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MAGNETIC ATTITUDE CONTROL FOR GRACE-LIKE MISSIONS

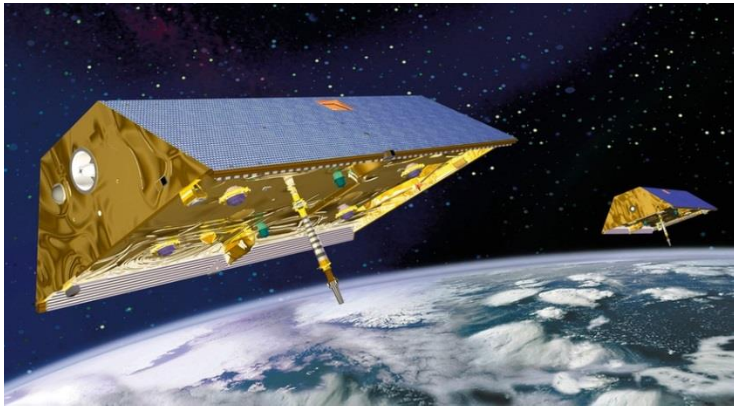
Yaroslav Mashtakov[1]
Mikhail Ovchinnikov[1]
Benny Rievers[2]
Meike List[2]
Florian Wöske[2]

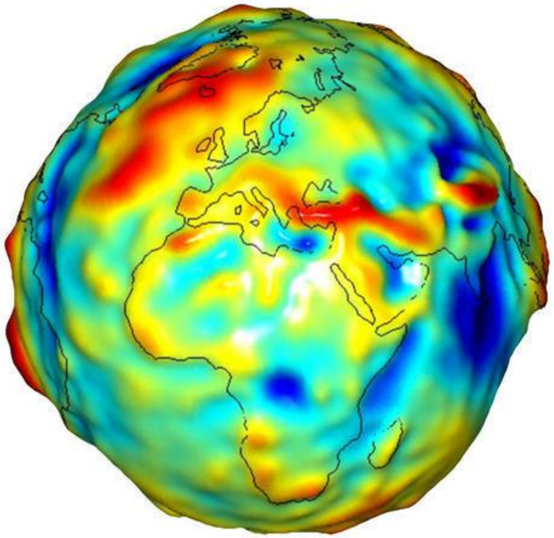
Keldysh Institute of Applied Mathematics of RAS[1]
Center of Applied Space Technology and Microgravity (ZARM) [2]

Introduction

GRACE mission:

- Launched 17 March 2002
- Two spacecrafts at the same polar orbit
- Conduct measurements of the Earth gravitational field
- Obtained data is used in oceanology, ice glaciers observation etc.
- Enhanced mission GRACE-FO was launched 22 May 2018





Introduction

- Microwave ranging device provide measurements of relative distance
- It is necessary to maintain line of sight between the satellites
- RW produce too much noise
- We can use only magnetorquers

Problem statement

We know:

- Orbital motion of each satellite
- Orbits are near circular and almost identical
- Distance between spacecrafts is about 200 km
- Spacecraft parameters

We have to:

- Maintain line of sight between the satellites
- Solar panels should always look away from the Earth

Reference motion

- Line of sight:

$$\mathbf{e}_1 = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\left\| \mathbf{r}_1 - \mathbf{r}_2 \right\|}$$

- Solar panels:

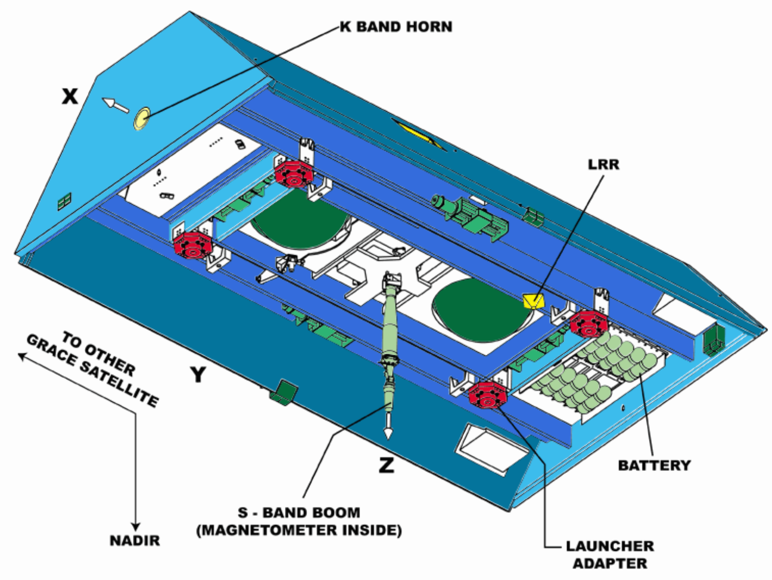
$$\mathbf{e}_3 = \frac{\mathbf{e}_1 (\mathbf{e}_1, \mathbf{r}_2) - \mathbf{r}_2}{\left\| \mathbf{r}_2 - \mathbf{e}_1 (\mathbf{e}_1, \mathbf{r}_2) \right\|}$$

