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Formulæ for computational transport simulations specified. Based on angular Sn and spatial finite element discretization have been applied to even-odd parity notation of transport equation. Also suggested several considerations on its solution techniques, such as new promising method of scattering source conversion or quasidiffusion method for inner iterations acceleration and Krylov methods for eigenvalue problem solution.

Keywords: *even-odd parity transport equation, finite elements, Sn-method, centered scattering source, quasidiffusion*

Коконков Никита Игоревич

Угловая Sn и конечно-элементная пространственная дискретизация чётно-нечётного уравнения переноса

Приведены формулы для численного моделирования переноса нейтральных частиц. Дискретизация чётно-нечётной системы уравнений переноса осуществлена на основе Sn-метода по углам и конечных элементов по пространству. Также приведены некоторые соображения насчёт способа её решения: новый многообещающий метод центрирования интеграла рассеяния и/или метод квазидиффузии для ускорения сходимости внутренних итераций и использование крыловских методов для решения стационарных задач.

Ключевые слова: *чётно-нечётная система уравнений переноса, конечные элементы, Sn-метод, центрированная форма интеграла рассеяния, квазидиффузия*

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Formalism

Base equations for computations is a system of even-odd parity transport equations in multigroup notation

$$\frac{1}{\upsilon_g}\frac{\partial\psi_g^+\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^-\left(\vec{\Omega}\right)}+\sigma^g\psi_g^+\left(\vec{\Omega}\right)=\int\limits_{\overleftarrow{\Omega}}\sigma_{gg}^+\left(\overleftarrow{\Omega}\right)\psi_g^+\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+ \\ +\sum_{g'\neq g}\int\limits_{\overleftarrow{\Omega}}\nu^{g'}\sigma_{g'g}^+\left(\overleftarrow{\Omega}\right)\psi_{g'}^+\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+\sum_{g'}\int\limits_{\overleftarrow{\Omega}}\nu^{g'f}\sigma_{g'fg}^+\left(\overleftarrow{\Omega}\right)\psi_{g'}^+\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+q_g, \tag{1e}$$

$$\frac{1}{\upsilon_g}\frac{\partial\psi_g^-\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^+\left(\vec{\Omega}\right)}+\sigma^g\psi_g^-\left(\vec{\Omega}\right)=\int\limits_{\overleftarrow{\Omega}}\sigma_{gg}^-\left(\overleftarrow{\Omega}\right)\psi_g^-\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+ \\ +\sum_{g'\neq g}\int\limits_{\overleftarrow{\Omega}}\nu^{g'}\sigma_{g'g}^-\left(\overleftarrow{\Omega}\right)\psi_{g'}^-\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+\sum_{g'}\int\limits_{\overleftarrow{\Omega}}\nu^{g'f}\sigma_{g'fg}^-\left(\overleftarrow{\Omega}\right)\psi_{g'}^-\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}, \tag{1o}$$

with boundary conditions

$$\psi_g^+\left(\vec{\Omega},\vec{r}\big|_{\partial V},t\right)+\psi_g^-\left(\vec{\Omega},\vec{r}\big|_{\partial V},t\right)=\psi_{\partial V}^g\left(\vec{\Omega},\vec{r}\big|_{\partial V},t\right),\overrightarrow{\partial V}\cdot\vec{\Omega}<0, \tag{2i}$$

$$\psi_g^+\left(\vec{\Omega},\vec{r}\big|_{\partial V},t\right)-\psi_g^-\left(\vec{\Omega},\vec{r}\big|_{\partial V},t\right)=\psi_{\partial V}^g\left(-\vec{\Omega},\vec{r}\big|_{\partial V},t\right),\overrightarrow{\partial V}\cdot\vec{\Omega}>0, \tag{2o}$$

and any initial conditions which can be introduced by end user.

The symbols in equations (1–2) defined as follows: υ_g – the group g velocity;
$\psi_g^{\pm}$ – the even-odd parity angular flux distribution as functions of time t, spatial coordinates $\vec{r}$, and angular direction $\vec{\Omega}$ for group g;
σ^g – the macroscopic total cross-section for group g;
$\sigma_{g'g}^{\pm}$ – the even-odd parity macroscopic differential (elastic) scattering cross-section from group g' and angular direction $\overleftarrow{\Omega}$ to group g and angular direction $\vec{\Omega}$;
$\sigma_{g'fg}^{\pm}$ – the even-odd parity macroscopic differential fission cross-section from group g' and angular direction $\overleftarrow{\Omega}$ to group g and angular direction $\vec{\Omega}$;
$\nu^{g'}$ – the mean number of neutrons produced due to elastic scattering in group g';
$\nu^{g'f}$ – the mean number of neutrons produced due to fission in group g';
q_g – the spatial extraneous source distribution in group g;
$\overrightarrow{\partial V}$ – the outward directed normal of boundary surface ∂V of convex region V;

$\psi_{\partial V}^{g}$ – the angular flux distribution at boundary surface as function of time t and inward angular direction $\vec{\Omega}$ for group g.

Abovementioned form is derived [1] from transport equation (15) by $\psi_g^{\pm}\left(\vec{\Omega}\right)=\psi_g\left(\vec{\Omega}\right)\pm\psi_g\left(-\vec{\Omega}\right)$ symmetrization of flux. Different approaches to calculation of group constants can be used with that system.

Also several notations of differential 'conservative' (from group g to the same group g) scattering cross-section – with and without expanding it in a Legendre polynomials series, second case by its turn splits into pointwise and tetra-linear in polar and azimuth angle. Application of those notations will be specified below.

Iterations Structure

Two iteration – inner (scattering source) and outer (fission source) – circles arose from solution of multigroup eigenvalue problem constituted by stationary form of (1–2):

$$\begin{gathered}\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^{-}\left(\vec{\Omega}\right)}+\sigma^{g}\psi_g^{+}\left(\vec{\Omega}\right)=\int\limits_{\overleftarrow{\Omega}}\sigma_{gg}^{+}\left(\overleftarrow{\Omega}\right)\psi_g^{+}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+\\+\sum_{g'\neq g}\int\limits_{\overleftarrow{\Omega}}\nu^{g'}\sigma_{g'g}^{+}\left(\overleftarrow{\Omega}\right)\psi_{g'}^{+}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+\sum_{g'}\int\limits_{\overleftarrow{\Omega}}\frac{\nu^{g'f}}{K}\sigma_{g'fg}^{+}\left(\overleftarrow{\Omega}\right)\psi_{g'}^{+}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+q_g,\end{gathered}\tag{3e}$$

$$\begin{gathered}\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^{+}\left(\vec{\Omega}\right)}+\sigma^{g}\psi_g^{-}\left(\vec{\Omega}\right)=\int\limits_{\overleftarrow{\Omega}}\sigma_{gg}^{-}\left(\overleftarrow{\Omega}\right)\psi_g^{-}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+\\+\sum_{g'\neq g}\int\limits_{\overleftarrow{\Omega}}\nu^{g'}\sigma_{g'g}^{-}\left(\overleftarrow{\Omega}\right)\psi_{g'}^{-}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+\sum_{g'}\int\limits_{\overleftarrow{\Omega}}\frac{\nu^{g'f}}{K}\sigma_{g'fg}^{-}\left(\overleftarrow{\Omega}\right)\psi_{g'}^{-}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega},\end{gathered}\tag{3o}$$

with stationary boundary conditions

$$\psi_g^{+}\left(\vec{\Omega},\vec{r}\big|_{\partial V}\right)+\psi_g^{-}\left(\vec{\Omega},\vec{r}\big|_{\partial V}\right)=\psi_{\partial V}^{g}\left(\vec{\Omega},\vec{r}\big|_{\partial V}\right),\ \overrightarrow{\partial V}\cdot\vec{\Omega}<0,\tag{4i}$$

$$\psi_g^{+}\left(\vec{\Omega},\vec{r}\big|_{\partial V}\right)-\psi_g^{-}\left(\vec{\Omega},\vec{r}\big|_{\partial V}\right)=\psi_{\partial V}^{g}\left(-\vec{\Omega},\vec{r}\big|_{\partial V}\right),\ \overrightarrow{\partial V}\cdot\vec{\Omega}>0,\tag{4o}$$

where K is the effective multiplication factor eigenvalue.

