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Tetrahedron formation maintenance via atmospheric drag control

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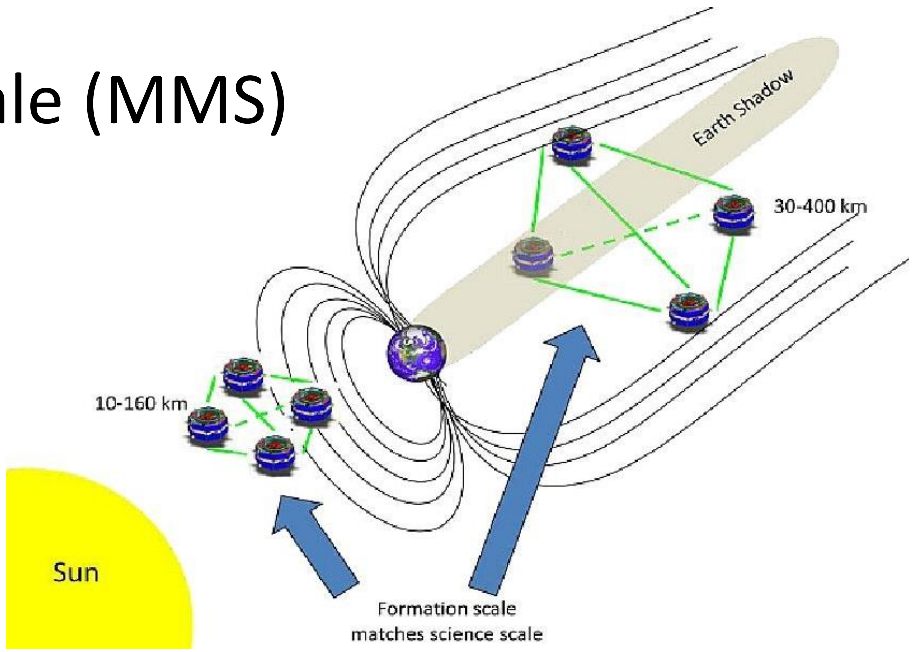
Introduction

Missions with tetrahedral formations

- Cluster/Cluster II
- The Auroral Lites
- Magnetospheric Multiscale (MMS)



Using tetrahedral formation

- Why?
  - Physical quantities change in time and space simultaneously
  - Even in linear approximation one needs to have four measurements to have a gradient
- Magnetosphere study
- Need to maintain the tetrahedron
  - Nondegenerate for all the lifetime of the mission
  - Possibly the closest to the regular one
- It is common and convenient to describe the tetrahedron using several scalar parameters

Problem statement

- Four satellites move on close LEOs
- One is moving along circular trajectory
- Need to obtain a reference orbit in order that the volume and shape of the corresponding tetrahedron maintain over time
- Size and shape must be formalized
- Also provide a simple control algorithm for several satellites to neglect perturbations

Tetrahedral configuration

- Quality of the tetrahedron is chosen to be meaningful in geometric sense and analytically analyzable

$$Q = 12\frac{(3|\mathrm{V}|)^{2/3}}{r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2} = 12\frac{(3|\mathrm{V}|)^{2/3}}{\mathrm{L}} = 3\frac{\sqrt[3]{\sigma_1^2\sigma_2^2\sigma_3^2}}{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

- Linearized HCW model, closed orbits described by

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0, \qquad x_i(t) = A_i \sin\nu + B_i \cos\nu,$$

$$\ddot{y} + 2n\dot{x} = 0, \qquad y_i(t) = 2A_i \cos\nu - 2B_i \sin\nu + C_i,$$

$$\ddot{z} + n^2 z = 0 \qquad z_i(t) = D_i \sin\nu + E_i \cos\nu,$$

- The main goal is to find such reference orbit that in passive motion in linearized model volume and quality of the tetrahedron remain constant

Quality/Volume conservation

The set of equations so that the tetrahedron does not change volume and quality in linear model

$$D_1 = aA_1 + bB_1, \quad D_2 = aA_2 + bB_2, \quad D_3 = aA_3 + bB_3,$$

$$E_1 = -bA_1 + aB_1, \quad E_2 = -bA_2 + aB_2, \quad E_3 = -bA_3 + aB_3,$$

$$3A_1B_1 + 3A_2B_2 + 3A_3B_3 - A_1B_2 - A_1B_3 - A_2B_1 - A_2B_3 - A_3B_1 - A_3B_2 = 0,$$

$$3\left(B_1^2 + B_2^2 + B_3^2 - A_1^2 - A_2^2 - A_3^2\right) + 2\left(A_1A_2 + A_1A_3 + A_2A_3 - B_1B_2 - B_1B_3 - B_2B_3\right) = 0,$$

