MAGNETIC ATTITUDE CONTROL FOR GRACE-LIKE MISSIONS

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The problem of attitude control law synthesis for a GRACE-like missions is considered. Due to restrictions imposed by payload only magnetorquers and thrusters can be used. Lyapunov-based controller is suggested. Control coef-ficients are obtained via Floquet theory. Suggested control then is tested in precise model of motion that includes gravity gradient, aerodynamic and solar radiation pressure torques.

INTRODUCTION

Precise measurements of the Earth gravitational field might provide a lot of useful data that are used in a wide variety of scientific applications, such as glaciers melting tracking and water redistribution at the Earth surface. In order to acquire this kind of measurements in 2002 the Gravity Recovery and Climate Experiment (GRACE) mission was launched. It consists of two satellites equipped with microwave ranging system that allows us to measure the distance between the satellites with high accuracy, which further can be used to estimate the gravitational field. However, for this system to be working it is necessary to maintain the line of sight between the satellites with the accuracy of about 0.15 deg. This problem becomes more critical for the GRACE-Follow-On mission, where laser interferometer is used for the distance measurements: it requires ten times better accuracy.[2] The problem of precise attitude control is complicated by the fact that reaction wheels cannot be installed on-board the satellites because they greatly affect the measurements. Therefore, only magnetorquers and gas thrusters can be used.

In present paper we suggest the Lyapunov-based attitude control algorithm that is implemented by magnetorquers. Its efficiency is very sensitive to the control parameters. They are chosen using Floquet theory and the assumption that the Earth magnetic field is represented by direct dipole model. Since the required accuracy is rather high, magnetic-only attitude control cannot provide it, so gas thrusters are included in the control loop.

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Suggested control law is tested using the real orbital motion of GRACE satellites and programming complex X-HPS developed by Center of Applied Space Technologies and Microgravity (ZARM) and German Aerospace Center (DLR).

It should be mentioned that the satellite platform was developed by Airbus Defense & Space, and, to the best knowledge of the authors, there is no open sources on how the GRACE attitude control actually works. Here we present our vision of how attitude control *might* be implemented on-board the satellite.

COORDINATE SYSTEMS

The following right-handed Cartesian coordinate systems are used in the paper:

- $OX_1X_2X_3$ – Inertial Frame (IF). Its origin is located in the Earth center of mass, OX_3 is aligned with the Earth rotation axis, OX_1 is directed to the vernal equinox of J2000 epoch.
- $O_aY_1Y_2Y_3$ – Orbital Frame (OF). Its origin is located in the satellite's center of mass, OY_3 is anti-parallel to its radius-vector, OY_2 is anti-parallel to orbital plane normal.
- $O_ax_1x_2x_3$ – Body Frame (BF) (Fig. 1). Its axes are the satellite principal axes of inertia, O_ax_1 corresponds to the ranging device line of sight.
- $O_aZ_1Z_2Z_3$ – Reference Frame (RF) (Fig. 2). It defines how the satellite should rotate.

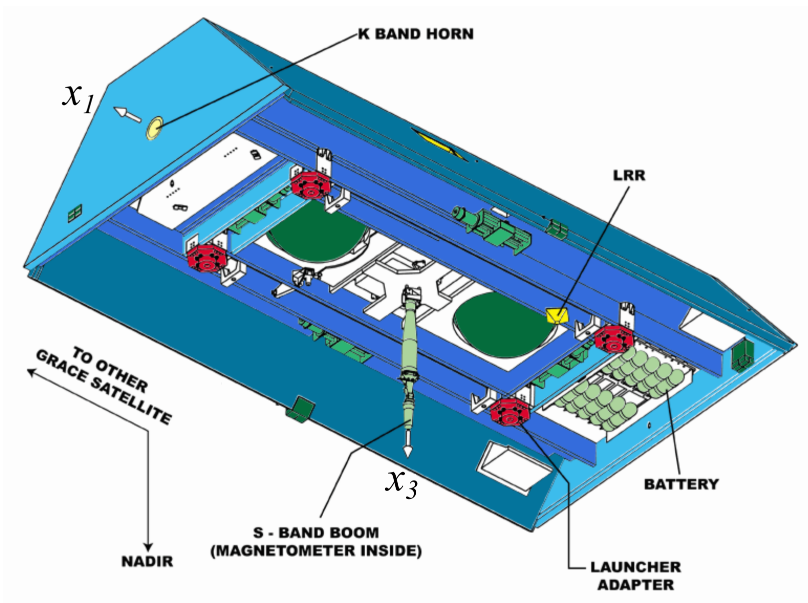


Figure 1: Body frame (credit to gfz-potsdam.de)

Let us obtain the exact expressions that describe reference motion, i.e. rotation matrix (from which the quaternion can be calculated), angular velocity and angular acceleration.

The main goal of the attitude control system is to maintain the line of sight between the two satellites. We will consider only the reference motion construction for the second satellite, for the first one it will be almost the same. Basis vectors of RF (Fig. 2) are chosen in the following way:

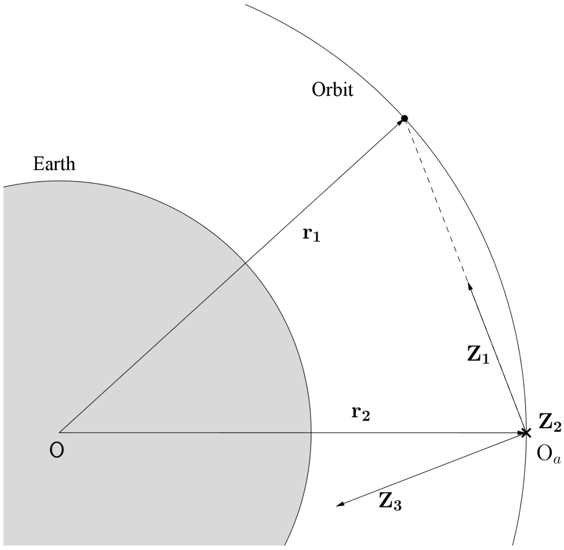


Figure 2: Reference Frame

$\mathbf{e}_1$ corresponds to the line of sight between two satellites, $\mathbf{e}_2$ is orthogonal to the line of sight and satellite radius-vector, $\mathbf{e}_3$ complements this system to the right orthogonal. Let $\mathbf{r}_1, \mathbf{r}_2, \mathbf{v}_1, \mathbf{v}_2$ be the position and velocity of the first and second satellites. Then basis vectors can be described as follows:

$$\mathbf{e}_1 = \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|}, \quad \mathbf{e}_3 = -\frac{\mathbf{r}_2 - \mathbf{e}_1(\mathbf{r}_2, \mathbf{e}_1)}{|\mathbf{r}_2 - \mathbf{e}_1(\mathbf{r}_2, \mathbf{e}_1)|}, \quad \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1.$$

