



Coupled Dynamics in a Formation of Satellites with Variable-Reflectivity Solar Sails

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Introduction

IKAROS - Reflection properties variation to control attitude

General idea

Solve attitude and relative two satellite motion control problem by means of solar sail

Goal

Get the elliptic relative orbit

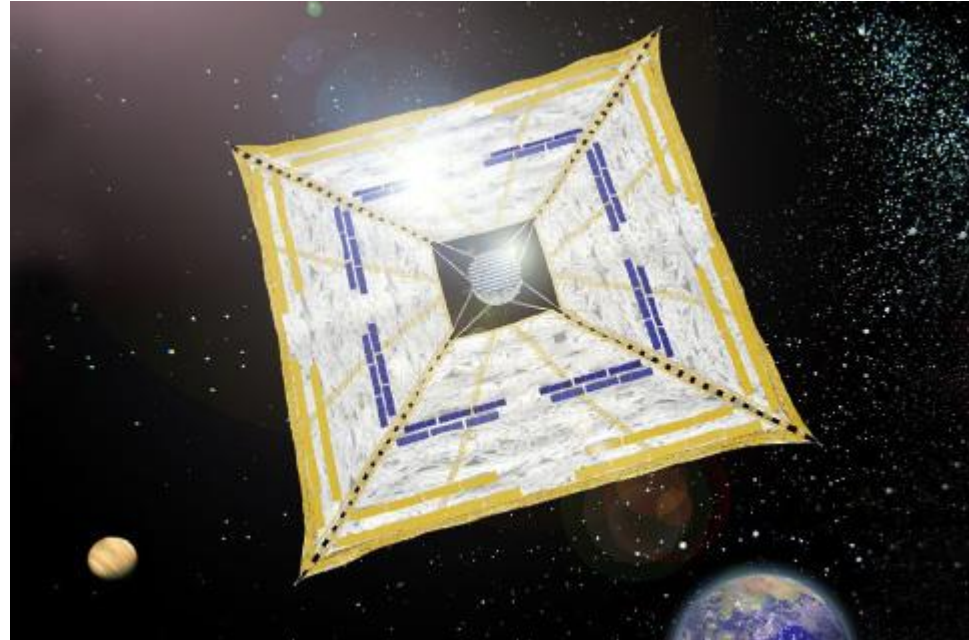
Assumptions

Satellites are identical

Specular reflection only

Center of mass motion: J_2 + solar radiation pressure (SRP)

Angular motion: Gravity gradient torque + SRP



IKAROS mission (global.jaxa.jp)

Motion equations

Orbital dynamics

$$\ddot{\mathbf{r}} = -\mu_E \frac{\mathbf{r}}{r^3} + \mathbf{g}, \quad \mathbf{g} = \mathbf{g}_{J_2} + \mathbf{g}_s, \quad \mathbf{g}_s = A(\mathbf{r}_s, \mathbf{n})((1-f)\mathbf{r}_s + 2f(\mathbf{r}_s, \mathbf{n})\mathbf{n}), \quad f = \frac{\int \alpha dS}{S}$$

Angular dynamics

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M}_s + \mathbf{M}_g, \quad \mathbf{M}_{\text{grav}} = 3\omega_0^2 \mathbf{e}_3 \times \mathbf{J}\mathbf{e}_3, \quad \mathbf{M}_s = \int \mathbf{r} \times d\mathbf{F}_s$$

Relative motion dynamics (control synthesis only)

$$(a_0 \ddot{\theta}_r) + 2\omega(a_0 \dot{\varphi}_r) = u_x,$$

$$(a_0 \ddot{\theta}_r) + \omega^2(a_0 \theta_r) = u_y,$$

$$\ddot{\rho} - 2\omega\dot{\rho} - 3\omega^2(a_0 \varphi_r) = u_z.$$

$$\mathbf{u} = \mathbf{0}$$

$$\Rightarrow$$

$$a_0 \theta_r = -3C_1 \omega t + 2C_2 \cos \omega t - 2C_3 \sin \omega t + C_4;$$

$$a_0 \varphi_r = C_5 \cos \omega t + C_6 \sin \omega t;$$

$$\rho = 2C_1 + C_2 \sin \omega t + C_3 \cos \omega t.$$

$$\mathbf{u} = \frac{\mathbf{F}_{s,2} - \mathbf{F}_{s,1}}{m}$$

Motion equations (relative motion parameterization)