The following inner iterations acceleration methods are applied (Appendixes B–C):

- Centered form of scattering source [4];
- Quasidiffusion method (auxiliary angular dimension reduced system) [5].

Angular Discretization

Two ways of Sn-angular discretization [2] are applied:

1) Piecewise-bilinear interpolation of angular flux distribution with pre-defined rectangle angular mesh (indices j, щ and k, э denote azimuth and polar nodes) for every angular direction gives for $\psi^{g\pm}_{jk}$ as function of spatial coordinates:

- With differential cross-sections in Legendre polynomial series (index l) expanding (quotients form is specified in Appendix D):

$$\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} G^{\alpha}_{\grave{j}\grave{k}}\frac{\partial\psi^{g-}_{\grave{j}\grave{k}}}{\partial\alpha}+\sigma^{g}_{jk}\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C_{\grave{j}\grave{k}}\psi^{g+}_{\grave{j}\grave{k}}-$$

$$-\sum_{l=2i}\sigma^{gg}_{l}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} H^{jk}_{l}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g+}_{\grave{\text{щ}}\grave{\text{э}}}=\sum_{g'\neq g}\sum_{l=2i}\nu^{g'}\sigma^{g'g}_{l}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} H^{jk}_{l}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}}+ \qquad (5e)$$

$$+\sum_{g'}\sum_{l=2i}\nu^{g'f}\sigma^{g'fg}_{l}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} H^{jk}_{l}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}}+W_{jk}q_{g},$$

$$\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} G^{\alpha}_{\grave{j}\grave{k}}\frac{\partial\psi^{g+}_{\grave{j}\grave{k}}}{\partial\alpha}+\sigma^{g}_{jk}\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C_{\grave{j}\grave{k}}\psi^{g-}_{\grave{j}\grave{k}}-$$

$$-\sum_{l=2i+1}\sigma^{gg}_{l}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} H^{jk}_{l}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g-}_{\grave{\text{щ}}\grave{\text{э}}}=\sum_{g'\neq g}\sum_{l=2i+1}\nu^{g'}\sigma^{g'g}_{l}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} H^{jk}_{l}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}}+ \qquad (5o)$$

$$+\sum_{g'}\sum_{l=2i+1}\nu^{g'f}\sigma^{g'fg}_{l}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} H^{jk}_{l}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}};$$

- With twice bi-linear notation of differential cross-sections (quotients form is specified in Appendix E):

$$\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} G^{\alpha}_{\grave{j}\grave{k}}\frac{\partial\psi^{g-}_{\grave{j}\grave{k}}}{\partial\alpha}+\sigma^{g}_{jk}\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C_{\grave{j}\grave{k}}\psi^{g+}_{\grave{j}\grave{k}}-$$

$$-\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\grave{k}}\sum_{\text{эщ}}H_{\text{эщ}}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\sigma^{\grave{j}\grave{k}+}_{gg\grave{\text{щ}}\grave{\text{э}}}\right)\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g+}_{\grave{\text{щ}}\grave{\text{э}}}\right)=W_{jk}q_{g}+$$

$$+\sum_{g'\neq g}\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\grave{k}}\sum_{\text{эщ}}H_{\text{эщ}}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'}\sigma^{\grave{j}\grave{k}+}_{g'g\grave{\text{щ}}\grave{\text{э}}}\right)\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}}\right)+ \qquad (6e)$$

$$+\sum_{g'}\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\grave{k}}\sum_{\text{эщ}}H_{\text{эщ}}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'f}\sigma^{\grave{j}\grave{k}+}_{g'fg\grave{\text{щ}}\grave{\text{э}}}\right)\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}}\right),$$

$$\sum_{\acute{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} G^{\alpha}_{\grave{j}\acute{k}}\frac{\partial\psi^{g+}_{\grave{j}\acute{k}}}{\partial\alpha}+\sigma^{g}_{jk}\sum_{\acute{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C_{\grave{j}\acute{k}}\psi^{g-}_{\grave{j}\acute{k}}-$$
$$-\sum_{\acute{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\acute{k}}\sum_{\text{эщ}}H_{\text{эщ}}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\sigma^{\grave{j}\acute{k}-}_{gg\grave{\text{щ}}\grave{\text{э}}}\right)\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g-}_{\grave{\text{щ}}\grave{\text{э}}}\right)=$$
$$=\sum_{g'\neq g}\sum_{\acute{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\acute{k}}\sum_{\text{эщ}}H_{\text{эщ}}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'}\sigma^{\grave{j}\acute{k}-}_{g'g\grave{\text{щ}}\grave{\text{э}}}\right)\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}}\right)+ \tag{6o}$$
$$+\sum_{g'}\sum_{\acute{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\acute{k}}\sum_{\text{эщ}}H_{\text{эщ}}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'f}\sigma^{\grave{j}\acute{k}-}_{g'fg\grave{\text{щ}}\grave{\text{э}}}\right)\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}}\right);$$

2) Discrete angles with pre-defined angular directions d's and corresponding angular quadrature coefficients C_d (both given by user, see Appendix F):

- With differential cross-sections in Legendre polynomial series expanding:

$$G^{\alpha}_{d}\frac{\partial\psi^{g-}_{d}}{\partial\alpha}+\sigma^{d}_{g}\psi^{g+}_{d}-\sum_{l=2i}\sigma^{gg}_{l}H^{l}_{d}\sum_{\text{ц}}C_{\text{ц}}H^{l}_{\text{ц}}\psi^{g+}_{\text{ц}}=$$
$$=\sum_{g'\neq g}\sum_{l=2i}\nu^{g'}\sigma^{g'g}_{l}H^{l}_{d}\sum_{\text{ц}}C_{\text{ц}}H^{l}_{\text{ц}}\psi^{g'+}_{\text{ц}}+\sum_{g'}\sum_{l=2i}\nu^{g'f}\sigma^{g'fg}_{l}H^{l}_{d}\sum_{\text{ц}}C_{\text{ц}}H^{l}_{\text{ц}}\psi^{g'+}_{\text{ц}}+q_{g}, \tag{7e}$$

$$G^{\alpha}_{d}\frac{\partial\psi^{g+}_{d}}{\partial\alpha}+\sigma^{d}_{g}\psi^{g-}_{d}-\sum_{l=2i+1}\sigma^{gg}_{l}H^{l}_{d}\sum_{\text{ц}}C_{\text{ц}}H^{l}_{\text{ц}}\psi^{g-}_{\text{ц}}=$$
$$=\sum_{g'\neq g}\sum_{l=2i+1}\nu^{g'}\sigma^{g'g}_{l}H^{l}_{d}\sum_{\text{ц}}C_{\text{ц}}H^{l}_{\text{ц}}\psi^{g'-}_{\text{ц}}+\sum_{g'}\sum_{l=2i+1}\nu^{g'f}\sigma^{g'fg}_{l}H^{l}_{d}\sum_{\text{ц}}C_{\text{ц}}H^{l}_{\text{ц}}\psi^{g'-}_{\text{ц}}; \tag{7o}$$

- Without differential cross-sections in Legendre polynomial series expanding:

$$G^{\alpha}_{d}\frac{\partial\psi^{g-}_{d}}{\partial\alpha}+\sigma^{d}_{g}\psi^{g+}_{d}-\sum_{\text{ц}}C_{\text{ц}}\sigma^{d+}_{gg\text{ц}}\psi^{g+}_{\text{ц}}=$$
$$=\sum_{g'\neq g}\sum_{\text{ц}}C_{\text{ц}}\nu^{g'}\sigma^{d+}_{g'g\text{ц}}\psi^{g'+}_{\text{ц}}+\sum_{g'}\sum_{\text{ц}}C_{\text{ц}}\nu^{g'f}\sigma^{d+}_{g'fg\text{ц}}\psi^{g'+}_{\text{ц}}+q_{g}, \tag{8e}$$

