KIAM Astrodynamics Day, June 28, 2019
Room 4 (formerly Room 26), Main Building

Station-Keeping in an Unstable High Near-Circular Lunar Orbit

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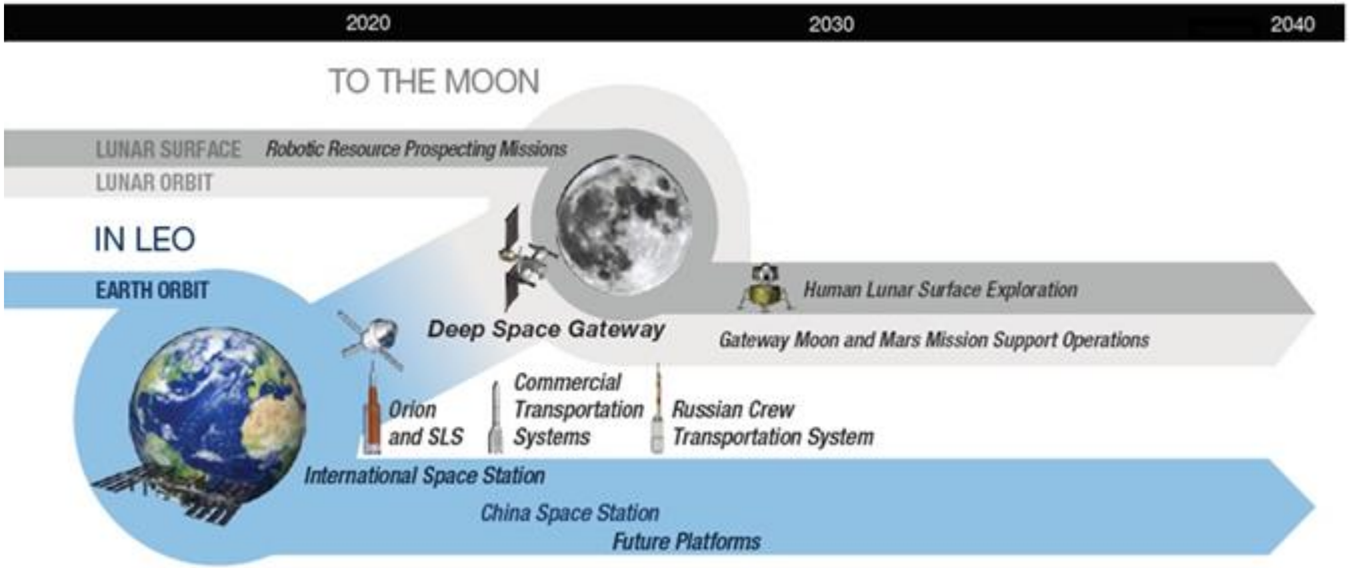
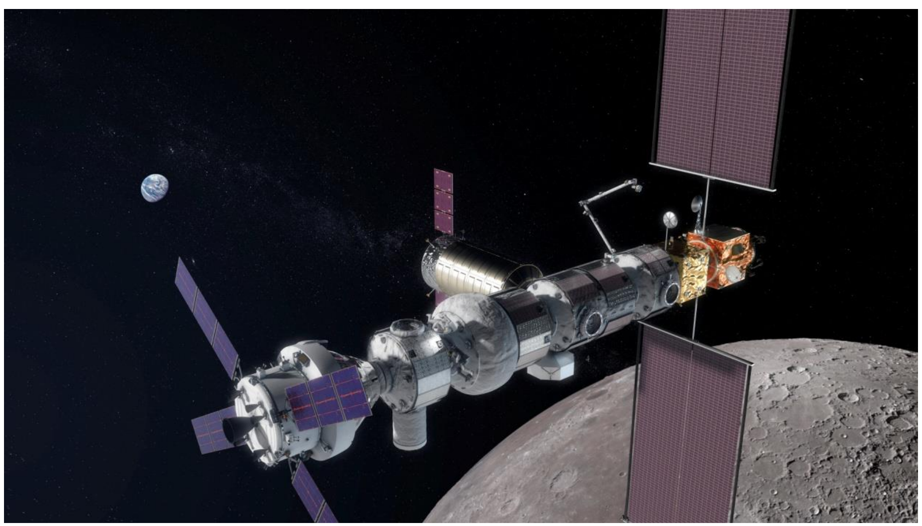
Lunar Orbital Platform-Gateway