$$a_0 \theta_r = 2B_2 \cos \psi_1 + B_3,$$

$$\rho = B_2 \sin \psi_1 + 2B_1,$$

$$(\dot{a}_0 \theta_r) = -2B_2 \omega \sin \psi_1 - 3B_1 \omega,$$

$$\dot{\rho} = B_2 \omega \cos \psi_1,$$

$$a_0 \phi_r = B_4 \cos \psi_2,$$

$$(\dot{a}_0 \phi_r) = -B_4 \omega \sin \psi_2.$$

Goal

Ellipse with center at (0,0) and semi-axes $2B_{20}$ and B_{20}

$$B_1 = 0, B_2 = B_{20}, B_3 = 0, B_4 = 0$$

$$\dot{B}_1 = \frac{1}{\omega} u_x,$$

$$\dot{B}_3 = 3B_1 \omega - \frac{2}{\omega} u_z,$$

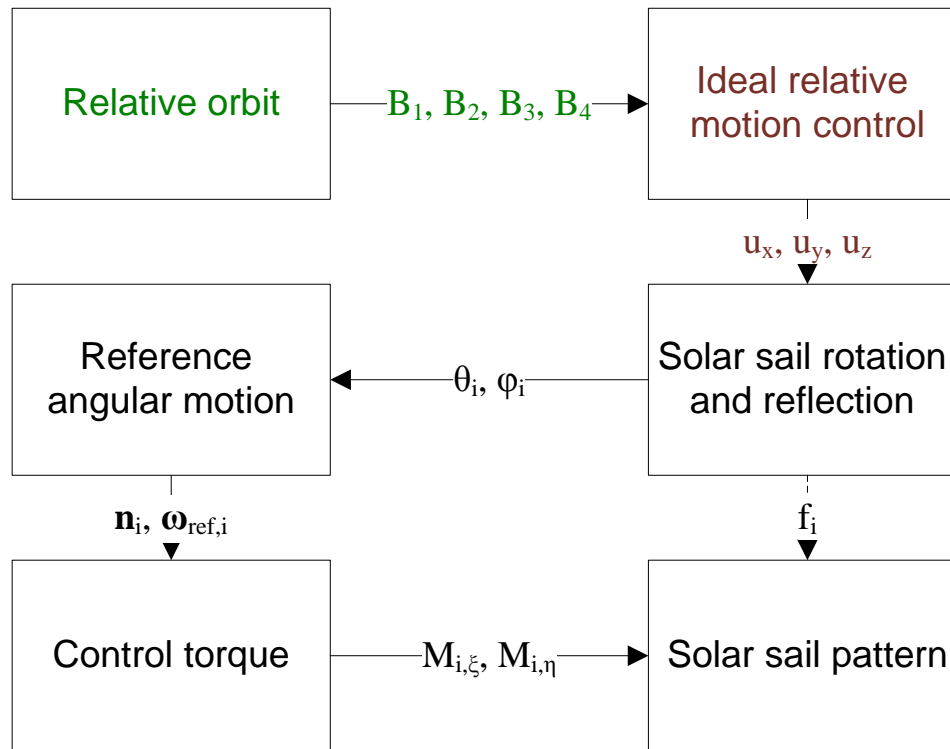
$$\dot{B}_2 = \frac{1}{\omega} (u_z \cos \psi_1 - 2u_x \sin \psi_1),$$

$$\dot{\psi}_1 = \omega - \frac{1}{B_2 \omega} (u_z \sin \psi_1 + 2u_x \cos \psi_1),$$

$$\dot{B}_4 = \frac{1}{\omega} u_y \cos \psi_2,$$

$$\dot{\psi}_2 = \omega - \frac{1}{\omega B_4} u_y \sin \psi_2.$$

Control synthesis scheme



Control force synthesis

Relative orbit center stabilization

$$V = \frac{1}{2} B_1^2 + \frac{1}{2} B_3^2$$

$$\dot{V} = B_1 \dot{B}_1 + B_3 \dot{B}_3 = \frac{1}{\omega} B_1 u_x + B_3 \left(3B_1 \omega - \frac{2}{\omega} u_z \right)$$

$$u_x = -k_1 B_1, \quad k_1 > 0,$$

$$u_z = \frac{1}{2} (3B_1 \omega^2 + k_2 \omega B_3), \quad k_2 > 0.$$

Relative orbit size stabilization (B_1 and B_3 is small)

