Degtyarev A.A., Hao-Chi Chang, Ovchinnikov M.Yu.

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KELDYSH INSTITUTE OF APPLIED MATHEMATICS
RUSSIAN ACADEMY OF SCIENCE

A.A.Degtyarev, Hao-Chi Chang,
M.Yu.Ovchinnikov

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A magnetic attitude control system, owing to absence of moving parts and certain simplicity of realization, is common used in practice. Such kind of systems require low power, are relatively lightweight and inexpensive. Due to these features the active magnetic attitude control system is one of the most common attitude control system implemented on board of small satellites. The serious limitation during their realization is a control torque vector direction singularity. Control systems with magnetic actuators combined with other active or passive control elements, for example with a gravitational boom and/or with flywheels, are more effective and more often are realized in practice. In the present work a dynamically symmetrical satellite equipped by a pitch flywheel and active magnetic control system is considered. The general approaches of magnetic control algorithm design are considered. Realization of the proposed method is illustrated by some special cases.

Key words: magnetic attitude control system, pitch flywheel, control algorithm design

Активное магнитное управление динамически симметричным спутником, оснащенным тангажным маховиком. А.А. Дегтярев, Хао-Ши Чанг, М.Ю. Овчинников. Препринт ИПМ им. М.В. Келдыша РАН, Москва, 31 страница, 17 рисунков, библиография: 12 наименований.

Магнитные системы ориентации конструктивно просты, просты в реализации, не потребляют значительных энергетических ресурсов спутника, имеют малую массу и недороги. Благодаря этому магнитные системы часто устанавливают на малых аппаратах. Серьезным ограничением при их реализации является тот факт, что управляющий механический магнитный момент всегда лежит в плоскости перпендикулярной вектору напряженности геомагнитного поля. Системы управления, в которых магнитные актюаторы сочетаются с другими активными или пассивными системами, например с гравитационными или маховичными системами, более эффективны и чаще всего реализуются на практике. В работе рассматривается реализация активного магнитного управления динамически симметричным спутником, оснащенным тангажным маховиком. Рассматриваются общие подходы к построению управления подобной системой. Реализация предложенного метода иллюстрируется на примере частных случаев.

Ключевые слова: активная магнитная система управления, тангажный маховик, разработка алгоритма управления

Introduction

Motion of the satellite equipped with a magnetic attitude control system (MACS) is considered[1]. The satellite is actuated by a set of mutually perpendicular magnetic coils. The concept is that interaction between the Earth's magnetic field and a magnetic dipole generated by the coils results in a mechanical torque used for attitude control. Such kind of systems, owing to absence of moving parts and certain simplicity of realization, are common used in practice for tumbling of a satellite, flywheel unloading, disturbed motion damping and even to provide a unique three-axis orientation. Magnetic control is attractive for small, low cost satellites in low Earth orbits. Magnetic systems require low power, are relatively lightweight and inexpensive.

Satellite three-axis orientation by active MACS only is considered, for example in [1-5]. However, realization of the control systems meets difficulties. The most serious one is a control torque vector direction limitation. It can be generated only perpendicular to the geomagnetic field vector induction.

Control systems with magnetic actuators combined with other active or passive control elements, for example with a gravitational boom and/or with flywheels, are more effective and more often are realized in practice [6-8]. However, there is a study [4] where advantage of "pure" MACS for three-axis orientation with respect to an orbital reference frame without the boom is shown.

In the present work a dynamically symmetrical satellite equipped by a pitch flywheel and active magnetic control system is considered. Flywheel provides one-axis satellite orientation as its angular momentum vector is collinear to the satellite's dynamic symmetry axis and perpendicular to Kepler's orbit plane. The magnetic coils provide three-axis orientation of the satellite.

The main idea of a considered magnetic control algorithm is a synthesis of two control components. The first component orients a satellite providing the Lyapunov stability of the attitude and the second one provides an asymptotical stability of equilibria and carries out oscillation damping. Both components of control have the same physical nature and are realized simultaneously by the same actuators. Here the general approaches of magnetic control algorithm design are considered. Realization of the proposed method is illustrated by some special cases.

1. Equations of motion

Consider the motion of a gyrostat-satellite subjected to gravitational and magnetic control torques. The latter appears due to interaction between the Earth magnetic field and a magnetic dipole moment generated by the coils. We introduce two right-hand Cartesian coordinate frames with origin in the satellite center of mass O. $OX_1X_2X_3$ is the orbital reference frame (ORF). The axis OX_3 is directed along

[1] The work is supported by the Russian Foundation for Basic Research (Grant 07-01-92001) and the National Scientific Council of Taiwan (Project RP07E03).

the radius-vector of the satellite center of mass with respect to center mass of the Earth; the axis OX_2 is alongwith the normal to the orbital plane, the axis OX_1 comprise completed right-hand reference frame. $Ox_1x_2x_3$ is the satellite body-fixed reference frame (BRF); Ox_k $(k=1,2,3)$ are the principal central axes of inertia of the satellite.

The orientation of the satellite BRF with respect to ORF is determined by the angles α, β and γ (see Figure 1).

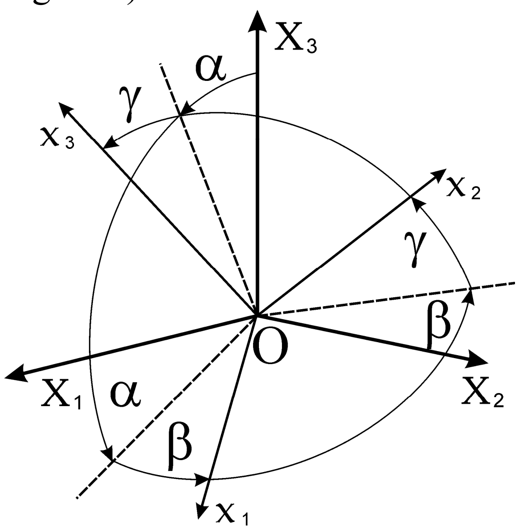

Figure 1. Orbital and body-fixed reference frames relation

The direction cosines of the axes Ox_k in the orbital reference frame $a_{ij}=\cos(X_i,x_j)$ are written as

$$
\begin{aligned}
&a_{11}=\cos\alpha\cos\beta, && a_{23}=-\cos\beta\sin\gamma,\\
&a_{12}=\sin\alpha\sin\gamma-\cos\alpha\sin\beta\cos\gamma, && a_{31}=-\sin\alpha\cos\beta,\\
&a_{13}=\sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma, && a_{32}=\cos\alpha\sin\gamma+\sin\alpha\sin\beta\cos\gamma, \qquad (1)\\
&a_{21}=\sin\beta, && a_{33}=\cos\alpha\cos\gamma-\sin\alpha\sin\beta\sin\gamma,\\
&a_{22}=\cos\beta\cos\gamma,
\end{aligned}
$$

Kinematic equations of the satellite attitude motion have the form

$$
\begin{aligned}
&p=(\dot\alpha+\omega_0)a_{21}+\dot\gamma=\overline{p}+\omega_0a_{21},\\
&q=(\dot\alpha+\omega_0)a_{22}+\dot\beta\sin\gamma=\overline{q}+\omega_0a_{22}, \qquad (2)\\
&r=(\dot\alpha+\omega_0)a_{23}+\dot\beta\cos\gamma=\overline{r}+\omega_0a_{23}.
\end{aligned}
$$

In the equations (2) p,q,r are the projections of the satellite angular velocity in the axes Ox_k; ω_0 is the angular velocity of the orbital motion of the satellite center of mass; The dot designates differentiation with respect to time.

Dynamic equations of the satellite attitude motion have the form

$$\hat{\boldsymbol{J}}\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times\hat{\boldsymbol{J}}\boldsymbol{\omega}+\boldsymbol{\omega}\times\boldsymbol{K}'+\frac{\tilde{d}\boldsymbol{K}}{dt}=\boldsymbol{M}_{gr}+\boldsymbol{M}_{magn}. \tag{3}$$

Here $\hat{\boldsymbol{J}}=diag(A,B,C)$ is the tensor of inertia of the satellite (A,B,C are its principal central moments of inertia), $\boldsymbol{\omega}$ is satellite angular velocity vector and $\boldsymbol{K}'$ is flywheels' (internal i.e. with respect to BRF) angular momentum vector. The projections of the satellite internal angular momentum in the axes Ox_k are as follows

$$\bar{h}_1=\sum_{j=1}^{n}J_j\alpha_j\dot{\varphi}_j\ ,\ \ \bar{h}_2=\sum_{j=1}^{n}J_j\beta_j\dot{\varphi}_j\ ,\ \ \bar{h}_3=\sum_{i=1}^{n}J_j\gamma_j\dot{\varphi}_j$$

where J_j is an axial moment of inertia of a corresponding flywheel, and α_j ,β_j ,γ_j are flywheel spin-axis orientation angles with respect to BRF. Gravitational torque with respect to axes Ox_k has the form

$$\boldsymbol{M}_{gr}=3\omega_0^2\begin{pmatrix}a_{31}\\ a_{32}\\ a_{33}\end{pmatrix}\times\hat{\boldsymbol{J}}\begin{pmatrix}a_{31}\\ a_{32}\\ a_{33}\end{pmatrix}.$$

The control magnetic torque affecting the satellite is

$$\boldsymbol{M}_{magn}=\boldsymbol{m}_{\Sigma}\times\boldsymbol{B}. \tag{4}$$

Here $\boldsymbol{m}_{\Sigma}$ is a total dipole moment produced by the coils mounted on the satellite, $\boldsymbol{B}$ is the induction vector of the Earth magnetic field.

