TETRAHEDRON FORMATION MAINTENANCE VIA ATMOSPHERIC DRAG CONTROL

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The problem of tetrahedron formation reference motion synthesis and its maintenance is considered. Linear Hill-Clohessy-Wiltshire equations are used to study passive relative motion of the satellites. Lyapunov-based control that increases the mission lifetime and utilizes atmospheric drag only is proposed.

INTRODUCTION

Utilization of small satellites can greatly reduce the cost and complexity of space mission. In addition, they make it easier to launch several satellites simultaneously. This, in turn, brings new possibilities, especially in the scientific missions that require simultaneous and distributed measurements. A good example of such mission is Magnetospheric MultiScale, when four satellites forming a tetrahedron carried out measurements of the Earth magnetic field.

In this paper we consider the similar to MMS mission problem, but at low near-circular Earth orbit. The main goal is to construct and maintain such relative motion of four satellites, that the tetrahedron formed by them preserves its shape and volume. This problem is divided into two parts. On the first step we construct such reference motion that in linear Hill-Clohessy-Wiltshire model the tetrahedron does not change. Several families of such orbits have been obtained.

However, in more precise model of motion under the influence of external disturbance, e.g. the effect of J2 spherical harmonic and nonlinear terms in equations of relative motion, obtained tetrahedrons degrade over time, and it is necessary to implement additional control in order to maintain the formation. Therefore, the second part of this paper is dedicated to the study of means of relative motion control. Since the satellites are supposed to be small, the most interesting approaches are the fuelless ones. Here we consider the atmospheric drag control that allows us to significantly increase lifetime of the mission. This control is implemented by changing the satellite attitude w.r.t. Orbital Frame, which, in turn, changes the satellite cross section area.

In order to synthesize the control law we utilize Hill-Clohessy-Wiltshire equations, but instead of using Cartesian coordinates to describe the relative motion we use combinations of the initial conditions of the solution. The main idea of the method is the same as whilst using osculating elements that describe orbital motion in disturbed two-body problem. This allows us to develop rather simple controller, which ensures the required relative orbit maintaining. It

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should be mentioned that the controller is developed using the simple drag model when the acceleration caused by the atmosphere is antiparallel to the satellite velocity. In order to verify developed control law the more precise model of motion, which includes J2 perturbation and more complex model of atmospheric drag, is used in the simulation.

PROBLEM STATEMENT AND MOTION MODEL

Let us consider the following problem

- Four satellites orbit passively on near circular orbits, major semiaxes of their orbits are equal.
- Four satellites together form a tetrahedron for which we want to define some mathematical equivalent of size and shape.
- We need to find the initial parameters for the satellites motion so that the tetrahedron does not change its size and shape over time at least approximately
- Also, the tetrahedron should never reach zero volume.
- Moreover, we want to construct a simple control algorithm to maintain shape of the tetrahedron in a presence of disturbances.

We use the following right-handed Cartesian reference frames:

Inertial Reference frame (IRF): its center $O_{\oplus}$ is at the Earth center of mass, the axis $O_{\oplus}Z$ is directed along the Earth axis of rotation, the axis $O_{\oplus}X$ is directed to the vernal equinox corresponding to the epoch J2000.

Orbital Reference Frame (ORF): its center O is at the one of satellites, the axis Ox is directed along the radius vector of the point O away from the Earth, the axis Oz is normal to the orbital plane and is directed along the orbital momentum.

The center of ORF is located in one of the satellites that is moving along the circular orbit. Without loss of generality we refer to this satellite as “the fourth”. Its motion in ORF is described by

$$\mathbf{r}_4(t) = \langle x_4(t), y_4(t), z_4(t) \rangle = \langle 0,0,0 \rangle.$$

Our assumptions allow us to describe the relative motion of other satellites using the linearized CW equations:

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - 3n^2 x = 0, \\ &\ddot{y} + 2n\dot{x} = 0, \\ &\ddot{z} + n^2 z = 0, \end{aligned} \tag{1}$$

where $n = \sqrt{\mu / \rho^3}$ is the mean motion, μ is the Earth gravitational parameter, ρ is the radius of circular orbit. Orbits major semiaxes being equal guarantees periodic motion of each satellite in ORF, so the tetrahedron size is bounded over time. This motion is described then by equations

$$\begin{aligned} &x_i(t) = A_i \sin\nu + B_i \cos\nu, \\ &y_i(t) = 2A_i \cos\nu - 2B_i \sin\nu + C_i, \\ &z_i(t) = D_i \sin\nu + E_i \cos\nu, \end{aligned} \tag{2}$$

where $\nu = nt$. Here A_i, B_i, C_i, D_i, E_i are constants depending on the initial values of motion, index i attains values 1, 2, 3. The motion of the fourth satellite is described by the same set of equations with all the constants being equal to zero.