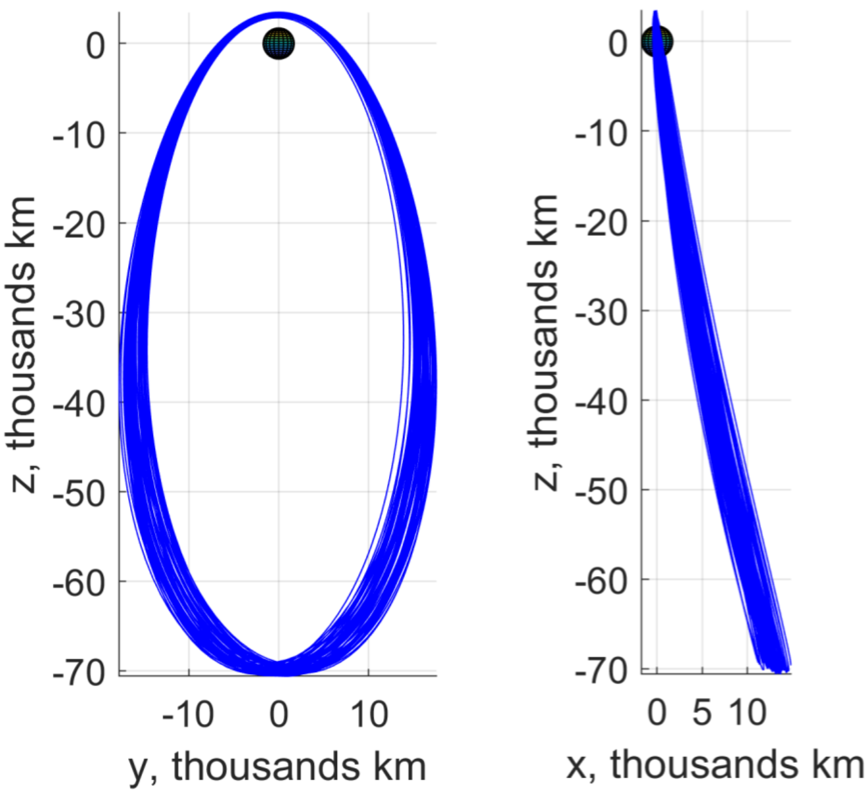


Credit: NASA

- Moon and deep space investigation
- Platform for future robotic and crewed lunar missions
- Space hub for interplanetary missions

Near Rectilinear Halo Orbits (NRHOs)

NRHOs – a class of halo orbits with a low minimal distance to the minor body



Properties of lunar NRHOs:

- Good shadow and radio visibility conditions
- Unstable but require low station-keeping costs
- Stationary visibility conditions of the lunar surface
- Apolune is above the lunar south pole

Southern NRHO L2 9:2 in the standard rotating frame
(perilune altitude = 1400 km, orbit period = 6.652 days)

High Circular Polar Orbits (HCPOs)