2. Geomagnetic field model

Following the Gauss theory

$$\boldsymbol{H}=-grad\,U_m \tag{5}$$

where $\boldsymbol{H}$ is the geomagnetic field intensity and U_m is its potential which can be written down in the form of decomposition on the spherical harmonious functions

$$U=R\sum_{n=1}^{\infty}\left(\frac{R_e}{r}\right)^{n+1}\sum_{m=0}^{n}\left(g_n^m\cos m\lambda+h_n^m\sin m\lambda\right)P_n^m(\cos\theta). \tag{6}$$

Here r,λ,θ are satellite's ~~s~~ center of mass spherical coordinates, g_n^m, h_n^m are given tabulated coefficients, $P_n^m(\cos\theta)$ are the associated Legendre functions.

Notation: the term "magnetic field" is applied to two various vector fields designated as $\boldsymbol{B}$ and $\boldsymbol{H}$. In the International System of Units (SI) $\boldsymbol{B}=\mu_0\boldsymbol{H}$ where $\mu_0=4\pi\times10^{-7}\left[N/A^2\right]$ is free space permeability.

The first terms of the harmonious series (6) permit a simple physical interpretation.

2.1. Inclined dipole model

First order terms (n=1) in series (6) constitute the potential of a dipole which is located in the center of the Earth and its dipole moment is equal to $\mu_m = 7.812 \times 10^{22}\ A \cdot m^2$. The dipole axis is tilted to the axis of the Earth rotation with angle $\delta_m = 168.5^o$. Vector $\boldsymbol{H}$ creates a conic surface in the Kenig reference frame with the origin in point O during its orbital motion. The first axis of the reference frame is collinear to the direction from the Earth center to the orbit ascending node and the third one is collinear to the spin axis of the Earth.

This geomagnetic field interpretation named ***Inclined Dipole Model***.

2.2. Direct dipole model

In theoretical study a simpler model of the geomagnetic field is used. Since the angle δ_m is insignificant and satellite's orbital period, as a rule, is much shorter than period of the Earth spin rotation, assuming that $\delta_m = 0$, obtain a ***Direct Dipole Model***. In this case the projections of the geomagnetic field intensity in the axes of ORF are [12]:

$$H_1 = \frac{\mu_m}{r^3}\cos u \sin i,\ H_2 = \frac{\mu_m}{r^3}\cos i,\ H_3 = -2\frac{\mu_m}{r^3}\sin u \sin i\,. \qquad (7)$$

Here r is orbit radius, i is orbit inclination and u is latitude argument. According to this model, the intensity vector varies both in size and direction and forms the conic surface which gets closed up for half of a satellites orbit. Value of the geomagnetic field intensity vector is as following

$$|\boldsymbol{H}| = \frac{\mu_m}{r^3}\sqrt{1 + 3\sin^2 i \sin^2 u}\,.$$

2.3. Averaged geomagnetic field model

Simpler model of the geomagnetic field is described in [12]. Vector $\boldsymbol{H}$ rotation velocity is considered constant and is equal to its average velocity along the orbit. Vector $\boldsymbol{H}$ magnitude is considered to be constant too. The given model is called ***averaged geomagnetic field model***. According to this interpretation, the vector $\boldsymbol{H}$ forms a direct circular cone. An angle between cone generator and orbit plane is equal to the orbit inclination (see Figure 2). The vector $\boldsymbol{H}$ terminus (end point) moves on the cone base with the constant angular velocity which is equal to double satellite orbital speed.

The projections of the geomagnetic field intensity vector in the axes of ORF are as follows

$$H_1 = \frac{\mu_m}{r^3}\cos u \sin i,\ H_2 = \frac{\mu_m}{r^3}\cos i,\ H_3 = -\frac{\mu_m}{r^3}\sin u \sin i\,. \qquad (8)$$

The vector $\boldsymbol{H}$ magnitude can be equal, in particular, to a simple mean from its minimum and maximum values

$$|\boldsymbol{H}| = \frac{\mu_m}{2r^3}\left(1+\sqrt{1+3\sin^2 i}\right) = const$$

or to an integral mean for one revolution period

$$|\boldsymbol{H}| = \frac{\mu_m}{\pi r^3}\int_0^{\pi}\sqrt{1+3\sin^2 i \sin^2 u}\,du = const.$$

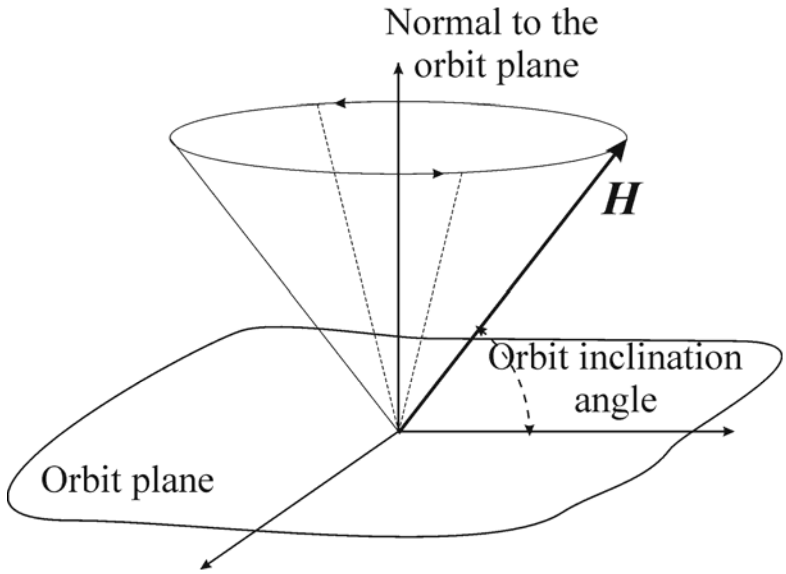

Figure 2. An averaged geomagnetic field model interpretation

3. Control low synthesis

The main idea of the considered magnetic control algorithm is a synthesis of two control components. The first component defines the required satellite orientation and provides its stability in Lyapunov's sense and the second one provides an asymptotical stability of equilibria and carries out oscillation damping. Both components of control have the same physical nature and are accomplished simultaneously by the same coils. So, we separate magnetic control torque (4) in two parts. The first one is a "restoring" torque and the second one is a "damping" torque,

$$\begin{aligned} &\boldsymbol{M}_{magn} = \boldsymbol{M}_{magn\_r} + \boldsymbol{M}_{magn\_d} = \boldsymbol{m}_r \times \boldsymbol{B} + \boldsymbol{m}_d \times \boldsymbol{B}, \\ &\boldsymbol{m}_\Sigma = \boldsymbol{m}_r + \boldsymbol{m}_d. \end{aligned} \tag{9}$$

The satellite equation of motion (3) can be considered as following

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x}, \boldsymbol{u}, t) \tag{10}$$

where $\boldsymbol{x}$ is the satellite state vector, $f(\boldsymbol{x})$ is a conservative autonomous term responsible for existence of the equilibrium and its Lyapunov's stability and $g(\boldsymbol{x}, \boldsymbol{u}, t)$ is a damping magnetic control

$$g(\boldsymbol{x}, \boldsymbol{u}, t) = \boldsymbol{m}_d \times \boldsymbol{B}\ .$$

Restoring dipole moment $\boldsymbol{m}_r$ can be considered as a constant for a short time. Taking into account this assumption we write conservative terms as follows

$$\dot{\boldsymbol{\omega}} = \hat{\boldsymbol{J}}^{-1}\left[-\boldsymbol{\omega}\times\hat{\boldsymbol{J}}\boldsymbol{\omega}-\boldsymbol{\omega}\times\boldsymbol{K}'-\frac{\tilde{d}\boldsymbol{K}'}{dt}+\boldsymbol{M}_{gr}+\boldsymbol{m}_r\times\boldsymbol{B}\right]. \tag{11}$$

For equations (11) and (2) the generalized integral of energy (the Jacobi integral)

$$J = T_2 - T_0 + \Pi$$

is as following in explicit form

$$\begin{aligned} J = {} & \frac{1}{2}\left(A\overline{p}^2 + B\overline{q}^2 + C\overline{r}^2\right) + \frac{3}{2}\omega_0^2\left[\left(A-C\right)a_{31}^2 + \left(B-C\right)a_{32}^2\right] + \\ & + \frac{1}{2}\omega_0^2\left[\left(B-A\right)a_{21}^2 + \left(B-C\right)a_{23}^2\right] - \omega_0\sum_{i=1}^{3}\overline{h}_i a_{2i} - \\ & - B_1\sum_{i=1}^{3}m_{ri}a_{1i} - B_2\sum_{i=1}^{3}m_{ri}a_{2i} - B_3\sum_{i=1}^{3}m_{ri}a_{3i} = const. \end{aligned} \tag{12}$$

Here m_{ri} are the projections of the restoring dipole moment $\boldsymbol{m}_r$ on the axes of the BRF.