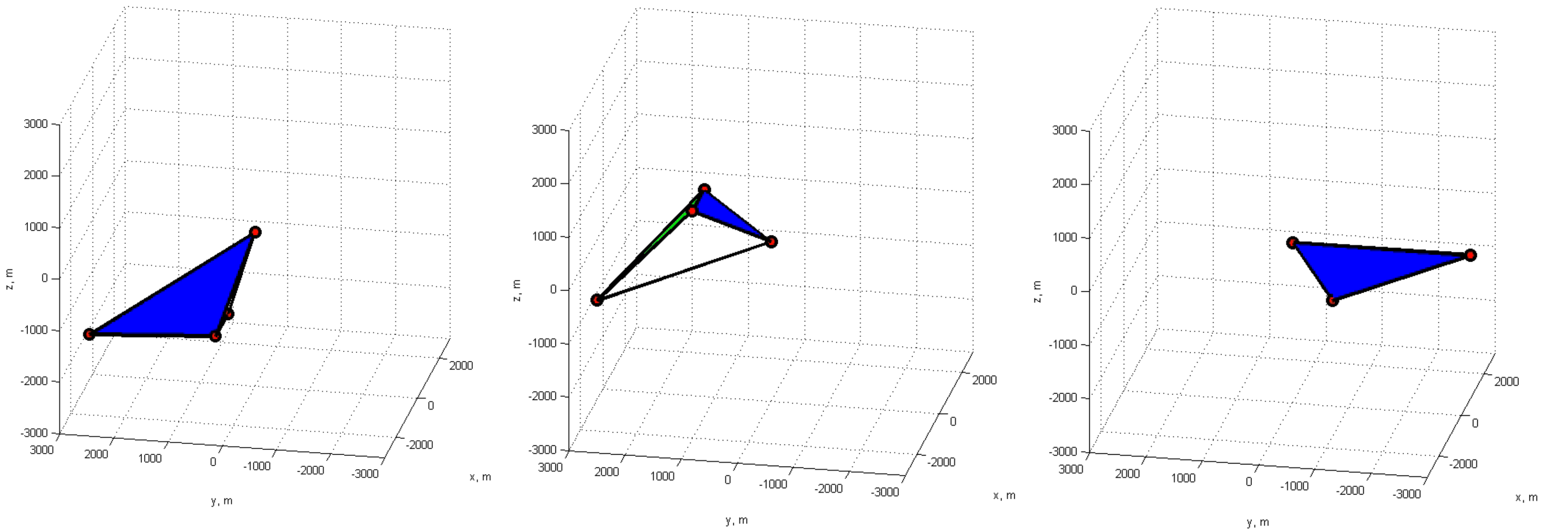
$$C_1(3A_1 - A_2 - A_3) + C_2(3A_2 - A_1 - A_3) + C_3(3A_3 - A_1 - A_2) = 0,$$

