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**KELDYSH INSTITUTE OF APPLIED MATHEMATICS**  
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**Staged multi-result supercompilation:  
filtering before producing**

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## **Sergei A. Grechanik, Ilya G. Klyuchnikov, Sergei A. Romanenko. Staged multi-result supercompilation: filtering before producing**

When applying supercompilation to problem-solving, multi-result supercompilation enables us to find the best solutions by generating a set of possible residual graphs of configurations that are then filtered according to some criteria. Unfortunately, the search space may be rather large. However, we show that the search can be drastically reduced by decomposing multi-result supercompilation into two stages. The first stage produces a compact representation for the set of residual graphs by delaying some graph-building operation. These operations are performed at the second stage, when the representation is interpreted, to actually produce the set of graphs. The main idea of our approach is that, instead of filtering a collection of graphs, we can analyze and clean its compact representation. In some cases of practical importance (such as selecting graphs of minimal size and removing graphs containing unsafe configurations) cleaning can be performed in linear time.

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## **С.А. Гречаник, И.Г. Ключников, С.А. Романенко. Стадированная многорезультатная суперкомпиляция: фильтрация результатов до их порождения**

В случае применения суперкомпиляции в области решения задач, многорезультатная суперкомпиляция позволяет обнаруживать наилучшие решения благодаря тому, что порождается некоторое множество возможных остаточных графов конфигураций, которое затем фильтруется в соответствии с некоторыми критериями. К сожалению, пространство поиска при этом может получаться весьма обширным. Однако, мы показываем, что можно значительно уменьшить объем поиска разложив процесс многорезультатной суперкомпиляции в композицию из двух стадий. На первой стадии порождается компактное представление множества остаточных графов, которое получается в результате задержки некоторых операций по построению графов. Эти операции выполняются на второй стадии, когда компактное представление интерпретируется, в результате чего и генерируется множество графов. Основная идея предлагаемого подхода состоит в том, что вместо фильтрации множества графов можно выполнять анализ и чистку его компактного представления. Во многих случаях, представляющих практический интерес (таких, как отбор графов минимального размера или отбрасывание графов, содержащих ненадежные конфигурации) чистка может быть выполнена за линейное время.

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# 1 Introduction

When applying supercompilation [41, 42, 45, 43, 44, 37, 38, 35, 33, 9, 21, 19] to problem-solving [30, 31, 32, 29, 28, 10, 12, 15, 14, 22], multi-result supercompilation enables us to find the best solutions by generating a set of possible residual graphs of configurations that are then filtered according to some criteria [13, 25, 23, 24].

Unfortunately, the search space may be rather large [16, 17]. However, we show that the search can be drastically reduced by decomposing multi-result supercompilation into two stages [39, 40]. The first stage produces a compact representation for the set of residual graphs by delaying some graph-building operation. These operations are performed at the second stage, when the representation is interpreted, to actually produce the set of graphs. The main idea of our approach is that, instead of filtering a collection of graphs, we can analyze and clean its compact representation. In some cases of practical importance (such as selecting graphs of minimal size and removing graphs containing unsafe configurations) cleaning can be performed in linear time.

## 2 Filtering before producing... How?

### 2.1 Multi-result supercompilation and filtering

A popular approach to problem solving is *trial and error*: (1) *generate* alternatives, (2) *evaluate* alternatives, (3) *select* the best alternatives.

Thus, when trying to apply supercompilation to problem solving we naturally come to the idea of *multi-result* supercompilation: instead of trying to guess, which residual graph of configurations is “the best” one, a multi-result supercompiler produces a collection of residual graphs.

Suppose we have a multi-result supercompiler `mrsc` and a filter `filter`. Combining them, we get a problem-solver

```
solver = filter ◦ mrsc
```

where `mrsc` is a general-purpose tool (at least to some extent), while `filter` incorporates some knowledge about the problem domain. A good feature of this design is its modularity and the clear separation of concerns: in ideal case, `mrsc` knows nothing about the problem domain, while `filter` knows nothing about supercompilation.

### 2.2 Fusion of supercompilation and filtering

However, the main problem with multi-result supercompilation is that it can produce millions of residual graphs! Hence, it seems to be a good idea to suppress the generation of the majority residual graphs “on the fly”, in the process of supercompilation. This can be achieved if the criteria `filter` is based upon are

“monotonic”: if some parts of a partially constructed residual graph are “bad”, then the completed residual graph is also certain to be a “bad” one.

We can exploit monotonicity by fusing `filter` and `mrsc` into a monolithic program

```
solver' = fuse filter mrsc
```

where `fuse` is an automatic tool (based, for example, on supercompilation), or just a postgraduate who has been taught (by his scientific adviser) to perform fusion by hand. :-)

An evident drawback of this approach is its non-modularity. Every time `filter` is modified, the fusion of `mrsc` and `filter` has to be repeated.

## 2.3 Staged supercompilation: multiple results seen as a residual program

Here we put forward an alternative approach which:

1. Completely separates supercompilation from filtering.
2. Enables filtering of partially constructed residual graphs.

Thus the technique is modular, and yet reduces the search space and consumed computational resources.