$$V = \frac{1}{2} B_1^2 + \frac{1}{2} B_3^2 + \frac{1}{2} (B_2 - B_{20})^2$$

$$\dot{V} = \frac{1}{\omega} (B_1 - 2(B_2 - B_0) \sin \psi_1) u_x +$$

$$+ \frac{1}{\omega} (-2B_3 + (B_2 - B_0) \cos \psi_1) u_z - 3B_1 B_3 \omega$$

$$u_{\max} > |3B_1 B_3 \omega^2| \quad \text{stability condition}$$

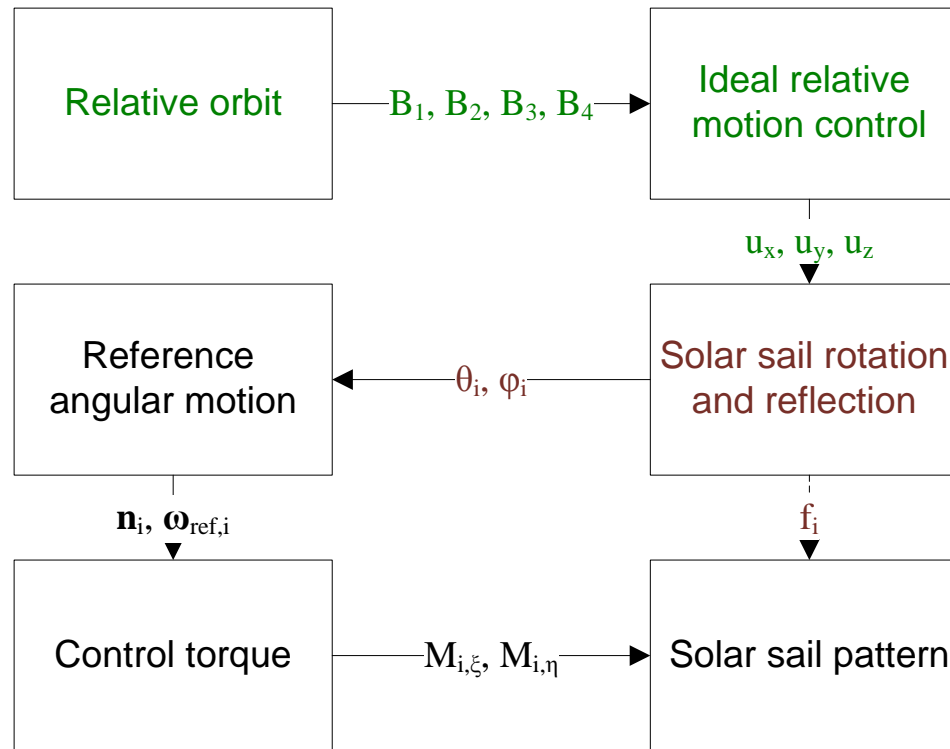
$$u_x = -k_3 (B_1 - 2(B_2 - B_0) \sin \psi_1), \quad k_3 > 0$$