$$C_1(3B_1 - B_2 - B_3) + C_2(3B_2 - B_1 - B_3) + C_3(3B_3 - B_1 - B_2) = 0.$$

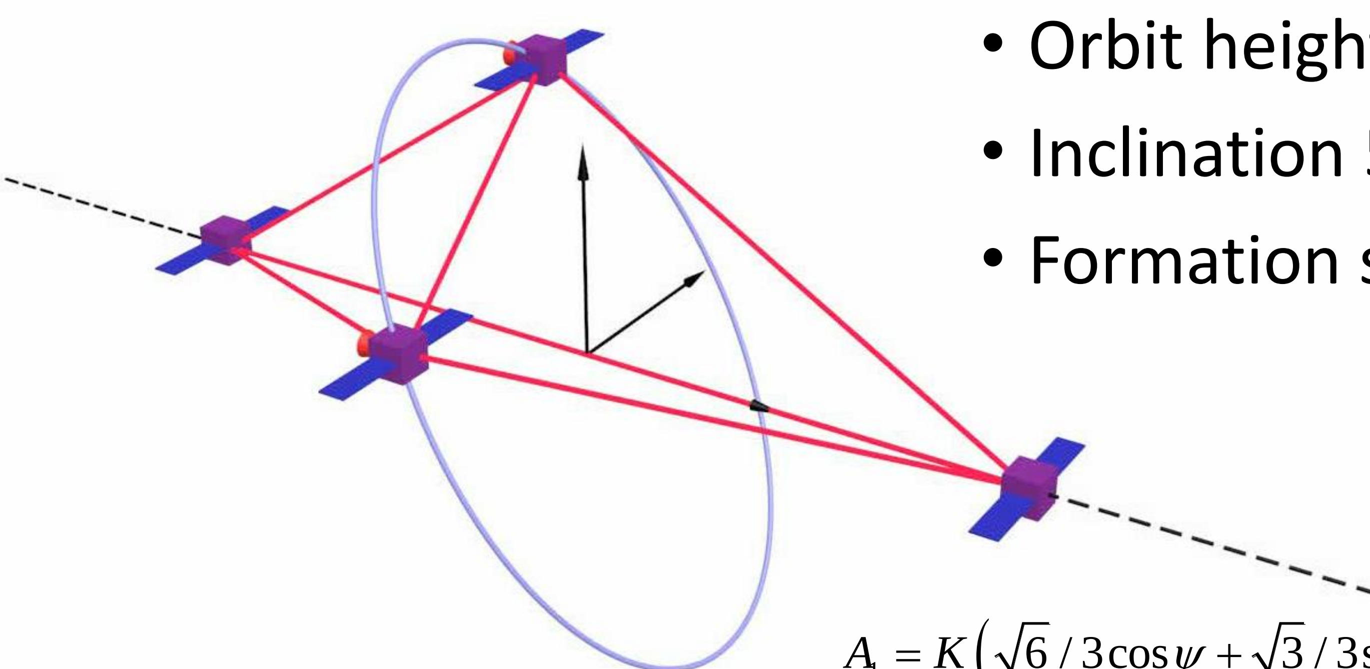
Quality/Volume conservation

Solutions are 7-parametric families

Some of the solutions are presented below



Focus on leader-follower formation



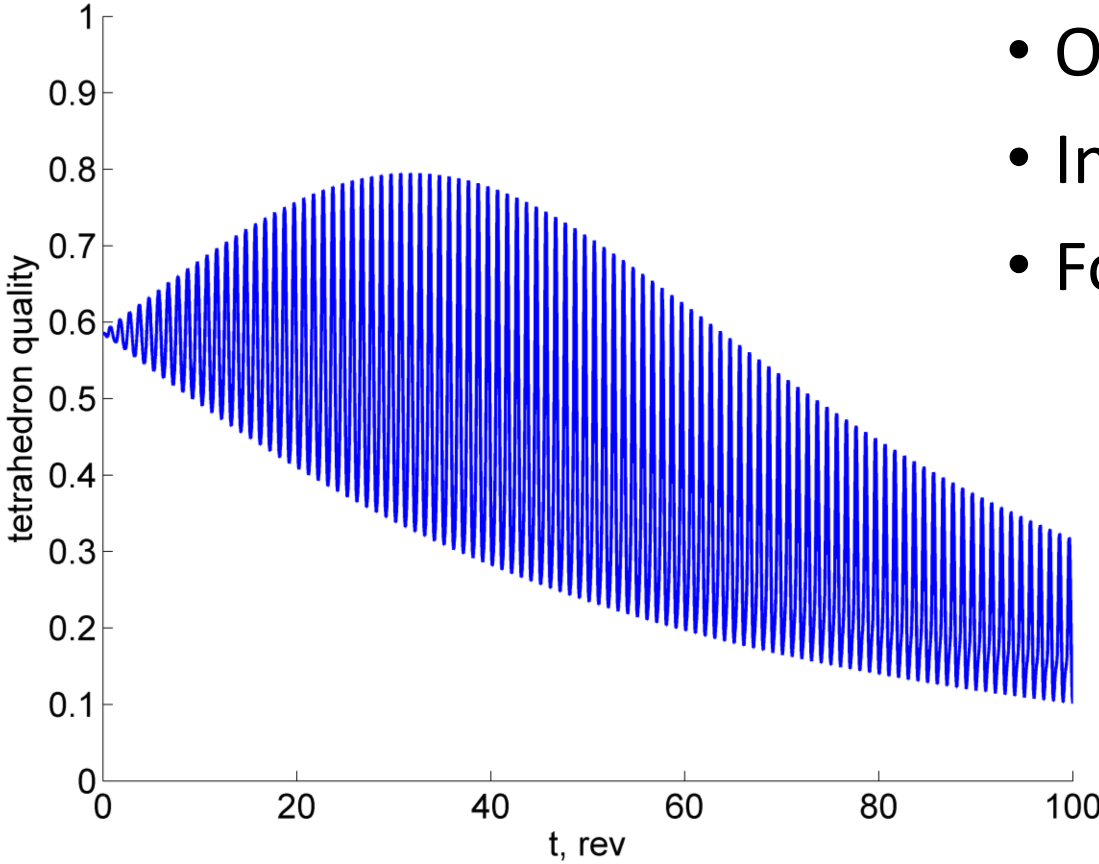
- Orbit height – 400 km
- Inclination 51.6 deg
- Formation size K = 1000 m