Our “recipe” is as follows. (1) Replace small-step supercompilation with big-step supercompilation. (2) Decompose supercompilation into two stages. (3) Consider the result of the first stage as a “program” to be interpreted by the second stage. (4) Transform the “program” to reduce the number of graphs to be produced.

### 2.3.1 Small-step $\Rightarrow$ big-step

Supercompilation can be formulated either in “small-step” or in “big-step” style. Small-step supercompilation proceeds by rewriting a graph of configurations. Big-step supercompilation is specified/implemented in compositional style: the construction of a graph amounts to constructing its subgraphs, followed by synthesizing the whole graph from its previously constructed parts. Multi-result supercompilation was formulated in small-step style [23, 24]. First of all, given a small-step multi-result supercompiler `mrsc`, we can refactor it, to produce a *big-step* supercompiler `naive-mrsc`.

### 2.3.2 Identifying Cartesian products

Now it turns out that, at some places, `naive-mrsc` calculates “Cartesian products”: if a graph  $g$  is to be constructed from  $k$  subgraphs  $g_1, \dots, g_k$ , `naive-mrsc` computes  $k$  sets of graphs  $gs_1, \dots, gs_k$  and then considers all possible  $g_i \in gs_i$  for  $i = 1, \dots, k$  and constructs corresponding versions of the graph  $g$ .

### 2.3.3 Staging: delaying Cartesian products

At this point the process of supercompilation can be decomposed into two stages

$$\text{naive-mrsc} \doteq \langle\langle\_ \rangle\rangle \circ \text{lazy-mrsc}$$

where  $\langle\langle\_ \rangle\rangle$  is a unary function, and  $f \doteq g$  means that  $f x = g x$  for all  $x$ .

At the first stage, `lazy-mrsc` generates a “lazy graph”, which, essentially, is a “program” to be “executed” by  $\langle\langle\_ \rangle\rangle$ . Unlike `naive-mrsc`, `lazy-mrsc` does not calculate Cartesian products immediately: instead, it outputs requests for  $\langle\langle\_ \rangle\rangle$  to calculate them at the second stage.

### 2.3.4 Fusing filtering with the generation of graphs

Suppose, `l` is a lazy graph produced by `lazy-mrsc`. By evaluating  $\langle\langle l \rangle\rangle$ , we can generate the same bag of graphs, as would have been produced by the original `naive-mrsc`.

However, usually, we are not interested in the whole bag  $\langle\langle l \rangle\rangle$ . The goal is to find “the best” or “most interesting” graphs. Hence, there should be developed some techniques of extracting useful information from a lazy graph  $l$  without evaluating  $\langle\langle l \rangle\rangle$  directly.

This can be formulated in the following form. Suppose that a function `filter` filters bags of graphs, removing “bad” graphs, so that

$$\text{filter} \langle\langle l \rangle\rangle$$

generates the bag of “good” graphs. Let `clean` be a transformer of lazy graphs such that

$$\text{filter} \circ \langle\langle\_ \rangle\rangle \doteq \langle\langle\_ \rangle\rangle \circ \text{clean}$$

which means that `filter`  $\langle\langle l \rangle\rangle$  and  $\langle\langle \text{clean } l \rangle\rangle$  always return the same collection of graphs.

In general, a lazy graph transformer `clean` is said to be a *cleaner* if for any lazy graph `l`

$$\langle\langle \text{clean } l \rangle\rangle \subseteq \langle\langle l \rangle\rangle$$

The nice property of cleaners is that they are *composable*: given `clean1` and `clean2`, `clean2 ∘ clean1` is also a cleaner.

## 2.4 Typical cleaners

Typical tasks are finding graphs of minimal size and removing graphs that contain “bad” configurations. It is easy to implement corresponding cleaners in a such a way that the lazy graph is traversed only once, in a linear time.

## 2.5 What are the advantages?

We get the following scheme:

$$\begin{aligned} \text{filter} \circ \text{naive-mrsc} &\doteq \\ \text{filter} \circ \langle\langle\_ \rangle\rangle \circ \text{lazy-mrsc} &\doteq \langle\langle\_ \rangle\rangle \circ \text{clean} \circ \text{lazy-mrsc} \end{aligned}$$

We can see that:

- The construction is modular: `lazy-mrsc` and `⟨⟨_⟩⟩` do not have to know anything about filtering, while `clean` does not have to know anything about `lazy-mrsc` and `⟨⟨_⟩⟩`.
- Cleaners are composable: we can decompose a sophisticated cleaner into a composition of simpler cleaners.
- In many cases (of practical importance) cleaners can be implemented in such a way that the best graphs can be extracted from a lazy graph in linear time.

## 2.6 Codata and corecursion: decomposing lazy-mrsc

By using codata and corecursion, we can decompose `lazy-mrsc` into two stages

$$\text{lazy-mrsc} \doteq \text{prune-cograph} \circ \text{build-cograph}$$

where `build-cograph` constructs a (potentially) infinite tree, while `prune-cograph` traverses this tree and turns it into a lazy graph (which is finite).

The point is that `build-cograph` performs driving and rebuilding configurations, while `prune-cograph` uses whistle to turn an infinite tree to a finite graph. Thus `build-cograph` knows nothing about the whistle, while `prune-cograph` knows nothing about driving and rebuilding. This further improves the modularity of multi-result supercompilation.