$$u_z = -k_4 (-2B_3 + (B_2 - B_0) \cos \psi_1), \quad k_4 > 0$$

Out-of-plane stabilization

$$u_y = -k_y B_4 \cos \psi_2, \quad k_y > 0$$

Control synthesis scheme



Sail attitude and reflection

Solar sail normal

$$\mathbf{n} = \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix}$$

Control for small θ_i

$$u_{x_s} = 2Af_2\theta_2 \cos \varphi_2 - 2Af_1\theta_1 \cos \varphi_1,$$

$$u_{y_s} = 2Af_2\theta_2 \sin \varphi_2 - 2Af_1\theta_1 \sin \varphi_1,$$

$$u_{z_s} = Af_2 - Af_1.$$

f=0 and f=1 zero torque
f=0.5 maximum torque

$$(f_1 - 0.5)^2 + (f_2 - 0.5)^2 \rightarrow \min$$

$$f_2 - f_1 = \frac{u_{z_s}}{A},$$

$$0 < f_{\min} \leq f_i \leq f_{\max} < 1, i = 1, 2.$$

$$f_1 = 0.5 - \frac{u_{z_s}}{2A} \quad f_{\min} = 0.25, f_{\max} = 0.75$$

$$f_2 = 0.5 + \frac{u_{z_s}}{2A} \quad -0.5 \leq \frac{u_{z_s}}{A} \leq 0.5$$

Maximum u_x and u_y

$$L = (f_2\theta_2 \cos \varphi_2 - f_1\theta_1 \cos \varphi_1)^2 +$$

$$+ (f_2\theta_2 \sin \varphi_2 - f_1\theta_1 \sin \varphi_1)^2.$$

$$\varphi_1 = \varphi_2, \theta_1\theta_2 < 0$$

$$\varphi_1 = \varphi_2 + \pi, \theta_1\theta_2 > 0$$

Minimum θ_i

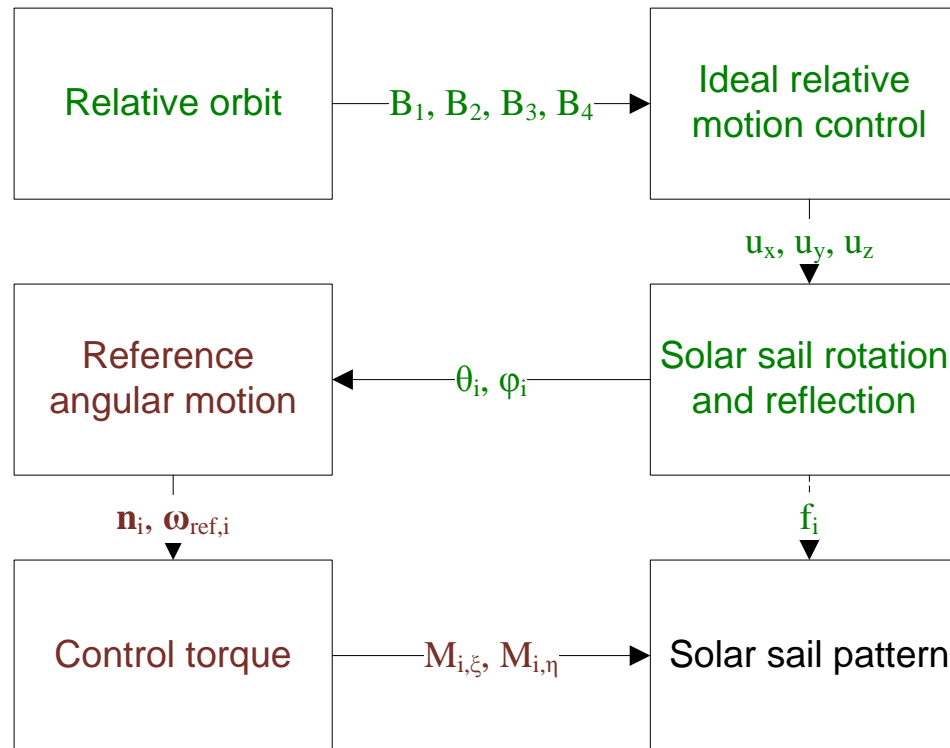
$$L = \theta_1^2 + \theta_2^2,$$

$$f_2\theta_2 - f_1\theta_1 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}$$

$$\theta_1 = -\frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A} \frac{f_1}{f_1^2 + f_2^2},$$

$$\theta_2 = \frac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A} \frac{f_2}{f_1^2 + f_2^2}.$$

Control synthesis scheme



Attitude motion control

SRP torque

$$\begin{pmatrix} M_\xi \\ M_\eta \\ M_\zeta \end{pmatrix} = \frac{\Phi_0}{c} \cos \theta \begin{pmatrix} -P \cos \theta \\ -Q \cos \theta \\ Q \sin \theta \sin \beta + P \sin \theta \cos \beta \end{pmatrix} \quad P = -\int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \eta \alpha d\xi d\eta \quad Q = -\int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \xi \alpha d\xi d\eta$$

Lyapunov control function

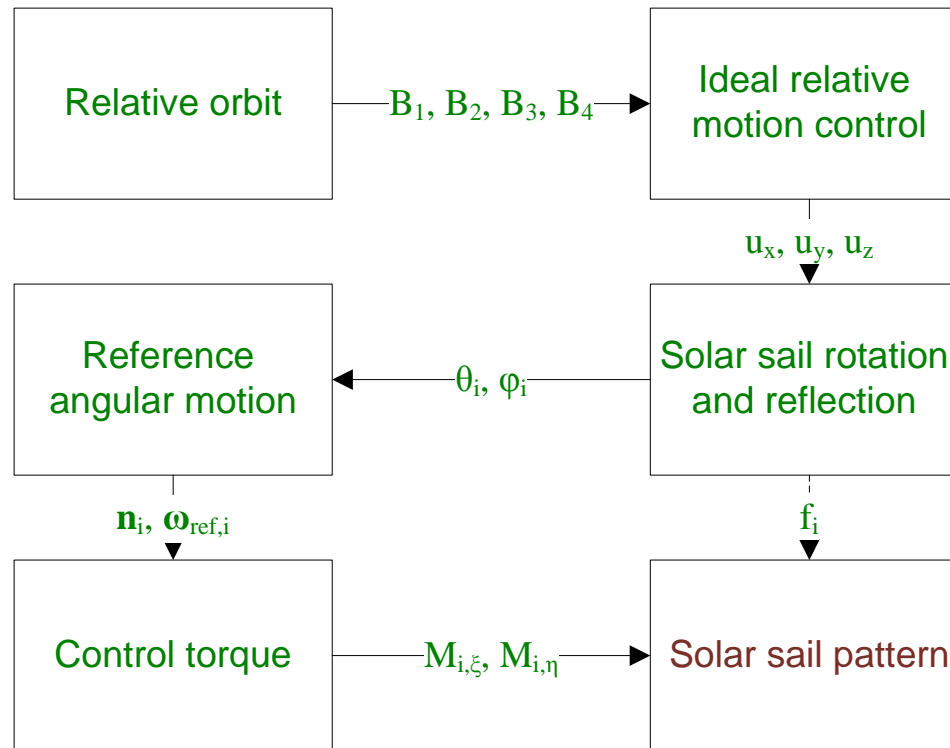
$$V = \frac{1}{2} (J_\xi \omega_{rel,1}^2 + J_\eta \omega_{rel,2}^2) + k_a \left(1 - \left(\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T, \mathbf{Bn} \right) \right)$$

$$\boldsymbol{\omega}_{rel} = \boldsymbol{\omega} - \boldsymbol{\omega}_{ref}, \quad \boldsymbol{\omega}_{ref} = \mathbf{n} \times \dot{\mathbf{n}}$$