SIZE AND SHAPE OF TETRAHEDRON

We now derive the conditions for the tetrahedron to preserve size and shape. In this section we do not use the fact that two satellites move along the same orbit, rather we define size and shape in the general case of tetrahedron.

The natural measure for the size of the tetrahedron is its volume $\mathbb{V}$. In ORF the volume has the form

$$\mathbb{V} = \frac{1}{6}\det\|\mathbf{r}_1 - \mathbf{r}_4, \mathbf{r}_2 - \mathbf{r}_4, \mathbf{r}_3 - \mathbf{r}_4\| = \frac{1}{6}\det\|\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\|.$$

Substituting $\mathbf{r}_i$ with values from (2) we obtain the volume as trigonometric polynomial of ν

$$6\mathbb{V} = P\sin^3\nu + Q\cos^3\nu + R\sin^2\nu\cos\nu + T\sin\nu\cos^2\nu$$
$$+U\sin^2\nu + V\cos^2\nu + W\sin\nu\cos\nu.$$

The coefficients in the polynomial depend on initial conditions, i.e. on A_i, B_i, C_i, D_i, E_i. For the volume $\mathbb{V}$ of the tetrahedron to remain constant it is necessary that

$$\text{P = Q = T = W = R = 0,}$$
$$\text{U = V.}$$

Under these conditions, the volume is equal to $\mathbb{V} = U/6$, hence they are also sufficient.

We want the tetrahedron to be non-degenerate. To simplify notation we combine constants A_i, B_i, C_i, D_i, E_i in (2) in vectors. Let $\mathbf{A} = \langle A_1, A_2, A_3\rangle$, vectors $\mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}$ are defined analogously.

With this notation and after appropriate simplifications conditions have the form

$$\begin{aligned}
&U = V \to (\mathbf{C}, \mathbf{D}, \mathbf{A}) = (\mathbf{C}, \mathbf{E}, \mathbf{B}),\\
&P = 0 \to (\mathbf{B}, \mathbf{A}, \mathbf{D}) = 0,\\
&Q = 0 \to (\mathbf{A}, \mathbf{E}, \mathbf{B}) = 0,\\
&W = 0 \to (\mathbf{C}, \mathbf{D}, \mathbf{B}) = (\mathbf{C}, \mathbf{A}, \mathbf{E}),\\
&R = 0 \to (\mathbf{B}, \mathbf{A}, \mathbf{E}) = 0,\\
&T = 0 \to (\mathbf{A}, \mathbf{D}, \mathbf{B}) = 0,
\end{aligned} \tag{3}$$

where $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ is mixed product of three vectors $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$.

If $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ or $\mathbf{E}$ is equal to zero, then $\mathbb{V} = 0$ that should be avoided. If none of these vectors are equal to zero, then from (3) we can derive that all four of them should be coplanar. If $\mathbf{A}$ and $\mathbf{B}$ are collinear, again $\mathbb{V} = 0$. If $\mathbf{A}$ and $\mathbf{B}$ are not collinear then they form a basis in a plane and coplanar $\mathbf{D}$, $\mathbf{E}$ are expressed as linear combinations

$$\mathbf{D} = a\mathbf{A} + b\mathbf{B},$$
$$\mathbf{E} = c\mathbf{A} + d\mathbf{B},$$

so that

$$b(\mathbf{C}, \mathbf{B}, \mathbf{A}) = c(\mathbf{C}, \mathbf{A}, \mathbf{B}),$$
$$a(\mathbf{C}, \mathbf{A}, \mathbf{B}) = d(\mathbf{C}, \mathbf{A}, \mathbf{B}),$$

which eventually leads to $b = -c$, $a = d$.

With such conditions the volume $\mathbb{V}$ could be calculated from the formula

$$\mathbb{V} = \frac{b}{6}(\mathbf{A}, \mathbf{C}, \mathbf{B}).$$

The coefficient b should not be equal to zero in all subsequent calculations.

Unlike the volume, the shape of the tetrahedron does not have simple geometric or algebraic interpretation, partially due to the fact that the tetrahedron is not fully described by lengths of its edges. We do not demand the similarity of the tetrahedron in each moment of time instead we use one parameter that depicts the shape of the tetrahedron in average. We also assume that conservation of this parameter implies conservation of the shape at least approximately. This parameter, which here and below is called the tetrahedron quality, is described as

$$\mathbb{Q} = 12\frac{(3\mathbb{V})^{2/3}}{\mathbb{L}}$$

Here $\mathbb{V}$ is the volume, $\mathbb{L}$ is the sum of squares of the tetrahedron edge lengths. For regular tetrahedron $\mathbb{Q}=1$ and for degenerate one (when four satellites lie in the same plane) $\mathbb{Q}=0$.