The generalized energy integral (12) can be considered as the Lyapunov function for equations (11) and we denote expression

$$W = \Pi - T_0 \tag{13}$$

as a changed potential energy and further we use it for search and stability analysis of equilibrium orientations of the equations conservative part (11).

3.1. Equilibrium orientation existence and stability

The satellite relative equilibrium orientations with respect to ORF are determined by stationary points of the function (13), i.e. by the solutions of the following equations

$$\frac{\partial W}{\partial \alpha} = 0, \frac{\partial W}{\partial \beta} = 0, \frac{\partial W}{\partial \gamma} = 0.$$

It is necessary to note that it is difficult to obtain full set of these equations' solutions (full set of possible satellite orientations) as a function of inertia and magnetic parameters.

On the other hand, we can formulate this problem as following. Let us search the restoring dipole moment of the magnetic coils which provides required orientation of the satellite for given inertia parameters and given parameters of the internal angular momentum. In some special (trivial) cases the required orientation existence of the equations (11) can be determined by the solution of a matrix equation

$$\hat{\boldsymbol{A}}\boldsymbol{m}\times\boldsymbol{B} = 0$$

where $\hat{\boldsymbol{A}}$ is a corresponding direction cosines matrix.

The satellite stationary motion is stable if stationary values of variables turn function (13) into a minimum. If the function W does not accept the minimum value in the stationary motion one can conclude instability of the stationary motion. To analyze an extremum of the function (13) let us obtain its quadratic form. Denote

$$\alpha = \alpha_0 + \bar{\alpha}, \quad \beta = \beta_0 + \bar{\beta}, \quad \gamma = \gamma_0 + \bar{\gamma}$$

where $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ are small deviations from the satellite equilibrium $\alpha = \alpha_0 = const$, $\beta = \beta_0 = const$, $\gamma = \gamma_0 = const$. Then the energy integral takes the form

$$A\bar{p}^2 + B\bar{q}^2 + C\bar{r}^2 + \omega_0^2\left(A_{\alpha\alpha}\bar{\alpha}^2 + A_{\beta\beta}\bar{\beta}^2 + A_{\gamma\gamma}\bar{\gamma}^2 + 2A_{\alpha\beta}\bar{\alpha}\bar{\beta} + \right.$$
$$\left. + 2A_{\beta\gamma}\bar{\beta}\bar{\gamma} + 2A_{\gamma\alpha}\bar{\gamma}\,\bar{\alpha}\right) + \Sigma^* = const.$$

Here the symbol Σ^* designates the terms of the third and higher order with respect to $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$,

$$\begin{aligned}
A_{\alpha\alpha} &= 3\left[(A-C)\left(\bar{a}_{11}^2 - \bar{a}_{31}^2\right) + (B-C)\left(\bar{a}_{12}^2 - \bar{a}_{32}^2\right)\right] + \\
&\quad + \frac{1}{2}\bar{B}_1\sum_{i=1}^{3} m_{ri}\bar{a}_{1i} + \frac{1}{2}\bar{B}_3\sum_{i=1}^{3} m_{ri}\bar{a}_{3i}, \\
A_{\beta\beta} &= \left[(B-A) - (B-C)\sin^2\gamma_0\right]\left(1 + 3\sin^2\alpha_0\right)\cos 2\beta_0 - \\
&\quad - \frac{3}{4}(B-C)\sin 2\alpha_0 \sin\beta_0 \sin 2\gamma_0 + h_1\bar{a}_{21} + h_2\bar{a}_{22} + h_3\bar{a}_{23} + \\
&\quad + \frac{1}{2}\bar{B}_1\left[m_{r1}\bar{a}_{11} - \bar{a}_{21}\cos\alpha_0\left(m_{r2}\cos\gamma_0 - m_{r3}\sin\gamma_0\right)\right] + \frac{1}{2}\bar{B}_2\sum_{i=1}^{3} m_{ri}\bar{a}_{2i} + \\
&\quad + \frac{1}{2}\bar{B}_3\left[m_{r1}\bar{a}_{31} + \bar{a}_{21}\sin\alpha_0\left(m_{r2}\cos\gamma_0 - m_{r3}\sin\gamma_0\right)\right], \\
A_{\gamma\gamma} &= (B-C)\left[\left(\bar{a}_{22}^2 - \bar{a}_{23}^2\right) - 3\left(\bar{a}_{32}^2 - \bar{a}_{33}^2\right)\right] + h_2\bar{a}_{22} + h_3\bar{a}_{23} + \\
&\quad + \frac{1}{2}\bar{B}_1\sum_{i=2}^{3} m_{ri}\bar{a}_{1i} + \frac{1}{2}\bar{B}_2\sum_{i=2}^{3} m_{ri}\bar{a}_{2i} + \frac{1}{2}\bar{B}_3\sum_{i=2}^{3} m_{ri}\bar{a}_{3i}, \qquad (14) \\
A_{\alpha\beta} &= -\frac{3}{2}(A-C)\sin 2\alpha_0 \sin 2\beta_0 + 3(B-C)\left(\bar{a}_{32}\cos\alpha_0 - \bar{a}_{12}\sin\alpha_0\right)\bar{a}_{22} \\
&\quad - \frac{1}{2}\bar{B}_1\left(m_{r1}\bar{a}_{21}\sin\alpha_0 + m_{r2}\bar{a}_{22}\sin\alpha_0 + m_{r3}\bar{a}_{31}\sin\gamma_0\right) - \\
&\quad - \frac{1}{2}\bar{B}_3\left(m_{r1}\bar{a}_{21}\cos\alpha_0 + m_{r2}\bar{a}_{11}\cos\gamma_0 + m_{r3}\bar{a}_{11}\sin\gamma_0\right), \\
A_{\beta\gamma} &= -\frac{1}{2}(B-C)\sin 2\beta_0 \sin 2\gamma_0 - 3(B-C)\left(\bar{a}_{33}\cos\gamma_0 - \bar{a}_{32}\sin\gamma_0\right)\bar{a}_{31} - \\
&\quad - h_2\sin\beta_0\sin\gamma_0 - h_3\bar{a}_{21}\cos\gamma_0 - \frac{1}{2}\bar{B}_1\bar{a}_{11}\left(m_{r2}\sin\gamma_0 + m_{r3}\cos\gamma_0\right) - \\
&\quad - \frac{1}{2}\bar{B}_2\bar{a}_{21}\left(m_{r2}\sin\gamma_0 - m_{r3}\cos\gamma_0\right) - \frac{1}{2}\bar{B}_3\sin\alpha_0\left(m_{r2}\bar{a}_{23} - m_{r3}\bar{a}_{22}\right),
\end{aligned}$$

$$A_{\gamma\alpha}=-3(B-C)(\bar{a}_{12}\bar{a}_{33}+\bar{a}_{13}\bar{a}_{32})-\frac{1}{2}\bar{B}_1(m_{r2}\bar{a}_{33}-m_{r3}\bar{a}_{32})+$$
$$+\frac{1}{2}\bar{B}_3(m_{r2}\bar{a}_{13}-m_{r3}\bar{a}_{12});$$
$$\bar{a}_{ij}=a_{ij}(\alpha_0,\beta_0,\gamma_0),\quad h_i=\bar{h}_i/\omega_0,\quad \bar{B}_i=B_i/\omega_0^2.$$

It follows from the Lyapunov theorem that solution $\alpha=\alpha_0$, $\beta=\beta_0$, $\gamma=\gamma_0$ is stable if the quadratic form
$$A_{\alpha\alpha}\bar{\alpha}^2+A_{\beta\beta}\bar{\beta}^2+A_{\gamma\gamma}\bar{\gamma}^2+2A_{\alpha\beta}\bar{\alpha}\bar{\beta}+2A_{\beta\gamma}\bar{\beta}\bar{\gamma}+2A_{\gamma\alpha}\bar{\gamma}\,\bar{\alpha}$$
is positive-definite, that is, the following inequalities
$$\begin{gathered}A_{\alpha\alpha}>0,\ A_{\alpha\alpha}A_{\beta\beta}-A_{\alpha\beta}^2>0,\\ A_{\alpha\alpha}A_{\beta\beta}A_{\gamma\gamma}+2A_{\alpha\beta}A_{\beta\gamma}A_{\alpha\gamma}-A_{\alpha\alpha}A_{\beta\gamma}^2-A_{\beta\beta}A_{\alpha\gamma}^2-A_{\gamma\gamma}A_{\alpha\beta}^2>0\end{gathered}\tag{15}$$
are valid. The choice of the coil restoring dipole moment components m_{ri} satisfying the condition (15) provides stability of considered orientation.