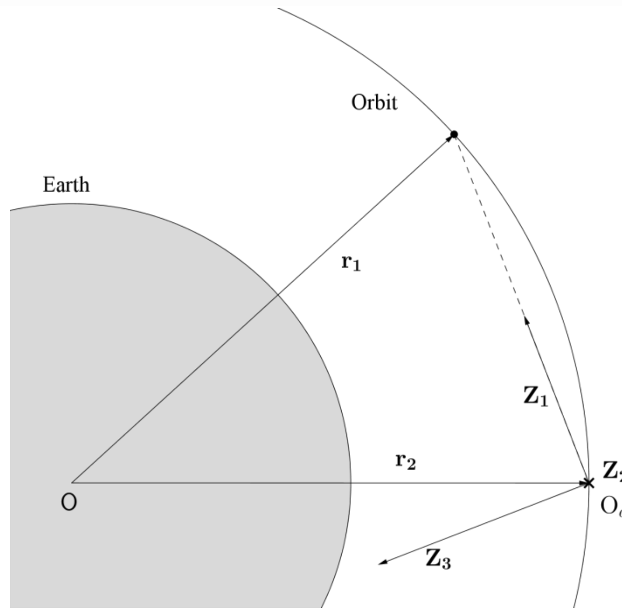
- Reference motions is given as a DCM:

$$\mathbf{D} = \left(\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3 \right)^T$$

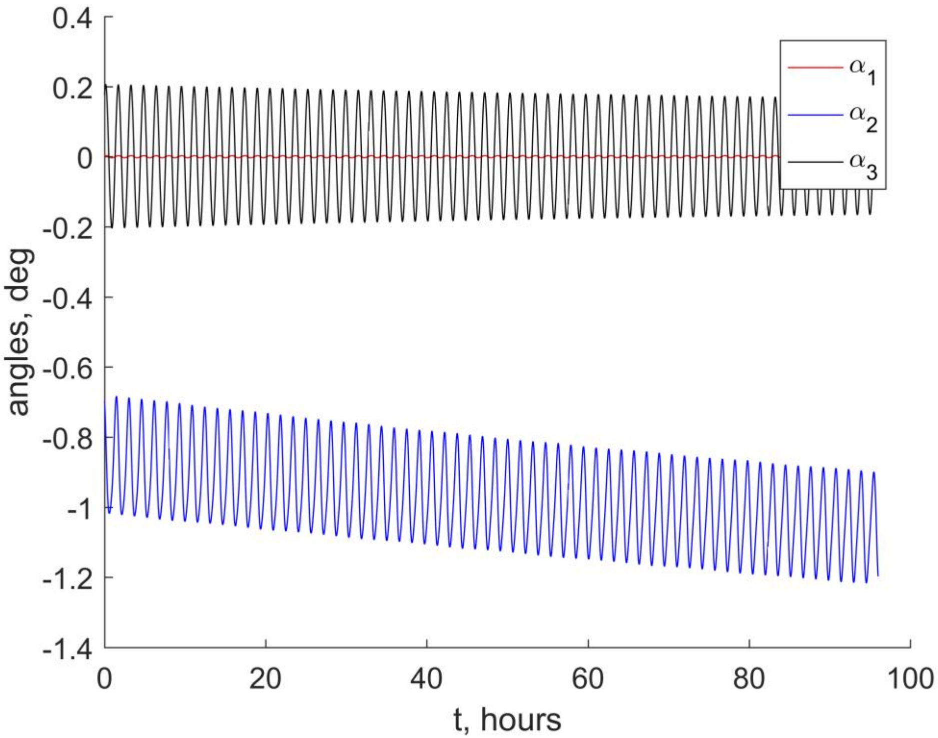
- Reference angular velocity from Poisson kinematics:

$$[\boldsymbol{\omega}]_\times = -\dot{\mathbf{D}} \mathbf{D}$$





Orbital Frame



Difference between Orbital and Reference Frames

- It is described as:

$$\mathbf{j}_3 = -\frac{\mathbf{r}_2}{\|\mathbf{r}_2\|}$$

$$\mathbf{j}_1 = \frac{\mathbf{v}_2 - \mathbf{j}_3(\mathbf{j}_3, \mathbf{v}_2)}{\|\mathbf{v}_2 - \mathbf{j}_3(\mathbf{j}_3, \mathbf{v}_2)\|}$$

$$\mathbf{j}_2 = \mathbf{j}_3 \times \mathbf{j}_1$$

- Orbital and reference frame are almost the same
- The problem is similar to stabilization of the spacecraft in Orbital Frame

Equations of motion

- Only GG and magnetic torque are considered

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} = 3\frac{\mu_E}{r^5}\mathbf{r} \times \mathbf{J}\mathbf{r} + \mathbf{m} \times \mathbf{B}$$

$$\dot{\mathbf{Q}} = \frac{1}{2}\mathbf{Q} \circ \boldsymbol{\omega}_{abs}$$

- Satellites are on the circular orbits with orbital angular velocity ω_0
- Motion is near the equilibrium in Orbital Frame
- Note: equilibrium is unstable (due to mission requirements – we have to minimize atmospheric drag)

Linearization

- Equations of relative motion

$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\omega}$$

$$\dot{\boldsymbol{\omega}} = \mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{A}_\omega \boldsymbol{\omega} - \mathbf{J}^{-1} [\mathbf{B}]_\times \mathbf{m}$$

- $[\mathbf{B}]_\times$ – skew symmetric cross product matrix

$$\mathbf{A}_\alpha = \mathrm{diag}\left(4\omega_0^2 \frac{C-B}{A},\ 3\omega_0^2 \frac{C-A}{B},\ \omega_0^2 \frac{A-B}{C} \right)$$

$$\mathbf{A}_\omega = \begin{pmatrix} 0 & 0 & \omega_0 \dfrac{C+A-B}{A} \\ 0 & 0 & 0 \\ \omega_0 \dfrac{B-C-A}{C} & 0 & 0 \end{pmatrix}$$

Control

$$\mathbf{M}_{ideal} = \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} - \mathbf{M}_{ext} - \mathbf{J}\left(\boldsymbol{\omega} \times \boldsymbol{\omega}_{ref}\right) + \mathbf{J}\dot{\boldsymbol{\omega}}_{ref} - k_s \mathbf{s} - k_\omega \boldsymbol{\omega}$$

$\boldsymbol{\omega}_{ref}$ — reference angular velocity

$\mathbf{M}_{ext}$ — external torque

$\boldsymbol{\omega}$ — relative angular velocity

$\mathbf{s}$ — vector part of relative quaternion

Lyapunov-based control

Advantages:

- Simple
- Ensures asymptotic stability

Disadvantages:

- Does not take into account magnetic control specifics
- Depends on two control parameters which are must be chosen adequately

Control

- Using magnetorquers create as mush torque as possible

$$\mathbf{m} = \frac{\mathbf{B} \times \mathbf{M}_{ideal}}{\left\|\mathbf{B}\right\|^2}$$

- Provided control torque

$$\mathbf{M}_{ctrl} = \mathbf{M}_{ideal} - \mathbf{e}_B\left(\mathbf{e}_B, \mathbf{M}_{ideal}\right) = -\left[\mathbf{e}_B\right]_\times \left[\mathbf{e}_B\right]_\times \mathbf{M}_{ideal}, \quad \mathbf{e}_B = \frac{\mathbf{B}}{\left\|\mathbf{B}\right\|}$$

- Linearized equations of motion

$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\omega}$$

$$\dot{\boldsymbol{\omega}} = \mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{A}_\omega \boldsymbol{\omega} + \mathbf{J}^{-1}\left[\mathbf{e}_B\right]_\times \left[\mathbf{e}_B\right]_\times \left(\mathbf{J}\mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{J}\mathbf{A}_\omega \boldsymbol{\omega} + k_\alpha \boldsymbol{\alpha} + k_\omega \boldsymbol{\omega}\right)$$

- Is it possible to make it asymptotically stable?