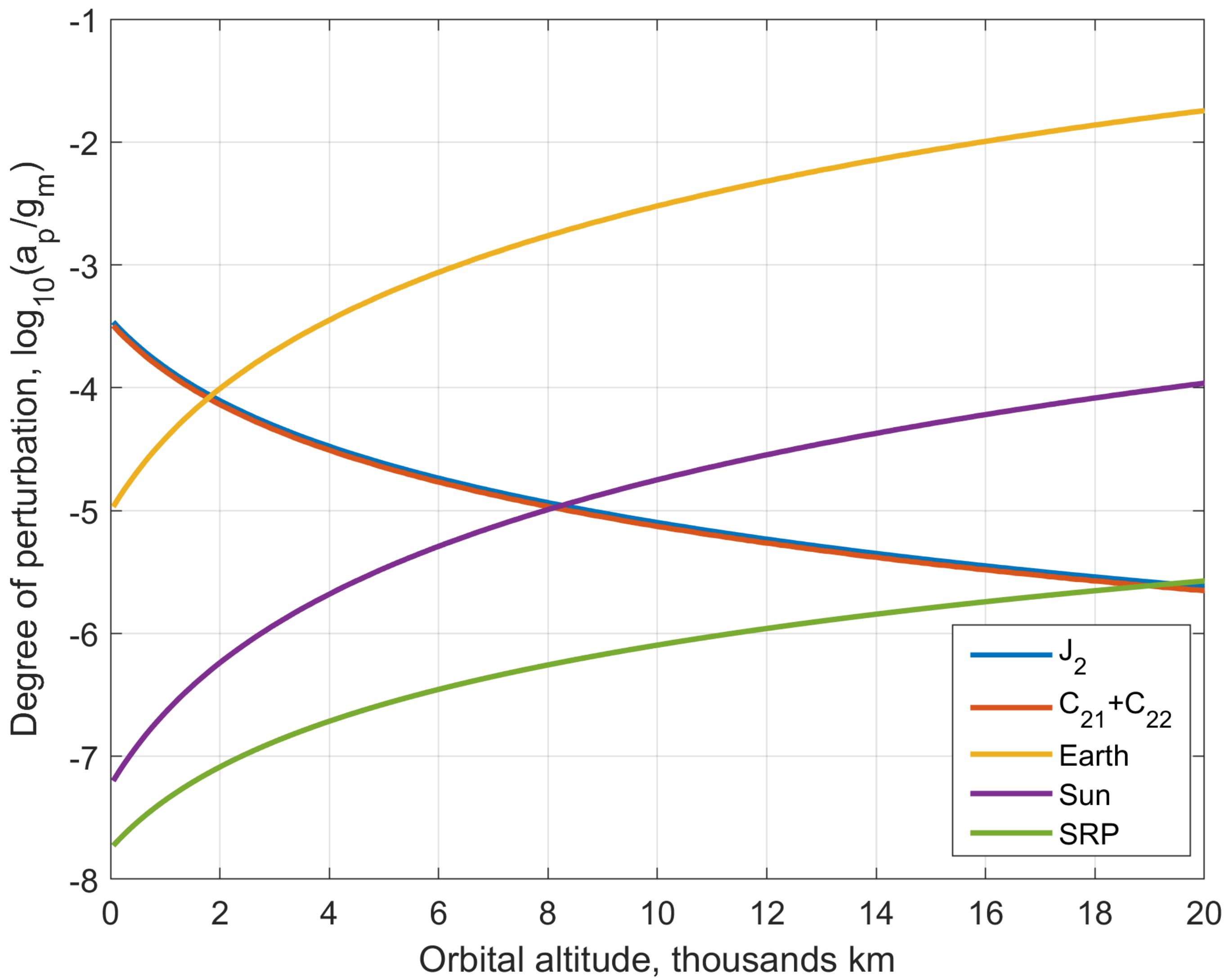
HCPOs are considered as an alternative to NRHOs for the orbital station:

- Polar regions visibility conditions are similar to those for the NRHOs

- Unlike NRHOs, these are orbitally stable orbits

- Transfers from low Earth orbits are cheaper than those to NRHOs

Perturbations at different altitudes



Double-averaged model

Lidov (1961), Kozai (1962) investigated the double-averaged model:

- Only third-body perturbation is taken into account
- The third body moves in a circular orbit around the central body
- Gauss equations in osculating elements are averaged w.r.t. orbital motion of the spacecraft and orbital motion of the third body

Double-averaged equations of motion

$$\frac{d\bar{\bar{a}}}{dt} = 0$$

$$\frac{d\bar{\bar{e}}}{dt} = 5B\bar{\bar{e}}\sqrt{1-\bar{\bar{e}}^2}\sin^2\bar{\bar{i}}\sin 2\bar{\bar{\omega}}$$

$$\frac{d\bar{\bar{i}}}{dt} = -\frac{5B\bar{\bar{e}}^2}{2\sqrt{1-\bar{\bar{e}}^2}}\sin 2\bar{\bar{i}}\sin 2\bar{\bar{\omega}}$$

$$\frac{d\bar{\bar{\Omega}}}{dt} = \frac{B}{\sqrt{1-\bar{\bar{e}}^2}}\left(5\bar{\bar{e}}^2\cos 2\bar{\bar{\omega}} - 3\bar{\bar{e}}^2 - 2\right)\cos\bar{\bar{i}}$$