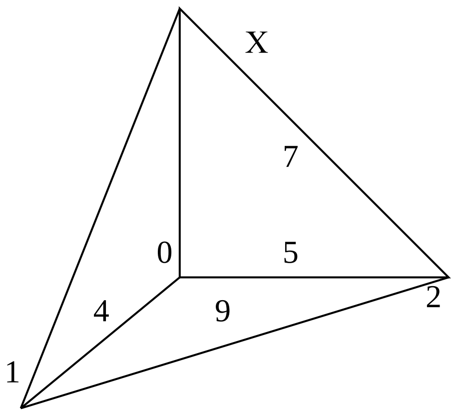
$$G^{\alpha}_{d}\frac{\partial\psi^{g+}_{d}}{\partial\alpha}+\sigma^{d}_{g}\psi^{g-}_{d}-\sum_{\text{ц}}C_{\text{ц}}\sigma^{d-}_{gg\text{ц}}\psi^{g-}_{\text{ц}}=$$
$$=\sum_{g'\neq g}\sum_{\text{ц}}C_{\text{ц}}\nu^{g'}\sigma^{d-}_{g'g\text{ц}}\psi^{g'-}_{\text{ц}}+\sum_{g'}\sum_{\text{ц}}C_{\text{ц}}\nu^{g'f}\sigma^{d-}_{g'fg\text{ц}}\psi^{g'-}_{\text{ц}}. \tag{8o}$$

Spatial Discretization

Finite element [3] spatial discretization on tetrahedron mesh with piecewise linear or quadratic basis function is applied to any of abovementioned angular discretization of equations (1) or (3). For every angular direction flux distribution is expanded in a basis functions series

$$\psi_{jk}^{g\pm}\left(\vec{r}\right)=\sum_{E}\psi_{jkE}^{g\pm}\Psi_{E}\left(\vec{r}\right), \tag{9}$$

where for each point E in spatial domain V basis function $\Psi_E\left(\vec{r}\right)=\sum_{\Xi\ni E}\Theta_E^{\Xi}\left(\vec{r}\right)$ and scalar function $\Theta_E^{\Xi}\left(\vec{r}\right)$ is polynomial in tetrahedrons Ξ that contain the E point for which $\Theta_E^{\Xi}\left(\vec{r}\right)\Big|_{\vec{r}\notin\Xi}=0$ and $\Theta_E^{\Xi}\left(\vec{r}_E\right)=1$.



Picture 1.
The tetrahedron and
numeration of points in it.

Cross-sections are taken to be constant within tetrahedron $\sigma\left(\vec{r}\right)\Big|_{\vec{r}\in\Xi}=\sigma_{\Xi}$, the same notion is used for extraneous source distribution $q\left(\vec{r}\right)\Big|_{\vec{r}\in\Xi}=q_{\Xi}$. Thus, (in inner points of spatial domain V) spatial discretization is completely described with five quotients for all combinations of points A and E in tetrahedron Ξ:

$$\mathit{\Gamma}_E^{\Xi A}=\int_{\Xi}\Theta_A^{\Xi}\left(\vec{r}\right)\overrightarrow{\nabla\Theta_E^{\Xi}\left(\vec{r}\right)}dV, \tag{10d}$$

$$У_E^{\Xi A}=\int_{\Xi}\Theta_E^{\Xi}\left(\vec{r}\right)\Theta_A^{\Xi}\left(\vec{r}\right)dV, \tag{10g}$$

$$И_g^{A}=\sum_{\Xi\ni A}q_{\Xi}^{g}\int_{\Xi}\Theta_A^{\Xi}\left(\vec{r}\right)dV, \tag{10s}$$

(the first one is a vector).

Computational Formulæ

1) Piecewise-bilinear interpolation of angular flux distribution with pre-defined rectangle angular mesh (indices j, щ and k, э denote azimuth and polar nodes) for every angular direction gives for expansion (9) coefficients $\psi_{jkE}^{g\pm}$:

- With differential cross-sections in Legendre polynomial series (index l) expanding (angular quotients form is specified in Appendix D):

$$
\begin{aligned}
&G_{00}^{\alpha 00}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{00E}^{g-}+C_{00}^{00}\sum_{\Xi\ni A}\sigma_{11}^{g\Xi}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{00E}^{g+}-\\
&-\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i}H_{l}^{00}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\sigma_{l\Xi}^{gg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g+}=\text{И}_{g}^{A}+\\
&+\sum_{g'\neq g}\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i}H_{l}^{00}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{l\Xi}^{g'g}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g'+}+\\
&+\sum_{g'}\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i}H_{l}^{00}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{l\Xi}^{g'fg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g'+},
\end{aligned}
\tag{11a}
$$

$$
\begin{aligned}
&G_{00}^{\alpha 00}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{00E}^{g+}+C_{00}^{00}\sum_{\Xi\ni A}\sigma_{11}^{g\Xi}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{00E}^{g-}-\\
&-\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1}H_{l}^{00}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\sigma_{l\Xi}^{gg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g-}=\\
&=\sum_{g'\neq g}\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1}H_{l}^{00}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{l\Xi}^{g'g}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g'-}+\\
&+\sum_{g'}\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1}H_{l}^{00}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{l\Xi}^{g'fg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g'-},
\end{aligned}
\tag{11b}
$$

$$
\begin{aligned}
&\sum_{`j=j-1}^{j}G_{`j0}^{\alpha j0}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{`j0E}^{g-}+\sum_{`j=j-1}^{j}C_{`j0}^{j0}\sum_{\Xi\ni A}\sigma_{j1}^{g\Xi}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`j0E}^{g+}-\\
&-\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i}H_{l}^{j0}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\sigma_{l\Xi}^{gg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g+}=W_{j0}\text{И}_{g}^{A}+\\
&+\sum_{g'\neq g}\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i}H_{l}^{j0}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{l\Xi}^{g'g}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g'+}+\\
&+\sum_{g'}\sum_{\text{эщ}}\sum_{`\text{э}=\text{э}-1}^{\text{э}}\sum_{`\text{щ}=\text{щ}-1}^{\text{щ}}\sum_{l=2i}H_{l}^{j0}B_{\text{щ}`\text{э}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{l\Xi}^{g'fg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{`\text{щ}`\text{э}E}^{g'+},
\end{aligned}
\tag{11c}
$$

$$\sum_{\grave{j}=j-1}^{j} G^{\alpha j0}_{\grave{j}0} \sum_{\Xi\ni A}\sum_{E\in\Xi} \mathit{\Gamma}^{\Xi A}_{aE}\psi^{g+}_{\grave{j}0E} + \sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\Xi\ni A}\sigma^{g\Xi}_{j1}\sum_{E\in\Xi} \text{У}^{\Xi A}_{E}\psi^{g-}_{\grave{j}0E} -$$
$$-\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H^{j0}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\sigma^{gg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g-}_{\grave{\text{щ}}\grave{\text{э}}E} =$$
$$= \sum_{g'\neq g}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H^{j0}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'}_{\Xi}\sigma^{g'g}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}E} +$$
$$+\sum_{g'}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H^{j0}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'f}_{\Xi}\sigma^{g'fg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}E}, \tag{11d}$$

$$\sum_{\grave{k}=k-1}^{k} G^{\alpha 0k}_{0\grave{k}} \sum_{\Xi\ni A}\sum_{E\in\Xi} \mathit{\Gamma}^{\Xi A}_{aE}\psi^{g-}_{0\grave{k}E} + \sum_{\grave{k}=k-1}^{k} C^{0k}_{0\grave{k}}\sum_{\Xi\ni A}\sigma^{g\Xi}_{1k}\sum_{E\in\Xi} \text{У}^{\Xi A}_{E}\psi^{g+}_{0\grave{k}E} -$$
$$-\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i} H^{0k}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\sigma^{gg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g+}_{\grave{\text{щ}}\grave{\text{э}}E} = W_{0k}\text{И}^A_g +$$
$$+ \sum_{g'\neq g}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i} H^{0k}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'}_{\Xi}\sigma^{g'g}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}E} +$$
$$+\sum_{g'}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i} H^{0k}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'f}_{\Xi}\sigma^{g'fg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}E}, \tag{11e}$$