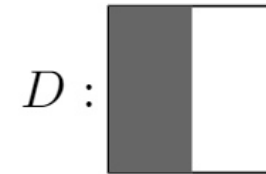
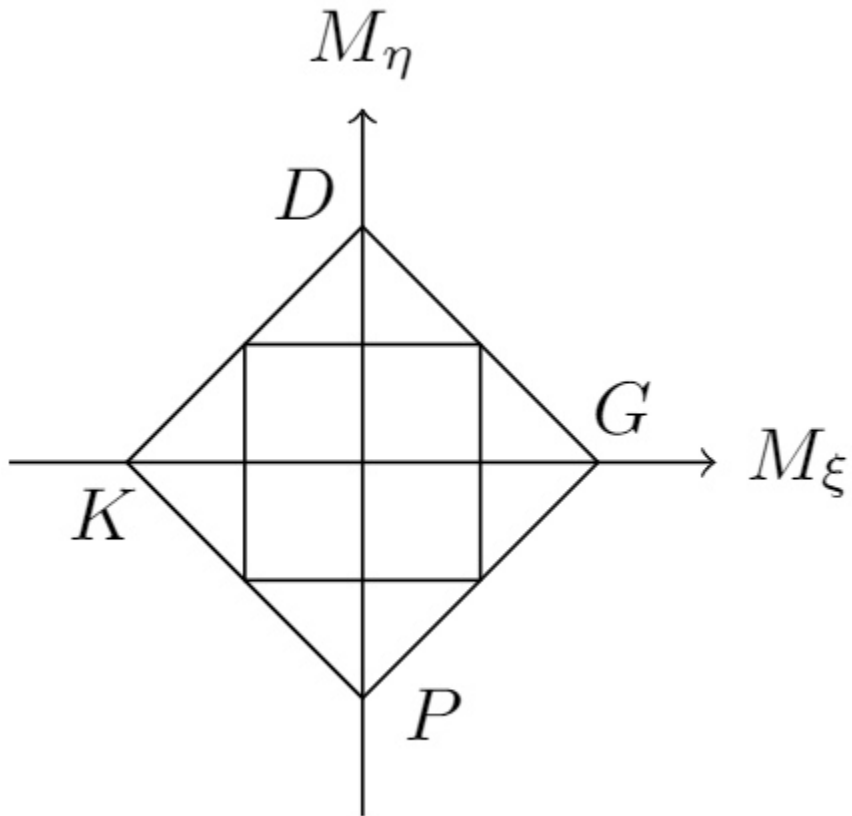
$$\dot{V} = \boldsymbol{\omega}_{rel}^T \left(\mathbf{J} \dot{\boldsymbol{\omega}}_{rel} + k_a \mathbf{Bn} \times \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T \right)$$

$$\mathbf{M}_{control} = \left(-k_\omega \boldsymbol{\omega}_{rel} - \mathbf{M}_{ext} + \boldsymbol{\omega} \times \mathbf{J} \boldsymbol{\omega} - \mathbf{J} \boldsymbol{\omega} \times \mathbf{B} \boldsymbol{\omega}_{ref} + \mathbf{J} \mathbf{B} \dot{\boldsymbol{\omega}}_{ref} - k_a \mathbf{Bn} \times \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T \right)_{\xi, \eta}$$