Hence, the matrix from IF to RF is

$$\mathbf{D}_{ref} = \begin{pmatrix} \mathbf{e}_1^T \\ \mathbf{e}_2^T \\ \mathbf{e}_3^T \end{pmatrix}.$$

Angular velocity is obtained using Poisson equations for direction cosine matrices:

$$[\boldsymbol{\omega}_{ref}]_\times = -\dot{\mathbf{D}}_{ref}\mathbf{D}_{ref}^T.$$

Here $[\cdot]_\times$ is skew symmetric matrix of cross product:

$$[\mathbf{a}]_\times = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}.$$

Time derivative of $\mathbf{D}_{ref}$ can be found using expressions for the basis vectors, i.e.

$$\dot{\mathbf{D}}_{ref} = \begin{pmatrix} \dot{\mathbf{e}}_1^T \\ \dot{\mathbf{e}}_2^T \\ \dot{\mathbf{e}}_3^T \end{pmatrix}.$$

They, in turn, are

$$\begin{aligned} \dot{\mathbf{e}}_1 &= \frac{\mathbf{v}_1 - \mathbf{v}_2 - \mathbf{e}_1(\mathbf{v}_1 - \mathbf{v}_2, \mathbf{e}_1)}{|\mathbf{r}_1 - \mathbf{r}_2|}, \\ \dot{\mathbf{e}}_3 &= \frac{\mathbf{h} - \mathbf{e}_3(\mathbf{h}, \mathbf{e}_3)}{|\mathbf{r}_2 - \mathbf{e}_1(\mathbf{r}_2, \mathbf{e}_1)|}, \\ \dot{\mathbf{e}}_2 &= \dot{\mathbf{e}}_3 \times \mathbf{e}_1 + \mathbf{e}_3 \times \dot{\mathbf{e}}_1. \end{aligned}$$

Here $\mathbf{h} = \mathbf{v}_2 - \dot{\mathbf{e}}_1 (\mathbf{r}_2, \mathbf{e}_1) - \mathbf{e}_1 [(\mathbf{v}_2, \mathbf{e}_1) + (\mathbf{r}_2, \dot{\mathbf{e}}_1)]$. Expressions for the components of $\boldsymbol{\omega}_{ref}$ then are

$$\begin{aligned}
\omega_{ref,1} &= \frac{1}{2}\Big[(\dot{\mathbf{e}}_2, \mathbf{e}_3) - (\dot{\mathbf{e}}_3, \mathbf{e}_2) \Big], \\
\omega_{ref,2} &= \frac{1}{2}\Big[(\dot{\mathbf{e}}_3, \mathbf{e}_1) - (\dot{\mathbf{e}}_1, \mathbf{e}_3) \Big], \\
\omega_{ref,3} &= \frac{1}{2}\Big[(\dot{\mathbf{e}}_1, \mathbf{e}_2) - (\dot{\mathbf{e}}_2, \mathbf{e}_1) \Big].
\end{aligned}$$

Reference angular acceleration might be found in the same way, but expressions for it are rather bulky. Since the reference angular motion is almost constant in OF, it can be determined as zero, otherwise it can be found numerically.

As was mentioned earlier, line of ranging device sight is principal axis of inertia. In addition, the satellites move along almost the same orbit and the distance between them is about 200 km. Taking into account expressions for basis vectors of the OF

$$\mathbf{j}_3 = -\frac{\mathbf{r}_2}{r_2}, \quad \mathbf{j}_2 = -\frac{\mathbf{r}_2 \times \mathbf{v}_2}{|\mathbf{r}_2 \times \mathbf{v}_2|}, \quad \mathbf{j}_1 = \mathbf{j}_2 \times \mathbf{j}_3,$$

we can notice that reference motion almost the same as stabilization in the vicinity of unstable equilibrium w.r.t. OF (difference between these motions is shown in Fig. 3 using Euler angles, rotation sequence $2-3-1$ with angles $\alpha_2, \alpha_3, \alpha_1$ respectively). Hence, in the next Section attitude control law will be designed to stabilize the satellite in equilibrium position w.r.t. OF. However, in the simulations differences between the OF and the RF are taken into account.