Similar to the volume derivation we derive the expression for $\mathbb{L}$:

$$\mathbb{L} = (\mathbf{r}_1-\mathbf{r}_2)^2 + (\mathbf{r}_1-\mathbf{r}_3)^2 + (\mathbf{r}_2-\mathbf{r}_3)^2 + \mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2.$$

After substitutions, reductions of terms and all simplifications the derivation for $\mathbb{L}$ is a trigonometric polynomial. In general the polynomial has the form

$$\mathbb{L} = P\cos^2\nu + Q\cos\nu\sin\nu + R\sin^2\nu + T\cos\nu + U\sin\nu + W$$

The necessary and sufficient conditions for conservation of $\mathbb{L}$ have the form

$$\begin{aligned} &Q = T = U = 0,\\ &P = R. \end{aligned}$$

Using volume conservation expressions we can simplify the equations. Finally, for the non-degenerate tetrahedron preserving its volume and quality (size and shape) vectors $\mathbf{A}$ and $\mathbf{B}$ must be non-collinear and the following expressions must be true

$$\begin{aligned} &\mathbf{D} = a\mathbf{A} + b\mathbf{B},\\ &\mathbf{E} = -b\mathbf{A} + a\mathbf{B}, \end{aligned}$$

$$\begin{aligned} &3A_1B_1 + 3A_2B_2 + 3A_3B_3 - A_1B_2 - A_1B_3 - A_2B_1 - A_2B_3 - A_3B_1 - A_3B_2 = 0,\\ &3\left(B_1^2 + B_2^2 + B_3^2 - A_1^2 - A_2^2 - A_3^2\right) + 2\left(A_1A_2 + A_1A_3 + A_2A_3\right) - 2\left(B_1B_2 - B_1B_3 - B_2B_3\right) = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} &C_1(3A_1 - A_2 - A_3) + C_2(3A_2 - A_1 - A_3) + C_3(3A_3 - A_1 - A_2) = 0,\\ &C_1(3B_1 - B_2 - B_3) + C_2(3B_2 - B_1 - B_3) + C_3(3B_3 - B_1 - B_2) = 0. \end{aligned}$$

To fully describe all possible configurations preserving volume and quality one should solve this system for unknown vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ and $\mathbf{E}$. This is the system of 10 equations with 15 variables and 2 parameters so the solutions are 7-parametric families. Two of the parameters are the constants a and b. One parameter should be proportional to the volume of the tetrahedron because enlarging or shrinking of the tetrahedron does not affect the quality. The fourth parameter is the initial phase of the motion, because $\nu = nt$ changes from 0 to 2π over time, so adding arbitrary number to the phase of all satellites in a group does not change the motion. The fifth parameter could be found using the following observation: vector $\mathbf{C}$ has three components, but only two last equations depend on them, so $\mathbf{C}$ could be found only up to a factor. This arbitrary factor is the fifth parameter – in our case it is the shift between two satellites orbiting on the same orbit. Note that the satellite renumbering does not affect the dynamics, so we refer two different solutions obtained from each other by renumbering the satellites to a single family of solutions. The geometric sense of two remaining parameters is described below.

PARTICULAR SOLUTIONS ANALYSIS

In a search for particular solutions we do the following variables change

$$A_1 = \alpha\cos\varphi, \quad B_1 = \alpha\sin\varphi,$$
$$A_2 = \beta\cos\psi, \quad B_2 = \beta\sin\psi,$$
$$A_3 = \gamma\cos\theta, \quad B_3 = \gamma\sin\theta,$$

here α,β,γ -- amplitudes of oscillations of first, second and third satellites in ORF respectively, φ,ψ,θ are the initial phases. Given α,β,γ one can find phases so two ratios between amplitudes (for example $\beta/\alpha,\ \gamma/\alpha$) are two remaining parameters.

In this paper we are focusing on leader-follower solution of the abovementioned equations: two satellites, the third and the forth are on the same orbit, so the third satellite rests in ORF. That means $\gamma = 0$ and

$$2\left(\alpha\beta\cos(\varphi+\psi)\right) = 3\left(\alpha^2\cos 2\varphi + \beta^2\cos 2\psi\right),$$
$$2\left(\alpha\beta\sin(\varphi+\psi)\right) = 3\left(\alpha^2\sin 2\varphi + \beta^2\sin 2\psi\right).$$

Moreover, $\nu = nt$ changes from 0 to 2π upon the motion, so we choose initial moment of time so that $\psi = 0$. The only non-degenerate solution is

$$\alpha = \beta = K,$$
$$\cos\varphi = \frac{1}{3}.$$

Here K represents the linear size of tetrahedron (average length of edge) and φ is the phase shift between the first and the second satellites. When ψ is again nonzero arbitrary phase angle we obtain initial conditions

$$A_1 = K\left(\sqrt{6}/3\cos\psi + \sqrt{3}/3\sin\psi\right),$$
$$A_2 = K\left(\sqrt{6}/3\cos\psi - \sqrt{3}/3\sin\psi\right),$$
$$A_3 = 0,$$
$$B_1 = K\left(-\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right),$$
$$B_2 = K\left(\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right),$$
$$B_3 = 0.$$

Substituting obtained relations in (4) we obtain

$$C_1 = C_2 = c, \quad C_3 = 2c$$

with arbitrary c. Combining it with

$$\mathbf{D} = a\mathbf{A} + b\mathbf{B},$$
$$\mathbf{E} = -b\mathbf{A} + a\mathbf{B},$$

we obtain a complete set of solutions to the problem with K,ψ,a,b,c being parameters.