3.2. Asymptotical stability of the equilibrium orientation

For the equations (11) the Lyapunov function exists. Taking into account damping torque and considering completed equation of motion (10), the function derivative has the form
$$\dot{V}=\left[\frac{\partial V}{\partial \mathbf{x}}\right]\dot{\mathbf{x}}=\left[\frac{\partial V}{\partial \mathbf{x}}\right]\left(f(\mathbf{x})+g(\mathbf{x},\mathbf{u},t)\right)=\left[\frac{\partial V}{\partial \mathbf{x}}\right]g(\mathbf{x},\mathbf{u},t).\tag{16}$$
Following the Lyapunov theorem the condition of the equilibrium orientation asymptotical stability has the form $\dot{V}<0$. In our case the system state vector is
$$\mathbf{x}=[p,q,r,\alpha,\beta,\gamma].\tag{17}$$
The damping control torque is as following
$$g(\mathbf{x},\mathbf{u},t)=[\mathbf{m}_d\times\mathbf{B},0,0,0].\tag{18}$$
So, it is enough to consider just satellite relative motion kinetic energy as the Lyapunov function candidate. In this case conditions (16)-(18) can be rewritten as
$$\dot{V}=\frac{\partial V}{\partial \bar{p}}\{\mathbf{m}_d\times\mathbf{B}\}_1+\frac{\partial V}{\partial \bar{q}}\{\mathbf{m}_d\times\mathbf{B}\}_2+\frac{\partial V}{\partial \bar{r}}\{\mathbf{m}_d\times\mathbf{B}\}_3<0.$$
Rewrite it in a detailed form

$$\dot{V} = \begin{pmatrix} A\overline{p} \\ B\overline{q} \\ C\overline{r} \end{pmatrix} \cdot \begin{pmatrix} B_3 \sum_{i=1}^{3} m_{di} a_{2i} - B_2 \sum_{i=1}^{3} m_{di} a_{3i} \\ B_1 \sum_{i=1}^{3} m_{di} a_{3i} - B_3 \sum_{i=1}^{3} m_{di} a_{1i} \\ B_2 \sum_{i=1}^{3} m_{di} a_{1i} - B_1 \sum_{i=1}^{3} m_{di} a_{2i} \end{pmatrix} < 0. \tag{19}$$

The choice of the coils damping dipole moment components m_{di} satisfying the condition (19) provides stability of the considered orientation.

4. Some special cases of the magnetic control synthesis

As mentioned earlier, it is not so easy to solve a problem of the magnetic control synthesis in general case. In this section some special cases which adopt the analytical solution and confirm the perceptivity of the chosen approach will be considered. Let us consider the following system configuration:

- the satellite is an axisymmetrical body ($A=C$);
- the flywheel spin axis coincides with the satellite dynamic symmetry axis, that is, $\overline{h}_1 = \overline{h}_3 = 0,\ \overline{h}_2 \neq 0$;
- magnetic coils are collinear to the axes of BRF.

We consider the satellite motion in a polar orbit, that is, $B_2 = 0$ assuming the averaged geomagnetic field model (8). The system state vector is supposed to be known at each moment of time.

It is well known that at a certain relationship between system parameters the gyrostat-satellite has equilibrium orientation. At the equilibrium the flywheel angular momentum is perpendicular to the orbit plane (see [9-11] where completed analysis of available gyrostat equilibria is given). The satellite in axisymmetrical configuration has one-parameter family of solutions. By other words, the flywheel provides one-axis orientation of the satellite. Magnetic torque (9) will be considered as a control torque to provide three-axis orientation of the satellite. We consider the required orientation

$$\alpha = \alpha_0, \quad \beta = 0, \quad \gamma = 0 \tag{20}$$

At the first stage we pay attention to the system with one degree of freedom, namely, to the rotated planar disk equipped with the orthogonal magnetic coils located in its plane. Vector $\boldsymbol{B}$ has the constant magnitude and rotates with constant angular velocity in a plane parallel to the disk. Further a method of the magnetic control synthesis providing asymptotical stability of the required orientation is considered.

4.1. One-axis controlled rotation

Consider the rotation of the axisymmetrical disk subjected to magnetic control torque which is resulted by interaction of the external magnetic field with a magnetic dipole moment generated by the coils the disk mounted on. $OX_1X_2X_3$ is the inertial reference frame (IRF) and $Ox_1x_2x_3$ is the disk-body reference frame (DRF). Ox_k $(k = 1,2,3)$ are the principal central axes of inertia of the disk. Axes OX_1 and Ox_1 coincide. External magnetic field vector $\boldsymbol{B}$ has the constant magnitude and rotates in the disk plane with constant angular velocity. The equation of motion in this case is

$$J\dot{\boldsymbol{\omega}} = \boldsymbol{m}_\Sigma \times \boldsymbol{B}$$

where J is an axial moment of inertia of the disk and $\boldsymbol{m}_\Sigma$ is a dipole moment of the disk. It is necessary to construct magnetic control (9) which provides an existence and stability of required orientation $\alpha = \alpha_0$.

First of all, let us consider the equilibria existence problem. Assuming the restoring dipole moment for a short time is a constant value, we write disk potential energy as

$$\Pi = (\boldsymbol{m}_r, \boldsymbol{B}) = -B_1 (m_{r1} cos\alpha + m_{r3} sin\alpha) - B_3 (-m_{r1} sin\alpha_0 + m_{r3} cos\alpha_0).$$

The condition of existence of equilibria $\alpha = \alpha_0$ has the form

$$\left.\frac{\partial \Pi}{\partial \alpha}\right|_{\alpha=\alpha_0} = B_1 (m_{r1} sin\alpha_0 - m_{r3} cos\alpha_0) + B_3 (m_{r1} cos\alpha_0 + m_{r3} sin\alpha_0) = 0.$$

Introducing following relationships

$$\begin{aligned} D_1 &= B_1 sin\alpha_0 + B_3 cos\alpha_0, \\ D_3 &= B_1 cos\alpha_0 - B_3 sin\alpha_0, \end{aligned} \tag{21}$$

we rewrite the condition of stability existence

$$m_{r1} D_1 = m_{r3} D_3. \tag{22}$$

Conditions of stability of required equilibria $\alpha = \alpha_0$ is a positiveness of the second order of potential energy function

$$\left.\frac{\partial^2 \Pi}{\partial \alpha^2}\right|_{\alpha=\alpha_0} = B_1 (m_{r1} cos\alpha_0 + m_{r3} sin\alpha_0) - B_3 (m_{r1} sin\alpha_0 - m_{r3} cos\alpha_0) > 0$$

which can be rewritten in the form

$$m_{r1} D_3 + m_{r1} D_1 > 0. \tag{23}$$

Thus, the conditions (22) and (23) determine values of the restoring dipole moment components which provide the Lyapunov stability of the required equilibria $\alpha = \alpha_0$. It is obvious that it is necessary to add these conditions to conditions of a physical reliability of the moment created

$$m_{ri} \le \bar{m}_r. \tag{24}$$

Here $\bar{m}_r$ is the maximum value of the dipole moment of the coil.

In Figure 3 and in the Table 1 the scheme of a choice of the dipole moments satisfying to conditions (22) - (24) is resulted.