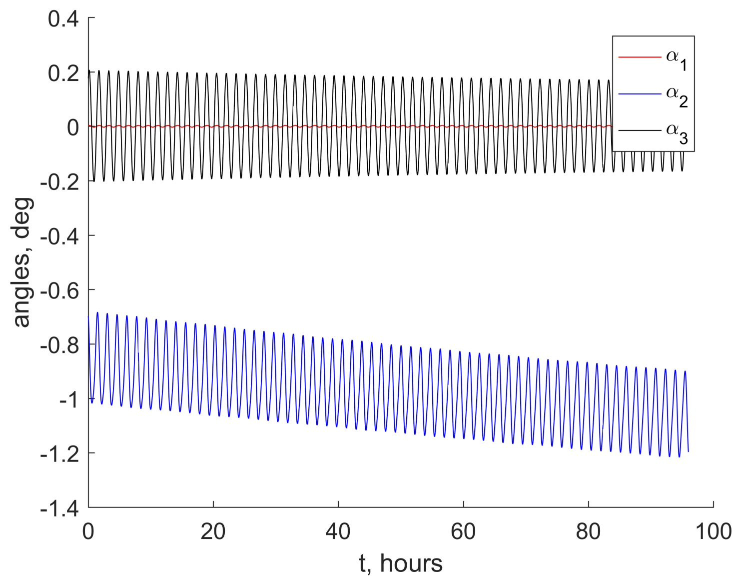


Figure 3: Difference between the Orbital and Reference Frames

ATTITUDE CONTROL

Lyapunov-based attitude control

There are plenty of algorithms that can be used for attitude control. The most interesting ones allow us to ensure the asymptotic stability of the reference motion. Hence they still will provide decent accuracy even in the presence of sufficiently small external disturbances that are not taken into account in the control loop (e.g. solar radiation pressure and aerodynamic torques which may be too complicated for on-board processing). One of such algorithms is based on the utilization of the Barbashin-Krasovskii-LaSalle principle.[4,5] Consider the equations of motion in general form

$$\begin{aligned} &\mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} = \mathbf{M}_{ctrl} + \mathbf{M}_{ext}, \\ &\dot{\mathbf{Q}} = \frac{1}{2}\mathbf{Q} \circ \boldsymbol{\omega}_{abs}. \end{aligned} \tag{1}$$

Here $\mathbf{J} = \text{diag}\,(A, B, C)$ is satellite tensor of inertia, $\boldsymbol{\omega}_{abs}$ is angular velocity w.r.t. IF, $\mathbf{M}_{ctrl}, \mathbf{M}_{ext}$ are control and external torques respectively, $\mathbf{Q} = (q_0, \mathbf{q})^T$ is the quaternion that describes transition from IF to BF, "$\circ$" is the quaternion multiplication:

$$\mathbf{Q} \circ \mathbf{N} = \begin{pmatrix} q_0 \\ \mathbf{q} \end{pmatrix} \circ \begin{pmatrix} \nu_0 \\ \boldsymbol{\nu} \end{pmatrix} = \begin{pmatrix} q_0\nu_0 - (\mathbf{q}, \boldsymbol{\nu}) \\ q_0\boldsymbol{\nu} + \nu_0\mathbf{q} + \mathbf{q} \times \boldsymbol{\nu} \end{pmatrix}$$

The reference motion is described by quaternion $\mathbf{R}$ from IF to RF and reference angular velocity $\boldsymbol{\omega}_{ref}$. In addition, for the reference motion Poisson equation

$$\dot{\mathbf{R}} = \frac{1}{2}\mathbf{R} \circ \boldsymbol{\omega}_{ref}$$

must be satisfied.

Let us chose the positive-definite Lyapunov-candidate function in the form

$$V = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_s(1 - s_0), \quad k_s = const > 0 \tag{2}$$

where $\boldsymbol{\omega}_{rel} = \boldsymbol{\omega}_{abs} - \tilde{\mathbf{S}} \circ \boldsymbol{\omega}_{ref} \circ \mathbf{S}$, $\mathbf{S} = (s_0, \mathbf{s}) = \tilde{\mathbf{R}} \circ \mathbf{Q}$ is quaternion from RF to BF, tilde denotes quaternion conjugation. In addition, $\dot{\mathbf{S}} = \frac{1}{2}\mathbf{S} \circ \boldsymbol{\omega}_{rel}$ that can be verified directly using the definition of $\mathbf{S}$. Time derivative of (2), in accordance with (1), is

$$\dot{V} = \left(\boldsymbol{\omega}_{rel}, \mathbf{J}\dot{\boldsymbol{\omega}}_{rel}\right) + k_s\left(\mathbf{s}, \boldsymbol{\omega}_{rel}\right).$$

Consider $\dot{\boldsymbol{\omega}}_{rel}$ in more details:

$$\begin{aligned} \dot{\boldsymbol{\omega}}_{rel} &= \dot{\boldsymbol{\omega}}_{abs} + \frac{1}{2}\boldsymbol{\omega}_{rel} \circ \tilde{\mathbf{S}} \circ \boldsymbol{\omega}_{ref} \circ \mathbf{S} - \frac{1}{2}\tilde{\mathbf{S}} \circ \boldsymbol{\omega}_{ref} \circ \mathbf{S} \circ \boldsymbol{\omega}_{rel} - \tilde{\mathbf{S}} \circ \dot{\boldsymbol{\omega}}_{ref} \circ \mathbf{S} \\ &= \dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{rel} \times \left(\tilde{\mathbf{S}} \circ \boldsymbol{\omega}_{ref} \circ \mathbf{S}\right) - \tilde{\mathbf{S}} \circ \dot{\boldsymbol{\omega}}_{ref} \circ \mathbf{S}. \end{aligned}$$

Let $\boldsymbol{\omega}_{ref}^{BF} = \tilde{\mathbf{S}} \circ \boldsymbol{\omega}_{ref} \circ \mathbf{S}$, i.e. it is reference angular velocity written in BF, and $\dot{\boldsymbol{\omega}}_{ref}^{BF} = \tilde{\mathbf{S}} \circ \dot{\boldsymbol{\omega}}_{ref} \circ \mathbf{S}$. Thus, time derivative of (2) can be rewritten as

$$\dot{V} = \left(\boldsymbol{\omega}_{rel}, \mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \mathbf{J}\boldsymbol{\omega}_{rel} \times \boldsymbol{\omega}_{ref}^{BF} - \mathbf{J}\dot{\boldsymbol{\omega}}_{ref}^{BF} + k_s\mathbf{s}\right).$$

In accordance with the already mentioned Barbashin-Krasovskii-LaSalle principle, in order to provide the asymptotic stability of the required motion, derivative of the Lyapunov-candidate function should be nonpositive and set of all the points where its derivative is equal to zero should not contain any whole trajectories except the required one. If

