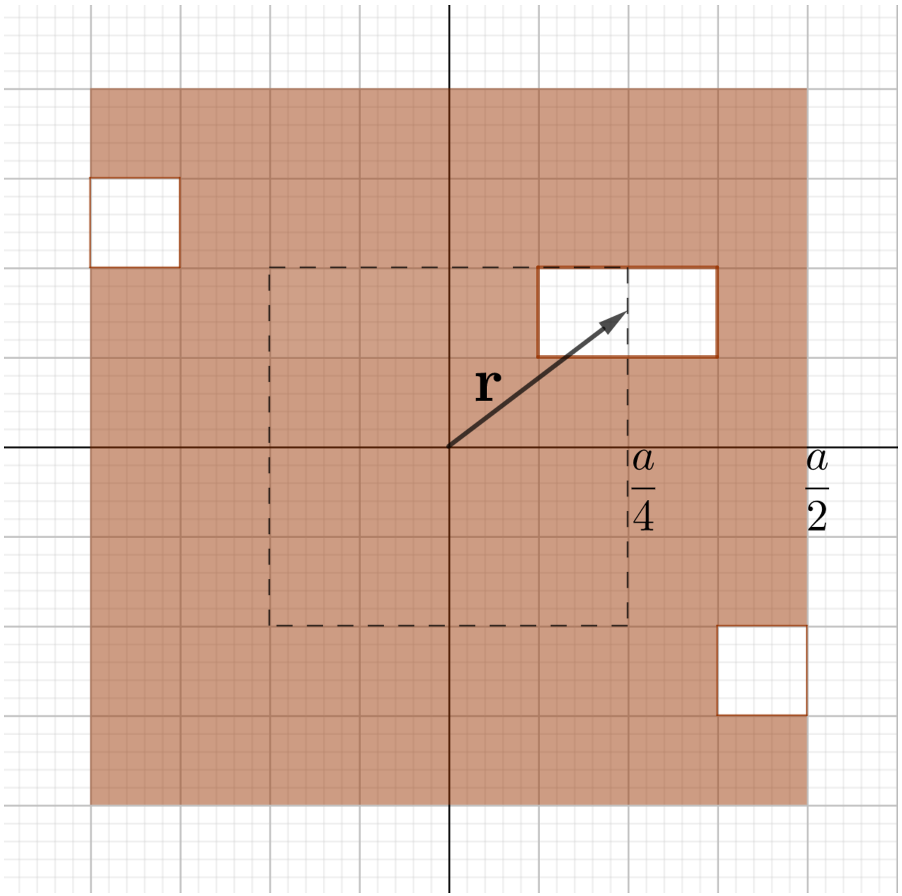
$$\mathbf{M}_{\mathrm{control}} = \left(-k_\omega\boldsymbol{\omega}_{\mathrm{rel}} - \mathbf{M}_{\mathrm{ext}} + \boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega} - \mathbf{J}\boldsymbol{\omega}\times\mathbf{B}\boldsymbol{\omega}_{\mathrm{ref}} + \mathbf{JB}\dot{\boldsymbol{\omega}}_{\mathrm{ref}} - k_{\mathrm{a}}\mathbf{Bn}\times\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}\right)_{\xi,\eta}$$

