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Satellite Relative Motion SDRE-based Control for Capturing a Noncooperative Tumbling Object

**Mahdi Reza Akhloumadi,
Danil Ivanov**

Moscow Institute of Physics and Technology (National research university)
Keldysh Institute of Applied Mathematics RAS

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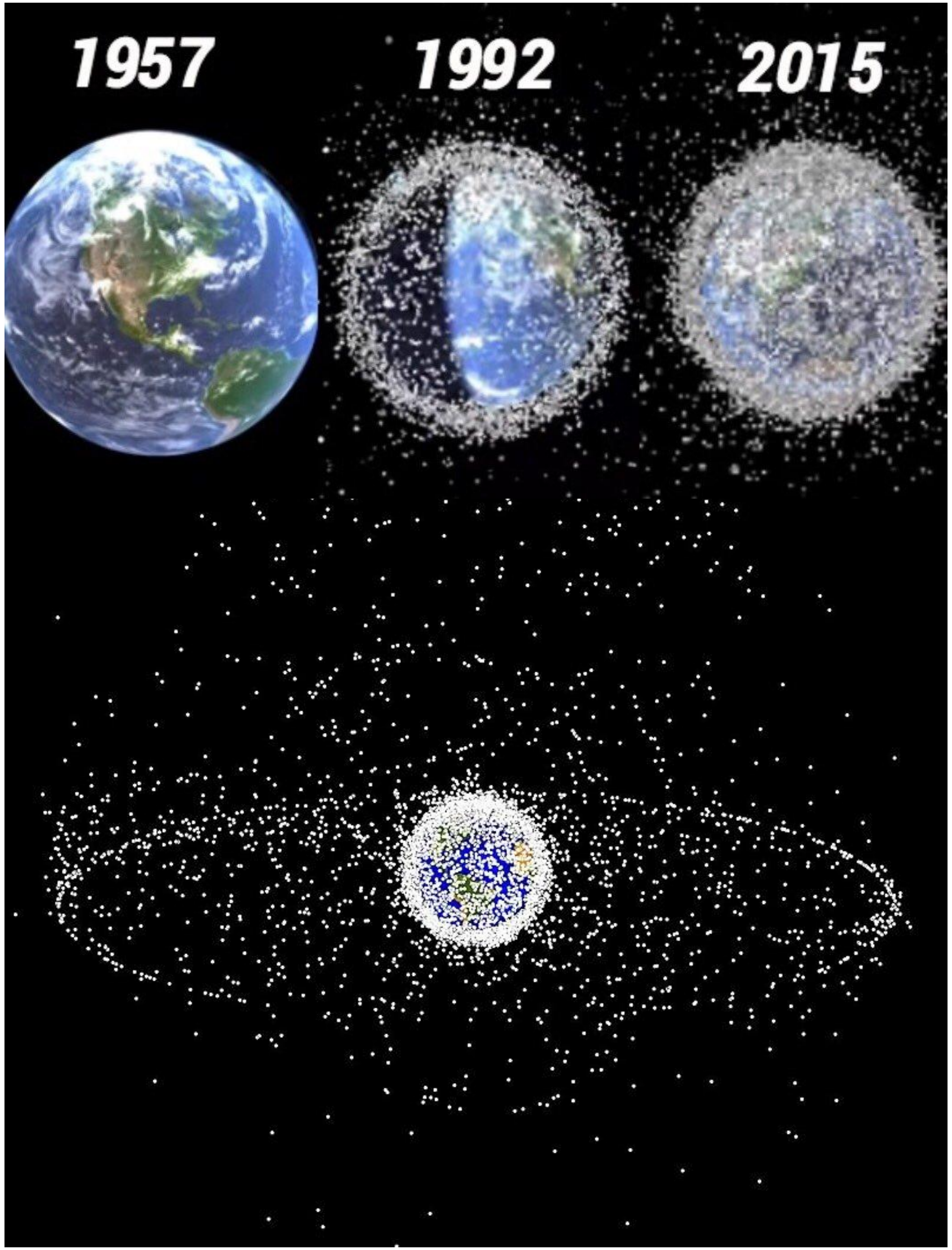
Content