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \mathbf{J}\left[\boldsymbol{\omega}_{rel} \times \boldsymbol{\omega}_{ref}^{BF}\right] - \mathbf{J}\dot{\boldsymbol{\omega}}_{ref}^{BF} + k_s\mathbf{s} = -k_w\boldsymbol{\omega}_{rel}, k_w = const > 0,$$

then this requirement is satisfied, hence reference motion is globally asymptotically stable. The expression for the control torque in this case is

$$\mathbf{M}_{ctrl} = \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - \mathbf{M}_{ext} - \mathbf{J}\left[\boldsymbol{\omega}_{rel} \times \boldsymbol{\omega}_{ref}^{BF}\right] + \mathbf{J}\dot{\boldsymbol{\omega}}_{ref}^{BF} - k_s\mathbf{s} - k_w\boldsymbol{\omega}_{rel}. \tag{3}$$

If we consider only two last terms of the suggested control law, we obtain the standard Proportional Derivative (PD) controller. Additional terms include information about external disturbances and allow us to track even time-variable reference motion with high accuracy. Estimations of the provided accuracy can be found in.[6]

Described above attitude control law is not the single one that can be derived using Lyapunov approach. Several examples of similar control laws are given in.[7,8]

Implementation of control torques using magnetorquers

In order to control the satellite attitude magnetorquers are used. The control torque in this case is

$$\mathbf{M}_{ctrl} = \mathbf{m} \times \mathbf{B},$$

where $\mathbf{m}$ is the magnetic dipole vector, generated by three non parallel magnetorquers, $\mathbf{B}$ is the external magnetic field. It is obvious that for any time there is a direction where the control torque cannot be generated. Hence, standard Lyapunov-based control could not be applied in this case. However, there are several approaches to use magnetorquers for orbital or inertial stabilization. Usually they are based on Proportional-Derivative controller,[9–11] when dipole vector is chosen as follows

$$\mathbf{m} = \mathbf{B} \times (-k_p\mathbf{s} - k_d\boldsymbol{\omega}_{rel})$$

There are several modifications of this control law (e.g.[12] when the main goal is to decrease the satellite angular velocity).

We will utilize the same idea, but will use not only the PD part of Lyapunov control law (3), but also additional terms. In this case the dipole moment should be chosen as follows:

$$\mathbf{m} = \frac{1}{(\mathbf{B}, \mathbf{B})}\mathbf{B} \times \mathbf{M}_{id},$$

where $\mathbf{M}_{id}$ corresponds to Eq.(3), the ideal torque desired by the controller. Then the generated control torque is

$$\mathbf{M}_{gen} = \mathbf{M}_{id} - \mathbf{e}_B(\mathbf{e}_B, \mathbf{M}_{id}) = -\left[\mathbf{e}_B\right]_\times \left[\mathbf{e}_B\right]_\times \mathbf{M}_{id}, \quad \mathbf{e}_B = \frac{\mathbf{B}}{\sqrt{(\mathbf{B}, \mathbf{B})}}. \tag{4}$$

This means that $\mathbf{M}_{gen}$ is the projection of $\mathbf{M}_{id}$ into the plain perpendicular to the external magnetic field $\mathbf{B}$. Thus no wasteful magnetic dipole $\mathbf{m}$ parallel to $\mathbf{B}$ is generated.

Simulation shows that if control coefficients are chosen appropriately, the satellite can be stabilized in the vicinity of even unstable equilibrium, though the accuracy of stabilization might be unacceptable for mission requirements. There is a technique based on Floquet theory application that allow us to appropriately choose control parameters.[10,13] Further we consider it in more details.

Satellite dynamics in the vicinity of equilibrium

In this subsection the satellite dynamics is described. Let the satellite moves along near-circular orbit, hence there are several equilibriums in OF. The main goal is to stabilize the specified one.

Equations of the satellite passive angular motion, considering gravity gradient torque, are

$$\begin{aligned}
&\mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} = 3\frac{\mu_E}{r^5}\mathbf{r} \times \mathbf{J}\mathbf{r}, \\
&\dot{\mathbf{D}} = -\left[\boldsymbol{\omega}_{abs}\right]_\times \mathbf{D}.
\end{aligned} \tag{5}$$

Here $\mathbf{J} = \operatorname{diag}(A, B, C)$ is the satellite tensor of inertia, $\boldsymbol{\omega}_{abs}$ is the angular velocity w.r.t. IF, μ_E is the Earth gravitational parameter, $\mathbf{r}$ is the satellite radius-vector, $\mathbf{D}$ is the rotation matrix from IF to BF such as $\mathbf{x}_{BF} = \mathbf{D}\mathbf{x}_{IF}$.

Since the satellite has to be stabilized for the whole period of mission, we can consider the case when it does not leave vicinity of equilibrium, hence equations of motion can be linearized. It is suitable to use relative variables to describe satellite motion: $\mathbf{A}$ is the rotation matrix from OF to BF, $\boldsymbol{\omega} = \boldsymbol{\omega}_{abs} - \mathbf{A}\boldsymbol{\omega}_0$ is the relative angular velocity, $\boldsymbol{\omega}_0 = (0, -\omega_0, 0)^T$ is the orbital angular velocity in OF. In addition, to parametrize the attitude Euler angles (rotation sequence $2-3-1$ with angles $\alpha_2, \alpha_3, \alpha_1$ respectively) are used. Linearization in the vicinity of equilibrium leads to

$$\begin{aligned}
\mathbf{J}\dot{\boldsymbol{\omega}} = &\mathbf{J}\left[\boldsymbol{\omega} \times \boldsymbol{\omega}_0\right] - \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega}_0 - \boldsymbol{\omega}_0 \times \mathbf{J}\boldsymbol{\omega} \\
&+ \left[\boldsymbol{\omega}_0 \times \mathbf{J}\left(\boldsymbol{\alpha} \times \boldsymbol{\omega}_\mathbf{0}\right) + \left(\boldsymbol{\alpha} \times \boldsymbol{\omega}_\mathbf{0}\right) \times \mathbf{J}\boldsymbol{\omega}_0\right] \\
&- 3\omega_0^2\left(\mathbf{e}_1 \times \mathbf{J}\left[\boldsymbol{\alpha} \times \mathbf{e}_1\right] + \left[\boldsymbol{\alpha} \times \mathbf{e}_1\right] \times \mathbf{J}\mathbf{e}_1\right)' \\
\dot{\mathbf{A}} = &-\left[\boldsymbol{\omega}\right]_\times.
\end{aligned}$$