$$\frac{d\bar{\bar{\omega}}}{dt} = \frac{B}{\sqrt{1-\bar{\bar{e}}^2}}\left[\left(5\cos^2\bar{\bar{i}} - 1 + \bar{\bar{e}}^2\right) + 5\left(1 - \bar{\bar{e}}^2 - \cos^2\bar{\bar{i}}\right)\cos 2\bar{\bar{\omega}}\right]$$

$$\frac{d\bar{\bar{M}}}{dt} = \frac{\mu^{1/2}}{\bar{\bar{a}}^{3/2}} - \frac{B}{3}\left[\left(3\bar{\bar{e}}^2 + 7\right)\left(3\cos^2\bar{\bar{i}} - 1\right) + 15\left(1 + \bar{\bar{e}}^2\right)\sin^2\bar{\bar{i}}\cos 2\bar{\bar{\omega}}\right]$$

First integrals:

$$C_1 = \left(1 - \bar{\bar{e}}^2\right)\cos^2\bar{\bar{i}},$$

$$C_2 = \bar{\bar{e}}^2\left(\frac{2}{5} - \sin^2\bar{\bar{i}}\sin^2\bar{\bar{\omega}}\right)$$

$$B = \frac{3}{8}\frac{k n_E^2 \bar{\bar{a}}^{3/2}}{\mu^{1/2}}$$

Lidov, M.L. *The evolution of orbits of artificial satellites of planets under the action of gravitational perturbations of external bodies*, Planetary and Space Science, Vol. 9, No. 10, 1962, pp. 719–759

Phase portrait of the reduced system

It is convenient to make a change of variables:

$$\bar{\bar{e}}_x = \bar{\bar{e}} \cos \bar{\bar{\omega}},\ \bar{\bar{e}}_y = \bar{\bar{e}} \sin \bar{\bar{\omega}}$$

$$\dot{\bar{\bar{e}}}_x = 6B\bar{\bar{e}}_y\sqrt{1 - \bar{\bar{e}}_x^2 - \bar{\bar{e}}_y^2}$$

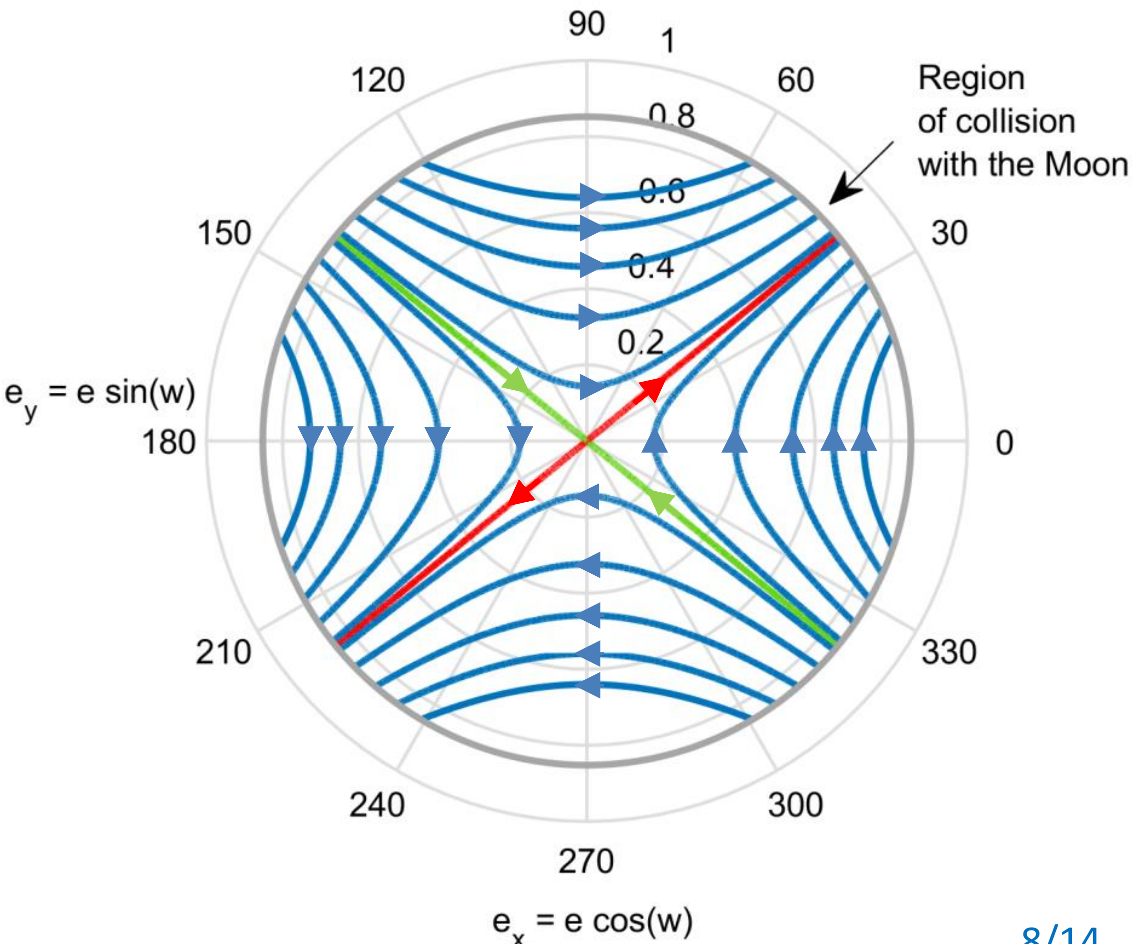
$$\dot{\bar{\bar{e}}}_y = 4B\bar{\bar{e}}_x\sqrt{1 - \bar{\bar{e}}_x^2 - \bar{\bar{e}}_y^2}$$