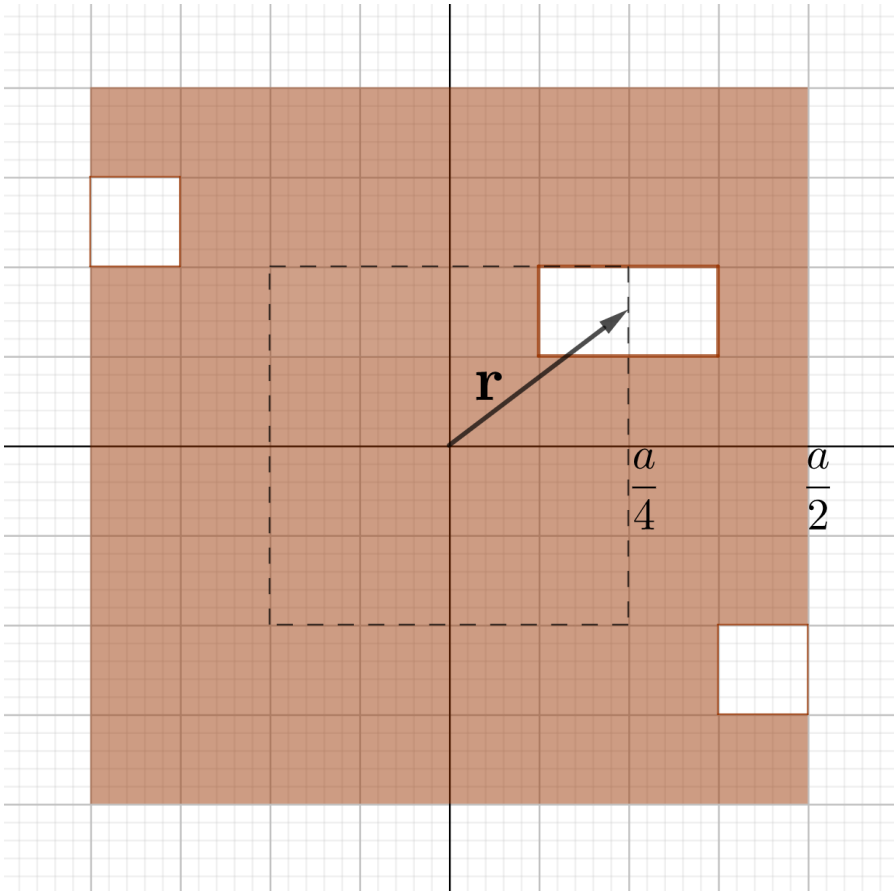
Control synthesis scheme



Solar sail pattern



Solar sail pattern



1. Center of rectangle

$$\frac{\xi}{\eta} = -\frac{M_{\eta}}{M_{\xi}},$$

$$\xi^2 + \eta^2 = \left(\frac{a}{4}\right)^2,$$

$$\text{sign}(\eta) = -\text{sign}(M_{\xi}).$$

2. Area

$$S_{rec} = \left| \frac{cF_{s,\zeta}}{\Phi_0 \cos^2 \theta} \right|, \quad F_{s,\zeta} = \sqrt{\frac{M_{\xi}^2 + M_{\eta}^2}{\xi^2 + \eta^2}} \Rightarrow N_0$$

3. Modifying pattern

$$N_{req} = f \cdot n^2, \quad 0.25n^2 \leq N \leq 0.75n^2$$

$$N_0 \leq N_{req} \Rightarrow \text{add cell pairs with zero torque}$$

Numerical example

Orbit size $R_{orb} = 9000 \text{ km}$

Relative orbit $\mathbf{r}_{rel} = (200 \ 100 \ 50) \text{ m}$

$\mathbf{v}_{rel} = (0.05 \ 0.5 \ 1) \text{ m/s}$

Mass $m = 10 \text{ kg}$

Size $5 \times 5 \text{ m}$

Inertia tensor $\mathbf{J} = \text{diag}(2.1 \ 2.1 \ 3.8) \text{ kg} \cdot \text{m}^2$

Angular velocity $\boldsymbol{\omega}_1 = (0.002 \ 0.003 \ 0.001) \text{ rad/s}$

$\boldsymbol{\omega}_2 = (0.001 \ 0.003 \ 0.002) \text{ rad/s}$

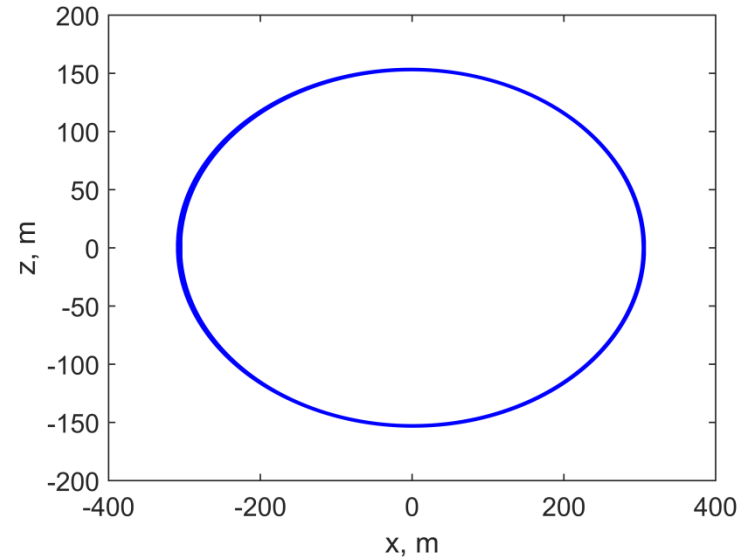
Control parameters $k_\omega = 0.02 \text{ N} \cdot \text{m} \cdot \text{s}$, $k_a = 10^{-4} \text{ N} \cdot \text{m}$