$$\sum_{\grave{k}=k-1}^{k} G^{\alpha 0k}_{0\grave{k}} \sum_{\Xi\ni A}\sum_{E\in\Xi} \mathit{\Gamma}^{\Xi A}_{aE}\psi^{g+}_{\grave{j}\grave{k}E} + \sum_{\grave{k}=k-1}^{k} C^{0k}_{0\grave{k}}\sum_{\Xi\ni A}\sigma^{g\Xi}_{1k}\sum_{E\in\Xi} \text{У}^{\Xi A}_{E}\psi^{g-}_{0\grave{k}E} -$$
$$-\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H^{0k}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\sigma^{gg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g-}_{\grave{\text{щ}}\grave{\text{э}}E} =$$
$$= \sum_{g'\neq g}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H^{0k}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'}_{\Xi}\sigma^{g'g}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}E} +$$
$$+\sum_{g'}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H^{0k}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'f}_{\Xi}\sigma^{g'fg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'-}_{\grave{\text{щ}}\grave{\text{э}}E}, \tag{11f}$$

$$\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} G^{\alpha jk}_{\grave{j}\grave{k}} \sum_{\Xi\ni A}\sum_{E\in\Xi} \mathit{\Gamma}^{\Xi A}_{aE}\psi^{g-}_{\grave{j}\grave{k}E} + \sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C^{jk}_{\grave{j}\grave{k}}\sum_{\Xi\ni A}\sigma^{g\Xi}_{jk}\sum_{E\in\Xi} \text{У}^{\Xi A}_{E}\psi^{g+}_{\grave{j}\grave{k}E} -$$
$$-\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i} H^{jk}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\sigma^{gg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g+}_{\grave{\text{щ}}\grave{\text{э}}E} =$$
$$= \sum_{g'\neq g}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i} H^{jk}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'}_{\Xi}\sigma^{g'g}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}E} +$$
$$+\sum_{g'}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i} H^{jk}_l B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}} \sum_{\Xi\ni A}\nu^{g'f}_{\Xi}\sigma^{g'fg}_{l\Xi}\sum_{E\in\Xi}\text{У}^{\Xi A}_E\psi^{g'+}_{\grave{\text{щ}}\grave{\text{э}}E} + W_{jk}\text{И}^A_g, \tag{11g}$$

$$\begin{aligned}
&\sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} G_{\grave{j}\grave{k}}^{\alpha jk}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{\grave{j}\grave{k}E}^{g+} + \sum_{\grave{k}=k-1}^{k}\sum_{\grave{j}=j-1}^{j} C_{\grave{j}\grave{k}}^{jk}\sum_{\Xi\ni A}\sigma_{jk}^{g\Xi}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{\grave{j}\grave{k}E}^{g-} - \\
&-\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H_{l}^{jk} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\sum_{\Xi\ni A}\sigma_{l\Xi}^{gg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g-} = \\
&=\sum_{g'\neq g}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H_{l}^{jk} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{l\Xi}^{g'g}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g'-} + \\
&+\sum_{g'}\sum_{\text{эщ}}\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}\sum_{l=2i+1} H_{l}^{jk} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{l\Xi}^{g'fg}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g'-};
\end{aligned} \tag{11h}$$

- With twice bi-linear notation of differential cross-sections (quotients form is specified in Appendix E):

$$\begin{aligned}
&G_{00}^{\alpha 00}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{00E}^{g-} + C_{00}^{00}\sum_{\Xi\ni A}\sigma_{11}^{g\Xi}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{00E}^{g+} - \\
&-C_{00}^{00}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\sigma_{gg\grave{\text{щ}}\grave{\text{э}}}^{00\Xi+}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g+}\right) = \text{И}_{g}^{A} + \\
&+\sum_{g'\neq g}C_{00}^{00}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\nu_{\Xi}^{g'}\sigma_{g'g\grave{\text{щ}}\grave{\text{э}}}^{00\Xi+}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g'+}\right) + \\
&+\sum_{g'}C_{00}^{00}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\nu_{\Xi}^{g'f}\sigma_{g'fg\grave{\text{щ}}\grave{\text{э}}}^{00\Xi+}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g'+}\right),
\end{aligned} \tag{12a}$$

$$\begin{aligned}
&G_{00}^{\alpha 00}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{00E}^{g+} + C_{00}^{00}\sum_{\Xi\ni A}\sigma_{11}^{g\Xi}\sum_{E\in\Xi}\text{У}_{E}^{\Xi A}\psi_{00E}^{g-} - \\
&-C_{00}^{00}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\sigma_{gg\grave{\text{щ}}\grave{\text{э}}}^{00\Xi-}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g-}\right) = \\
&=\sum_{g'\neq g}C_{00}^{00}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\nu_{\Xi}^{g'}\sigma_{g'g\grave{\text{щ}}\grave{\text{э}}}^{00\Xi-}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g'-}\right) + \\
&+\sum_{g'}C_{00}^{00}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\nu_{\Xi}^{g'f}\sigma_{g'fg\grave{\text{щ}}\grave{\text{э}}}^{00\Xi-}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}} B_{\grave{\text{щ}}\grave{\text{э}}}^{\text{щэ}}\text{У}_{E}^{\Xi A}\psi_{\grave{\text{щ}}\grave{\text{э}}E}^{g'-}\right),
\end{aligned} \tag{12b}$$

$$\sum_{\grave{j}=j-1}^{j} G^{\alpha j0}_{\grave{j}0}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma^{\Xi A}_{aE}\psi^{g-}_{\grave{j}0E}+\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\Xi\ni A}\sigma^{g\Xi}_{j1}\sum_{E\in\Xi}\text{У}^{\Xi A}_{E}\psi^{g+}_{\grave{j}0E}-$$

$$-\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\sigma^{\grave{j}0\Xi+}_{gg\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g+}_{\grave{\text{щ}}\text{э}E}\right)=W_{j0}\text{И}^{A}_{g}+ \tag{12c}$$

$$+\sum_{g'\neq g}\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'}_{\Xi}\sigma^{\grave{j}0\Xi+}_{g'g\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g'+}_{\grave{\text{щ}}\text{э}E}\right)+$$

$$+\sum_{g'}\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'f}_{\Xi}\sigma^{\grave{j}0\Xi+}_{g'fg\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g'+}_{\grave{\text{щ}}\text{э}E}\right),$$

$$\sum_{\grave{j}=j-1}^{j} G^{\alpha j0}_{\grave{j}0}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma^{\Xi A}_{aE}\psi^{g+}_{\grave{j}0E}+\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\Xi\ni A}\sigma^{g\Xi}_{j1}\sum_{E\in\Xi}\text{У}^{\Xi A}_{E}\psi^{g-}_{\grave{j}0E}-$$

$$-\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\sigma^{\grave{j}0\Xi-}_{gg\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g-}_{\grave{\text{щ}}\text{э}E}\right)= \tag{12d}$$

$$=\sum_{g'\neq g}\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'}_{\Xi}\sigma^{\grave{j}0\Xi-}_{g'g\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g'-}_{\grave{\text{щ}}\text{э}E}\right)+$$

$$+\sum_{g'}\sum_{\grave{j}=j-1}^{j} C^{j0}_{\grave{j}0}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'f}_{\Xi}\sigma^{\grave{j}0\Xi-}_{g'fg\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g'-}_{\grave{\text{щ}}\text{э}E}\right),$$

$$\sum_{\grave{k}=k-1}^{k} G^{\alpha 0k}_{0\grave{k}}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma^{\Xi A}_{aE}\psi^{g-}_{\grave{j}\grave{k}E}+\sum_{\grave{k}=k-1}^{k} C^{0k}_{0\grave{k}}\sum_{\Xi\ni A}\sigma^{g\Xi}_{1k}\sum_{E\in\Xi}\text{У}^{\Xi A}_{E}\psi^{g+}_{\grave{j}\grave{k}E}-$$