Characteristic exponents:

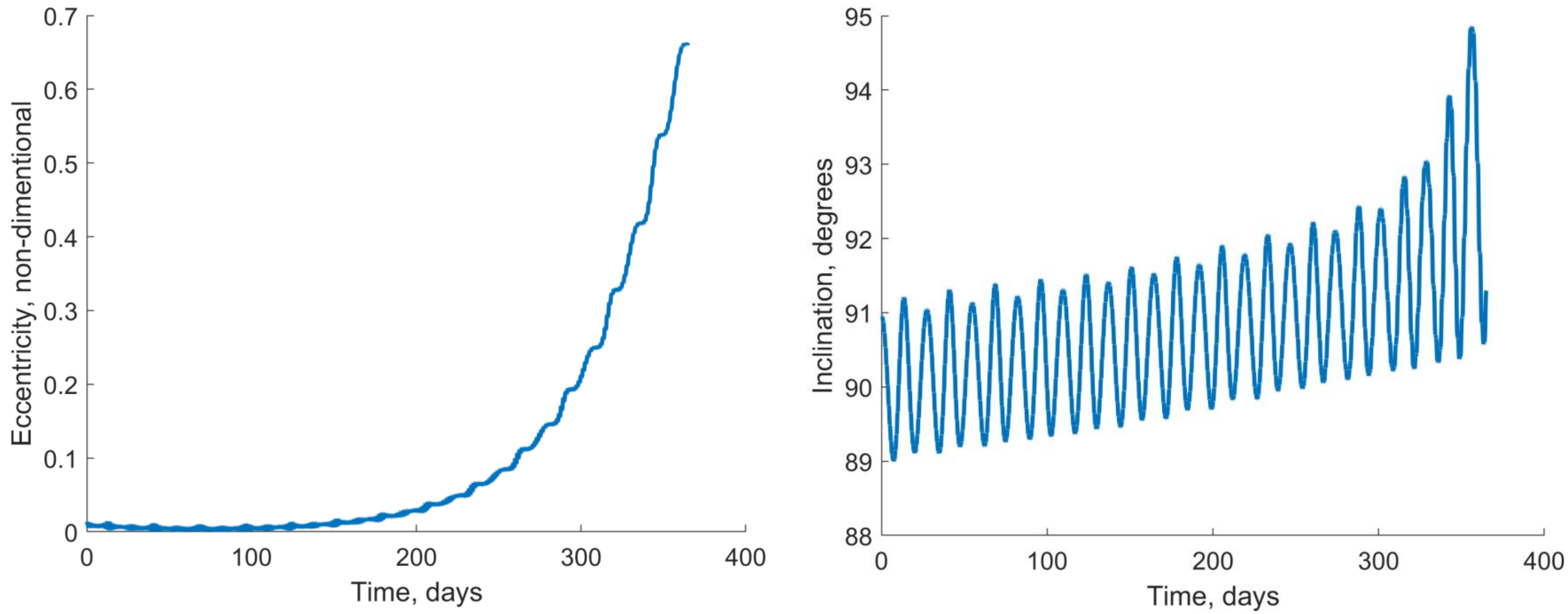
$$\lambda_{1,2} = \pm\sqrt{24}B$$

Characteristic time:

$1/\lambda_1 = 4.29 \cdot 10^7 c \approx 50$ days

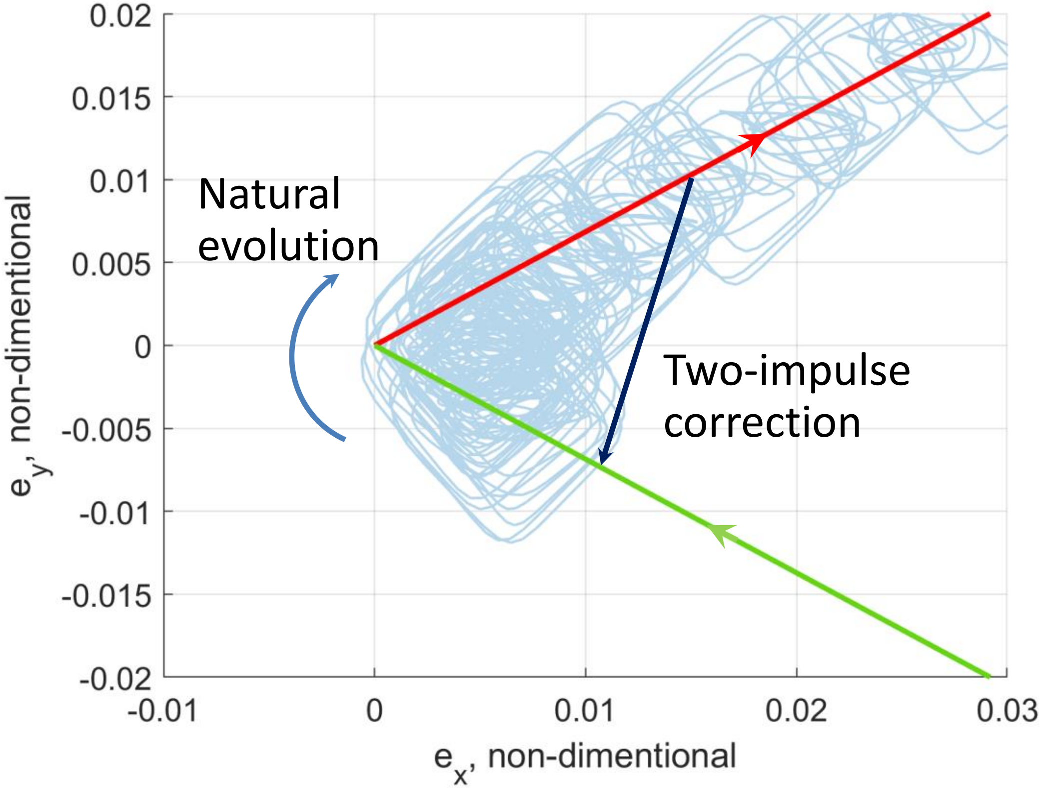


Osculating elements w/o station keeping



Initial values of the orbital elements:
a = 11,745 km, e = 0.01, i = 90.95^o, Ω = 0^o, ω = 342.75^o, M = 338.22^o

Station-keeping of high lunar orbits



- The station-keeping approach is based on targeting the stable manifold
- Two-impulse maneuvers are separated by regular time spans
- Each two-impulse maneuver is optimal in the sense of total ΔV

Station-keeping algorithm

- Given a phase state $\boldsymbol{x}_1$ at the moment t_1, consider the moment $t_2 = t_1 + T$, where T (periodicity of corrections) is fixed and the same in every station-keeping cycle
- Calculate two impulses $\Delta \boldsymbol{v}_1$ at the moment τ_1 and $\Delta \boldsymbol{v}_2$ at the moment τ_2 such that

$$J = \Delta v_1 + \Delta v_2 \to \min \quad \text{s.t.}$$

optimization variables:

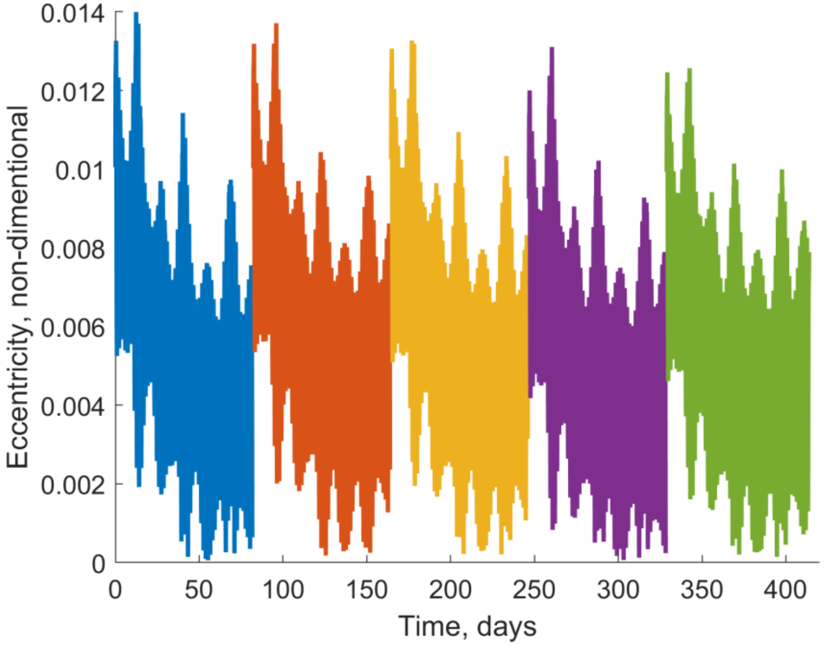
$$\Delta \boldsymbol{v}_1,\ \Delta \boldsymbol{v}_2,\ \tau_1,\ \tau_2$$

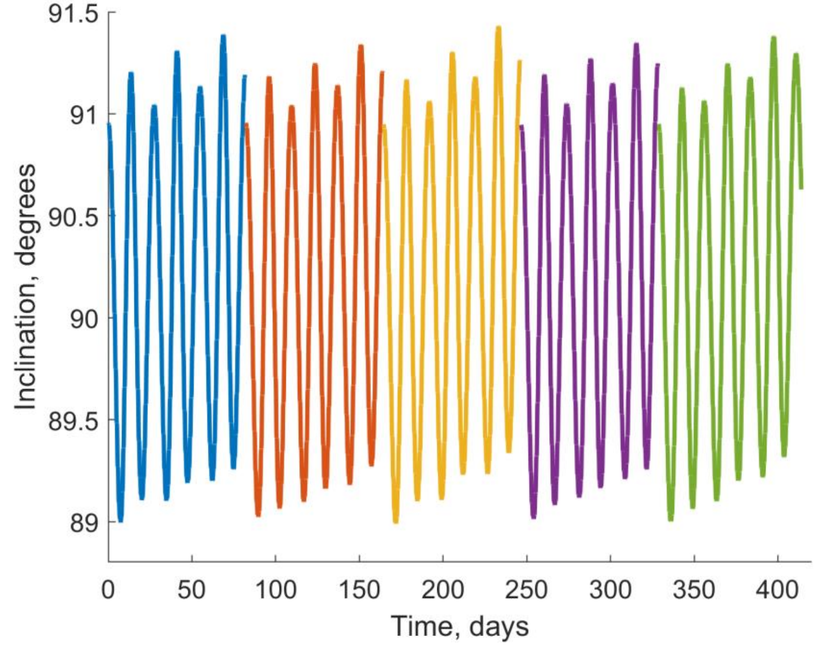
$$e_x(\tau_2) = e_{x0}$$
$$e_y(\tau_2) = e_{y0}$$
$$a(\tau_2) = a_0$$
$$M(\tau_2) = M_0$$
$$i(\tau_2) = i_0$$

$$\tau_1 \geq t_2 - P/2$$
$$\tau_2 \leq t_2 + P/2$$
$$\tau_2 \geq \tau_1$$

- Integrate the equations of motion from t_1 to τ_2 applying the impulses; now, τ_2 is new t_1