## 2.7 Cleaning before whistling

Now it turns out that some cleaners can be pushed over `prune-cograph`!

Suppose `clean` is a lazy graph cleaner and `clean $\infty$`  a cograph cleaner, such that

$$\text{clean} \circ \text{prune-cograph} \doteq \text{prune-cograph} \circ \text{clean}\infty$$

then

$$\begin{aligned} \text{clean} \circ \text{lazy-mrsc} &\doteq \\ \text{clean} \circ \text{prune-cograph} \circ \text{build-cograph} &\doteq \\ \text{prune-cograph} \circ \text{clean}\infty \circ \text{build-cograph} & \end{aligned}$$

The good thing is that `build-cograph` and `clean $\infty$`  work in a lazy way, generating subtrees by demand. Hence, evaluating

⟨⟨ `prune-cograph`  $\circ$  (`clean $\infty$`  (`build-cograph` `c`)) ⟩⟩

is likely to be less time and space consuming than directly evaluating

⟨⟨ `clean` (`lazy-mrsc` `c`) ⟩⟩

### 3 A model of big-step multi-result supercompilation

We have formulated and implemented in Agda [2] an idealized model of big-step multi-result supercompilation [1]. This model is rather abstract, and yet it can be instantiated to produce runnable supercompilers. By the way of testing, the abstract model has been instantiated to produce a multi-result supercompiler for counter systems [16].

#### 3.1 Graphs of configurations

Given an initial configuration  $c$ , a supercompiler produces a list of “residual” graphs of configurations:  $g_1, \dots, g_k$ .

Graphs of configurations are supposed to represent “residual programs” and are defined in Agda (see `Graphs.agda`) [2] in the following way:

```
data Graph (C : Set) : Set where
  back  : ∀ (c : C) → Graph C
  forth : ∀ (c : C) (gs : List (Graph C)) → Graph C
```

Technically, a `Graph C` is a tree, with `back` nodes being references to parent nodes.

A graph’s nodes contain configurations. Here we abstract away from the concrete structure of configurations. In this model the arrows in the graph carry no information, because, this information can be kept in nodes. (Hence, this information is supposed to be encoded inside “configurations”.)

To simplify the machinery, back-nodes in this model of supercompilation do not contain explicit references to parent nodes. Hence, `back c` means that  $c$  is foldable to a parent configuration (perhaps, to several ones).

- Back-nodes are produced by folding a configuration to another configuration in the history.
- Forth-nodes are produced by
  - decomposing a configuration into a number of other configurations (e.g. by driving or taking apart a let-expression), or
  - by rewriting a configuration by another one (e.g. by generalization, introducing a let-expression or applying a lemma during two-level supercompilation).

### 3.2 “Worlds” of supercompilation

The knowledge about the input language a supercompiler deals with is represented by a “world of supercompilation”, which is a record that specifies the following.

- **Conf** is the type of “configurations”. Note that configurations are not required to be just expressions with free variables! In general, they may represent sets of states in any form/language and as well may contain any *additional* information.
- $\_ \sqsubseteq \_$  is a “foldability relation”.  $c \sqsubseteq c'$  means that  $c$  is foldable to  $c'$ . (In such cases  $c'$  is usually said to be “more general” than  $c$ .)
- $\_ \sqsubseteq? \_$  is a decision procedure for  $\_ \sqsubseteq \_$ . This procedure is necessary for implementing supercompilation in functional form.
- $\_ \Rightarrow$  is a function that gives a number of possible decompositions of a configuration. Let  $c$  be a configuration and  $cs$  a list of configurations such that  $cs \in c \Rightarrow$ . Then  $c$  can be “reduced to” (or “decomposed into”) configurations  $cs$ .

Suppose that driving is deterministic and, given a configuration  $c$ , produces a list of configurations  $c \Downarrow$ . Suppose that rebuilding (generalization, application of lemmas) is non-deterministic and  $c \curvearrowright$  is the list of configurations that can be produced by rebuilding. Then (in this special case)  $\_ \Rightarrow$  can be implemented as follows:

$$c \Rightarrow = [ c \Downarrow ] ++ \text{map } [ \_ ] (c \curvearrowright)$$

- **whistle** is a “bar whistle” [6] (see `BarWhistle.agda`) that is used to ensure termination of functional supercompilation.

Thus we have the following definition in Agda:

```
record ScWorld : Set1 where

  field
    Conf : Set
    _ ⊆ _ : (c c' : Conf) → Set
    _ ⊆? _ : (c c' : Conf) → Dec (c ⊆ c')
    _ ⇒ : (c : Conf) → List (List Conf)
    whistle : BarWhistle Conf

  open BarWhistle whistle public

  History : Set
  History = List Conf
```

```
Foldable : ∀ (h : History) (c : Conf) → Set
Foldable h c = Any (λ c' h' c) h
```

```
foldable? : ∀ (h : History) (c : Conf) → Dec (Foldable h c)
foldable? h c = Any.any (λ c' h' c) h
```

Note that, in addition to (abstract) fields, there are a few concrete type and function definitions.