Here $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)^T$, $\mathbf{e}_1 = \mathbf{r}/r = (0, 0, -1)^T$ in OF. This result is obtained using the linearized expression $A \approx \mathbf{I}_3 - [\boldsymbol{\alpha}]_\times$ and omitting all the second order infinitesimals. After the mathematics, relative motion equations are

$$\begin{aligned}
\dot{\boldsymbol{\alpha}} &= \boldsymbol{\omega}, \\
\dot{\boldsymbol{\omega}} &= \begin{pmatrix} 4\omega_0^2\frac{(C-B)}{A} & 0 & 0 \\ 0 & 3\omega_0^2\frac{(C-A)}{B} & 0 \\ 0 & 0 & \omega_0^2\frac{(A-B)}{C} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \\
&+ \begin{pmatrix} 0 & 0 & \omega_0\frac{(C+A-B)}{A} \\ 0 & 0 & 0 \\ \omega_0\frac{(B-C-A)}{C} & 0 & 0 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}.
\end{aligned} \tag{6}$$

For further analysis we will use linear time-independent equations (6). It should be noticed that for some cases utilization of quaternions is more suitable. In the vicinity of equilibrium relation between quaternions and Euler angles (with the accuracy up to the second order infinitesimals) is

$$\begin{aligned}
s_0 &\approx 1, \\
\mathbf{s} &\approx \frac{1}{2}\boldsymbol{\alpha}.
\end{aligned}$$

Floquet analysis of controlled relative motion

There are different models of magnetic field that can be used for analytical and/or numerical study of satellite motion. For the analytical study we use quite simple model where the Earth magnetic field is generated by so-called direct dipole: it is located in the Earth center of mass and antiparallel to its rotation axis. This model is not very accurate, but it has one very important property: if a satellite moves along a circular orbit, the expression for magnetic field in OF is periodical. In addition, the difference between magnetic field in this model and more complicated one (e.g. IGRF) is rather small.[14] Therefore, if controller provides asymptotic stability in direct dipole model, then we can expect that in the full model it still will give a decent performance.

Expression for the induction vector of magnetic field of direct dipole in OF is

$$\mathbf{B}^{OF} = \frac{\mu_B}{r^3}\begin{pmatrix}\cos u \sin i \\ -\cos i \\ 2\sin u \sin i\end{pmatrix} \tag{7}$$

where $\mu_B = 7.812 \cdot 10^6 \text{ km}^3 \cdot \text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-1}$ is the Earth magnetic constant, r is distance between the Earth center of mass and the satellite, u is latitude argument, i is orbit inclination. Since satellite moves along circular keplerian orbit, $i = const, r = const, u = \omega_0 t + u_0$. Using this expression for magnetic field (7), implemented control torque (4) and linear equations (6) we obtain the following equations for the controlled motion:

$$\begin{aligned}&\dot{\boldsymbol{\alpha}} = \boldsymbol{\omega},\\ &\dot{\boldsymbol{\omega}} = \mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{A}_\omega \boldsymbol{\omega} + \mathbf{J}^{-1}\left[\mathbf{e}_B^{OF}\right]_\times \left[\mathbf{e}_B^{OF}\right]_\times \left(\mathbf{J}\mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{J}\mathbf{A}_\omega \boldsymbol{\omega} + k_\alpha \boldsymbol{\alpha} + k_w \boldsymbol{\omega}\right).\end{aligned} \tag{8}$$

Here $k_\alpha = \frac{1}{2}k_s$ and the following notations are used:

$$\mathbf{A}_\alpha = \begin{pmatrix} 4\omega_0^2 \frac{(C-B)}{A} & 0 & 0 \\ 0 & 3\omega_0^2 \frac{(C-A)}{B} & 0 \\ 0 & 0 & \omega_0^2 \frac{(A-B)}{C}\end{pmatrix},$$

$$\mathbf{A}_\omega = \begin{pmatrix} 0 & 0 & \omega_0 \frac{(C+A-B)}{A} \\ 0 & 0 & 0 \\ \omega_0 \frac{(B-C-A)}{C} & 0 & 0\end{pmatrix}.$$

Obtained equations are periodical with a period $T = \frac{2\pi}{\omega_0}$, where ω_0 is the orbital angular velocity. Hence, it is possible to apply Floquet theory[4,5] to study its motion.

Consider fundamental matrix $\mathbf{\Phi}$ of system (8) that was obtained using initial conditions $\mathbf{\Phi}(0) = \mathbf{I}_6$ where $\mathbf{I}_6$ is 6×6 identity matrix. Let ρ_k be the roots of the characteristic equation

$$\det\left(\mathbf{\Phi}(T) - \rho \mathbf{I}_6\right) = 0.$$

If for any k Re $(\ln \rho_k) < 0$, then the equilibrium $(\boldsymbol{\alpha}, \boldsymbol{\omega})^T = \mathbf{0}$ of the linear system is asymptotically stable. Moreover, the smaller $\max$ (Re $\ln \rho_k$), the faster system converges to equilibrium. Hence, the most appropriate control parameters can be obtained by solving the minimization problem

$$\max_k \text{Re}(\ln \rho_k) \to \min. \tag{9}$$

Two scalar parameters k_α, k_ω, can be replaced by two diagonal matrices $\mathbf{K}_\alpha, \mathbf{K}_w$ for better tuning:

$$k_\alpha \boldsymbol{\alpha} + k_w \boldsymbol{\omega} \to \mathbf{K}_\alpha \boldsymbol{\alpha} + \mathbf{K}_w \boldsymbol{\omega},$$

so there are six control parameters to be determined. It should be noticed that obtained control law will ensure asymptotic stability of the linear periodic system (8). However, using a more precise model for disturbances and non idealized relative motion, $(\boldsymbol{\alpha}, \boldsymbol{\omega})^T = \mathbf{0}$ might not be asymptotically stable. On the other hand, since the main disturbance caused by the gravity gradient (GG) torque is taken into account, and differences between the simplified model of magnetic field and the real one is rather small, it is likely that the obtained control law can provide decent accuracy.