Controlled evolution of eccentricity and inclination





Periodicity of corrections: 81.966 days

Change of station-keeping costs with correction maneuver frequency

Periodicity of corrections, days	Mean annual station-keeping cost, m/s/year	Standard deviation, m/s/year
54.644	25.088	3.656
81.966	20.547	1.785
109.288	22.190	3.941
136.610	23.923	3.480
163.932	30.528	5.648

Navigation errors (3σ): 1 km, 1 cm/s
Impulse magnitude error (3σ): 1%
Impulse direction error (3σ): 3 deg
Minimal impulse magnitude allowed: 1.5 mm/s

The corresponding station-keeping costs for the NRHO L_2 9:2:
0.246 ± 0.024 m/s/year

Conclusion

- Stability properties of high near-circular polar orbits around the Moon are investigated.
- The near-circular orbits with an altitude of 10,000 km are unstable: the eccentricity increases, whereas the mean semi-major axis is constant, which leads to collision with the Moon. The characteristic time of the eccentricity vector evolution is approximately 50 days.
- Based on the double-averaged dynamics analysis, an algorithm of station-keeping is designed and implemented in code.
- Annual station-keeping costs exceed 20 m/s, this is by 1-2 orders higher than for the near-rectilinear halo orbit L_2 9:2. The station-keeping of high circular orbits requires 1 correction in 2-4 months. To compare, near rectilinear orbits require 1-2 correction in a week.

Backup

Delaunay elements

Canonical variables in the two-body problem (Delaunay elements):

$$\mathbf{x} = \begin{pmatrix} l \\ g \\ h \end{pmatrix}$$

– true anomaly

– argument of pericenter

– longitude of the ascending node

$$\mathbf{X} = \begin{pmatrix} L \\ G \\ H \end{pmatrix} = \begin{pmatrix} \sqrt{\mu a} \\ L\sqrt{1-e^2} \\ G\cos i \end{pmatrix}$$

$$\mathcal{H}_K = -\frac{\mu^2}{2L^2}$$ – two-body Hamiltonian

Lie-Deprit's transform

- Canonical transformation $\varphi : (\mathbf{y}, \mathbf{Y}) \to (\mathbf{x}, \mathbf{X})$

$$\mathcal{H}(\mathbf{x}, \mathbf{X}; \varepsilon) = \sum_{n \geq 0} \frac{\varepsilon^n}{n!} \mathcal{H}_n^0(\mathbf{x}, \mathbf{X}) \longrightarrow \mathcal{H}'(\mathbf{y}, \mathbf{Y}; \varepsilon) = \sum_{n=0}^{k} \frac{\varepsilon^n}{n!} \mathcal{H}_0^n(\mathbf{Y}) + \sum_{n>k} \frac{\varepsilon^n}{n!} \mathcal{H}_0^n(\mathbf{y}, \mathbf{Y})$$

- Lie-Deprit's method: $W(\mathbf{x}, \mathbf{X}; \varepsilon) = \sum_{n \geq 0} \frac{\varepsilon^n}{n!} W_{n+1}(\mathbf{x}, \mathbf{X})$

$$\mathbf{x} = \mathbf{y} + \sum_{n \geq 0} \frac{\varepsilon^n}{n!} L_W^n(\mathbf{y}) \qquad \mathbf{y} = \mathbf{x} + \sum_{n \geq 0} \frac{\varepsilon^n}{n!} L_{(-W)}^n(\mathbf{x}) \qquad L_W = \{-; W\},\ L_{(-W)} = \{-; -W\}$$

$$\mathbf{X} = \mathbf{Y} + \sum_{n \geq 0} \frac{\varepsilon^n}{n!} L_W^n(\mathbf{Y}) \quad \mathbf{Y} = \mathbf{X} + \sum_{n \geq 0} \frac{\varepsilon^n}{n!} L_{(-W)}^n(\mathbf{X})$$

$\{-; -\}$ – Poisson bracket

Generating function can be found from recurrent formulas:

$$\mathcal{H}_j^i = \mathcal{H}_{j+1}^{i-1} + \sum_{k=0}^{j} C_j^k \{\mathcal{H}_{j-k}^{i-1}; W_{k+1}\} \quad \Leftrightarrow \quad \{W_n; \mathcal{H}_0^0\} = \mathcal{Q}_n - \mathcal{H}_0^n$$