$$A_1 = K\left(\sqrt{6}/3\cos\psi + \sqrt{3}/3\sin\psi\right),$$
$$A_2 = K\left(\sqrt{6}/3\cos\psi - \sqrt{3}/3\sin\psi\right),$$
$$A_3 = 0,$$
$$B_1 = K\left(-\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right),$$
$$B_2 = K\left(\sqrt{3}/3\cos\psi + \sqrt{6}/3\sin\psi\right),$$
$$B_3 = 0.$$

$$C_1 = C_2 = K\sqrt{\frac{5}{3}},$$
$$C_3 = 2K\sqrt{\frac{5}{3}}$$

$$\mathbf{D} = \sqrt{5}\mathbf{B},$$
$$\mathbf{E} = -\sqrt{5}\mathbf{A}$$

Focus on leader-follower formation



- Orbit height – 400 km
- Inclination 51.6 deg
- Formation size – 1000 m

Need to control formation

Using curvilinear coordinates

More natural to use and to describe relative motion

$$\frac{d^2}{dt^2}\rho - 2n\frac{d}{dt}\left(a_0\varphi\right) - 3n^2\rho = 0$$

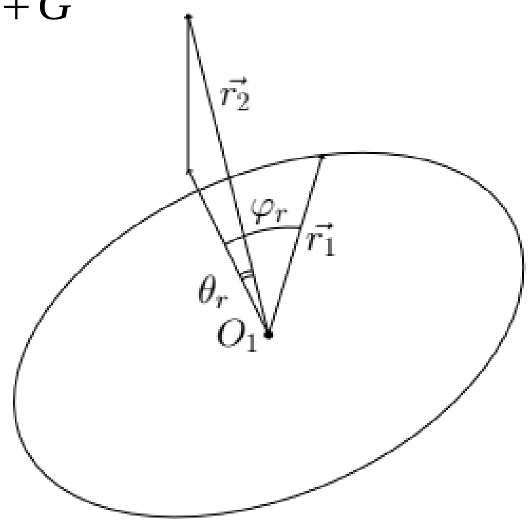
$$\frac{d^2}{dt^2}\left(a_0\varphi\right) + 2n\frac{d}{dt}\rho = 0$$

$$\frac{d^2}{dt^2}\left(a_0\theta\right) + n^2\left(a_0\theta\right) = 0$$

$$\rho = A\cos\left(nt+\gamma\right) + 2C,$$

$$a_0\varphi = -2A\sin\left(nt+\gamma\right) - 3Cnt + G$$

$$a_0\theta = B\sin\left(nt+\delta\right)$$



We use the same set of initial conditions for leader-follower formation

Need to maintain amplitudes A,B and phase differences $\gamma_1 - \gamma_2 = \delta_1 - \delta_2 = \mathrm{acos}(1/3)$

Osculating coordinates

New variables:

$$\rho = A\sin\psi + 2C,$$

$$\frac{d}{dt}\rho = An\cos\psi,$$

$$a_0\theta = 2A\cos\psi + D,$$

$$\frac{d}{dt}\left(a_0\theta\right) = -2An\sin\psi - 3C\omega,$$

$$a_0\varphi = B\sin\lambda,$$

$$\frac{d}{dt}\left(a_0\varphi_0\right) = Bn\cos\lambda.$$

The system is described by

$$\dot{A} = \frac{1}{n}\Big(\left(u_x + g_x\right)\cos\psi - 2\left(u_y + g_y\right)\sin\psi\Big),$$

$$\dot{B} = \frac{1}{n}\left(u_z + g_z\right)\cos\lambda,$$

$$\dot{C} = \frac{1}{n}\left(u_y + g_y\right),$$

$$\dot{D} = -3Cn - \frac{2}{n}\left(u_x + g_x\right),$$

$$\dot{\lambda} = n - \frac{1}{nB}\left(u_z + g_z\right)\sin\lambda.$$

$$\dot{\psi} = n - \frac{1}{An}\Big(\left(u_x + g_x\right)\sin\psi + 2\left(u_y + g_y\right)\cos\psi\Big).$$