$k_1 = k_3 = k_4 = 20$, $k_2 = 10^{-6} \text{ s}^{-1}$

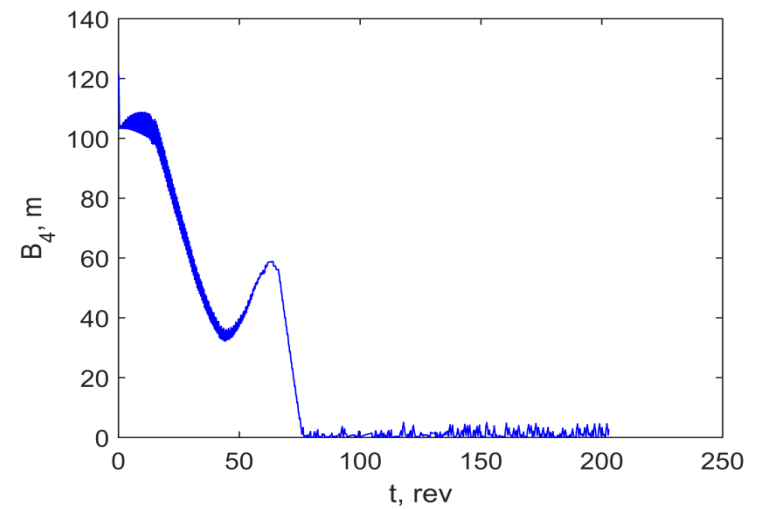
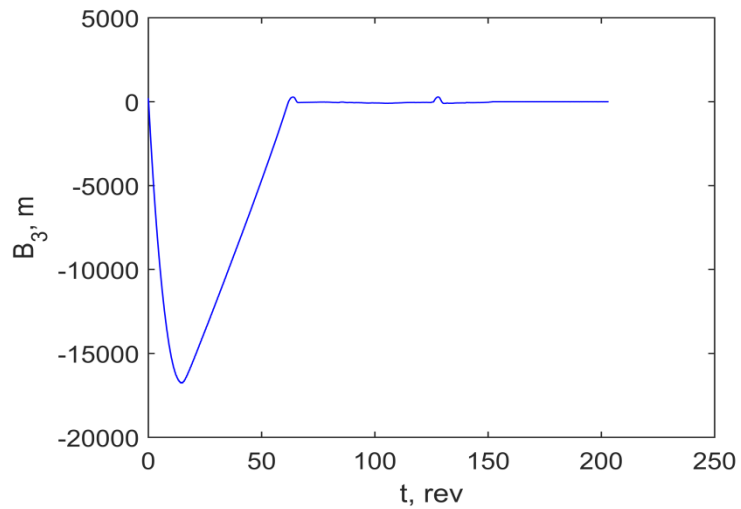
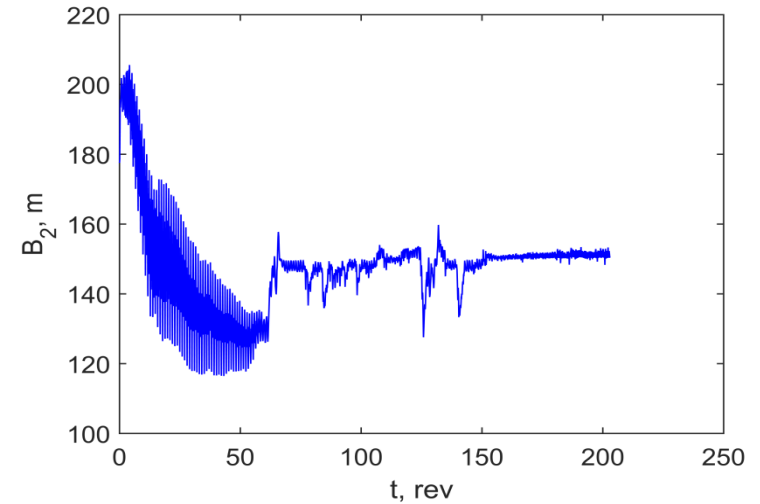
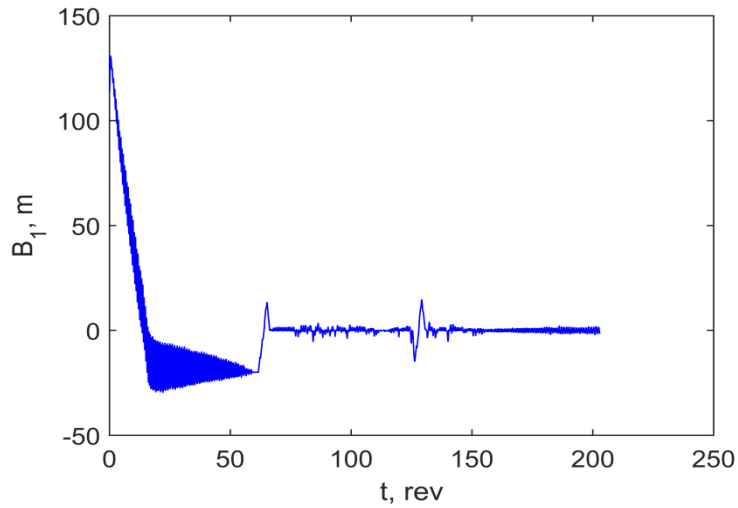
Switch condition $B_1 B_3 < 1 \text{ m}^2$