$$-\sum_{\grave{k}=k-1}^{k} C^{0k}_{0\grave{k}}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\sigma^{0\grave{k}\Xi+}_{gg\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g+}_{\grave{\text{щ}}\text{э}E}\right)=W_{0k}\text{И}^{A}_{g}+ \tag{12e}$$

$$+\sum_{g'\neq g}\sum_{\grave{k}=k-1}^{k} C^{0k}_{0\grave{k}}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'}_{\Xi}\sigma^{0\grave{k}\Xi+}_{g'g\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g'+}_{\grave{\text{щ}}\text{э}E}\right)+$$

$$+\sum_{g'}\sum_{\grave{k}=k-1}^{k} C^{0k}_{0\grave{k}}\sum_{\text{эщ}}H_{\text{эщ}}\sum_{\Xi\ni A}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\nu^{g'f}_{\Xi}\sigma^{0\grave{k}\Xi+}_{g'fg\grave{\text{щ}}\grave{\text{э}}}\right)\sum_{E\in\Xi}\left(\sum_{\grave{\text{э}}=\text{э}-1}^{\text{э}}\sum_{\grave{\text{щ}}=\text{щ}-1}^{\text{щ}}B^{\text{щэ}}_{\grave{\text{щ}}\grave{\text{э}}}\text{У}^{\Xi A}_{E}\psi^{g'+}_{\grave{\text{щ}}\text{э}E}\right),$$

$$\begin{aligned}&\sum_{ҟ=k-1}^{k} G^{\alpha 0k}_{0ҟ} \sum_{\Xi\ni A}\sum_{E\in\Xi} \Gamma^{\Xi A}_{aE}\psi^{g+}_{0ҟE} + \sum_{ҟ=k-1}^{k} C^{0k}_{0ҟ}\sum_{\Xi\ni A}\sigma^{g\Xi}_{1k}\sum_{E\in\Xi} У^{\Xi A}_{E}\psi^{g-}_{0ҟE} - \\ &- \sum_{ҟ=k-1}^{k} C^{0k}_{0ҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\sigma^{0ҟ\Xi-}_{gg`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g-}_{`щ`эE}\right) = \\ &= \sum_{g'\neq g}\sum_{ҟ=k-1}^{k} C^{0k}_{0ҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\nu^{g'}_{\Xi}\sigma^{0ҟ\Xi-}_{g'g`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g'-}_{`щ`эE}\right) + \\ &+ \sum_{g'}\sum_{ҟ=k-1}^{k} C^{0k}_{0ҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\nu^{g'f}_{\Xi}\sigma^{0ҟ\Xi-}_{g'fg`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g'-}_{`щ`эE}\right),\end{aligned} \tag{12f}$$

$$\begin{aligned}&\sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} G^{\alpha jk}_{`jҟ} \sum_{\Xi\ni A}\sum_{E\in\Xi} \Gamma^{\Xi A}_{aE}\psi^{g-}_{`jҟE} + \sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{\Xi\ni A}\sigma^{g\Xi}_{jk}\sum_{E\in\Xi} У^{\Xi A}_{E}\psi^{g+}_{`jҟE} - \\ &- \sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\sigma^{`jҟ\Xi+}_{gg`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g+}_{`щ`эE}\right) = \\ &= \sum_{g'\neq g}\sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\nu^{g'}_{\Xi}\sigma^{`jҟ\Xi+}_{g'g`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g'+}_{`щ`эE}\right) + \\ &+ \sum_{g'}\sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\nu^{g'f}_{\Xi}\sigma^{`jҟ\Xi+}_{g'fg`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g'+}_{`щ`эE}\right) + \\ &+ W_{jk}И^{A}_{g},\end{aligned} \tag{12g}$$

$$\begin{aligned}&\sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} G^{\alpha jk}_{`jҟ} \sum_{\Xi\ni A}\sum_{E\in\Xi} \Gamma^{\Xi A}_{aE}\psi^{g+}_{`jҟE} + \sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{\Xi\ni A}\sigma^{g\Xi}_{jk}\sum_{E\in\Xi} У^{\Xi A}_{E}\psi^{g-}_{`jҟE} - \\ &- \sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\sigma^{`jҟ\Xi-}_{gg`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g-}_{`щ`эE}\right) = \\ &= \sum_{g'\neq g}\sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\nu^{g'}_{\Xi}\sigma^{`jҟ\Xi-}_{g'g`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g'-}_{`щ`эE}\right) + \\ &+ \sum_{g'}\sum_{ҟ=k-1}^{k}\sum_{`j=j-1}^{j} C^{jk}_{`jҟ}\sum_{эщ} H_{эщ}\sum_{\Xi\ni A}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}\nu^{g'f}_{\Xi}\sigma^{`jҟ\Xi-}_{g'fg`щ`э}\right)\sum_{E\in\Xi}\left(\sum_{`э=э-1}^{э}\sum_{`щ=щ-1}^{щ} B^{щэ}_{`щ`э}У^{\Xi A}_{E}\psi^{g'-}_{`щ`эE}\right);\end{aligned} \tag{12h}$$

2) Discrete angles with pre-defined angular directions d's and corresponding angular quadrature coefficients C_d (both given by user, see Appendix F):

- With differential cross-sections in Legendre polynomial series expanding:

$$G_d^{\alpha}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{dE}^{g-}+\sum_{\Xi\ni A}\sigma_g^{d\Xi}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{dE}^{g+}-$$
$$-\sum_{l=2i}H_d^l\sum_{ц}C_ц H_ц^l\sum_{\Xi\ni A}\sigma_{l\Xi}^{gg}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g+}=И_g^A+$$
$$+\sum_{g'\neq g}\sum_{l=2i}H_d^l\sum_{ц}C_ц H_ц^l\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{l\Xi}^{g'g}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'+}+$$
$$+\sum_{g'}\sum_{l=2i}H_d^l\sum_{ц}C_ц H_ц^l\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{l\Xi}^{g'f}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'+} \tag{13e}$$

$$G_d^{\alpha}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{dE}^{g+}+\sum_{\Xi\ni A}\sigma_g^{d\Xi}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{dE}^{g-}-$$
$$-\sum_{l=2i+1}H_d^l\sum_{ц}C_ц H_ц^l\sum_{\Xi\ni A}\sigma_{l\Xi}^{gg}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g-}=$$
$$=\sum_{g'\neq g}\sum_{l=2i+1}H_d^l\sum_{ц}C_ц H_ц^l\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{l\Xi}^{g'g}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'-}+$$
$$+\sum_{g'}\sum_{l=2i+1}H_d^l\sum_{ц}C_ц H_ц^l\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{l\Xi}^{g'f}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'-} \tag{13o}$$

- Without differential cross-sections in Legendre polynomial series expanding:

$$G_d^{\alpha}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{dE}^{g-}+\sum_{\Xi\ni A}\sigma_g^{d\Xi}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{dE}^{g+}-\sum_{ц}C_ц\sum_{\Xi\ni A}\sigma_{ggц}^{d\Xi+}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g+}=$$
$$=\sum_{g'\neq g}\sum_{ц}C_ц\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{g'gц}^{d\Xi+}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'+}+\sum_{g'}\sum_{ц}C_ц\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{g'fц}^{d\Xi+}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'+}+ \tag{14e}$$
$$+И_g^A$$

$$G_d^{\alpha}\sum_{\Xi\ni A}\sum_{E\in\Xi}\Gamma_{aE}^{\Xi A}\psi_{dE}^{g+}+\sum_{\Xi\ni A}\sigma_g^{d\Xi}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{dE}^{g-}-\sum_{ц}C_ц\sum_{\Xi\ni A}\sigma_{ggц}^{d\Xi-}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g-}=$$
$$=\sum_{g'\neq g}\sum_{ц}C_ц\sum_{\Xi\ni A}\nu_{\Xi}^{g'}\sigma_{g'gц}^{d\Xi-}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'-}+\sum_{g'}\sum_{ц}C_ц\sum_{\Xi\ni A}\nu_{\Xi}^{g'f}\sigma_{g'fц}^{d\Xi-}\sum_{E\in\Xi}y_E^{\Xi A}\psi_{цE}^{g'-} \tag{14o}$$

Conclusions

With boundary conditions (4) relationships (11) or (12) or (13) or (14) form complete system of linear equations from which solution of the effective multiplication factor eigenvalue problem could be obtained by Krylov methods. Introducing time dependence and time differentiation of angular flux distribution in (11–14), coupling each one with boundary conditions (2) one could obtain system of ODE's.