Atmospheric control algorithm

[illegible]

- Asymptotically stable controller
- We neglect additional accelerations
- But acceleration is only along velocity vector

$$\dot{A} = \frac{-2u_y \sin\psi}{n},$$

$$\dot{C} = \frac{u_y}{n},$$

$$\dot{D} = -3Cn.$$

Atmospheric control algorithm

First step – shift adjust and drift elimination

$$C_{ref} = 0, \quad D_{ref} = D_0$$

- Direct Lyapunov method

$$V = C^2 + k_D\left(D - D_{ref}\right)^2, \quad k_D > 0.$$

- Control

$$u_y = 3n^2 k_D\left(D - D_{ref}\right) - k_c C$$

Second step – phasing

- Control only if it helps to phase properly

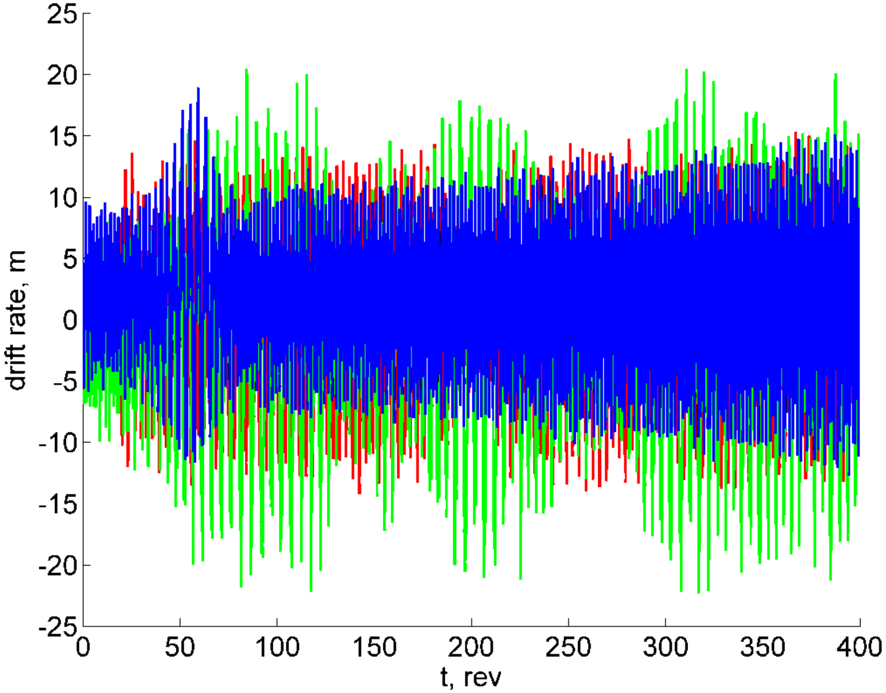
$$\text{if} \quad \psi_2 - \psi_1 > \text{acos}(1/3),$$

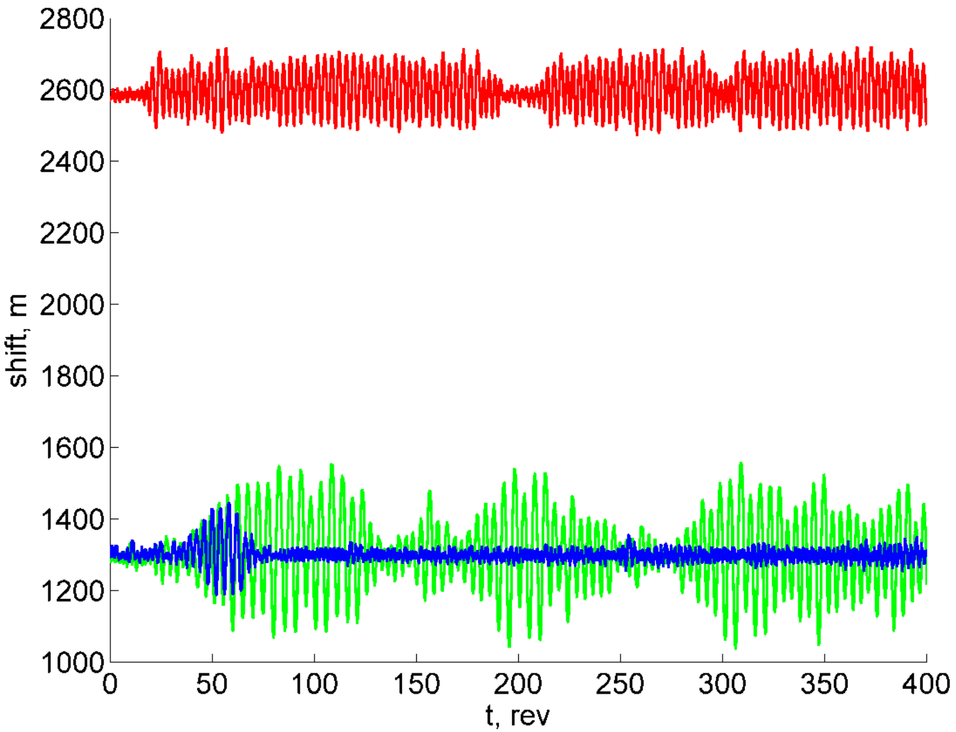
$$\text{then control only if } u_y \cos\psi_1 < 0.$$