Thruster torque calculation

In the GRACE case, it is not possible to ensure the necessary attitude accuracy using magnetorquers only, because for any time there is a direction in which no control can be applied and additional disturbances that are affecting the satellite. In order to maintain the necessary accuracy an additional attitude control system that consists of several thrusters is applied.

Thrusters provide very large torque in short period, hence their influence on the satellite angular motion is almost the same as immediate change of angular velocity. This can be used for the thruster torque calculation: when the relative attitude reaches allowed boundaries, thrusters change angular velocity along the corresponding axis to the desired one. It can be chosen as constant for every axis, i.e.

$$\omega_{des,i} = -\text{sign}(q_{rel,i})\gamma_i$$

where γ_i is constant, $q_{rel,i}$ is i-th component of vector part of the relative quaternion. Control torque then can be calculated as

$$M^{thr}_{ctrl,i} = J_{ii}\left(\omega_{des,i} - \omega_{rel,i}\right).$$

Since there are many disturbances affecting the satellite (solar radiation pressure, atmospheric drag etc.) that cannot be calculated on-board, it might be reasonable to adjust γ_i. For example, it can be changed in the following way:

$$\gamma_i(t_k) = f(t_k, t_{k-1})\gamma_i(t_{k-1})$$

where

$$f(t_k, t_{k-1}) = \begin{cases} 1, & \text{if } t_k - t_{k-1} \geq T \\ 1 + a\left(T - (t_k - t_{k-1})\right), & \text{if } (q_{rel,i}(t_k) q_{rel,i}(t_{k-1})) > 0 \\ 1 - a\left(T - (t_k - t_{k-1})\right), & \text{if } (q_{rel,i}(t_k) q_{rel,i}(t_{k-1})) < 0 \end{cases}$$

Here a, T are constants. Using this adjustment we ensure that if two consequent firings along the same axis are in the same direction, the fire rate increases. If they are in the different directions, the fire rate decreases.

SIMULATION RESULTS

In order to enable a test of the performance of the developed Attitude Determination and Control System (ADCS) algorithms, a GRACE-like simulation was set up by using the eXtended High Performance Satellite dynamics simulator (XHPS) which has been developed within the framework of the German collaborative research center geo-Q.[20] The XHPS is a derivative of the HPS jointly developed by the Center of Applied Space Technology and Microgravity (ZARM) and the German Aerospace Center (DLR) for more than 10 years. It has been successfully used for the simulation of various space missions such as MICROSCOPE, Gravity Probe-B and Galileo. Within XHPS, a satellite mission scenario can be assembled by means of a modular MATLAB/SIMULINK database employing C/C++ codes to model the satellite dynamics. This includes the interaction with Earths gravity field, the influence of non-gravitational effects[21–24] such as solar radiation pressure, albedo radiation, infrared radiation, thermal radiation pressure, atmospheric drag and the characteristics of the spacecraft itself. By means of scenario-specific actuator, sensor and controller models the operation of the ADCS can be simulated with inclusion of a realistic satellite dynamics model. The implementation of the perturbations acting on the trajectory and attitude of the satellite includes a detailed geometrical model of the respective satellite, thus effectively treating the influence of reflections, material parameter distribution and shadowing effects. Therefore, a very realistic modeled dynamical behavior of the satellite can be realized. By specifying the sensor model outputs according to the real satellite specifications, the simulations can be used to create so called mock data sets which resemble the real data sets in terms of protocol architecture, data rates etc.

For the study in this paper, we implemented the GRACE satellite scenario in XHPS with the following parameters, thus providing a simulation testbed for the developed ADCS algorithms:

- Satellite orbital motion corresponds to the one of GRACE mission: orbit inclination is about 89 deg, orbit radius is 6 862 km and satellite separation is about 200 km. Initial conditions for both satellites are taken from real GRACE data from year 2006

- Tensor of inertia is $\mathbf{J} = \text{diag}(110.4, 580.5, 649.5)\ \text{kg} \cdot \text{m}^2$

- Maximum dipole moment of magnetorquers is 50 A·m^2

- Aerodynamic drag, solar radiation pressure and GG torque from spherical harmonic gravity field are included into the simulation, but only the simplified GG torque is included in the control loop.

- Attitude measurements are provided by two independent orthogonal star trackers. Covariance matrix of the noise is $\text{diag}(5^2, 5^2, 40^2)\ \text{arcsec}^2$

- Angular velocity measurements noise is $\text{diag}(5^2, 5^2, 5^2)(\text{arcsec}/\text{sec})^2$

Two different Kalman Filters were implemented. The first one utilizes measurements from the star tracker only, and the second one also uses measurements of the angular velocity provided by gyro. In Figures 5 and 6 results of the Kalman filtering are presented. The comparison shows that the filter process clearly reduces the error of the observations. There is almost no difference between them, because gyro measurements are too rough in comparison with the star trackers, hence utilization of its measurements have very little impact on the efficiency of the Kalman Filter.