1. Space debris
2. Motion and control algorithm of removal
3. Results of the study
4. Conclusion

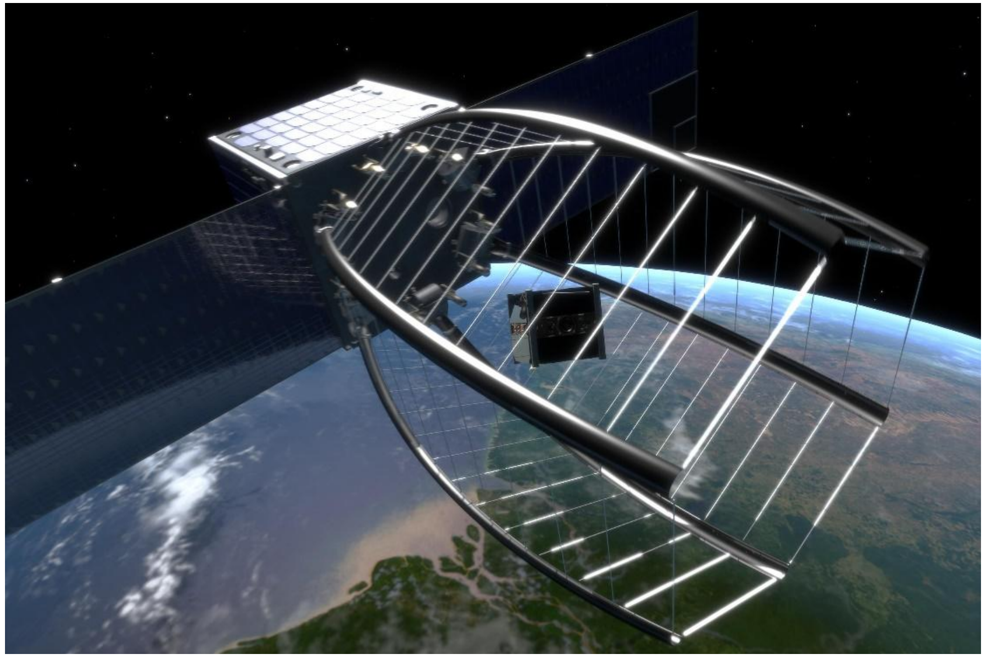


Introduction

- Space Debris
- Treat of space debris
- Kessler effect
- Importance of debris removal

Different Solutions

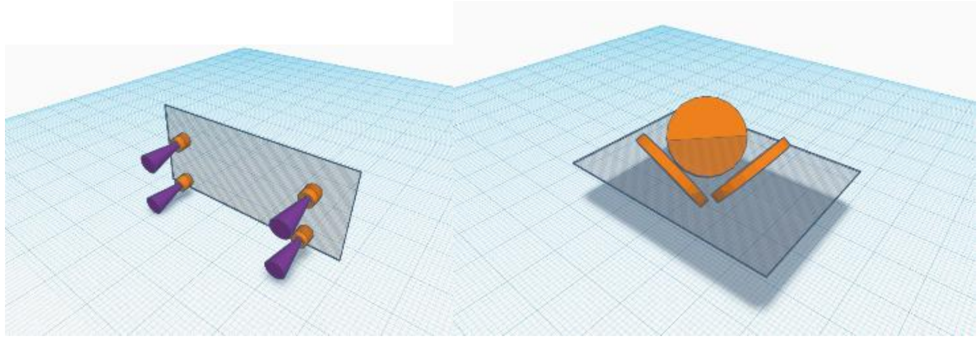
1. Protect spacecraft
2. Destroy or deorbit debris using lasers
3. Remove debris
   - Passive removal
   - Active removal



Problem Statement

Assumed:

- Uncontrolled tumbling object
- Relative motion of the object is known
- Spacecraft is equipped with
  - Thrusters
  - Reaction wheels
  - Capturing system
- The thrusters have misalignment
- The reaction wheels have limited maximal angular momentum



It is necessary

- to develop a relative motion control algorithm to capture the object
- to study the possibility to capture the object