How to choose coefficients?

- System is linear and time dependent
- We will use simple direct dipole model
- In this model system is periodical
- Floquet theory might be applied for stability analysis of the system

$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\omega}$$

$$\dot{\boldsymbol{\omega}} = \mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{A}_\omega \boldsymbol{\omega} + \mathbf{J}^{-1} \left[\mathbf{e}_B\right]_\times \left[\mathbf{e}_B\right]_\times \left(\mathbf{J}\mathbf{A}_\alpha \boldsymbol{\alpha} + \mathbf{J}\mathbf{A}_\omega \boldsymbol{\omega} + k_\alpha \boldsymbol{\alpha} + k_\omega \boldsymbol{\omega}\right)$$

- It allow us to choose k_α, k_ω appropriately
- For better tuning they can be replaced by diagonal matrices

$$k_\alpha \to \operatorname{diag}\left(k_{\alpha,1}, k_{\alpha,2}, k_{\alpha,3}\right)$$

$$k_\omega \to \operatorname{diag}\left(k_{\omega,1}, k_{\omega,2}, k_{\omega,3}\right)$$

Floquet theory

- Obtain monodromy matrix
- Find its eigenvalues ρ_k
- For asymptotic stability

$$\max_k \left(\mathrm{Re} \left(\ln \rho_k \right) \right) < 0$$

- $\max_k \left(\mathrm{Re} \left(\ln \rho_k \right) \right)$ corresponds to the convergence rate
- We get optimization problem

$$\max_k \left(\mathrm{Re} \left(\ln \rho_k \right) \right) \to \min$$

- Solve it numerically

Gas thrusters

- Magnetic coils cannot provide necessary accuracy
- We will use cold gas thrusters
- They are turned on only when satellite reaches allowed accuracy borders
- Denote

$$\omega_{des,k} = -s_k \gamma_k, \quad \gamma_k(t_k) = f(t_k, t_{k-1}) \gamma_k(t_{k-1})$$

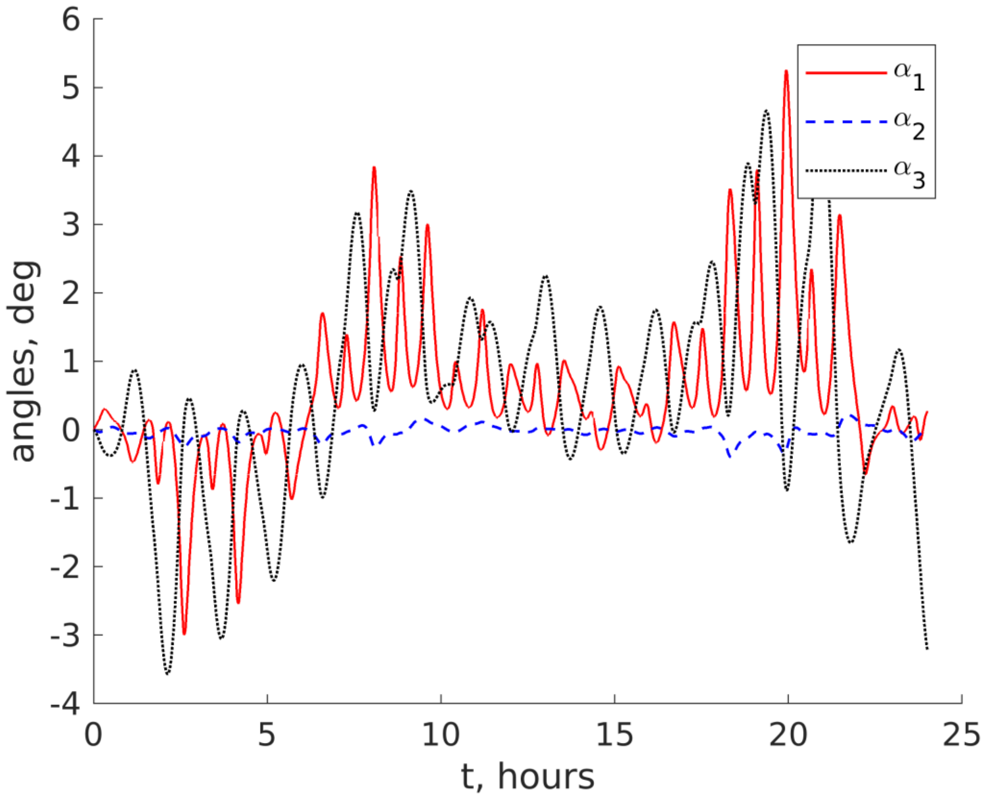
$$f(t_k, t_k) = \begin{cases} 1, & if \ (t_k - t_{k-1}) > T \\ 1 + a(T - (t_k - t_{k-1})), & if \ \ firing\ in\ the\ same\ direction \\ 1 - a(T - (t_k - t_{k-1})), & if \ \ firing\ in\ the\ opposite\ direction \end{cases}$$