Maximum control $u_{max} = 10^{-6} \text{ N}$

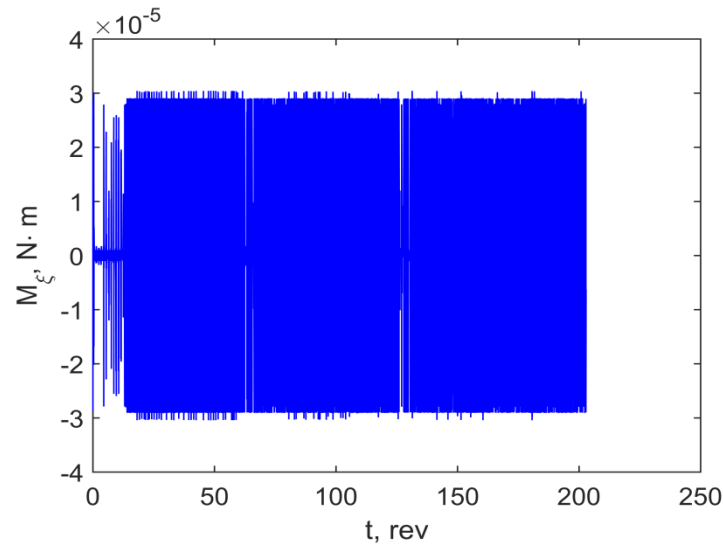
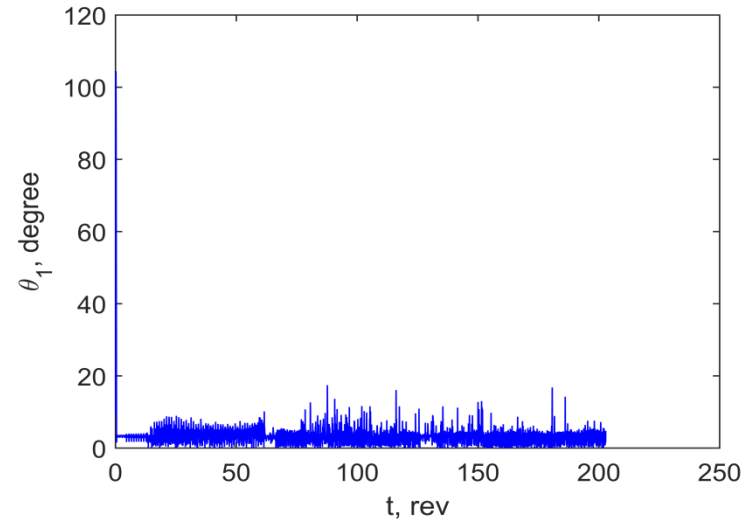
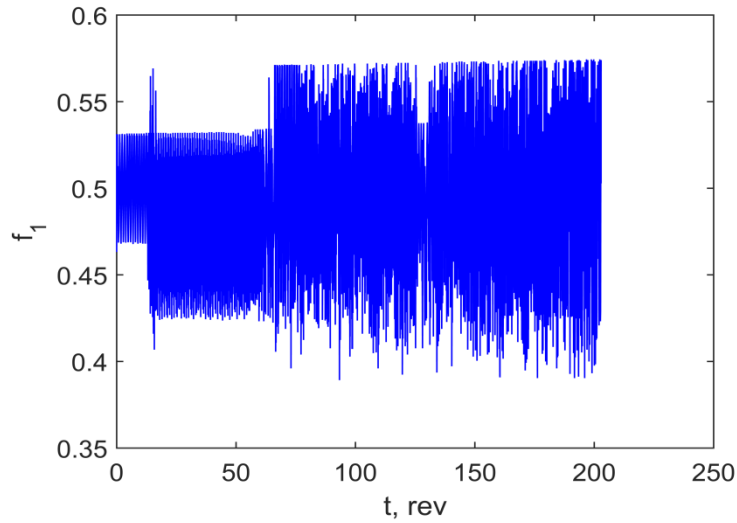
$M_{trq} = 3 \times 10^{-5} \text{ N} \cdot \text{m}$



Numerical example (B-parameters)



Numerical example (control)



Conclusion

It is shown that it is possible to stabilize elliptic relative orbit by means of the SRP only

The control synthesis scheme include the control of the angular and relative motion simultaneously

The relative motion is provided by the SRP force and control source is the solar sail attitude and variable reflectivity

The desired solar sail attitude is also proved by the SRP (torque)