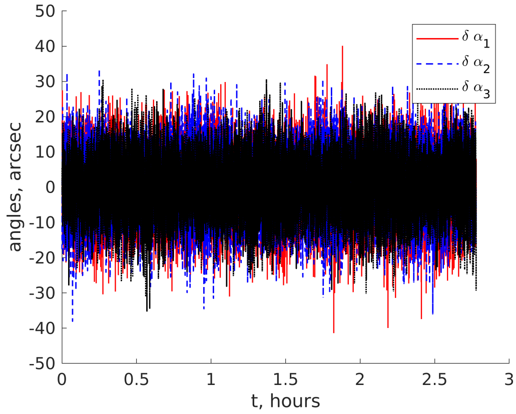
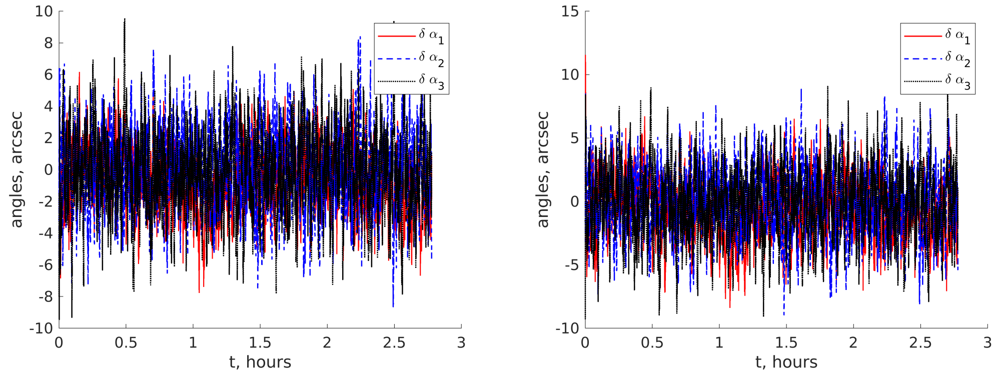


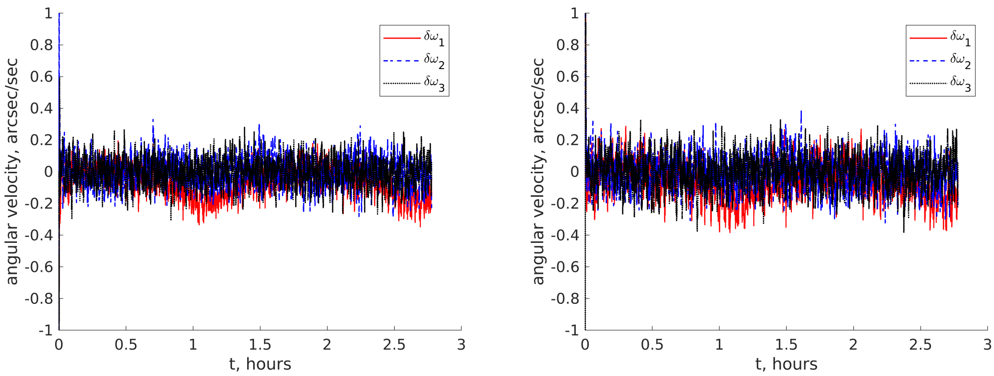
Figure 4: Attitude measurement accuracy provided by star trackers



(a) Star tracker only

(b) Star tracker and gyro

Figure 5: Filtered attitude measurement accuracy

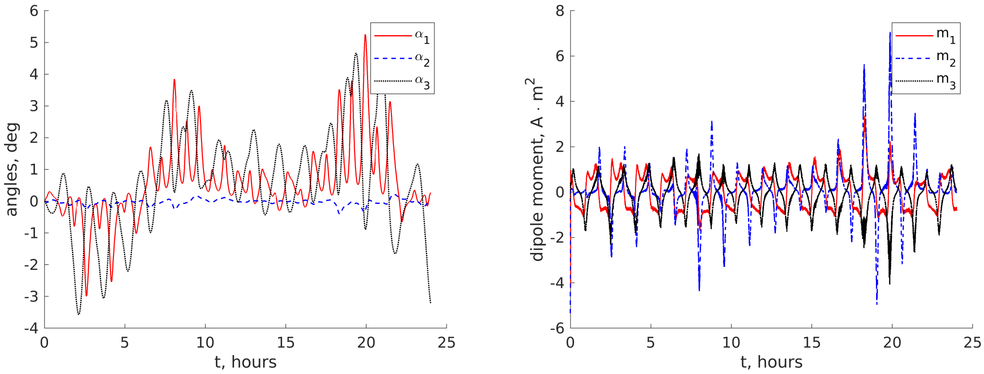


(a) Star Tracker only

(b) Star Tracker and gyro

Figure 6: Filtered angular velocity measurement accuracy

In Figure 7 and results of purely magnetic attitude control utilization are shown. Control parameters obtained using Floquet theory are $\mathbf{K}_\alpha \approx \mathrm{diag}(0.0012, 0.0030, -0.0005)\mathrm{Nm}$ and $\mathbf{K}_\omega \approx \mathrm{diag}(1.05, 3.1, 0.33)\mathrm{Nms}$. It should be noticed that one of the control parameters is negative, which is counter-intuitive. Despite this fact, provided accuracy is in the range of about 5 degree, which is rather good for magnetic attitude control but does not satisfy mission requirements.

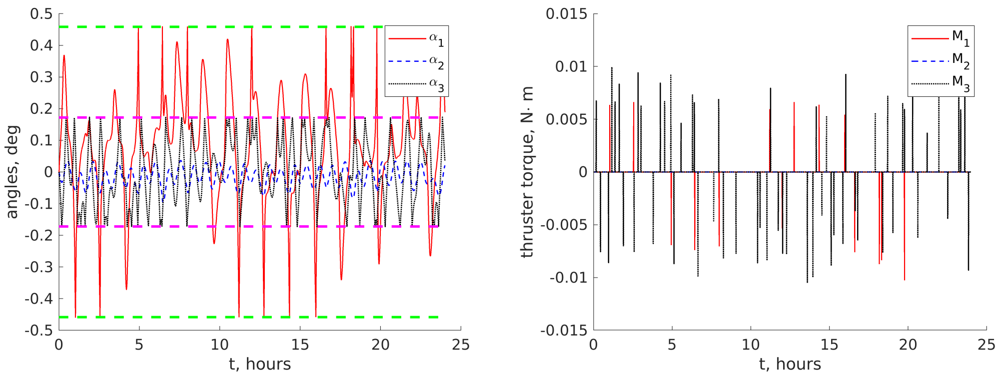


(a) Attitude accuracy

(b) Dipole moment of magnetorquers

Figure 7: Simulation results of the Lyapunov-based attitude control without thrusters

In addition to the purely magnetic attitude control two additional scenarios that utilize thrusters were considered. The first one corresponds to the initial GRACE mission, the attitude accuracy requirements are $\|\alpha_{2,3}\| \leq 0.17$ deg and $\|\alpha_1\| \leq 0.46$ deg. Results of the simulation are presented in Figure 8.