- `History` is a list of configuration that have been produced in order to reach the current configuration.
- `Foldable h c` means that `c` is foldable to a configuration in the history `h`.
- `foldable? h c` decides whether `Foldable h c`.

If we need labeled edges in the graph of configurations, the labels can be hidden inside configurations. (Recall that “configurations” do not have to be just symbolic expressions, as they can contain any additional information.)

### 3.3 A relational specification of big-step non-deterministic supercompilation

In `BigStepSc.agda` there is given a relational definition of non-deterministic supercompilation in terms of two relations

```
infix 4 _⊢NDSC_↔_ _⊢NDSC*_↔_

data _⊢NDSC_↔_ : ∀ (h : History) (c : Conf)
  (g : Graph Conf) → Set
_⊢NDSC*_↔_ : ∀ (h : History) (cs : List Conf)
  (gs : List (Graph Conf)) → Set
```

which are defined with respect to a world of supercompilation.

Let `h` be a history, `c` a configuration and `g` a graph. Then `h ⊢NDSC c ↔ g` means that `g` can be produced from `h` and `c` by non-deterministic supercompilation.

Let `h` be a history, `cs` a list of configurations, `gs` a list of graphs, and `length cs = length gs`. Then `h ⊢NDSC* cs ↔ gs` means that each `g ∈ gs` can be produced from the history `h` and the corresponding `c ∈ cs` by non-deterministic supercompilation. Or, in Agda:

```
h ⊢NDSC* cs ↔ gs = Pointwise.Rel (λ c h c) cs gs
```

`⊢NDSC_↔_` is defined by two rules

```

data _ $\vdash$ NDSC_ $\leftrightarrow$ _ where
  ndsc-fold :  $\forall$  {h : History} {c}
    (f : Foldable h c)  $\rightarrow$ 
    h  $\vdash$ NDSC c  $\leftrightarrow$  back c
  ndsc-build :  $\forall$  {h : History} {c}
    {cs : List (Conf)} {gs : List (Graph Conf)}
    ( $\neg$ f :  $\neg$  Foldable h c)
    (i : cs  $\in$  c  $\Rightarrow$ )
    (s : (c :: h)  $\vdash$ NDSC* cs  $\leftrightarrow$  gs)  $\rightarrow$ 
    h  $\vdash$ NDSC c  $\leftrightarrow$  forth c gs

```

The rule `ndsc-fold` says that if `c` is foldable to a configuration in `h` there can be produced the graph `back c` (consisting of a single back-node).

The rule `ndsc-build` says that there can be produced a node `forth c gs` if the following conditions are satisfied.

- *c* is not foldable to a configuration in the history `h`.
- `c  $\Rightarrow$`  contains a list of configurations `cs`, such that `(c :: h)  $\vdash$ NDSC* cs  $\leftrightarrow$  gs`.

Speaking more operationally, the supercompiler first decides how to decompose `c` into a list of configurations `cs` by selecting a `cs  $\in$  c`. Then, for each `c  $\in$  cs` the supercompiler produces a graph `g`, to obtain a list of graphs `gs`, and builds the graph `c  $\leftrightarrow$  forth c gs`.

### 3.4 A relational specification of big-step multi-result supercompilation

The main difference between multi-result and non-deterministic supercompilation is that multi-result uses a *whistle* (see `Whistles.agda`) in order to ensure the finiteness of the collection of residual graphs.

In `BigStepSc.agda` there is given a relational definition of multi-result supercompilation in terms of two relations

```

infix 4 _ $\vdash$ MRSC_ $\leftrightarrow$ _ _ $\vdash$ MRSC*_ $\leftrightarrow$ _

data _ $\vdash$ MRSC_ $\leftrightarrow$ _ :  $\forall$  (h : History) (c : Conf)
  (g : Graph Conf)  $\rightarrow$  Set
_ $\vdash$ MRSC*_ $\leftrightarrow$ _ :  $\forall$  (h : History) (cs : List Conf)
  (gs : List (Graph Conf))  $\rightarrow$  Set

```

Again, `_ $\vdash$ MRSC*_ $\leftrightarrow$ _` is a “point-wise” version of `_ $\vdash$ MRSC_ $\leftrightarrow$ _`:

```

h  $\vdash$ MRSC* cs  $\leftrightarrow$  gs = Pointwise.Rel (_ $\vdash$ MRSC_ $\leftrightarrow$ _ h) cs gs

```

`_ $\vdash$ MRSC_ $\leftrightarrow$ _` is defined by two rules

```

data _ $\vdash$ MRSC_ $\leftrightarrow$ _ where
  mrsc-fold  :  $\forall$  {h : History} {c}
    (f : Foldable h c)  $\rightarrow$ 
    h  $\vdash$ MRSC c  $\leftrightarrow$  back c
  mrsc-build :  $\forall$  {h : History} {c}
    {cs : List Conf} {gs : List (Graph Conf)}
    ( $\neg$ f :  $\neg$  Foldable h c)
    ( $\neg$ w :  $\neg$   $\zeta$  h)  $\rightarrow$ 
    (i : cs  $\in$  c  $\Rightarrow$ )
    (s : (c :: h)  $\vdash$ MRSC* cs  $\leftrightarrow$  gs)  $\rightarrow$ 
    h  $\vdash$ MRSC c  $\leftrightarrow$  forth c gs

```

We can see that  $\_ \vdash$ NDSC $\leftrightarrow$ \_ and  $\_ \vdash$ MRSC $\leftrightarrow$ \_ differ only in that there is an additional condition  $\neg \zeta h$  in the rule `mrsc-build`.