Solar sail pattern





Solar sail pattern



1. Center of rectangle

$$\frac{\xi}{\eta} = -\frac{M_\eta}{M_\xi},$$

$$\xi^2 + \eta^2 = \left(\frac{a}{4}\right)^2,$$

$$\mathrm{sign}(\eta) = -\mathrm{sign}(M_\xi).$$

2. Area

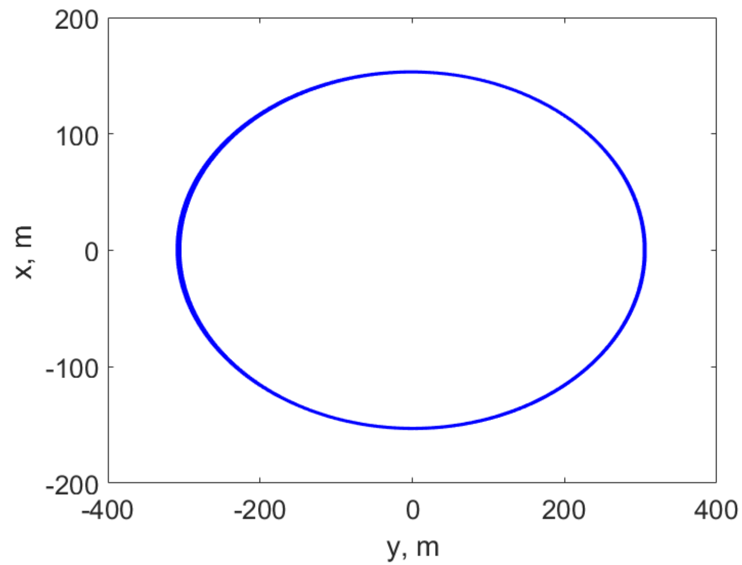
$$S_{rec} = \left|\frac{cF_{s,\zeta}}{\Phi_0 \cos^2\theta}\right|, \; F_{s,\zeta} = \sqrt{\frac{M_\xi^2 + M_\eta^2}{\xi^2 + \eta^2}} \Rightarrow N_0$$

3. Modifying pattern

$$N_{\mathrm{req}} = f \cdot n^2, \quad 0.25n^2 \le N \le 0.75n^2$$

$N_0 \le N_{req} \Rightarrow$ add cell pairs with zero torque

Simulation results



Orbit size $R_{orb} = 9000\ km$

Relative orbit $\mathbf{r}_{\mathrm{rel}} = \begin{pmatrix}200 & 100 & 50\end{pmatrix}\ \mathrm{m}$

$\mathbf{v}_{\mathrm{rel}} = \begin{pmatrix}0.05 & 0.5 & 1\end{pmatrix}\ \mathrm{m/s}$

Mass $m = 10\,kg$

Size $5\times 5\ \mathrm{m}$

Inertia tensor $\mathbf{J} = diag\begin{pmatrix}2.1 & 2.1 & 3.8\end{pmatrix}\ kg\cdot m^2$

Angular velocity $\boldsymbol{\omega}_1 = \begin{pmatrix}0.002 & 0.003 & 0.001\end{pmatrix}\ rad/s$

$\boldsymbol{\omega}_2 = \begin{pmatrix}0.001 & 0.003 & 0.002\end{pmatrix}\ rad/s$

Control parameters $k_\omega = 0.02\ N\cdot m\cdot s,\ k_a = 10^{-4}\ N\cdot m$

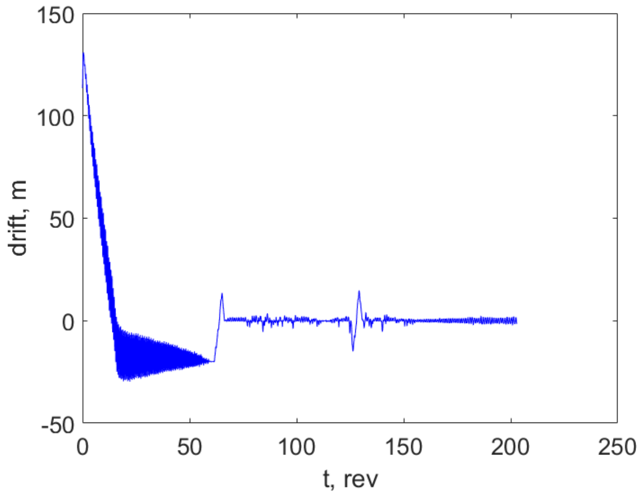
$k_1 = k_3 = k_4 = 20,\ k_2 = 10^{-6}\ s^{-1}$