Appendix A: The Transport Equation

The first-order non self-adjoint basic transport equation is [2]

$$\frac{1}{\upsilon_g}\frac{\partial\psi^g\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi^g\left(\vec{\Omega}\right)}+\sigma^g\psi^g\left(\vec{\Omega}\right)= \\ =\sum_{g'}\int_{\overleftarrow{\Omega}}\left(\nu^{g'}\sigma^{g'g}\left(\overleftarrow{\Omega}\right)+\nu^{g'f}\sigma^{g'fg}\left(\overleftarrow{\Omega}\right)\right)\psi^{g'}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+q_g, \tag{15}$$

where the macroscopic total cross-section for group g is

$$\sigma^g=\sigma^{ga}+\sigma_0^{gg}+\sigma_0^{gfg}. \tag{16}$$

Macroscopic differential (elastic) scattering and/or fission cross-sections from group g' and angular direction $\overleftarrow{\Omega}$ to group g and angular direction $\vec{\Omega}$ cold be expanded in Legendre polynomial series:

$$\sigma^{g'g}=\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\sigma_l^{g'g}\,\mathrm{P}_l\left(\mu\right) \tag{17a}$$

with expansion coefficients

$$\sigma_l^{g'fg}=\int_{-1}^{1}\sigma^{g'fg}\,\mathrm{P}_l\left(\mu\right)d\mu=\int_{\vec{\Omega}}\sigma^{g'fg}\,\mathrm{P}_l\left(\mu\right)d\vec{\Omega} \tag{17b}$$

and

$$\sigma^{g'fg}=\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\sigma_l^{g'fg}\,\mathrm{P}_l\left(\mu\right) \tag{18a}$$

with expansion coefficients

$$\sigma_l^{g'fg}=\int_{-1}^{1}\sigma^{g'fg}\,\mathrm{P}_l\left(\mu\right)d\mu=\int_{\vec{\Omega}}\sigma^{g'fg}\,\mathrm{P}_l\left(\mu\right)d\vec{\Omega} \tag{18b}$$

correspondingly.

Scattering and fission term in (16) are obtained by summation over all original angular directions $\overleftarrow{\Omega}$ and destined groups g

$$\sum_g\int_{\overleftarrow{\Omega}}\sigma^{g'g}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}=\sum_g\sigma_0^{g'g}=\sigma_0^{g'g'} \tag{19}$$

and

$$\sum_g\int_{\overleftarrow{\Omega}}\sigma^{g'fg}\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}=\sum_g\sigma_0^{g'fg}=\sigma_0^{g'fg'}. \tag{20}$$

Boundary conditions for (15) is

$$\psi^{g}\left(\vec{\Omega},\vec{r}\big|_{\partial V},t\right)=\psi_{\partial V}^{g}\left(\vec{\Omega},t\right),\ \overrightarrow{\partial V}\cdot\vec{\Omega}<0. \tag{21}$$

Appendix B: Centered Form of Scattering Source

System of even-odd parity equations (1) could be expressed in form with so-called centered scattering source [4]

$$\begin{aligned}&\frac{1}{\upsilon_g}\frac{\partial\psi_g^{+}\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^{-}\left(\vec{\Omega}\right)}+\varsigma_g^{+}\psi_g^{+}\left(\vec{\Omega}\right)=\\&=\int_{\bar{\Omega}}\sigma_{gg}^{+}\left(\bar{\Omega}\right)\psi_g^{+}\left(\bar{\Omega}\right)d\bar{\Omega}-\sigma_0^{gg}\psi_g^{+}\left(\vec{\Omega}\right)+\\&+\sum_{g'\neq g}\int_{\bar{\Omega}}\nu^{g'}\sigma_{g'g}^{+}\left(\bar{\Omega}\right)\psi_{g'}^{+}\left(\bar{\Omega}\right)d\bar{\Omega}+\sum_{g'}\int_{\bar{\Omega}}\nu^{g'f}\sigma_{g'fg}^{+}\left(\bar{\Omega}\right)\psi_{g'}^{+}\left(\bar{\Omega}\right)d\bar{\Omega}+q_g\end{aligned}\tag{22e}$$

$$\begin{aligned}&\frac{1}{\upsilon_g}\frac{\partial\psi_g^{-}\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^{+}\left(\vec{\Omega}\right)}+\varsigma_g^{-}\psi_g^{-}\left(\vec{\Omega}\right)=\\&=\int_{\bar{\Omega}}\sigma_{gg}^{-}\left(\bar{\Omega}\right)\psi_g^{-}\left(\bar{\Omega}\right)d\bar{\Omega}-\sigma_1^{gg}\psi_g^{-}\left(\vec{\Omega}\right)+\\&+\sum_{g'\neq g}\int_{\bar{\Omega}}\nu^{g'}\sigma_{g'g}^{-}\left(\bar{\Omega}\right)\psi_{g'}^{-}\left(\bar{\Omega}\right)d\bar{\Omega}+\sum_{g'}\int_{\bar{\Omega}}\nu^{g'f}\sigma_{g'fg}^{-}\left(\bar{\Omega}\right)\psi_{g'}^{-}\left(\bar{\Omega}\right)d\bar{\Omega}\end{aligned}\tag{22o}$$

where $\varsigma_g^{+}=\sigma^{ga}+\sigma_0^{gfg}$ and $\varsigma_g^{-}=\sigma^{ga}+\sigma_0^{gg}+\sigma_0^{gfg}-\sigma_1^{gg}$. By expansion differential cross sections in Legendre polynomial series theorem alternative notation of (22) obtained:

$$\begin{aligned}&\frac{1}{\upsilon_g}\frac{\partial\psi_g^{+}\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^{-}\left(\vec{\Omega}\right)}+\varsigma_g^{+}\psi_g^{+}\left(\vec{\Omega}\right)=\\&=\sum_{l=2k}\varsigma_l^{gg}\sum_{m=-l}^{l}\mathrm{Y}_{lm}\left(\vec{\Omega}\right)\int_{\bar{\Omega}}\tilde{\mathrm{Y}}_{lm}\left(\bar{\Omega}\right)\psi_g^{+}\left(\bar{\Omega}\right)d\bar{\Omega}+q_g+\\&+\sum_{g'\neq g}\sum_{l=2k}\nu^{g'}\sigma_l^{g'g}\sum_{m=-l}^{l}\mathrm{Y}_{lm}\left(\vec{\Omega}\right)\int_{\bar{\Omega}}\tilde{\mathrm{Y}}_{lm}\left(\bar{\Omega}\right)\psi_{g'}^{+}\left(\bar{\Omega}\right)d\bar{\Omega}+\\&+\sum_{g'}\sum_{l=2k}\nu^{g'f}\sigma_l^{g'fg}\sum_{m=-l}^{l}\mathrm{Y}_{lm}\left(\vec{\Omega}\right)\int_{\bar{\Omega}}\tilde{\mathrm{Y}}_{lm}\left(\bar{\Omega}\right)\psi_{g'}^{+}\left(\bar{\Omega}\right)d\bar{\Omega}\end{aligned}\tag{23e}$$