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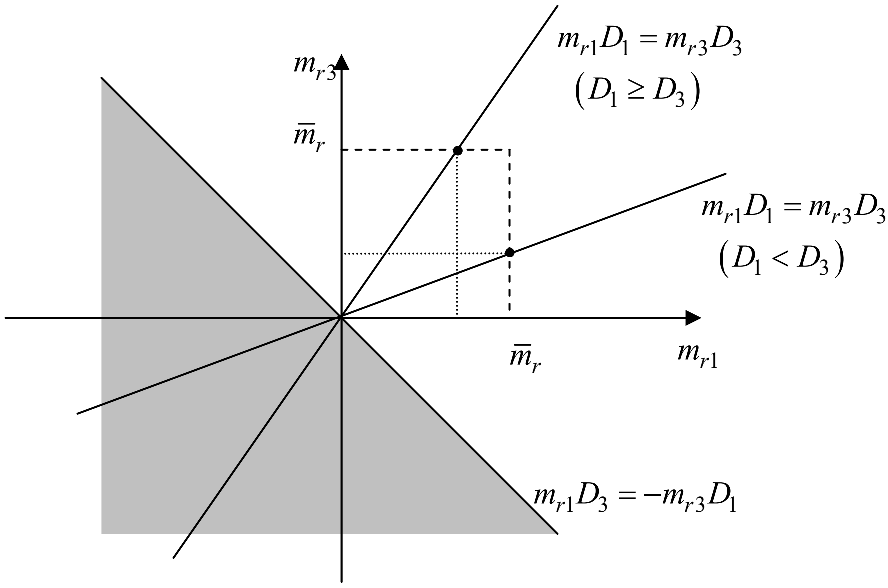

Figure 3. An algorithm of restoring dipole moment determination

Table 1. Restoring dipole components determination

$D_1 > 0,\quad D_3 > 0$		$D_1 > 0,\quad D_3 < 0$	
$D_1 \geq D_3$	$D_1 < D_3$	$\lvert D_1\rvert \geq \lvert D_3\rvert$	$\lvert D_1\rvert < \lvert D_3\rvert$
$m_{r3} = \bar{m}_r$	$m_{r1} = \bar{m}_r$	$m_{r3} = \bar{m}_r$	$m_{r1} = -\bar{m}_r$
$D_1 < 0,\quad D_3 < 0$		$D_1 < 0,\quad D_3 > 0$	
$\lvert D_1\rvert \geq \lvert D_3\rvert$	$\lvert D_1\rvert < \lvert D_3\rvert$	$\lvert D_1\rvert \geq \lvert D_3\rvert$	$\lvert D_1\rvert < \lvert D_3\rvert$
$m_{r3} = -\bar{m}_r$	$m_{r1} = -\bar{m}_r$	$m_{r3} = -\bar{m}_r$	$m_{r1} = \bar{m}_r$

The disk kinetic energy can be considered as a Lyapunov function, i.e. $\tilde{V} = \frac{1}{2}J\omega^2$. Its first derivative in accordance with (16) is as following

$$\dot{\tilde{V}} = \frac{\partial \tilde{V}}{\partial \omega}\left[\boldsymbol{m}_d \times \boldsymbol{B}\right]_2 =$$

$$= J\omega\left[m_{d1}\left(-B_1 sin\alpha - B_3 cos\alpha\right) + m_{d3}\left(B_1 cos\alpha - B_3 sin\alpha\right)\right].$$

Asymptotical stability condition $\dot{\tilde{V}} < 0$ is satisfied if the expressions

$$m_{d1} = -\bar{m}_d sign\left[J\omega\left(-B_1 sin\alpha - B_3 cos\alpha\right)\right],$$
$$m_{d3} = -\bar{m}_d sign\left[J\omega\left(B_1 cos\alpha - B_3 sin\alpha\right)\right] \quad (25)$$

are valid. It is a reason to choose current value of the magnitude of the damping magnetic moment of each coil to be proportional to the value of current angular velocity of the body, that is,

$$\bar{m}_d = k_d |\omega|.$$

The latter equality is actually equivalent to the requirement of the magnetic torque $\boldsymbol{m}_d \times \boldsymbol{B}$ proportionality to a current disk angular velocity (called viscous damping) and can be obtained from condition $\dot{\tilde{V}} = -\omega^2$.

4.1.1 Results of numerical study

In this section some results of numerical simulation of the disk motion affected by the control dipole moments (22), (23) and (25) are presented for the following values of the satellite parameters $J = 0.1875\text{kg}\cdot\text{m}^2$, $|\vec{B}| = 40 \times 4\pi \times 10^{-7} N/(Am)$, $\omega_{\vec{B}} = 0.01s^{-1}$, $\bar{m}_r = 1\text{A}\cdot\text{m}^2$. The initial conditions are $\alpha_{init} = 10^0$, $\dot{\alpha}_{init} = 0$. The required disk orientation is $\alpha_0 = 30^0$. In Figures 4 and 5 the synthesized restoring and damping coil dipole moments are presented. Behavior of the system state vector affected by the control torque $\boldsymbol{M}_{magn} = [\boldsymbol{m}_r + \boldsymbol{m}_d] \times \boldsymbol{B}$ is shown in Figure 6.

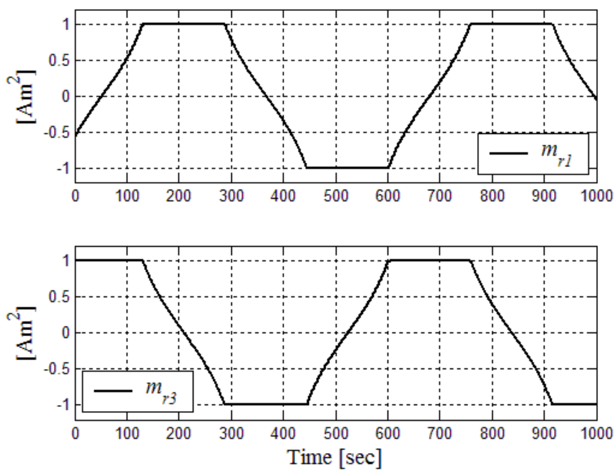

Figure 4. Restoring dipole moments

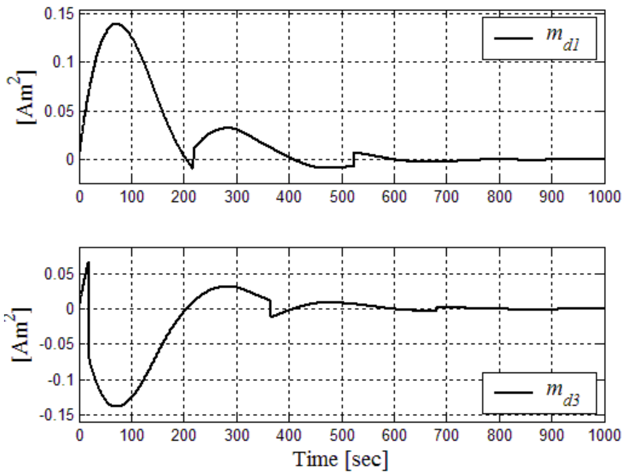

Figure 5. Damping dipole moments

Remark 1

In case of the restoring dipole moment in equilibrium orientation has an opposite direction with respect to the external magnetic field vector (restoring dipole moment components have an opposite sign as it is shown in Figure 4) another stable equilibrium orientation $\bar{\alpha}_0 = \alpha_0 - \pi$ exists. It obviously follows from conditions of existence and stability of the required orientation resulted above. In Figures 7- 9 this situation is presented.

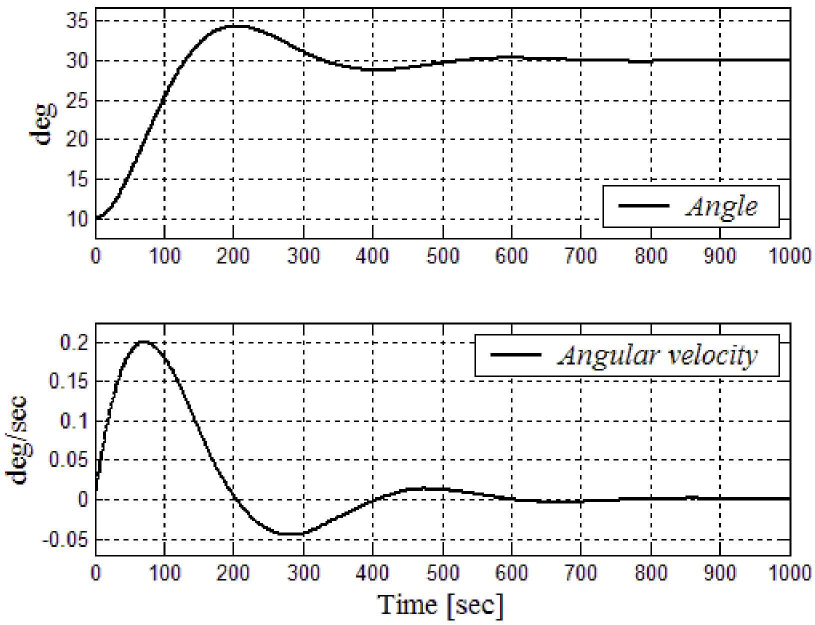

Figure 6. System state vector behavior (required orientation is α_0)