Coupled Motion Equations

Relative rotational motion

$$I_T \dot{\boldsymbol{\omega}}^T = I_T D(\mathbf{q}) I_C^{-1}\mathbf{S} - I_T \boldsymbol{\omega}_T^T \times \boldsymbol{\omega}^T + [\boldsymbol{\omega}_T^T \times I_T \boldsymbol{\omega}_T^T]$$

where

$$\mathbf{S} = -D(\mathbf{q})^{-1}\left(\boldsymbol{\omega}^T + \boldsymbol{\omega}_T^T\right) \times I_C D(\mathbf{q})^{-1}\left(\boldsymbol{\omega}^T + \boldsymbol{\omega}_T^T\right) -$$

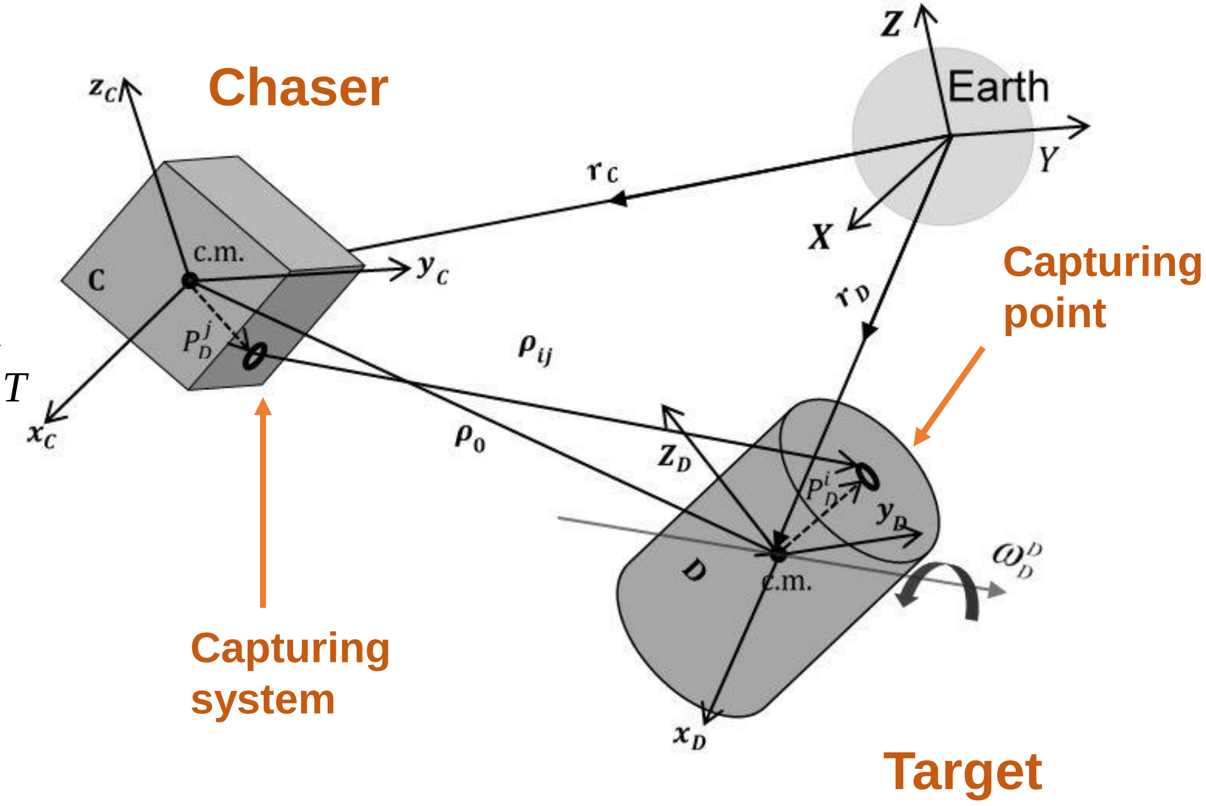
$$- D(\mathbf{q})^{-1}\left(\boldsymbol{\omega}^T + \boldsymbol{\omega}_T^T\right)\mathbf{h}_{WC} - \dot{\mathbf{h}}_{WC} + \mathbf{T}_C + \mathbf{N}_C + \mathbf{N}_T$$

Relative translational motion

$$\ddot{x}_{ij} - 2\omega_{OT}\dot{y}_{ij} - \dot{\omega}_{OT} y_{ij} - 3\omega_{OT}^2 x_{ij} = a_x + p_x$$

$$\ddot{y}_{ij} + 2\omega_{OT}\dot{x}_{ij} + \dot{\omega}_{OT} x_{ij} = a_y + p_y$$

$$\ddot{z}_{ij} + \omega_{OT}^2 z_{ij} = a_z + p_z$$



SDRE-based Control Algorithm

Dynamical system:

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}(t)\right) + \mathbf{g}\left(\mathbf{x}(t), \mathbf{u}(t)\right)$$

Functional to be minimized:

$$J = \frac{1}{2}\int_0^{t_f} \left[\mathbf{x}(t)^T Q\, \mathbf{x}(t) + \mathbf{u}(t)^T R\, \mathbf{u}(t) \right] dt,$$

Linearization:

$$f\left(\mathbf{x}\right) = A\left(\mathbf{x}\right)\mathbf{x},$$

$$g\left(\mathbf{x},\mathbf{u}\right) = B\left(\mathbf{x},\mathbf{u}\right)\mathbf{u}.$$

State Dependent Riccati Equation:

$$P\left(\mathbf{x},\mathbf{u}\right) A\left(\mathbf{x}\right) + A^T\left(\mathbf{x}\right) P\left(\mathbf{x},\mathbf{u}\right) - P\left(\mathbf{x},\mathbf{u}\right) B\left(\mathbf{x},\mathbf{u}\right) R^{-1} B^T\left(\mathbf{x},\mathbf{u}\right) P\left(\mathbf{x},\mathbf{u}\right) + Q = 0$$

Optimal control law

$$\boxed{\mathbf{u}\left(\mathbf{x}\right) = -R^{-1} B^T\left(\mathbf{x},\mathbf{u}\right) P\left(\mathbf{x},\mathbf{u}\right)\mathbf{x}}$$

Numerical Simulation

System parameters and initial conditions:

$$I_T = I_C = 2.2\ I_{3\times3}\,\text{kg}\cdot\text{m}^2$$

$$m = 50\,\text{kg}$$

$$\mathbf{q}_0 = [0,0,0,1]^T$$

$$\boldsymbol{\omega}_T^T = [10,-10,20]^T\,\text{deg/ s}$$

$$\boldsymbol{\rho}_0 = \mathbf{r}_0 = [x_0, y_0, z_0]^T = [50,27,100]^T\ \text{m}$$

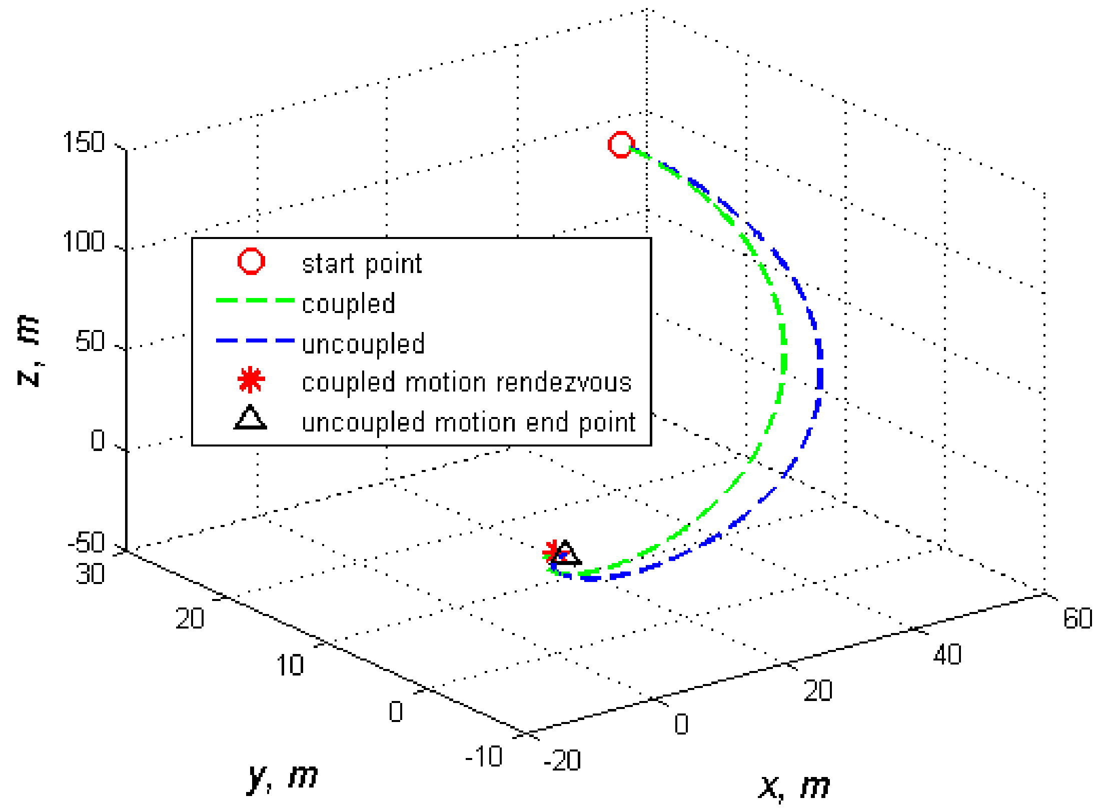
$$\dot{\boldsymbol{\rho}}_0 = \dot{\mathbf{r}}_0 = [0,\text{-}2,0]^T\ \text{m / s}$$