$$\frac{1}{\upsilon_g}\frac{\partial\psi_g^-\left(\vec{\Omega}\right)}{\partial t}+\vec{\Omega}\cdot\overrightarrow{\nabla\psi_g^+\left(\vec{\Omega}\right)}+\varsigma_g^-\psi_g^-\left(\vec{\Omega}\right)=$$
$$=\sum_{l=2k+1}\varsigma_l^{gg}\sum_{m=-l}^{l}\mathrm{Y}_{lm}\left(\vec{\Omega}\right)\int_{\vec{\Omega}}\tilde{\mathrm{Y}}_{lm}\left(\overleftarrow{\Omega}\right)\psi_g^-\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+$$
$$+\sum_{g'\neq g}\sum_{l=2k+1}\nu^{g'}\sigma_l^{g'g}\sum_{m=-l}^{l}\mathrm{Y}_{lm}\left(\vec{\Omega}\right)\int_{\vec{\Omega}}\tilde{\mathrm{Y}}_{lm}\left(\overleftarrow{\Omega}\right)\psi_{g'}^-\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}+ \tag{23o}$$
$$+\sum_{g'}\sum_{l=2k+1}\nu^{g'f}\sigma_l^{g'fg}\sum_{m=-l}^{l}\mathrm{Y}_{lm}\left(\vec{\Omega}\right)\int_{\vec{\Omega}}\tilde{\mathrm{Y}}_{lm}\left(\overleftarrow{\Omega}\right)\psi_{g'}^-\left(\overleftarrow{\Omega}\right)d\overleftarrow{\Omega}$$

where $\varsigma_l^{gg}=\sigma_l^{gg}-\sigma_{l-[l/2]}^{gg}$.

Appendix C: Quasidiffusion Method

According to original approach [5] first equation of quasidiffusion system is derived by integrating (15) over angular direction and second one is derived by integrating product of (15) and angular direction over angular direction. The same is done for boundary conditions (21). Thus auxiliary angular dimension reduced system is

$$\frac{1}{\upsilon_g}\frac{\partial U^g}{\partial t}+\nabla\vec{W}^g+\sigma^g U^g=\sum_{g'}\left(\nu^{g'}\sigma_0^{g'g}+\nu^{g'f}\sigma_0^{g'fg}\right)U^{g'}+4\pi q_g, \tag{24s}$$

$$\frac{1}{\upsilon_g}\frac{\partial\vec{W}^g}{\partial t}+\overrightarrow{\nabla\left(\underset{\leftrightarrow}{D}^g U^g\right)}+\sigma^g\vec{W}^g=\sum_{g'}\left(\nu^{g'}\sigma_1^{g'g}\left(\overleftarrow{\Omega}\right)+\nu^{g'f}\sigma_1^{g'fg}\left(\overleftarrow{\Omega}\right)\right)\vec{W}^{g'} \tag{24v}$$

with boundary conditions

$$\overrightarrow{\partial V}\cdot\vec{W}^g\left(\vec{r}\big|_{\partial V},t\right)-W_{\partial V}^g\left(\vec{r}\big|_{\partial V},t\right)=\left(U^g\left(\vec{r}\big|_{\partial V},t\right)-U_{\partial V}^g\left(\vec{r}\big|_{\partial V},t\right)\right)b^g. \tag{25}$$

The symbols in equations (24–25) defined as follows:
$U^g=\int_{\vec{\Omega}}\psi^g\left(\vec{\Omega}\right)d\vec{\Omega}$ – the scalar flux distribution as function of spatial coordinates $\vec{r}$ and time t for group g and $U_{\partial V}^g$ is the scalar flux distribution outside spatial domain;
$\vec{W}^g=\int_{\vec{\Omega}}\psi^g\left(\vec{\Omega}\right)\vec{\Omega}d\vec{\Omega}$ – the vector flux distribution as function of spatial coordinates $\vec{r}$ and time t for group g and $W_{\partial V}^g$ is the normal to border vector flux distribution outside;

$\underset{\leftrightarrow}{D}^g = \underset{\longleftrightarrow}{\left\{ D^g_{\alpha\beta} \right\}}$ – the quasidiffusion tensor as functions of spatial coordinates $\vec{r}$ and time t for group g which coordinates defined from relationship

$$D^g_{\alpha\beta} U^g = \int_{\vec{\Omega}} \Omega_\alpha \Omega_\beta \psi^g \left(\vec{\Omega} \right) d\vec{\Omega}; \qquad (26)$$

b^g – the boundary relation is defined as follows

$$b^g = \int_{\overrightarrow{\partial V} \cdot \vec{\Omega} > 0} \psi^g \left(\vec{\Omega}, \vec{r}\big|_{\partial V}, t \right) \overrightarrow{\partial V} \cdot \vec{\Omega} d\vec{\Omega} \Big/ \int_{\overrightarrow{\partial V} \cdot \vec{\Omega} > 0} \psi^g \left(\vec{\Omega}, \vec{r}\big|_{\partial V}, t \right) d\vec{\Omega}. \qquad (27)$$

Finite element spatial discretization of (24) may be derived in the same manner as in (9), additionally for (26) noting $\left(D^g_{\alpha\beta} U^g \right) = \sum_E \Psi_E \left(\vec{r} \right) \int_{\vec{\Omega}} \Omega_\alpha \Omega_\beta \psi^g_E \left(\vec{\Omega} \right) d\vec{\Omega}$ one may obtain

$$\begin{aligned} &\frac{1}{\upsilon_g} \sum_{\Xi \ni A} \sum_{E \in \Xi} У^{\Xi A}_E \frac{\partial U^g_E}{\partial t} + \sum_{\Xi \ni A} \sum_{E \in \Xi} \Gamma^{\Xi A}_E \vec{W}^g_E + \sum_{\Xi \ni A} \sigma^g_\Xi \sum_{E \in \Xi} У^{\Xi A}_E U^g_E = \\ &= \sum_{g'} \sum_{\Xi \ni A} \left(\nu^{g'}_\Xi \sigma^{g'g}_{0\Xi} + \nu^{g'f}_\Xi \sigma^{g'fg}_{0\Xi} \right) \sum_{E \in \Xi} У^{\Xi A}_E U^{g'}_E + 4\pi И^A_g, \end{aligned} \qquad (28s)$$

$$\begin{aligned} &\frac{1}{\upsilon_g} \sum_{\Xi \ni A} \sum_{E \in \Xi} У^{\Xi A}_E \frac{\partial \vec{W}^g_E}{\partial t} + \sum_{\Xi \ni A} \sum_{E \in \Xi} \underset{\leftrightarrow}{D}^g_E U^g_E \Gamma^{\Xi A}_E + \sum_{\Xi \ni A} \sigma^g_\Xi \sum_{E \in \Xi} У^{\Xi A}_E \vec{W}^g_E = \\ &= \sum_{g'} \sum_{\Xi \ni A} \left(\nu^{g'}_\Xi \sigma^{g'g}_{0\Xi} + \nu^{g'f}_\Xi \sigma^{g'fg}_{0\Xi} \right) \sum_{E \in \Xi} У^{\Xi A}_E \vec{W}^{g'}_E \end{aligned} \qquad (28v)$$

in inner points of spatial domain V.

Appendix D

$\mathrm{G}_R \left(\mu_{k-1}, \mu_k \right) = \left(2\mu_{k-1}{}^2 - 3\mu_{k-1}\mu_k - 2 \right) \sqrt{1 - \mu_{k-1}{}^2} + 3\left(\arcsin\left(\mu_k \right) - \arcsin\left(\mu_{k-1} \right) \right) \mu_k +$
$+ \left(\mu_k{}^2 + 2 \right) \sqrt{1 - \mu_k{}^2}, \quad \mathrm{G}_L \left(\mu_{k-1}, \mu_k \right) = 2\mu_{k-1} + \mu_k,$

$\mathrm{G}_C \left(\varphi_{j-1}, \varphi_j \right) = \cos\left(\varphi_{j-1} \right) - \cos\left(\varphi_j \right) - \left(\varphi_j - \varphi_{j-1} \right) \sin\left(\varphi_{j-1} \right),$

$\mathrm{G}_S \left(\varphi_{j-1}, \varphi_j \right) = \sin\left(\varphi_{j-1} \right) - \sin\left(\varphi_j \right) - \left(\varphi_j - \varphi_{j-1} \right) \cos\left(\varphi_{j-1} \right),$