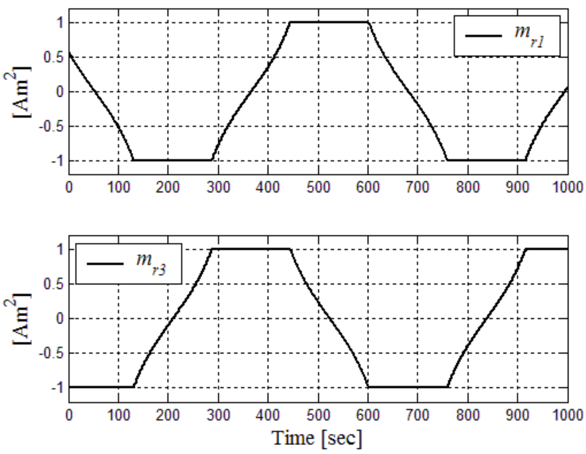

Figure 7. Restoring dipole moments

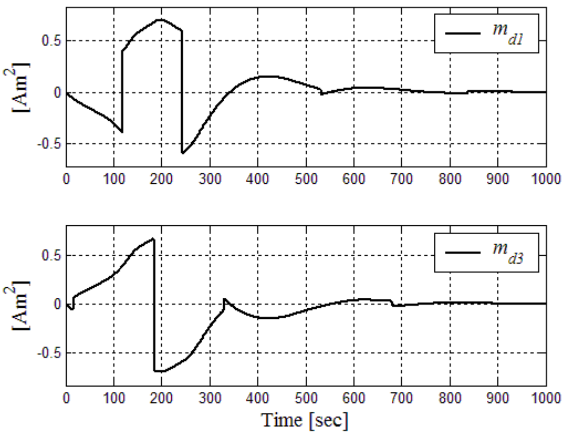

Figure 8. Damping dipole moments

Remark 2

If just one component of the restoring dipole moment (m_{r1} or m_{r3}) has the opposite sign then the disk spins "permanently". It is necessary to carry out additional analytical research of this motion type and establish the relationship between restoring and damping magnetic torques (between corresponding dipole moments). In Figures 10-12 the permanent disk rotation under constructed control is illustrated in case m_{r1} has opposite sign and in Figures 13-15 the same situation is considered if m_{r3} has opposite sign.

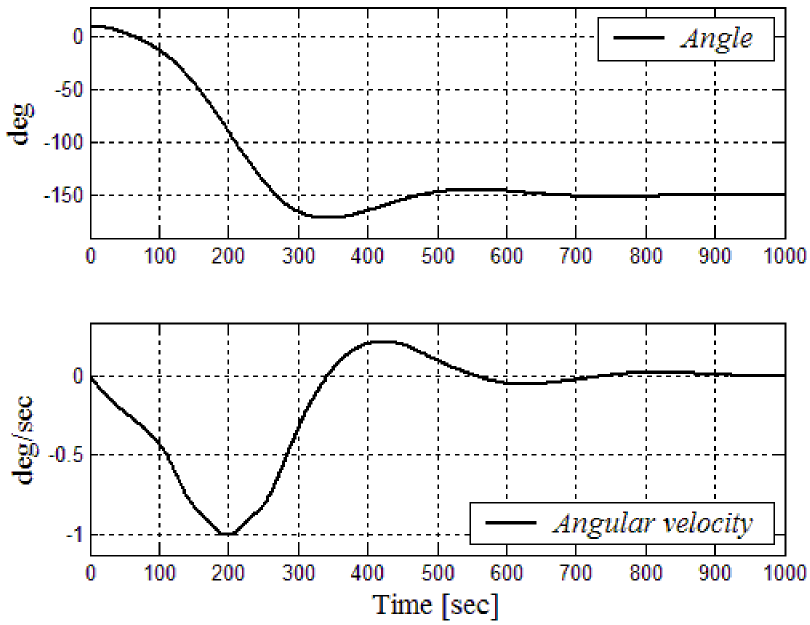

Figure 9. System state vector behavior (required orientation is $\alpha_0 - \pi$)

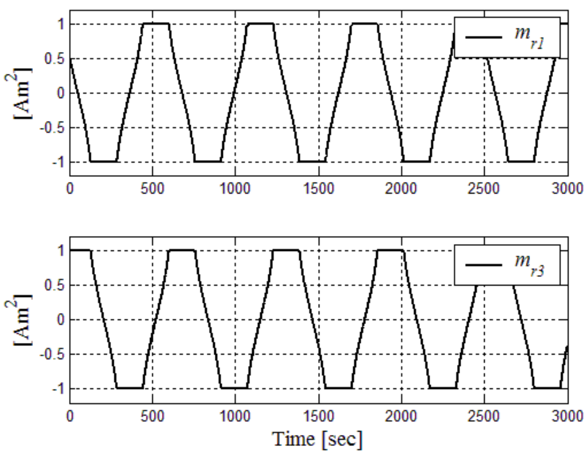

Figure 10. Restoring dipole moments

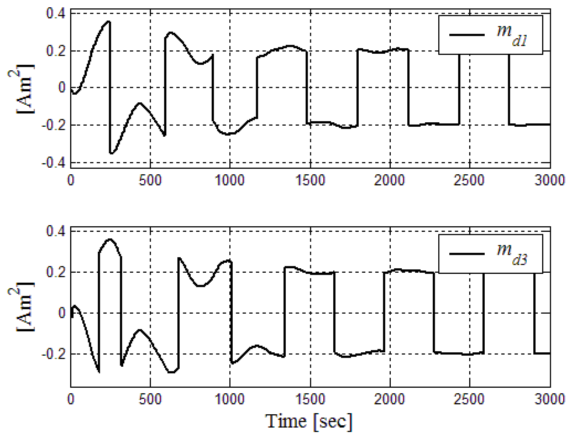

Figure 11. Damping dipole moments

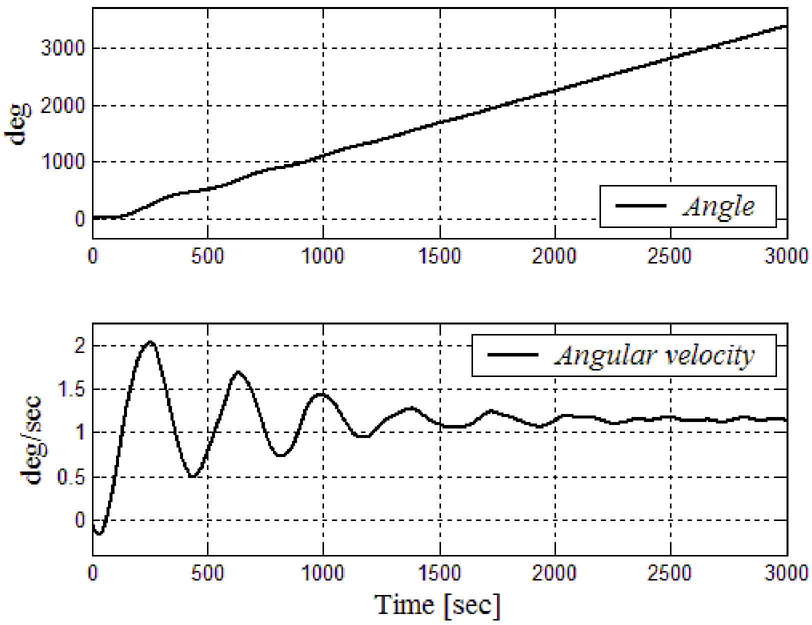

Figure 12. System state vector behavior (m_{r1} has opposite sign)

Remark 3

The additional analysis of the synthesized magnetic dipole moments $\boldsymbol{m}_r$ and $\boldsymbol{m}_d$ (from the point of view of their interrelationship and with current angular velocity) is necessary for control algorithm optimization at various chosen functional.