The predicate  $\zeta$  is provided by the whistle,  $\zeta h$  meaning that the history `h` is “dangerous”. Unlike the rule `ndsc-build`, the rule `mrsc-build` is only applicable when  $\neg \zeta h$ , i.e. the history `h` is not dangerous.

Multi-result supercompilation is a special case of non-deterministic supercompilation, in the sense that any graph produced by multi-result supercompilation can also be produced by non-deterministic supercompilation:

```

MRSC $\rightarrow$ NDSC :  $\forall$  {h : History} {c g}  $\rightarrow$ 
  h  $\vdash$ MRSC c  $\leftrightarrow$  g  $\rightarrow$  h  $\vdash$ NDSC c  $\leftrightarrow$  g

```

A proof of this theorem can be found in `BigStepScTheorems.agda`.

### 3.5 Bar whistles

Now we are going to give an alternative definition of multi-result supercompilation in form of a total function `naive-mrsc`. The termination of `naive-mrsc` is guaranteed by a “whistle”.

In our model of big-step supercompilation whistles are assumed to be “inductive bars” [6] and are defined in Agda in the following way.

First of all, `BarWhistles.agda` contains the following declaration of `Bar D h`:

```

data Bar {A : Set} (D : List A  $\rightarrow$  Set) :
  (h : List A)  $\rightarrow$  Set where
  now   : {h : List A} (bz : D h)  $\rightarrow$  Bar D h
  later : {h : List A} (bs :  $\forall$  c  $\rightarrow$  Bar D (c :: h))  $\rightarrow$  Bar D h

```

At the first glance, this declaration looks as a puzzle. But, actually, it is not as mysterious as it may seem.

We consider sequences of elements (of some type `A`), and a predicate `D`. If `D h` holds for a sequence `h`, `h` is said to be “dangerous”.

`Bar D h` means that either (1) `h` is dangerous, i.e. `D h` is valid right now (the rule `now`), or (2) `Bar D (c :: h)` is valid for *all* possible `c :: h` (the rule

later). Hence, for any continuation  $c :: h$  the sequence will *eventually* become dangerous.

The subtle point is that if `Bar D []` is valid, it implies that *any* sequence will eventually become dangerous.

A *bar whistle* is a record (see `BarWhistles.agda`)

```
record BarWhistle (A : Set) : Set1 where
  field
    ⚡      : (h : List A) → Set
    ⚡::    : (c : A) (h : List A) → ⚡ h → ⚡ (c :: h)
    ⚡?    : (h : List A) → Dec (⚡ h)

    bar[] : Bar ⚡ []
```

where

- $\text{⚡}$  is a predicate on sequences,  $\text{⚡ } h$  meaning that the sequence  $h$  is dangerous.
- $\text{⚡}::$  postulates that if  $\text{⚡ } h$  then  $\text{⚡ } (c :: h)$  for all possible  $c :: h$ . In other words, if  $h$  is dangerous, so are all continuations of  $h$ .
- $\text{⚡}?$  says that  $\text{⚡}$  is decidable.
- `bar []` says that any sequence eventually becomes dangerous. (In Coquand’s terms, `Bar ⚡` is required to be “an inductive bar”.)

### 3.6 A function for computing Cartesian products

The functional specification of big-step multi-result supercompilation considered in the following section is based on the function `cartesian`:

```
cartesian : ∀ {A : Set} (xss : List (List A)) → List (List A)
```

`cartesian` takes as input a list of lists  $xss$  (see `Util.agda`). Each list  $xs \in xss$  represents the set of possible values of the correspondent component.

Namely, suppose that  $xss$  has the form  $xs_1, xs_2, \dots, xs_k$ . Then `cartesian` returns a list containing all possible lists of the form  $x_1 :: x_2 :: \dots :: x_k :: []$  where  $x_i \in xs_i$ . In Agda, this property of `cartesian` is formulated as follows:

```
∈*↔∈cartesian :
  ∀ {A : Set} {xs : List A} {yss : List (List A)} →
    Pointwise.Rel _∈_ xs yss ↔ xs ∈ cartesian yss
```

A proof of the theorem `∈*↔∈cartesian` can be found in `Util.agda`.

### 3.7 A functional specification of big-step multi-result supercompilation

A functional specification of big-step multi-result supercompilation is given in the form of a total function (in `BigStepSc.agda`) that takes the initial configuration `c` and returns a list of residual graphs:

```
naive-mrsc : (c : Conf) → List (Graph Conf)
naive-mrsc' : ∀ (h : History) (b : Bar ↯ h) (c : Conf) →
              List (Graph Conf)
```

```
naive-mrsc c = naive-mrsc' [] bar [] c
```

`naive-mrsc` is defined in terms of a more general function `naive-mrsc'`, which takes more arguments: a history `h`, a proof `b` of the fact `Bar ↯ h`, and a configuration `c`.