Third step – amplitudes A, B

- The same, but if phasing is correct

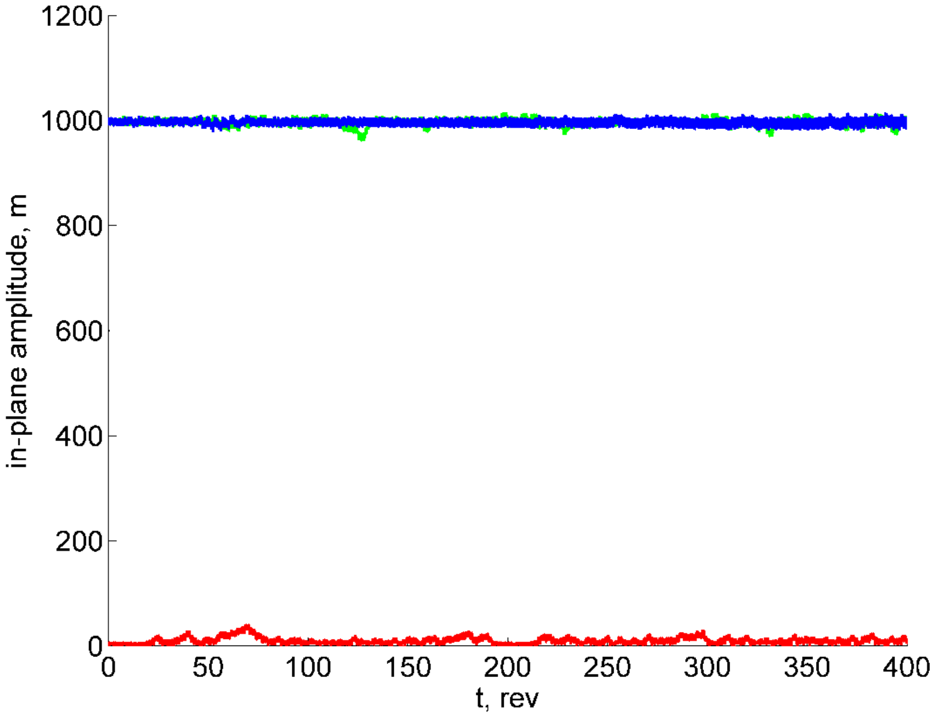
Atmospheric control algorithm

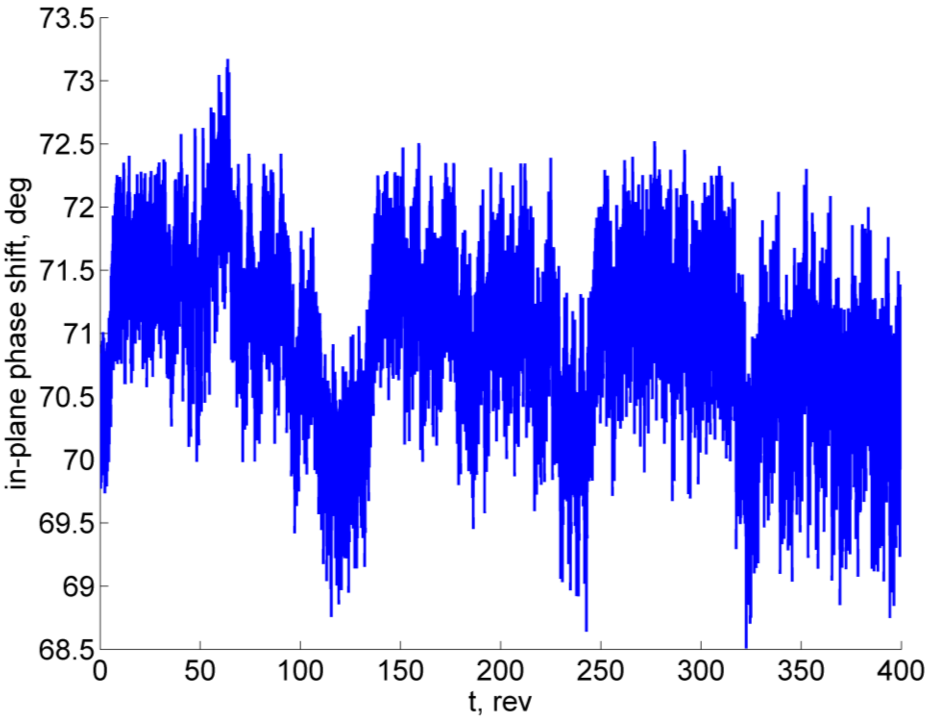




Drift and shift are controllable

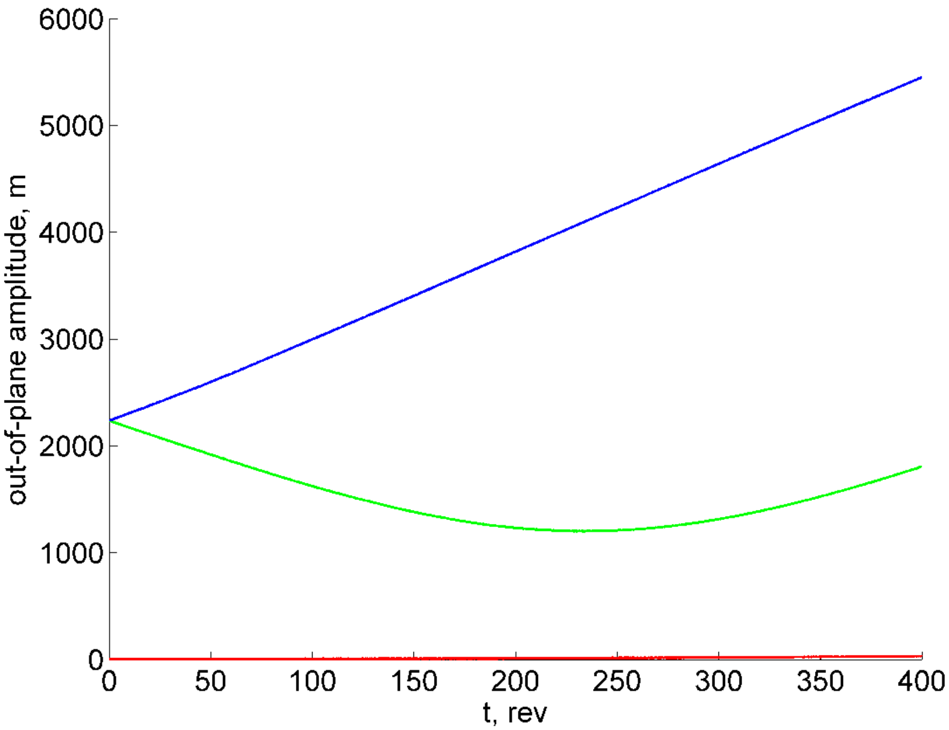
Atmospheric control algorithm

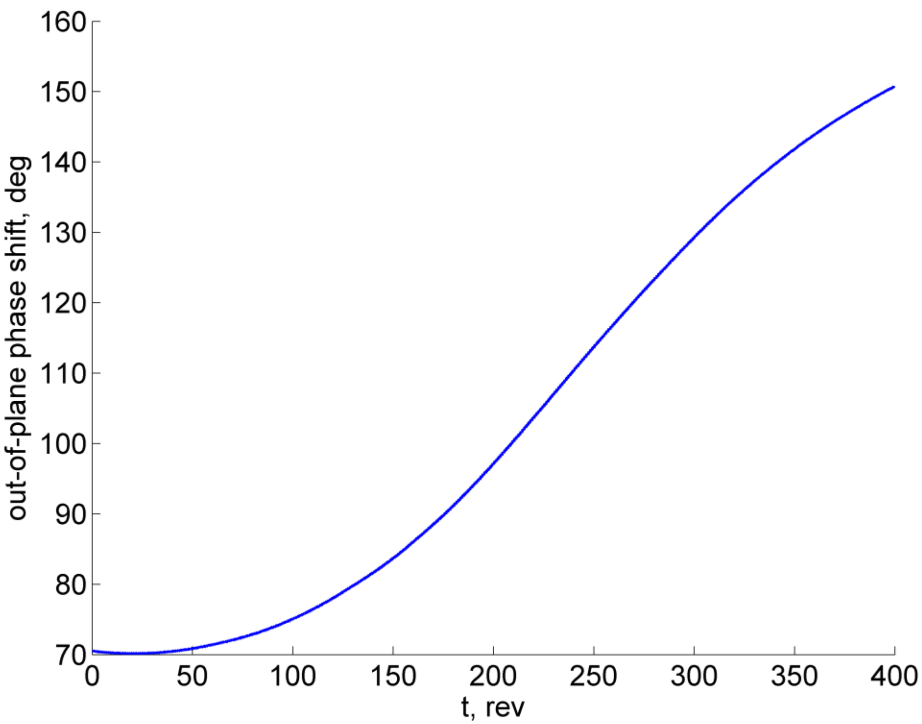




In-plane motion is fully controllable

Atmospheric control algorithm