- $M_{ctrl,k}^{thr} = \dfrac{1}{\Delta t_{thr}} J_{ii} \left(\omega_{des,k} - \omega_{rel,k} \right)$

Simulation results

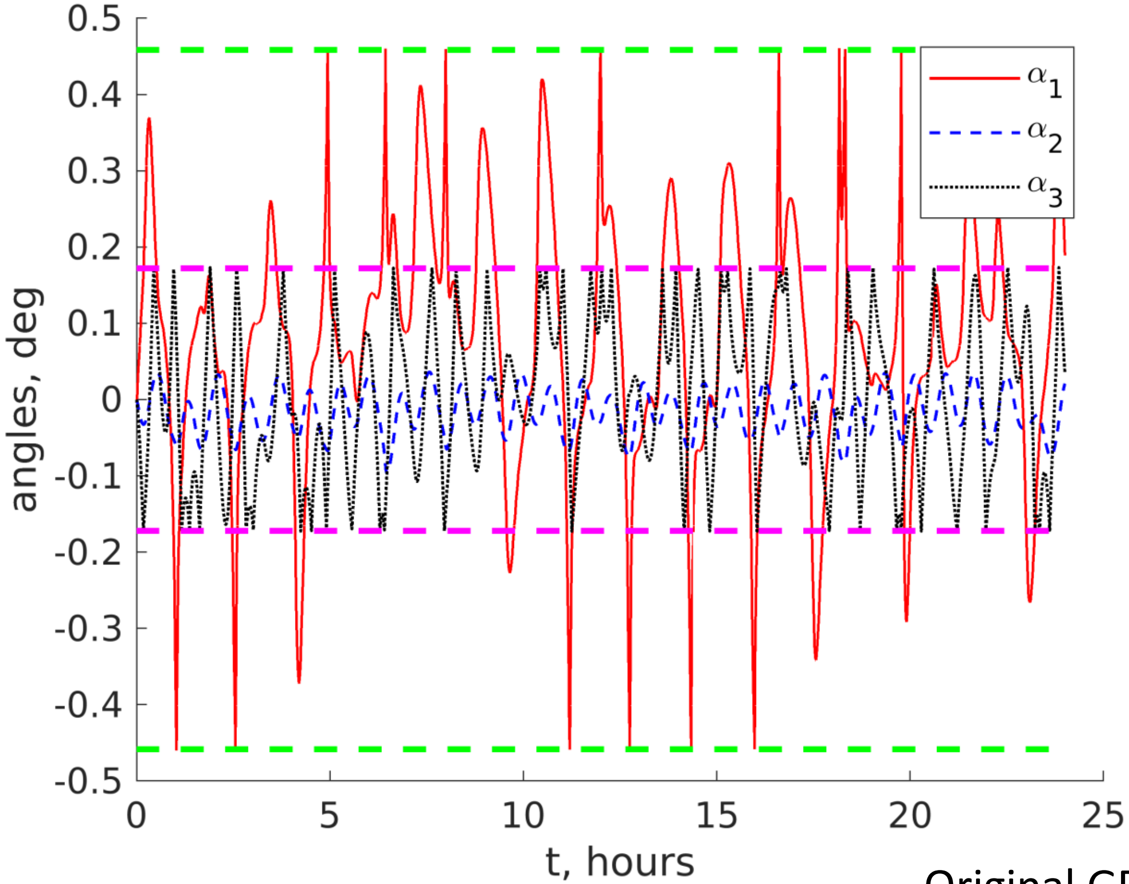
- Orbital motion corresponds to the real GRACE mission (polar orbit, height about 500 km)
- Atmospheric drag, solar radiation pressure, gravity gradient and magnetic torques are all included in the simulation
- Sensor’s noise also considered (Kalman filtration for startracker and gyros)
- Different mission scenarios: pure magnetic control, GRACE accuracy requirements, GRACE-FO accuracy requirements
- Simulation is carried out using eXtended High Performance satellite dynamics Simulator (XHPS) developed by ZARM and DLR