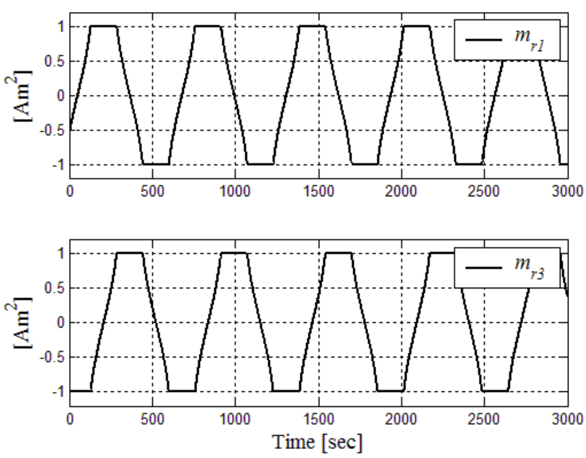

Figure 13. Restoring dipole moments

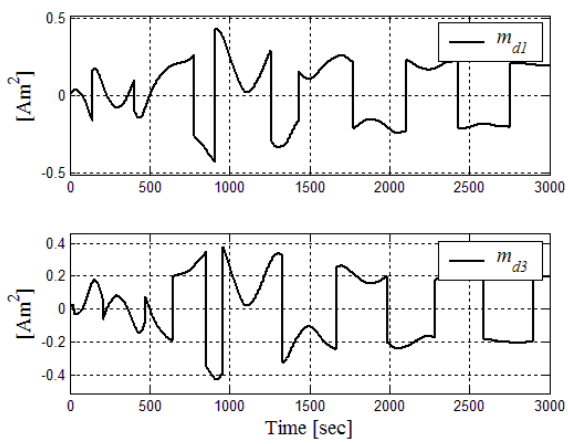

Figure 14. Damping dipole moments

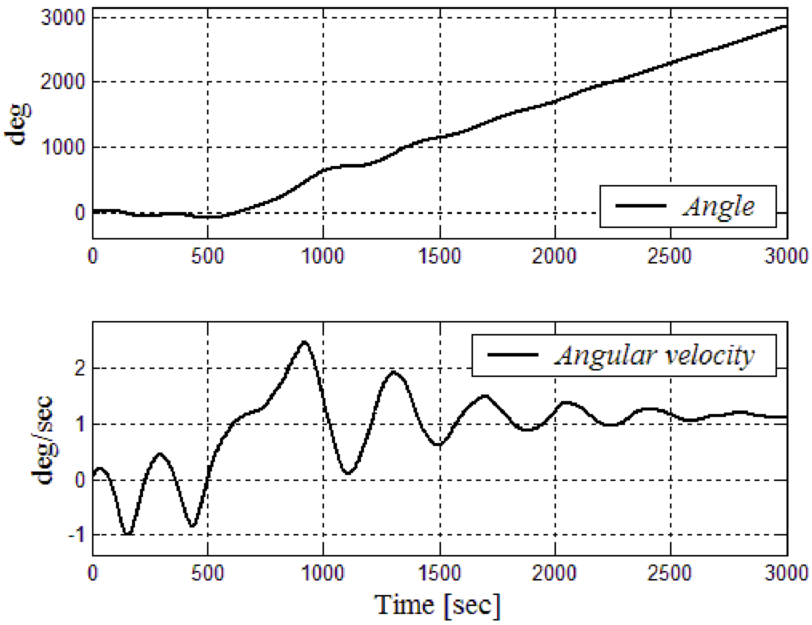

Figure 15. System state vector behavior (m_{r3} has opposite sign)

4.2. Motion of a dynamically symmetrical satellite

Now let us consider the satellite motion using task simplification introduced in section 4. The equation of motion (11) in this case takes the form

$$\begin{aligned}
&A\dot{p}+(A-B)qr-3\omega_0^2(A-B)a_{32}a_{33}-\bar{h}_2 r+\\
&+m_{r3}(B_1a_{21}+B_3a_{23})-m_{r2}(B_1a_{31}+B_3a_{33})=0,\\
&B\dot{q}+m_{r1}(B_1a_{31}+B_3a_{33})-m_{r3}(B_1a_{11}+B_3a_{13})=0,\\
&A\dot{r}+(B-A)pq-3\omega_0^2(B-A)a_{31}a_{32}+\bar{h}_2 p+\\
&+m_{r2}(B_1a_{11}+B_3a_{13})-m_{r1}(B_1a_{21}+B_3a_{23})=0;
\end{aligned} \qquad (26)$$

and the generalized energy integral (12) is written as follows

$$J=\frac{1}{2}\left(A\left(\bar{p}^2+\bar{r}^2\right)+B\bar{q}^2\right)+\frac{3}{2}\omega_0^2(B-A)a_{32}^2+$$

$$+\frac{1}{2}\omega_0^2(A-B)a_{22}^2-\omega_0\bar{h}_2a_{22}-B_1\sum_{i=1}^{3}m_{ri}a_{1i}-B_3\sum_{i=1}^{3}m_{ri}a_{3i}=const\ .$$

4.2.1 The conditions of required orientation existence

The conditions of required orientation (20) existence have the simplest form and can be obtained using matrix equation

$$\hat{\boldsymbol{A}}\boldsymbol{m}_r\times\boldsymbol{B}=\boldsymbol{0}$$

where $\hat{\boldsymbol{A}}$ is a corresponding direction cosines matrix

$$\hat{\mathbf{A}} = \begin{vmatrix} cos\alpha_0 & 0 & sin\alpha_0 \\ 0 & 1 & 0 \\ -sin\alpha_0 & 0 & cos\alpha_0 \end{vmatrix}.$$

The-equations defining conditions of equilibrium existence have the form

$$B_3 m_{r2} + B_2 \left(m_{r1} sin\alpha_0 - m_{r3} cos\alpha_0 \right) = 0;$$

$$B_1 \left(m_{r3} cos\alpha_0 - m_{r1} sin\alpha_0 \right) - B_3 \left(m_{r1} cos\alpha_0 + m_{r3} sin\alpha_0 \right) = 0;$$

$$B_2 \left(m_{r1} cos\alpha_0 + m_{r3} sin\alpha_0 \right) - B_1 m_{r2} = 0.$$

Assuming a polar orbit, these equations can be resulted in

$$\frac{m_{r1}}{m_{r3}} = \frac{B_1 cos\alpha_0 - B_3 sin\alpha_0}{B_1 sin\alpha_0 + B_3 cos\alpha_0}; m_{r2} = 0. \tag{27}$$

Taking into account the introduced expressions (21), the conditions (27) can be rewritten in the form (22). Thus, an existence of the required orientation (20) is provided with a determination of corresponding values of the restoring dipole magnetic moments of the coils m_{ri} satisfying the conditions (27).

4.2.2 Stability conditions of required orientation

Factors of the square-law form of the changed potential energy (14) are as follows

$$\begin{aligned}
&A_{\alpha\alpha} = \frac{1}{2} m_{r1} \left(B_1 \cos\alpha_0 - B_3 \sin\alpha_0 \right) + \frac{1}{2} m_{r3} \left(B_1 \sin\alpha_0 + B_3 \cos\alpha_0 \right), \\
&A_{\beta\beta} = \left(B - A \right) \left(1 + 3\sin^2\alpha_0 \right) + h_2 + \frac{1}{2} m_{r1} \left(\bar{B}_1 \cos\alpha_0 - \bar{B}_3 \sin\alpha_0 \right), \\
&A_{\gamma\gamma} = \left(B - A \right) \left(1 + 3\cos^2\alpha_0 \right) + h_2 + \frac{1}{2} m_{r3} \left(\bar{B}_1 \sin\alpha_0 + \bar{B}_3 \cos\alpha_0 \right), \\
&A_{\alpha\beta} = 0, \\
&A_{\beta\gamma} = \frac{3}{2} (B - A) \sin 2\alpha_0 - \frac{1}{2} m_{r3} \left(\bar{B}_1 \cos\alpha_0 - \bar{B}_3 \sin\alpha_0 \right), \\
&A_{\gamma\alpha} = 0
\end{aligned} \tag{28}$$

and the conditions of its positive definiteness (15) take a form of following inequalities

$$\begin{aligned}
&m_{r1} D_3 + m_{r2} D_1 > 0, \\
&\left(B - A \right) \left(1 + 3\sin^2\alpha_0 \right) + h_2 + m_{r1} \bar{D}_3 > 0, \\
&\left[\left(B - A \right) \left(1 + 3\sin^2\alpha_0 \right) + h_2 + \frac{1}{2} m_{r1} \bar{D}_3 \right] \left[\left(B - A \right) \left(1 + 3\cos^2\alpha_0 \right) + h_2 + \frac{1}{2} m_{r3} \bar{D}_1 \right] - \\
&\qquad - \left[\frac{3}{2} (B - A) \sin 2\alpha_0 - \frac{1}{2} m_{r3} \bar{D}_3 \right] > 0.
\end{aligned} \tag{29}$$

Let us notice that the first inequality in (29) together with the equilibrium existence condition (27) are completely equivalent to the corresponding conditions obtained by consideration of the one-axis rotation problem.

4.2.3 Asymptotical stability of required orientation

The asymptotical stability condition (19) can be rewritten in the form

$$\dot{V} = \sum_{i=1}^{3} m_{di} \left[A\overline{p}B_3 a_{2i} + B\overline{q}\left(B_1 a_{3i} - B_3 a_{1i}\right) - \overline{r}B_1 a_{2i} \right] < 0 .$$

The damping magnetic moments of the coils providing negativity of the Lyapunov function derivative, for example, can be chosen as

$$m_{di} = -\overline{m}_d sign\left[A\overline{p}B_3 a_{2i} + B\overline{q}\left(B_1 a_{3i} - B_3 a_{1i}\right) - \overline{r}B_1 a_{2i} \right]. \tag{30}$$

It is a simplest way of damping dipole moments definition. However, it is inconvenient to apply the derived approach into practice and it is necessary to search for other approaches to the expression (30) analysis. The choice of the damping magnetic moments of the coils which creates the mechanical torque proportional to relative angular velocity in this case is rather difficult (maybe it is even impossible).