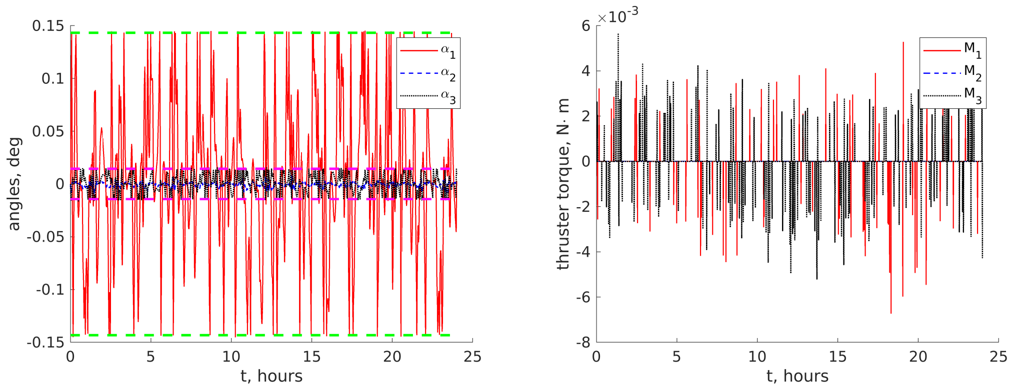
(a) Attitude accuracy

(b) Thruster torques

Figure 8: Simulation results of the Lyapunov-based attitude control with thrusters

The amount of firings is around 70 per day: 20 along the first axis and 50 along the third one. Second axis, which is orthogonal to the orbit plane, does not require any firings because it is controllable using magnetorquers only: the magnetic field is almost in the orbit plane, hence we can always produce the control torque in the direction orthogonal to the orbit plane. This is in good accordance with real GRACE data, but developed controller is able to drastically reduce the amount of firings: during the mission, it was about several hundreds per day.

The second scenario that was simulated corresponds to the GRACE-FO mission. It utilizes a laser interferometer to determine the relative distance between the two satellites. Hence, attitude accuracy requirements are much higher: $\|\alpha_{2,3}\| \leq 0.014$ deg and $\|\alpha_1\| \leq 0.14$ deg. In this case Lyapunov-based attitude control (with coefficients obtained using Floquet theory) shows unacceptable results: it require more than 500 firings to maintain the required accuracy. However, “hand tuning” of control coefficients for the Lyapunov-based algorithm can greatly improve the efficiency: only around 200 firings will be required – 60 along the first axis and 150 along the third one. Second axis still does not require additional thruster firings. Control coefficients in this case are $\mathbf{K}_\alpha \approx \text{diag}(0.03, 0.4, 0.45)$Nm and $\mathbf{K}_\omega \approx \text{diag}(2, 10, 15)$Nms. Simulation results are shown in the Fig. 9. However, in this case attitude control is sensitive to the accuracy of relative angular velocity



(a) Attitude accuracy (b) Thruster torques

Figure 9: Lyapunov-based attitude control with thrusters for GRACE-FO

measurements. Even with the Kalman Filter utilization, accuracy of this measurements is around 0.2 arcsec/sec, and the relative angular velocity is around 0.5 arcsec/sec (see Fig. 10). This also may explain the additional star camera and an improved inertial measurement unit (IMU) with laser gyros for the real GRACE-FO spacecrafts. Peaks in Fig. 10b appear periodically due to the presence of external disturbances (solar radiation pressure and aerodynamic torques) that are not included in the Kalman Filter model of motion.

CONCLUSION

In this paper the problem of hybrid magnetic/thrusters attitude control was considered. Lyapunov-based algorithm was suggested, and special technique of control parameters selection, which is based on Floquet theory utilization, was suggested.

Simulations showed that utilization of suggested technique in the case of original GRACE can reduce amount of thruster firings However, for the GRACE-FO requirements this approach fails, and it is necessary to adjust control parameters manually to reduce the amount of firings.

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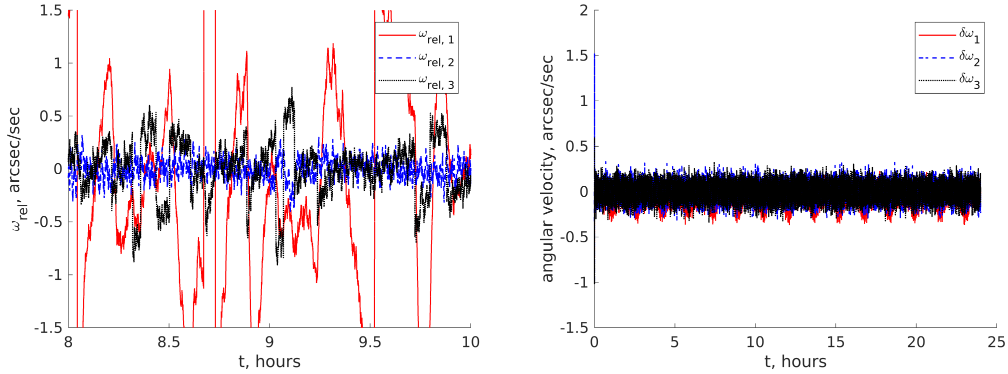

(a) Filtered measurements (b) Measurements accuracy after filtering

Figure 10: Relative angular velocity

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