In this case

$$\mathbb{V} = \frac{b}{6}(\mathbf{A},\mathbf{C},\mathbf{B}) = cK^2 b\frac{2\sqrt{2}}{9},$$
$$\mathbb{L} = \frac{8}{3}K^2(a^2+b^2+5) + 8c^2.$$

As being expected, neither volume, nor quality depends on ψ in linear case.

The maximum of quality is achieved when $a = 0, b = \pm\sqrt{5}, c = \pm K\sqrt{\frac{5}{3}}$ and is equal to $\mathbb{Q}_{max} = \frac{1}{\sqrt[3]{5}}$

ATMOSPHERIC DRAG CONTROL ALGORITHM

Simulations show that for high orbits the degradation of the tetrahedron quality is rather small, so active control of relative orbits is not necessary. However, at LEO tetrahedron degradation rate leaves much to be desired so some control algorithm is necessary.

At LEO, especially for small satellites, one of the most convenient and cheap way of relative motion control is to utilize atmospheric drag. In order to construct the efficient controller, we use curvilinear relative coordinates. Equations of motion can be described in the following way:

$$\begin{aligned}
&\frac{d^2}{dt^2}\rho_i - 2n\frac{d}{dt}(a_0\varphi_i) - 3n^2\rho_i = 0,\\
&\frac{d^2}{dt^2}(a_0\varphi_i) + 2n\frac{d}{dt}\rho_i = 0,\\
&\frac{d^2}{dt^2}(a_0\theta_i) + n^2(a_0\theta_i) = 0,
\end{aligned}$$

where $a_0 = |\mathbf{R}_4|$, i.e. it is the orbit radius of the satellite placed in ORF origin, θ_i, φ_i are the angles that define relative satellite position (see Figure 1). It should be noticed that they are obtained for the Keplerian motion, when the forth satellite moves along circular orbit. These equations are similar to (1), and for the formation with small relative orbits their solution describe the same relative orbits. However, these coordinates describe wider formation much better, because they allow us to take into account that satellites actually move along almost circular orbits. If we include disturbances $\mathbf{g}$ and control $\mathbf{u}$, the system became

$$\begin{aligned}
&\frac{d^2}{dt^2}\rho_i - 2n\frac{d}{dt}(a_0\varphi_i) - 3n^2\rho_i = u_x + g_x,\\
&\frac{d^2}{dt^2}(a_0\varphi_i) + 2n\frac{d}{dt}\rho_i = u_y + g_y,\\
&\frac{d^2}{dt^2}(a_0\theta_i) + n^2(a_0\theta_i) = u_z + g_z,
\end{aligned}$$

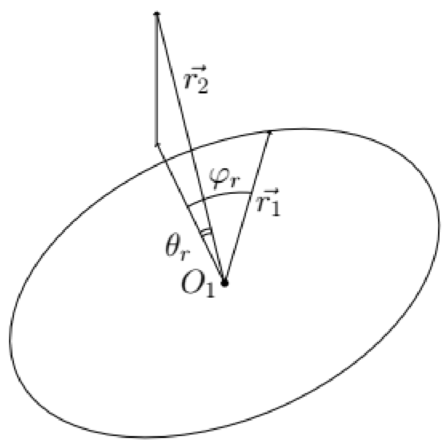

Figure 1 Curvilinear parameters

Introduce new variables $A, B, C, D, \lambda, \psi$:

$$\begin{aligned}
&\rho = A\sin\psi + 2C,\\
&\frac{d}{dt}\rho = An\cos\psi,\\
&a_0\theta = 2A\cos\psi + D,\\
&\frac{d}{dt}(a_0\theta) = -2An\sin\psi - 3C\omega,\\
&a_0\varphi = B\sin\lambda,\\
&\frac{d}{dt}(a_0\varphi_0) = Bn\cos\lambda.
\end{aligned}$$