4.2.4 Results of numerical integration.

In this section the results of numerical simulation of the satellite motion affected by the control dipole moments (27), (28) and (30) are presented for the following values of parameters

$$\mathrm{A} = 0.0938\left[\mathrm{kg\,m^2}\right]; \mathrm{B} = 0.1875\left[\mathrm{kg\,m^2}\right]; \omega_0 = 0.0017\left[\mathrm{rad/sec}\right];$$

$$\left|\vec{\mathrm{B}}\right| = 40 \times 4\pi \times 10^{-7}\left[\frac{\mathrm{N}}{\mathrm{A\,m}}\right], \quad \omega_{\vec{\mathrm{B}}} = 2\omega_0\left[\mathrm{rad/sec}\right]; \qquad .$$

$$\overline{\mathrm{m}}_\mathrm{r} = 1\left[\mathrm{A\,m^2}\right], \quad \overline{\mathrm{m}}_\mathrm{d} = 0.05\left[\mathrm{A\,m^2}\right]; \overline{\mathrm{h}}_2 = \frac{4}{3}\mathrm{B}\omega_0$$

Satellite initial orientation is defined by

$$\alpha = 10^{\hat{\imath}}, \quad \beta = 7^{\hat{\imath}}, \quad \gamma = 4^{\hat{\imath}} .$$

Satellite required orientation is defined by

$$\alpha = 30^{\hat{\imath}}, \quad \beta = 0^{\hat{\imath}}, \quad \gamma = 0^{\hat{\imath}} .$$

It is necessary to note the numerical solution of the inequalities (29) shows that last two inequalities at considered values of the system parameters are valid at any time. Thus, the method for determination of the restoring dipole moments is equivalent to the method obtained at the consideration of one-axis rotation motion (see Figure 3 and Table 1).

In Figure 16 the direction cosine a_{11} (the cosine of the angle between the first axes of orbital and body-fixed reference frames) and the direction cosine a_{22} (angle between the second axes) are shown. In Figure 17 the satellite absolute angular velocity components are presented. One can see that the constructed control provides the required orientation. However, fast oscillations of the velocity components are observed. First of all, it is involved with imperfect determination of the damping dipole moment. The special attention will be paid to this problem solution within next study.

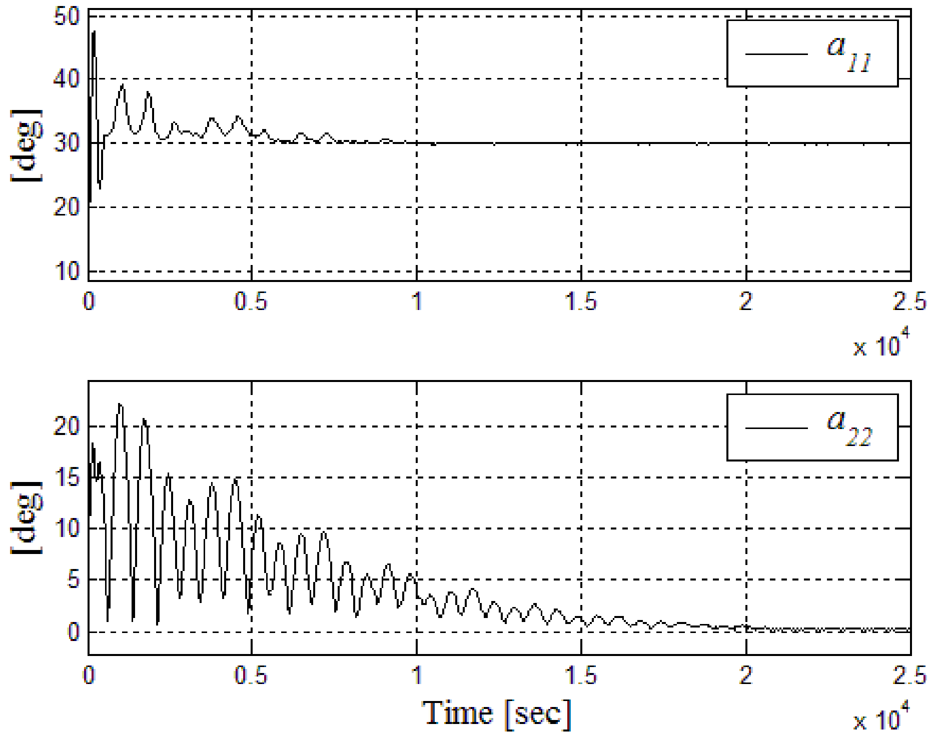

Figure 16. Required orientation providing

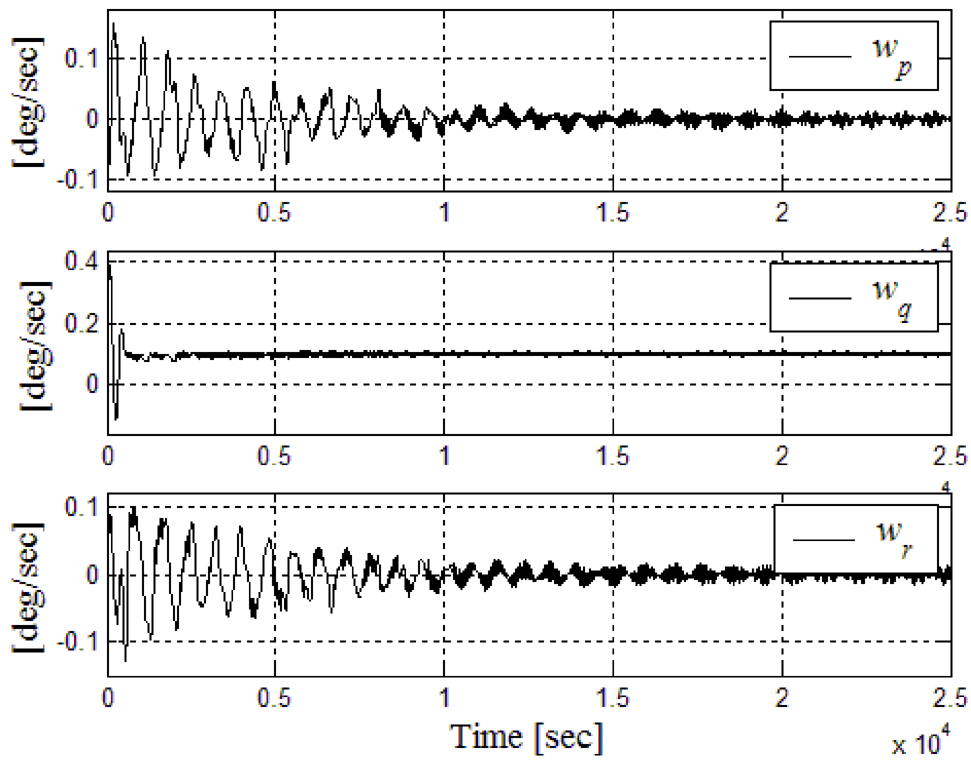

Figure 17. Angular velocity damping

Conclusion

In the present paper the dynamically symmetrical satellite equipped by the pitch flywheel and active magnetic control system was considered.

Magnetic control algorithm is based on the idea of synthesis of two control components. The first component provides the required satellite orientation and its Lyapunov stability. The second one provides an asymptotical stability. Both components of control have the same physical nature and are realized simultaneously by the same actuators.

The general approaches for magnetic control algorithm construction were considered. The conditions of the required orientation existence and stability are obtained. Some algorithms defining the coils dipole moments which provide asymptotical stability of the required equilibria are proposed.

Realization of the developed method is illustrated by some special cases. The pitch flywheel provides one-axis orientation of the dynamically symmetrical satellite, namely, provides a direction of an axis of dynamical symmetry along with the normal to the orbit plane. Active magnetic control in this case is used for the satellite unequivocal orientation in the orbit plane and for damping of small fluctuations in a vicinity of the required orientation. For this reason at the first stage of this work the one-axis rotation problem was considered. Rotation of dynamically symmetrical disk with respect to the fixed axis is investigated. The disk is equipped by the magnetic coils which dipole moment interacts with an external magnetic field developing the control torque. The vector of intensity of the geomagnetic field has the constant magnitude and rotates in a disk plane with a constant angular velocity. Based on the Lyapunov stability theory the magnetic control algorithm providing the asymptotical stability of the disk required orientation has been synthesized for the considered simplified problem. It is shown, that the similar control law can be used to provide a permanent rotation of the disk.

The developed methods of magnetic control synthesis were applied to more complex problem. Motion of dynamically symmetrical gyrostat-satellite under the action of gravitational and control magnetic torques was considered. Magnetic control was used to provide three-axis satellite orientation with respect to ORF or, that is more exact, to provide the unequivocal required orientation in orbit plane. The satellite motion was considered in a polar circular orbit in the averaged geomagnetic field. The obtained results of numerical simulation of the satellite motion affected by the synthesized control showed availability of the considered approach.

Here are the directions of further research:

- the determination of an optimal relationship between the restoring and the damping magnetic torques and establishing of their explicit dependence for a current satellite angular velocity.
- the development of the proposed control algorithm in more difficult cases (non-polar orbit; direct dipole model of the geomagnetic field);
- the analysis of an algorithm efficiency at piecewise continuous control.

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