$$\boldsymbol{\rho}_{i1} = [1,1,0]^T\ \text{m}$$

$$\boldsymbol{\rho}_{jo} = [1,0,1]^T\ \text{m}$$

Orbital elements of the target:

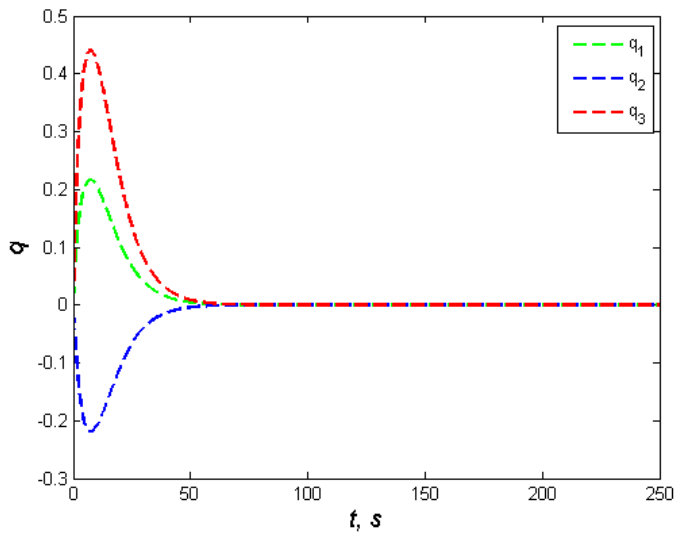
$$a = 7128\ \text{km};\ e = 0.03;\ i = 70^\circ;\ \Omega = 50^\circ; \omega = 80^\circ$$



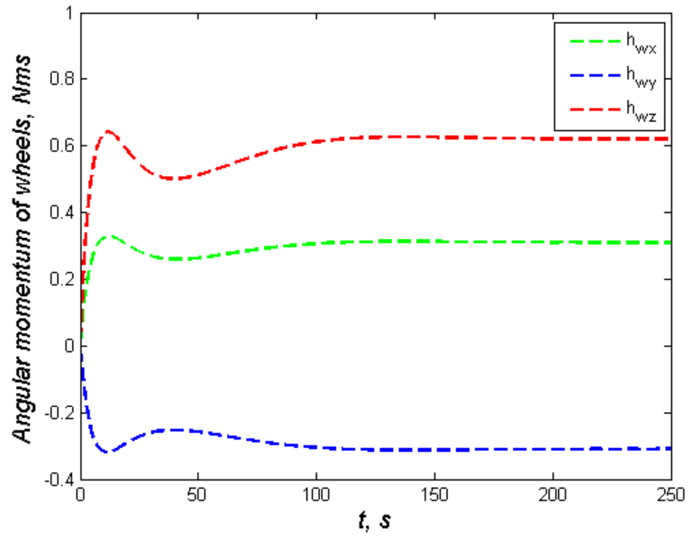
Translational trajectory of spacecraft with respect to the debris