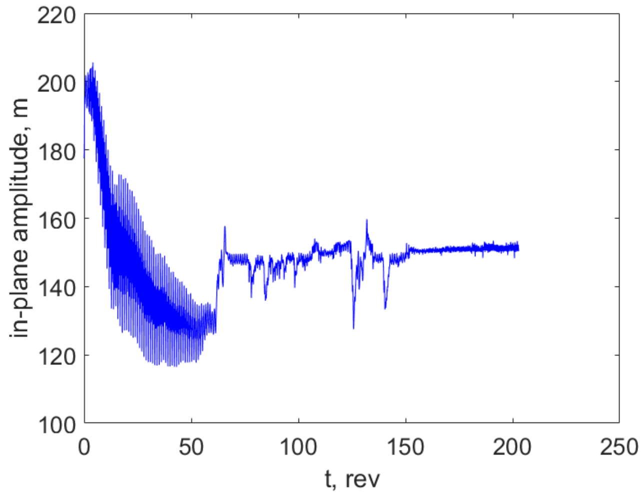
Switch condition $CD < 1\ m^2$

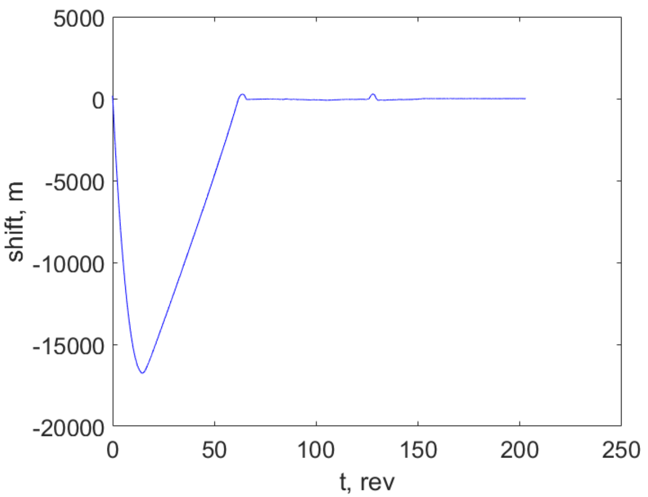
Maximum control $u_{\max} = 10^{-6}\ N$

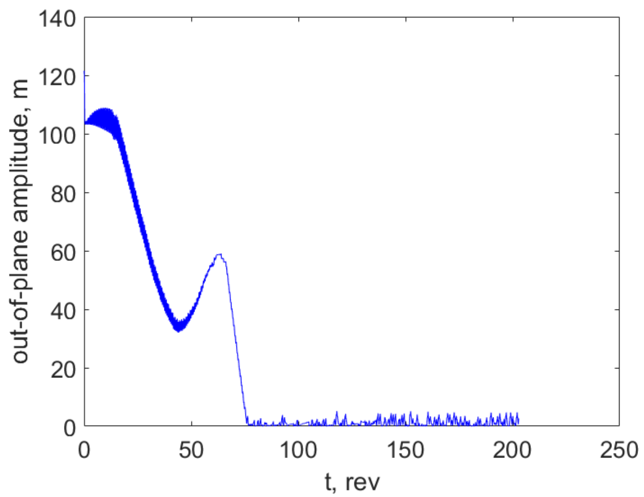
$M_{trq} = 3\times 10^{-5}\ N\cdot m$

Simulation results

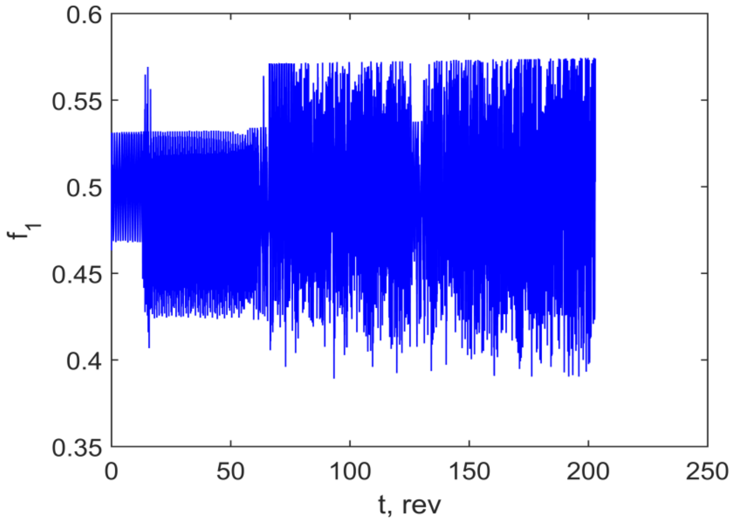


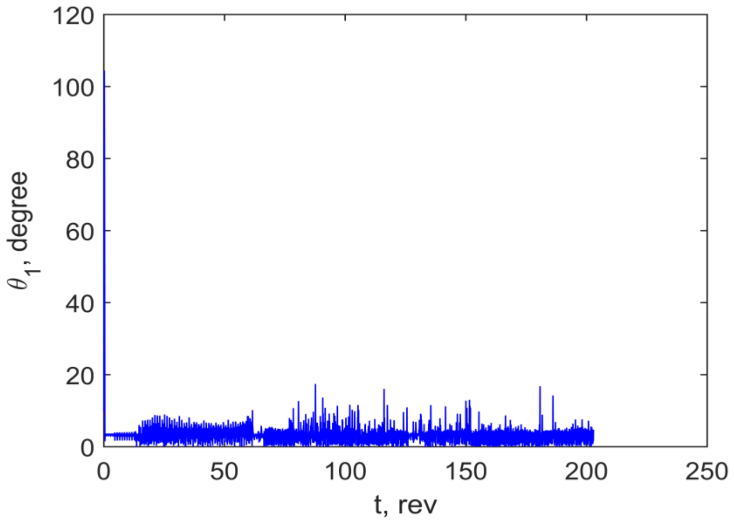


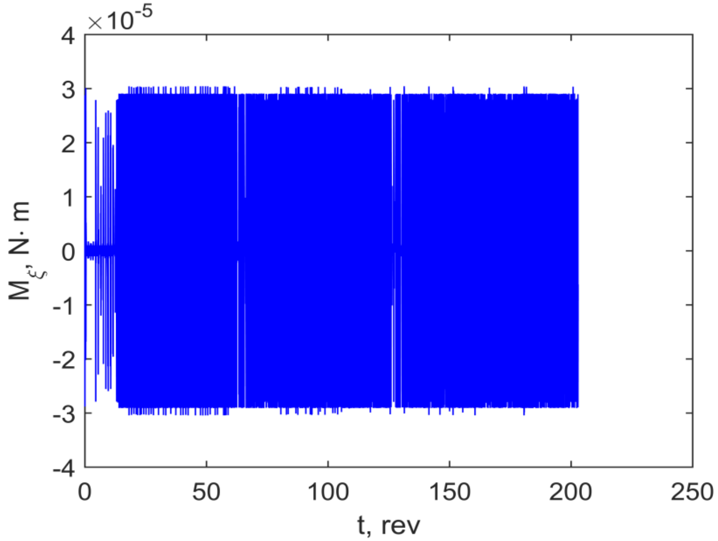




Simulation results







Conclusion

It is shown that it is possible to stabilize elliptic relative orbit by means of the SRP only

The control synthesis scheme include the control of the angular and relative motion simultaneously

The relative motion is provided by the SRP force and control source is the solar sail attitude and variable reflectivity

The desired solar sail attitude is also provided by the SRP torque

This work is supported by RFBR Grant No. 17-01-00449, 18-31-20014