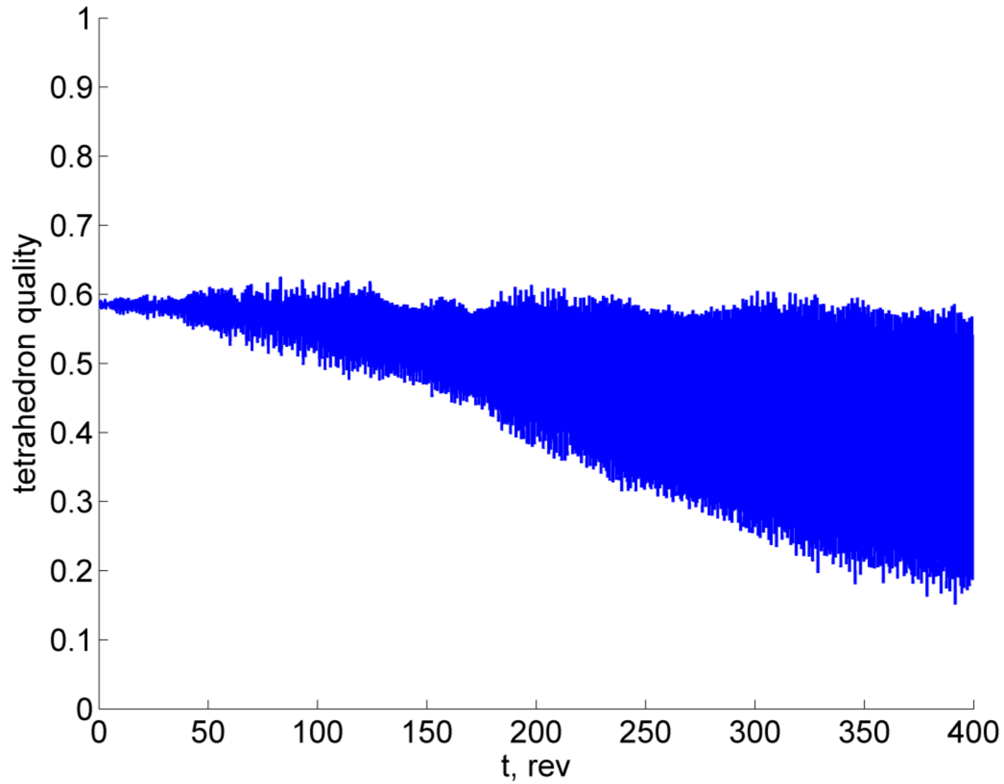




Out-of-plane motion is not controllable

Atmospheric control algorithm

- Tetrahedron configuration conserving quality in linear model
- Tetrahedron size K = 1000 m
- Orbit height = 400 km
- Masses = 5 kg
- Area = 0.15 m^2
- Inclination = 51.6 deg
- Disturbances – J2 and atmosphere



Conclusions

- The reference orbit for 4 satellites conserving volume and shape of the tetrahedron is found in linear model on circular orbit
- Suggested Lyapunov-based controller can effectively control relative in-plane motion of the satellites, thus slowing degradation rate of the tetrahedron
- The out-of-plane motion is almost uncontrollable, because the lift force is too small
- It is possible to decelerate the degradation rate of relative parameters for several weeks

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Questions?