Note that `naive-mrsc` calls `naive-mrsc'` with the empty history and has to supply a proof of the fact `Bar ↯ []`. But this proof is supplied by the whistle!

```
naive-mrsc' h b c with foldable? h c
... | yes f = [ back c ]
... | no ¬f with ↯? h
... | yes w = []
... | no ¬w with b
... | now bz with ¬w bz
... | ()
naive-mrsc' h b c | no ¬f | no ¬w | later bs =
  map (forth c)
      (concat (map (cartesian ∘ map
                    (naive-mrsc' (c :: h) (bs c))) (c ⇒)))
```

The definition of `naive-mrsc'` is straightforward.

- If `c` is foldable to the history `h`, a back-node is generated and the function terminates.
- Otherwise, if `↯ h` (i.e. the history `h` is dangerous), the function terminates producing no graphs.
- Otherwise, `h` is not dangerous, and the configuration `c` can be decomposed. (Also there are some manipulations with the parameter `b` that will be explained later.)
- Thus `c ⇒` returns a list of lists of configurations. The function considers each `cs ∈ c ⇒`, and, for each `c' ∈ cs` recursively calls itself in the following way: `naive-mrsc' (c :: h) (bs c) c'` producing a list of residual graphs `gs'`. So, `cs` is transformed into `gss`, a list of lists of graphs. Note that `length cs = length gss`.

- Then the function computes cartesian product `cartesian gss`, to produce a list of lists of graphs. Then the results corresponding to each  $cs \in c \Rightarrow$  are concatenated by `concat`.
- At this moment the function has obtained a list of lists of graphs, and calls `map (forth c)` to turn each graph list into a forth-node.

The function `naive-mrsc` is correct (sound and complete) with respect to the relation  $\_ \vdash \text{MRSC} \_ \leftrightarrow \_$ :

$$\begin{aligned} & \vdash \text{MRSC} \leftrightarrow \Leftrightarrow \text{naive-mrsc} : \\ & \{c : \text{Conf}\} \{g : \text{Graph Conf}\} \rightarrow \\ & \quad [] \vdash \text{MRSC } c \leftrightarrow g \Leftrightarrow g \in \text{naive-mrsc } c \end{aligned}$$

A proof of this theorem can be found in `BigStepScTheorems.agda`.

### 3.8 Why `naive-mrsc'` always terminates?

The problem with `naive-mrsc'` is that in the recursive call

$$\text{naive-mrsc}' (c :: h) (\text{bs } c) c'$$

the history grows ( $h$  becomes  $c :: h$ ), and the configuration is replaced with another configuration of unknown size ( $c$  becomes  $c'$ ). Hence, these parameters do not become “structurally smaller”.

But Agda’s termination checker still accepts this recursive call, because the second parameter does become smaller (`later bs` becomes `bs c`). Note that the termination checker considers `bs` and `bs c` to be of the same “size”. Since `bs` is smaller than `later bs` (a constructor is removed), and `bs` and `bs c` are of the same size, `bs c` is “smaller” than `later bs`.

Thus purpose of the parameter `b` is to persuade the termination checker that the function terminates. If `lazy-mrsc` is reimplemented in a language in which the totality of functions is not checked, the parameter `b` is not required and can be removed.

## 4 Staging big-step multi-result supercompilation

We can decompose the process of supercompilation into two stages

$$\text{naive-mrsc} \doteq \langle\langle\_ \rangle\rangle \circ \text{lazy-mrsc}$$

where  $\langle\langle\_ \rangle\rangle$  is a unary function, and  $f \doteq g$  means that  $f x = g x$  for all  $x$ .

At the first stage, `lazy-mrsc` generates a “lazy graph”, which, essentially, is a “program” to be “executed” by  $\langle\langle\_ \rangle\rangle$ .

## 4.1 Lazy graphs of configurations

A `LazyGraph C` represents a finite set of graphs of configurations (whose type is `Graph C`).

```
data LazyGraph (C : Set) : Set where
  ∅      : LazyGraph C
  stop  : (c : C) → LazyGraph C
  build : (c : C) (lss : List (List (LazyGraph C))) → LazyGraph C
```

A lazy graph is a tree whose nodes are “commands” to be executed by the interpreter  $\ll\_ \gg$ .

The exact semantics of lazy graphs is given by the function  $\ll\_ \gg$ , which calls auxiliary functions  $\ll\_ \gg^*$  and  $\ll\_ \gg \Rightarrow$  (see `Graphs.agda`).

```
 $\ll\_ \gg$  : {C : Set} (l : LazyGraph C) → List (Graph C)
 $\ll\_ \gg^*$  : {C : Set} (ls : List (LazyGraph C)) → List (List (Graph C))
 $\ll\_ \gg \Rightarrow$  : {C : Set} (lss : List (List (LazyGraph C))) →
  List (List (Graph C))
```

Here is the definition of the main function  $\ll\_ \gg$ :

```
 $\ll \emptyset \gg$  = []
 $\ll \text{stop } c \gg$  = [ back c ]
 $\ll \text{build } c \text{ lss} \gg$  = map (forth c)  $\ll \text{lss} \gg \Rightarrow$ 
```

It can be seen that  $\emptyset$  means “generate no graphs”, `stop` means “generate a back-node and stop”.