$W_{00} = 1, \quad W_{j0} = \left(\varphi_j - \varphi_{j-1} \right), \quad W_{0k} = \left(\mu_k - \mu_{k-1} \right), \quad W_{jk} = W_{j0} W_{0k}, \quad C^{00}_{00} = 1, \quad C^{j0}_{j0} = W_{j0}/2,$

$C^{0k}_{0k} = W_{0k}/2, \quad C^{jk}_{jk} = W_{jk}/4, \quad G^{00}_{00} = \left(0 \quad 0 \quad -1 \right)^T, \quad G^{j0}_{j0} = \left(0 \quad 0 \quad W_{j0}/2 \right)^T,$

$$G_{0k-1}^{0k}=\frac{1}{6}\begin{pmatrix}\mathrm{G}_R\left(\mu_{k-1},\mu_k\right)/W_{0k}\\ 0\\ W_{0k}\,\mathrm{G}_L\left(\mu_{k-1},\mu_k\right)\end{pmatrix},\ G_{j-1k-1}^{jk}=\frac{1}{6}\begin{pmatrix}\mathrm{G}_C\left(\varphi_{j-1},\varphi_j\right)\mathrm{G}_R\left(\mu_{k-1},\mu_k\right)/W_{jk}\\ \mathrm{G}_S\left(\varphi_{j-1},\varphi_j\right)\mathrm{G}_R\left(\mu_{k-1},\mu_k\right)/W_{jk}\\ W_{jk}\,\mathrm{G}_L\left(\mu_{k-1},\mu_k\right)/2\end{pmatrix},$$

$$G_{0k}^{0k}=\frac{1}{6}\begin{pmatrix}\mathrm{G}_R\left(\mu_k,\mu_{k-1}\right)/W_{0k}\\ 0\\ W_{0k}\,\mathrm{G}_L\left(\mu_k,\mu_{k-1}\right)\end{pmatrix},\ G_{jk-1}^{jk}=\frac{1}{6}\begin{pmatrix}\mathrm{G}_C\left(\varphi_j,\varphi_{j-1}\right)\mathrm{G}_R\left(\mu_{k-1},\mu_k\right)/W_{jk}\\ \mathrm{G}_S\left(\varphi_j,\varphi_{j-1}\right)\mathrm{G}_R\left(\mu_{k-1},\mu_k\right)/W_{jk}\\ W_{jk}\,\mathrm{G}_L\left(\mu_{k-1},\mu_k\right)/2\end{pmatrix},$$

$$G_{j-1k}^{jk}=\frac{1}{6}\begin{pmatrix}\mathrm{G}_C\left(\varphi_{j-1},\varphi_j\right)\mathrm{G}_R\left(\mu_k,\mu_{k-1}\right)/W_{jk}\\ \mathrm{G}_S\left(\varphi_{j-1},\varphi_j\right)\mathrm{G}_R\left(\mu_k,\mu_{k-1}\right)/W_{jk}\\ W_{jk}\,\mathrm{G}_L\left(\mu_k,\mu_{k-1}\right)/2\end{pmatrix},\ G_{jk}^{jk}=\frac{1}{6}\begin{pmatrix}\mathrm{G}_C\left(\varphi_j,\varphi_{j-1}\right)\mathrm{G}_R\left(\mu_k,\mu_{k-1}\right)/W_{jk}\\ \mathrm{G}_S\left(\varphi_j,\varphi_{j-1}\right)\mathrm{G}_R\left(\mu_k,\mu_{k-1}\right)/W_{jk}\\ W_{jk}\,\mathrm{G}_L\left(\mu_k,\mu_{k-1}\right)/2\end{pmatrix},$$

$$\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}=\frac{1}{W_{jk}W_{\text{щэ}}}\frac{2}{\delta_{0m}+1}\frac{(l-m)!}{(l+m)!},$$

$$H_l^{00}B_{\text{щ}-1\text{э}-1}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\,\mathrm{P}_{lm}(-1)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\cos(m\varphi)d\varphi,$$

$$H_l^{00}B_{\text{щ}-1\text{э}}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\,\mathrm{P}_{lm}(-1)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\cos(m\varphi)d\varphi,$$

$$H_l^{00}B_{\text{щэ}-1}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\,\mathrm{P}_{lm}(-1)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi-\varphi_{\text{щ}-1})\cos(m\varphi)d\varphi,$$

$$H_l^{00}B_{\text{щэ}}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\,\mathrm{P}_{lm}(-1)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi-\varphi_{\text{щ}-1})\cos(m\varphi)d\varphi,$$

$$H_l^{0k}B_{\text{щ}-1\text{э}-1}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\left(\int_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\cos(m\varphi)d\varphi,$$

$$H_l^{0k}B_{\text{щ}-1\text{э}}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\left(\int_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\cos(m\varphi)d\varphi,$$

$$H_l^{0k}B_{\text{щэ}-1}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\left(\int_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi-\varphi_{\text{щ}-1})\cos(m\varphi)d\varphi,$$

$$H_l^{0k}B_{\text{щэ}}^{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}_{\substack{jk\\ \text{щэ}}}^{lm}\left(\int_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi-\varphi_{\text{щ}-1})\cos(m\varphi)d\varphi,$$

$$H_l^{j0}B^{\text{щэ}}_{\text{щ}-1\text{э}-1}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\,\mathrm{P}_{lm}(-1)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{j0}B^{\text{щэ}}_{\text{щ}-1\text{э}}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\,\mathrm{P}_{lm}(-1)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{j0}B^{\text{щэ}}_{\text{щэ}-1}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\,\mathrm{P}_{lm}(-1)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{j0}B^{\text{щэ}}_{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\,\mathrm{P}_{lm}(-1)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{jk}B^{\text{щэ}}_{\text{щ}-1\text{э}-1}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\left(\int\limits_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{jk}B^{\text{щэ}}_{\text{щ}-1\text{э}}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\left(\int\limits_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{jk}B^{\text{щэ}}_{\text{щэ}-1}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\left(\int\limits_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu_{\text{э}}-\mu)\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi,$$

$$H_l^{jk}B^{\text{щэ}}_{\text{щэ}}=\sum_{m=0}^{l}\mathrm{I}^{lm}_{\substack{jk\\ \text{щэ}}}\left(\int\limits_{\mu_{k-1}}^{\mu_k}\mathrm{P}_{lm}(\mu)d\mu\right)\left(\int\limits_{\mu_{\text{э}-1}}^{\mu_{\text{э}}}(\mu-\mu_{\text{э}-1})\mathrm{P}_{lm}(\mu)d\mu\right)\int\limits_{\varphi_{\text{щ}-1}}^{\varphi_{\text{щ}}}(\varphi_{\text{щ}}-\varphi)\int\limits_{\varphi_j}^{\varphi_j}\big(\cos(m\varpi)\cos(m\varphi)+\sin(m\varpi)\sin(m\varphi)\big)d\varpi d\varphi.$$

Appendix E

$H_{щэ} = W_{щэ}/36,\quad B^{щэ}_{щ-1э-1} = 4,\quad B^{щэ}_{щ-1э} = 1,\quad B^{щэ}_{щэ-1} = 1,\quad B^{щэ}_{щэ} = 4$

All other coefficients are the same as in previous section.

Appendix F

For discrete angles discretization angular directions d's may be taken arbitrary. Approximation of integral over solid angle by summation over all directions $\int_{\vec{\Omega}} f\left(\vec{\Omega}\right) d\vec{\Omega} \approx \sum_d C_d f\left(\vec{\Omega}_d\right)$ is the basic requirement for them. In the simplest case corresponding angular quadrature coefficients satisfy relation $4\pi C_d = \int_{\Delta\vec{\Omega}_d} d\vec{\Omega}$. Way to form rather good set of angular directions and corresponding coefficients for quadrature could be found in [6].

The other coefficients in (7–8) and (13–14) are $G_d = \vec{\Omega}_d$ and $H^l_d H^l_ц = \sum_{m=-l}^{l} \Upsilon_{lm}\left(\vec{\Omega}_d\right) \tilde{\Upsilon}_{lm}\left(\overleftarrow{\Omega}_ц\right)$.

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