Numerical Simulation

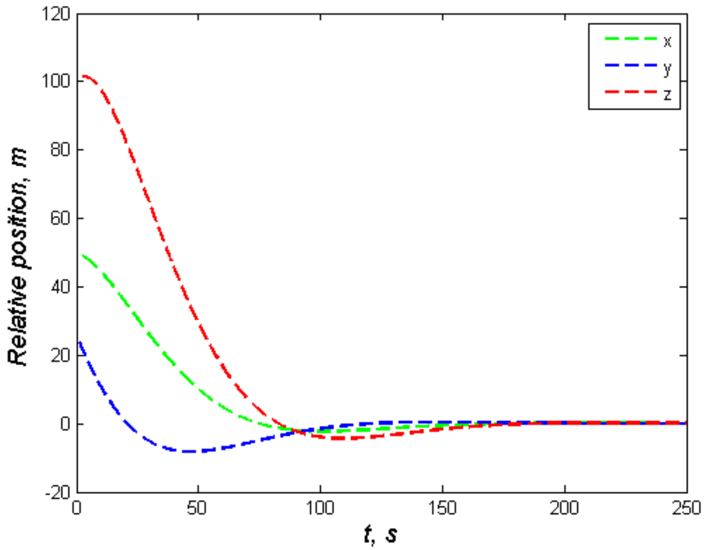
- Relative attitude and relative position of the points coincide after the maneuver
- Due to target angular velocity and thrusters misalignment the reaction wheels accumulated angular momentum
- It is necessary to study of the acceptability area of the system parameters



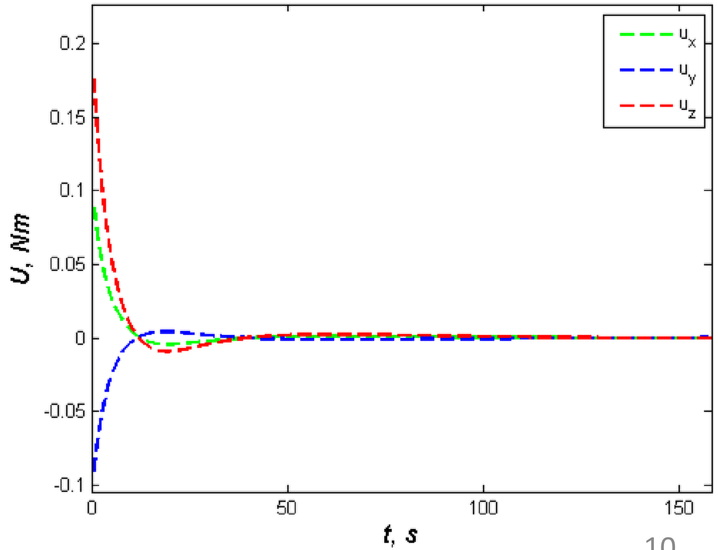
Relative attitude



Angular momentum of reaction wheels



Relative position



Thrusters force

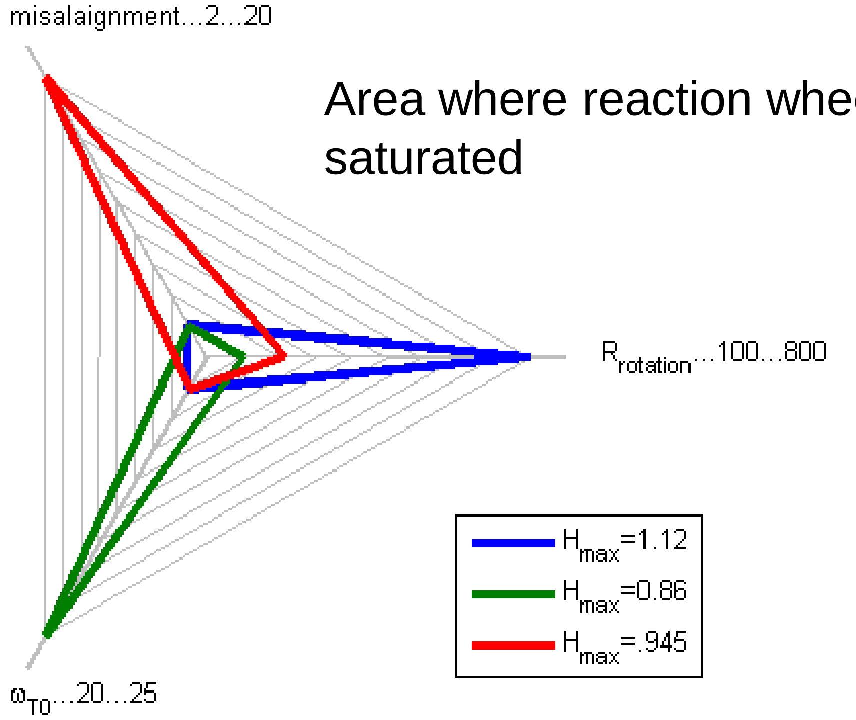
Study of the Acceptability Area

Consider parameters:

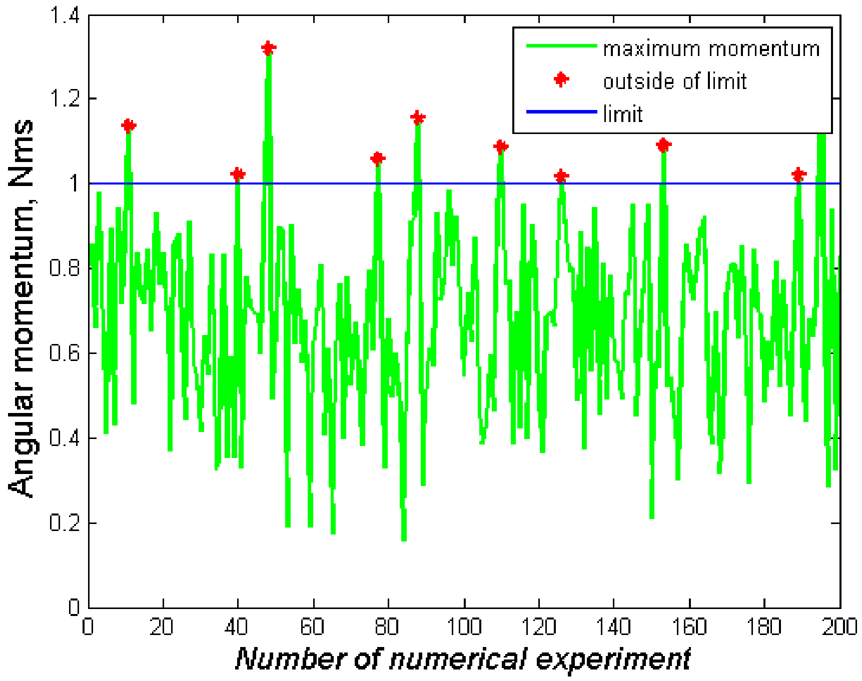
- Maximum of reaction wheels angular momentum
- Thrusters misalignment
- Angular velocity of object
- Control algorithm parameters
- Tensor of inertia of the object

Random parameters for the Monte-Carlo simulations:

- angular velocity of target
- misalignment of thrusters
- algorithm parameter



Area where reaction wheels are not saturated



Maximal angular momentum of reaction wheels

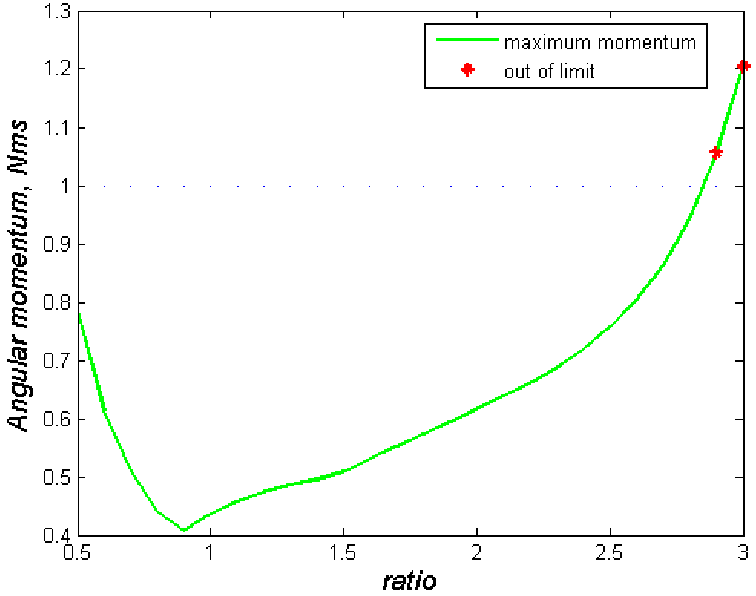
Case of Cylindrical Target

Target tensor of inertia:

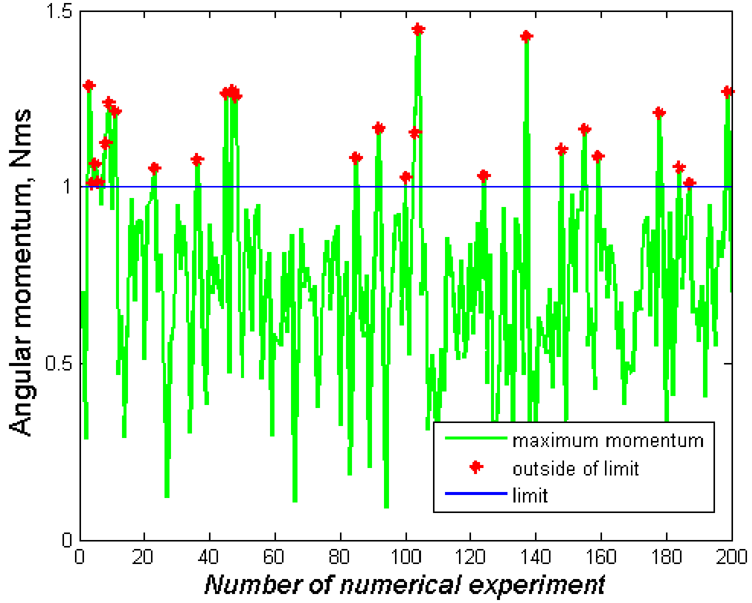
$$\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} = \begin{bmatrix} ratio * I & 0 & 0 \\ 0 & ratio * I & 0 \\ 0 & 0 & I \end{bmatrix}$$

- The reaction wheels provide tracking the target angular motion
- Increasing the elongation of the target body (ratio more then 1) or reducing it to a flat body (ratio tends to 0.5) leads to more required reaction wheels angular velocity at the same others simulation parameters

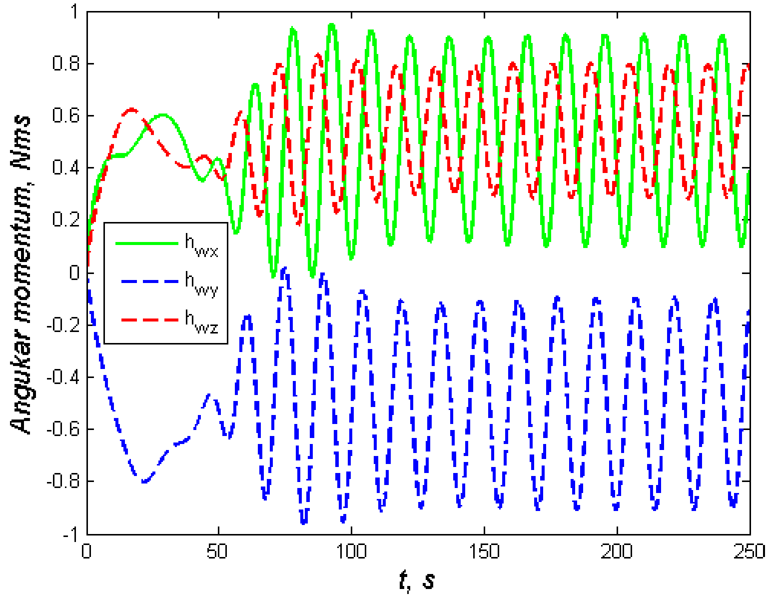
Dependence of the maximal reaction wheels angular momentum on inertia moments ration



Maximal angular momentum



Reaction wheels angular momentum



Conclusions

- SDRE-based control algorithm for a close range proximity to non-corporative debris is proposed
- The dependence of reaction wheels saturation on system and control parameters is obtained
- The effect of inertia tensor of target on reaction wheels saturation is studied
- The proposed technique allows to determine whether it is possible to track and capture a specific debris with given motion control restrictions of the chaser spacecraft

Thank you for attention!