The most interesting case is a build-node `build c lss`, where `c` is a configuration and `lss` a list of lists of lazy graphs. Recall that, in general, a configuration can be decomposed into a list of configurations in several different ways. Thus, each `ls`  $\in$  `lss` corresponds to a decomposition of `c` into a number of configurations  $c_1, \dots, c_k$ . By supercompiling each  $c_i$  we get a collection of graphs that can be represented by a lazy graph  $ls_i$ .

The function  $\ll\_ \gg^*$  considers each lazy graph in a list of lazy graphs `ls`, and turns it into a list of graphs:

```
 $\ll [] \gg^*$  = []
 $\ll l :: ls \gg^*$  =  $\ll l \gg$  ::  $\ll ls \gg^*$ 
```

The function  $\ll\_ \gg \Rightarrow$  considers all possible decompositions of a configuration, and for each decomposition computes all possible combinations of subgraphs:

```
 $\ll [] \gg \Rightarrow$  = []
 $\ll ls :: lss \gg \Rightarrow$  = cartesian  $\ll ls \gg^*$  ++  $\ll lss \gg \Rightarrow$ 
```

There arises a natural question: why  $\langle\langle\_ \rangle\rangle^*$  is defined by explicit recursion, while it does exactly the same job as would do  $\text{map } \langle\langle\_ \rangle\rangle$ ? The answer is that Agda's termination checker does not accept  $\text{map } \langle\langle\_ \rangle\rangle$ , because it cannot see that the argument in the recursive calls to  $\langle\langle\_ \rangle\rangle$  becomes structurally smaller. For the same reason  $\langle\langle\_ \rangle\rangle \Rightarrow$  is also defined by explicit recursion.

## 4.2 A functional specification of lazy multi-result supercompilation

Given a configuration  $c$ , the function `lazy-mrsc` produces a lazy graph.

```
lazy-mrsc : (c : Conf) → LazyGraph Conf
```

`lazy-mrsc` is defined in terms of a more general function `lazy-mrsc'`

```
lazy-mrsc' : ∀ (h : History) (b : Bar ↯ h)
```

```
  (c : Conf) → LazyGraph Conf
```

```
lazy-mrsc c = lazy-mrsc' [] bar[] c
```

The general structure of `lazy-mrsc'` is very similar (see Section 3) to that of `naive-mrsc'`, but, unlike `naive-mrsc`, it does not build Cartesian products immediately.

```
lazy-mrsc' h b c with foldable? h c
```

```
... | yes f = stop c
```

```
... | no ¬f with ↯? h
```

```
... | yes w = ∅
```

```
... | no ¬w with b
```

```
... | now bz with ¬w bz
```

```
... | ()
```

```
lazy-mrsc' h b c | no ¬f | no ¬w | later bs =
```

```
  build c (map (map (lazy-mrsc' (c :: h) (bs c))) (c ⇒))
```

Let us compare the most interesting parts of `naive-mrsc` and `lazy-mrsc`:

```
map (forth c)
```

```
  (concat (map (cartesian ○
```

```
    map (naive-mrsc' (c :: h) (bs c))) (c ⇒)))
```

```
...
```

```
build c (map (map (lazy-mrsc' (c :: h) (bs c))) (c ⇒))
```

Note that `cartesian` disappears from `lazy-mrsc`.

## 4.3 Correctness of lazy-mrsc and $\langle\langle\_ \rangle\rangle$

`lazy-mrsc` and  $\langle\langle\_ \rangle\rangle$  are correct with respect to `naive-mrsc`. In Agda this is formulated as follows:

$\text{naive}\equiv\text{lazy} : (c : \text{Conf}) \rightarrow \text{naive-mrsc } c \equiv \ll \text{lazy-mrsc } c \gg$

In other words, for any initial configuration  $c$ ,  $\ll \text{lazy-mrsc } c \gg$  returns the same list of graphs (the same configurations in the same order!) as would return  $\text{naive-mrsc } c$ .

A formal proof of  $\text{naive}\equiv\text{lazy}$  can be found in `BigStepScTheorems.agda`.

## 5 Cleaning lazy graphs

A function `clean` is said to be a “cleaner” if for any lazy graph  $l$

$$\ll \text{clean } l \gg \subseteq \ll l \gg$$

Suppose that a function `filter` filters bags of graphs, removing “bad” graphs, so that

$$\text{filter } \ll l \gg$$

generates the bag of “good” graphs. Let `clean` be a cleaner such that

$$\text{filter} \circ \ll\_ \gg \doteq \ll\_ \gg \circ \text{clean}$$

Then we can replace filtering of graphs with cleaning of lazy graphs

$$\text{filter} \circ \text{naive-mrsc} \doteq \ll\_ \gg \circ \text{clean} \circ \text{lazy-mrsc}$$

In `Graphs.agda` there are defined a number of filters and corresponding cleaners.