Their derivatives in accordance with equations of motion can be written as follows:

$$\begin{aligned}
&\dot{A} = \frac{1}{n}\left((u_x + g_x)\cos\psi - 2(u_y + g_y)\sin\psi\right),\\
&\dot{B} = \frac{1}{n}(u_z + g_z)\cos\lambda,\\
&\dot{C} = \frac{1}{n}(u_y + g_y),\\
&\dot{D} = -3Cn - \frac{2}{n}(u_x + g_x),\\
&\dot{\lambda} = n - \frac{1}{nB}(u_z + g_z)\sin\lambda.\\
&\dot{\psi} = n - \frac{1}{An}\left((u_x + g_x)\sin\psi + 2(u_y + g_y)\cos\psi\right).
\end{aligned}$$

The physical meaning of new variables is closely related to the meaning of CW constants: the projective relative trajectory is close to the ellipse with semiaxes A and $2A$, the oscillations in the Oz direction has the amplitude close to B, the trajectory drifts from the origin of ORF with rate C and initial shift D. Also, λ and ψ correspond to the position of the satellite on the relative orbit.

The relative orbit size changes slowly with respect to shift and drift that can change rapidly and with increasing rate. So the first step of a control algorithm should nullify drift and set shift to a desired value

$$C_{ref} = 0, \quad D_{ref} = D_0.$$

In order to increase lifetime of the satellite it would be useful to suggest fuelless control algorithm. For example, we can use atmospheric drag. It means that control can be applied only along the velocity vector of the satellite, i.e. $u_x = u_z = 0$. Here we do not take into account the possible reflection of air molecules from the satellite body, i.e. aerodynamic lift – we will consider it as a disturbance and, as well as other disturbances, will not include it in control synthesis. Hence, simplified system dynamics is

$$\begin{aligned}
&\dot{A} = \frac{-2u_y\sin\psi}{n},\\
&\dot{C} = \frac{u_y}{n},\\
&\dot{D} = -3Cn.
\end{aligned}$$

For control construction we use Lyapunov direct method. Consider the Lyapunov-candidate function

$$V = C^2 + k_D\left(D - D_{ref}\right)^2, \quad k_D > 0.$$

Its derivative is

$$\dot{V} = C\left(\frac{u_y}{n} - 3nk_D\left(D - D_{ref}\right)\right).$$

If $\dot{V} = -k_c C, \quad k_c > 0$, the derivative is non-positive and equal to zero on set containing only one whole trajectory $C = 0, D = D_{ref}$. According to Barbashin-Krasovski-LaSalle theorem the control

$$u_y = 3n^2 k_D\left(D - D_{ref}\right) - k_c C$$

provides asymptotic stability of a desired motion. However, due to the presence of external disturbances this control will only ensure that drift and shift of the orbit are within the vicinity of the required ones.

It should be noted that the main goal of the control is to preserve the tetrahedron, therefore not only shift and drift must preserved, but also relative ellipse size and difference between phases should be equal to a certain value. We consider so-called “leader-follower” formation, when two satellites lie almost on the same orbit (one in the origin of ORF, the other is shifted along the OY axis), and the other two satellites move along the same relative orbit with different phases. To be exact, as usual the forth satellite is in the origin of ORF, the third one is shifted so

$$D_3 = D_0, \ A_3 = 0, \ B_3 = 0,$$

and the remaining two satellites are on the same orbit

$$D_{1,2} = 0.5D_0, \ A_{1,2} = A_0, \ B_{1,2} = B_0.$$

In addition, phase shift between them must be

$$\psi_2 - \psi_1 = \lambda_2 - \lambda_1 = \text{acos}\frac{1}{3}.$$

In order to ensure this motion, we suggest simple idea: when shift and drift are within acceptable vicinity of the required values, control should work only when it helps to achieve correct phase shift between the satellites. For example, if $\psi_2 - \psi_1 > \text{acos}(1/3)$, it is necessary to increase $\dot{\psi}_1$, therefore control is applied only if

$$u_y \cos\psi_1 < 0.$$

Relative orbit radius is controlled in the same way, but only when both the phase shift and drift are in the vicinity of the required ones. It should be noted that we consider here the control that is applied only along the velocity of the satellite, therefore direction along orbit normal is uncontrollable.

Control implementation

In order to provide the necessary control atmospheric drag is utilized. It creates the force which is antiparallel to the satellite velocity, and some component that is aligned with the normal to satellite surface (so-called atmospheric lift), which is usually one order of magnitude lesser than drag force. Further we will suppose that satellites are the thin plates that have an area S and normal to their surface n_i. In this case drag force can be described in the following way:

$$\mathbf{F} = -\rho_{atm}\left(\mathbf{V}_i, \mathbf{n}_i\right)\left(\left(1-\varepsilon\right)\mathbf{V}_i + 2\varepsilon\left(\mathbf{V}_i, \mathbf{n}_i\right)\mathbf{n}_i + \left(1-\varepsilon\right)\alpha\mathbf{n}_i\right)S,$$

where ρ_{atm} is atmospheric density, $\mathbf{V}_i$ is satellite velocity w.r.t. the atmosphere, ε, α are some parameters that describe what part of the flow is absorbed or reflected.