Results



Purely magnetic attitude control

Results

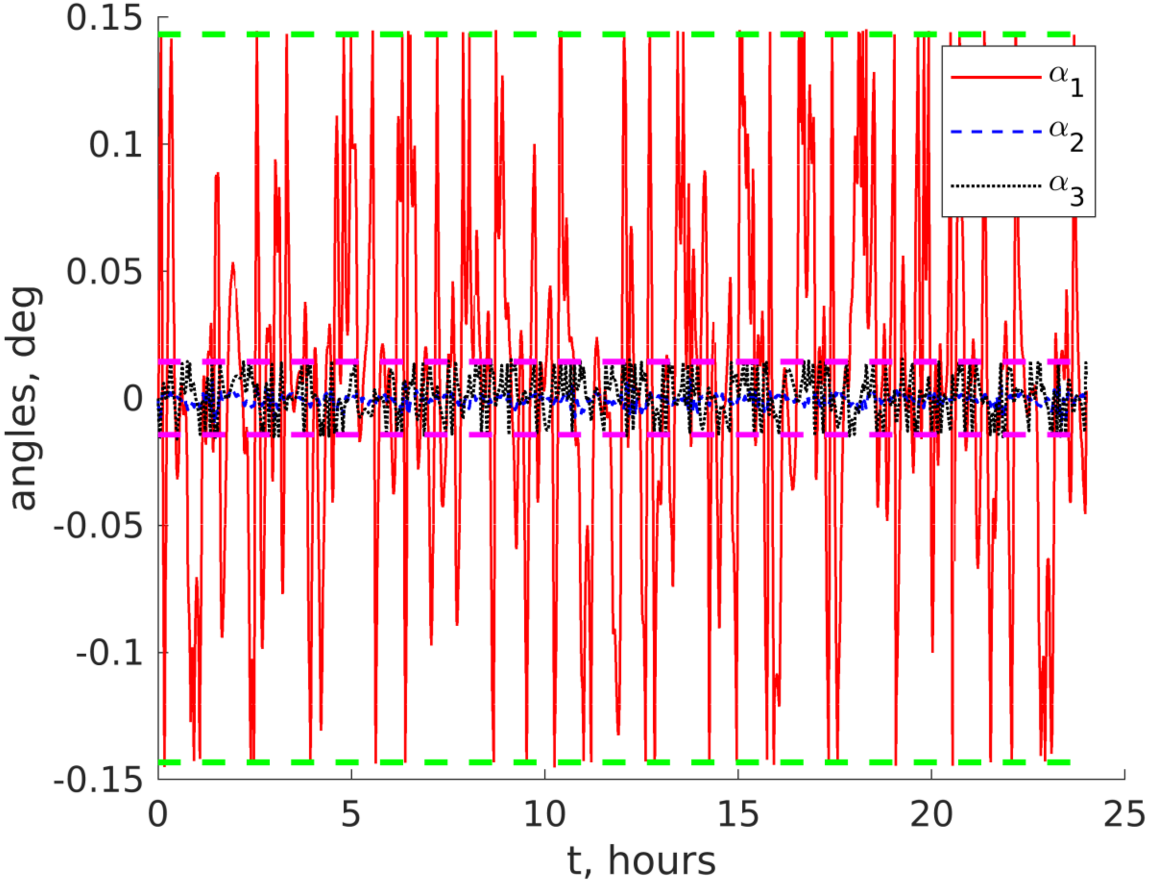


Magnetic coils + gas thrusters
GRACE requirements:
0.46 deg along line of sight
0.17 deg along other axes

Amount of firings per day:
20, 0, 50

Original GRACE required ~300 firings per day

Results



Magnetic coils + gas thrusters
GRACE-FO requirements:
0.14 deg along line of sight
0.014 deg along other axes

Hand tuning of the coefficients is necessary

Amount of firings per day:
60, 0, 150

Summary

- Problem of attitude control for GRACE-like mission is considered
- Lyapunov based attitude control is suggested
- Method of control coefficients selection is proposed (works for pure magnetic control and low accuracy requirements)
- For an extreme accuracy maintaining “hand tuning" is necessary

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