In according with suggested controller, it is necessary to provide control in both directions: along and antiparallel to velocity of the chief satellite. Since $u_{y,i} = f_{y,i} - f_{y,c}$, where $f_{y,i}$ denotes drag force that affects the i-th satellite, and $f_{y,c}$ is drag force that affects chief satellite, the required control might be provided in the following way. Chief's normal in the ORF is described as

$$n_c = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right),$$

which means that drag force affecting it is equal to $0.5 f_{max}$, where f_{max} denotes the maximum possible drag force (here we omit the reflected part of the flow: as was mentioned earlier, it is rather small). Therefore, since $f_{y,i}$ lies in the interval $[-f_{max}, 0]$, the resulting control $u_{y,i}$ lies in the interval $\left[-\frac{f_{max}}{2}, \frac{f_{max}}{2}\right]$. Hence, control which must be produce is chosen in the following way:

$$u_{y,i} = \begin{cases} u_{y,i} & if \;\; |u_{y,i}| < \dfrac{f_{max}}{2} \\ \dfrac{f_{max}}{2}\operatorname{sign}(u_{y,i}) & if \;\; |u_{y,i}| \geq \dfrac{f_{max}}{2} \end{cases}$$

It should be noted that $u_{y,i}$ not fully defines the satellite normal attitude: it defines only the angle between the velocity direction and $\mathbf{n}_i$. To fully describe it we will use the following technique: since there is also a lift force (which might affect the motion along orbit normal), we will choose the $\mathbf{n}_i$ direction in such a way, that lift force helps to preserve the necessary out-of-plain motion amplitude and phase shift, i.e. it will lie in the OYZ plane.

SIMULATION RESULTS

In order to test suggested control technique the following problem was simulated.

- Satellites are the 3U CubeSats with a drop-down solar panels.
- Mass is 5 kg.
- Area is 0.15 m^2.
- All the satellites move under Newtonian gravity field, as well as J2 and atmospheric drag perturbations.
- Formation size K is 1 km.

In Figure 2 different relative osculating parameters are presented. They correspond to the passive motion in the Newtonian field with J2 perturbation (without atmospheric drag). As we can see, part of them stays almost the same (e.g. in-plane phase shift and amplitude), but some of them changes drastically, such as relative shift and out-of-plane parameters. It leads to fast degradation of the tetrahedron.

In figure 3 the same parameters are presented, but now the control using atmospheric drag is utilized. As we can see, in this case all in-plane parameters are almost the same as necessary, and only out-of-plane motion is different. As we can see, in this case tetrahedron quality is almost preserved.

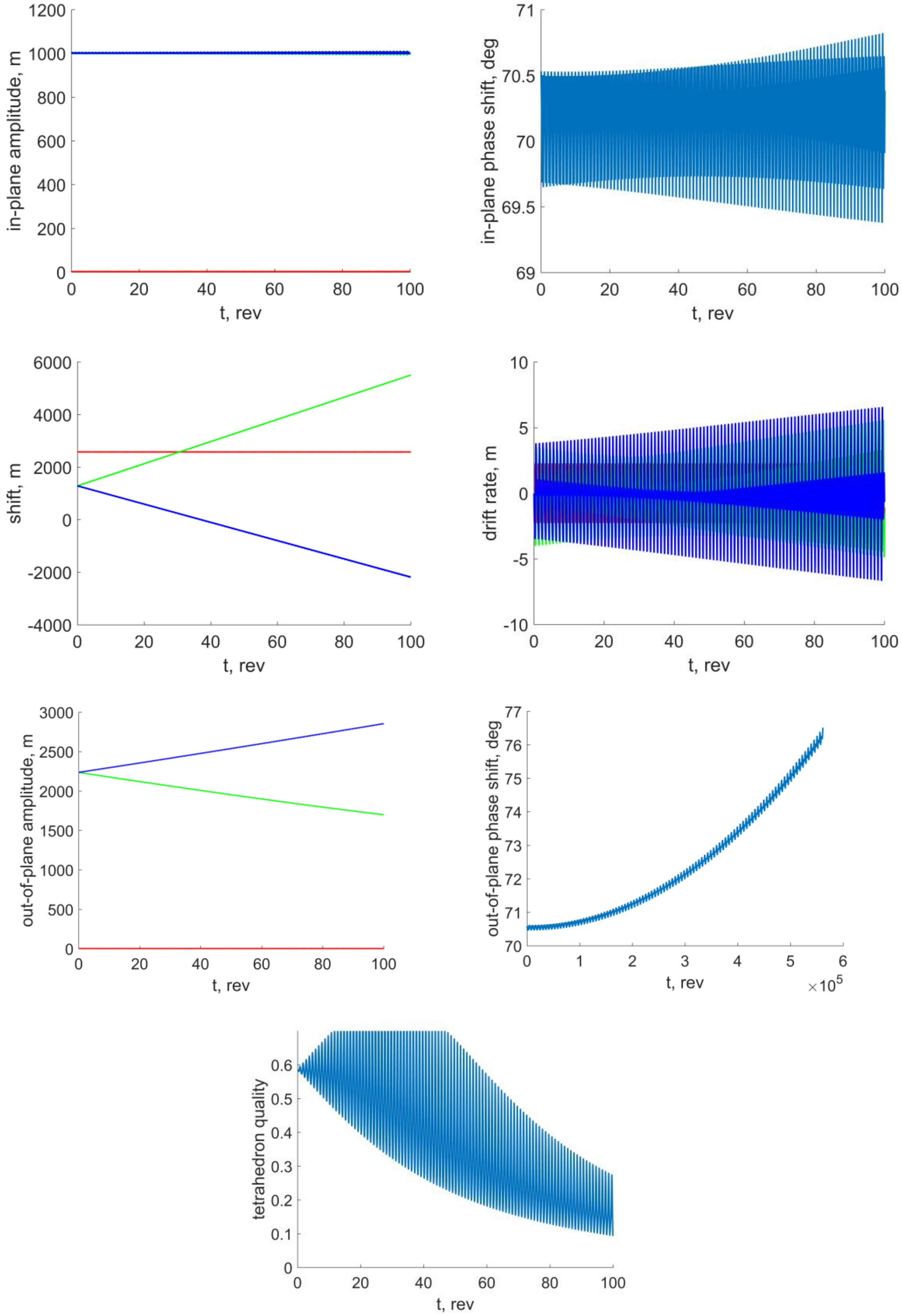

Figure 2 Relative motion without control

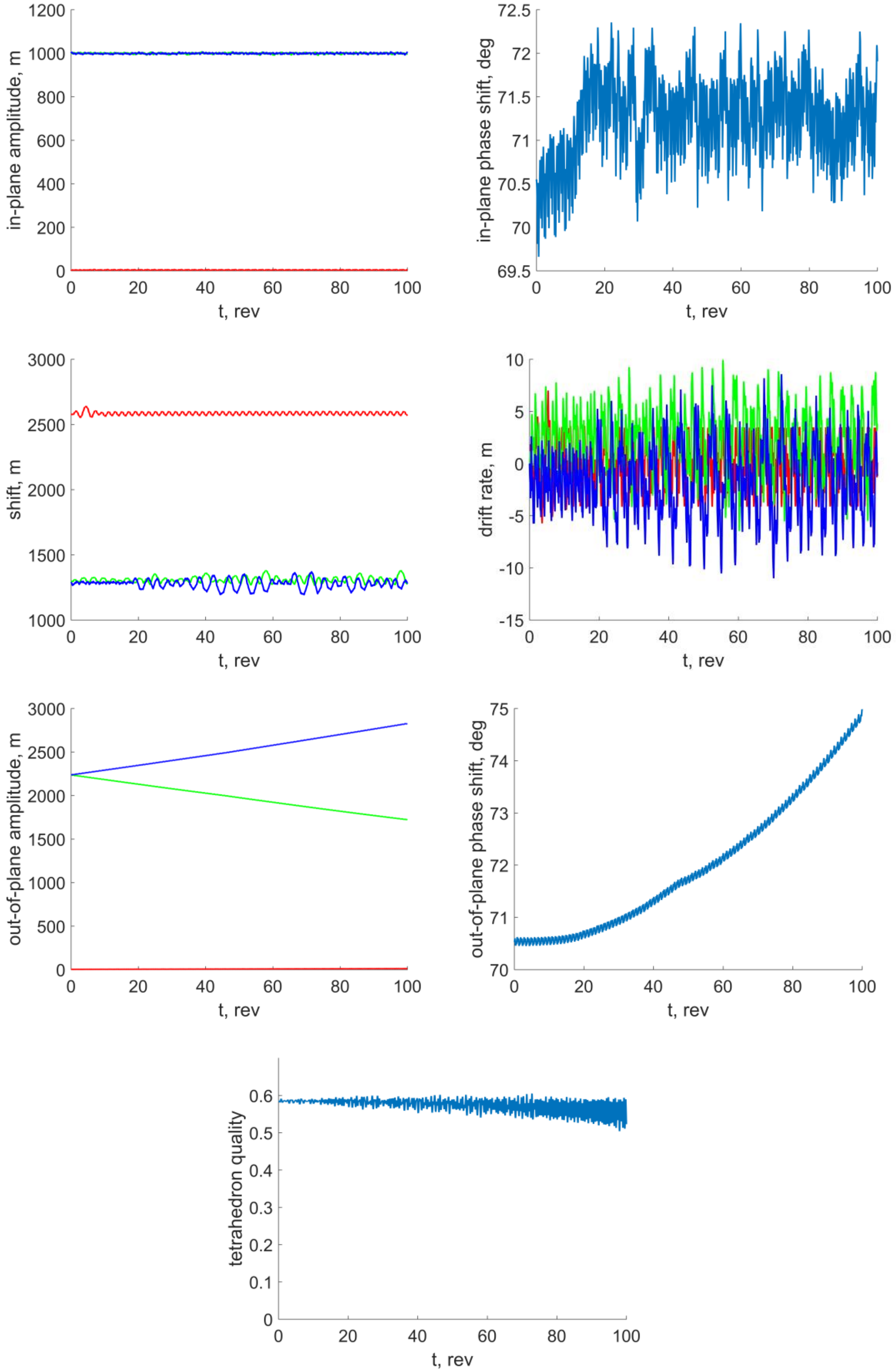

Figure 3 Relative motion under control

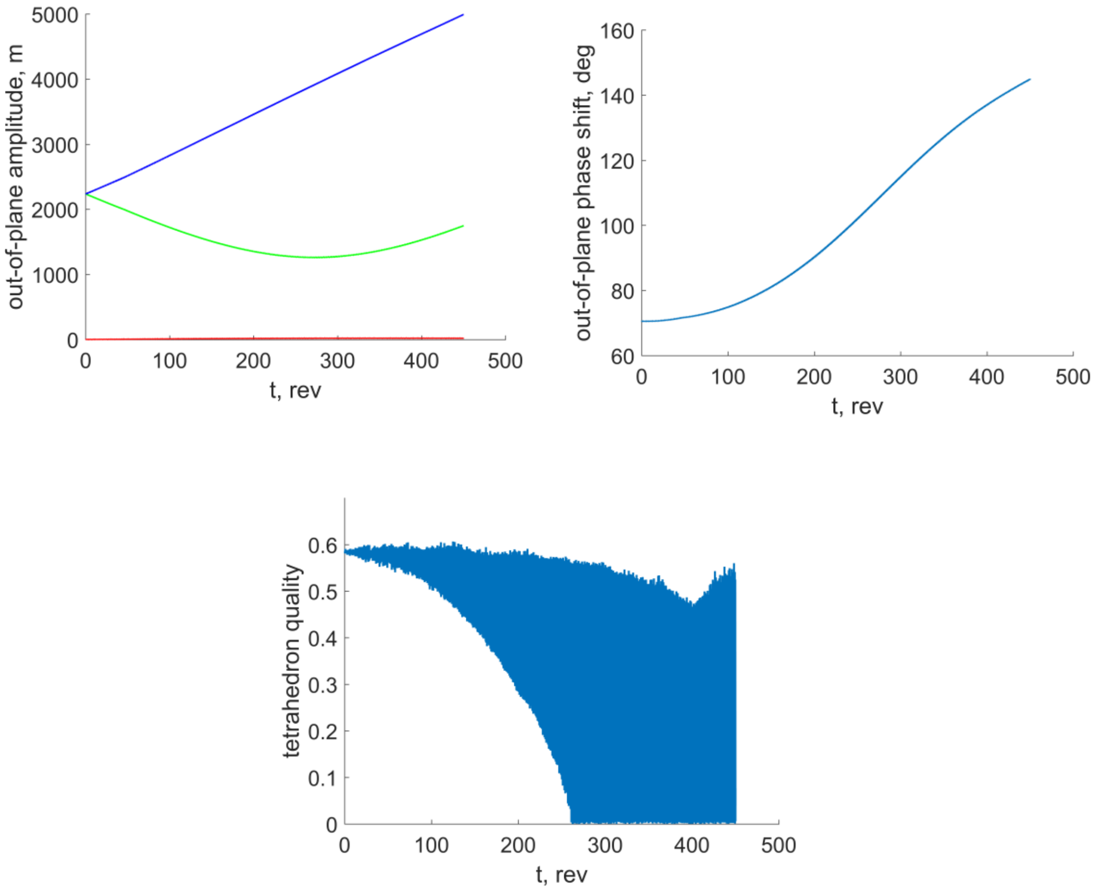

Figure 4 Tetrahedron quality evolution for long period of time

DISCUSSION AND CONCLUSION

Simulation shows that suggested technique can effectively control relative in-plane motion of the satellites, thus slowing degradation rate of the tetrahedron. It should be noted that out-of-plane motion is almost uncontrollable, because the lift force is too small. Therefore, even though it is possible to decelerate the degradation rate of relative parameters along this direction, they still degrade. This, in turn, leads to degradation of the tetrahedron: as we can see in Figure 4, after a month it degrades completely, therefore additional control (e.g. using thrusters) is required.

ACKNOWLEDGMENTS

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