### 5.1 Filter `fl-bad-conf` and cleaner `cl-bad-conf`

$\text{fl-bad-conf} : \{C : \text{Set}\} (\text{bad} : C \rightarrow \text{Bool}) (\text{gs} : \text{List } (\text{Graph } C)) \rightarrow \text{List } (\text{Graph } C)$

$\text{cl-bad-conf} : \{C : \text{Set}\} (\text{bad} : C \rightarrow \text{Bool}) (l : \text{LazyGraph } C) \rightarrow \text{LazyGraph } C$

Configurations represent states of a computation process. Some of these states may be “bad” with respect to the problem that is to be solved by means of super-compilation.

Given a predicate `bad` that returns `true` for “bad” configurations, `fl-bad-conf bad gs` removes from `gs` the graphs that contain at least one “bad” configuration.

The cleaner `cl-bad-conf` corresponds to the filter `fl-bad-conf`. `cl-bad-conf` exploits the fact that “badness” is monotonic, in the sense that a single “bad” configuration spoils the whole graph.

`cl-bad-conf` is correct with respect to `fl-bad-conf`:

```
cl-bad-conf-correct : {C : Set} (bad : C → Bool) →
  ⟨⟨_⟩⟩ ∘ cl-bad-conf bad ≐ fl-bad-conf bad ∘ ⟨⟨_⟩⟩
```

A formal proof of this theorem is given in `GraphsTheorems.agda`.

It is instructive to take a look at the implementation of `cl-bad-conf` in `Graphs.agda`, to get the general idea of how cleaners are really implemented:

```
cl-bad-conf : {C : Set} (bad : C → Bool) (l : LazyGraph C) →
  LazyGraph C
```

```
cl-bad-conf⇒ : {C : Set} (bad : C → Bool)
  (lss : List (List (LazyGraph C))) → List (List (LazyGraph C))
```

```
cl-bad-conf* : {C : Set} (bad : C → Bool)
  (ls : List (LazyGraph C)) → List (LazyGraph C)
```

```
cl-bad-conf bad ∅ = ∅
cl-bad-conf bad (stop c) =
  if bad c then ∅ else (stop c)
cl-bad-conf bad (build c lss) =
  if bad c then ∅ else (build c (cl-bad-conf⇒ bad lss))
```

```
cl-bad-conf⇒ bad [] = []
cl-bad-conf⇒ bad (ls :: lss) =
  cl-bad-conf* bad ls :: (cl-bad-conf⇒ bad lss)
```

```
cl-bad-conf* bad [] = []
cl-bad-conf* bad (l :: ls) =
  cl-bad-conf bad l :: cl-bad-conf* bad ls
```

## 5.2 Cleaner `cl-empty`

`cl-empty` is a cleaner that removes subtrees of a lazy graph that represent empty sets of graphs.

```
cl-empty : {C : Set} (l : LazyGraph C) → LazyGraph C
```

`cl-bad-conf` is correct with respect to `⟨⟨_⟩⟩`:

```
cl-empty-correct : ∀ {C : Set} (l : LazyGraph C) →
  ⟨⟨ cl-empty l ⟩⟩ ≐ ⟨⟨ l ⟩⟩
```

A formal proof of this theorem is given in `GraphsTheorems.agda`.

## 5.3 Cleaner `cl-min-size`

The function `cl-min-size`

`cl-min-size` :  $\forall \{C : \text{Set}\} (l : \text{LazyGraph } C) \rightarrow \mathbb{N} \times \text{LazyGraph } C$

takes as input a lazy graph `l` and returns either  $(0, \emptyset)$ , if `l` contains no graphs, or a pair  $(k, l')$ , where `l'` is a lazy graph, representing a single graph `g'` of minimal size `k`.

The details of how `cl-min-size` is implemented can be found in `Graphs.agda`.

A good thing about `cl-min-size` is it cleans any lazy graph `l` in linear time with respect to the size of `l`.

## 6 Codata and corecursion: cleaning before whistling

By using codata and corecursion, we can decompose `lazy-mrsc` into two stages

$$\text{lazy-mrsc} \stackrel{\circ}{=} \text{prune-cograph} \circ \text{build-cograph}$$

where `build-cograph` constructs a (potentially) infinite tree, while `prune-cograph` traverses this tree and turns it into a lazy graph (which is finite).

### 6.1 Lazy cographs of configurations

A `LazyCograph C` represents a (potentially) infinite set of graphs of configurations whose type is `Graph C` (see `Cographs.agda`).

```
data LazyCograph (C : Set) : Set where
  ∅      : LazyCograph C
  stop  : (c : C) → LazyCograph C
  build : (c : C)
          (lss : ∞(List (List (LazyCograph C)))) → LazyCograph C
```

Note that `LazyCograph C` differs from `LazyGraph C` the evaluation of `lss` in `build-nodes` is delayed.

### 6.2 Building lazy cographs

Lazy cographs are produced by the function `build-cograph`

```
build-cograph : (c : Conf) → LazyCograph Conf
```

which can be derived from the function `lazy-mrsc` by removing the machinery related to whistles.

`build-cograph` is defined in terms of a more general function `build-cographs'`.

```
build-cograph' : (h : History) (c : Conf) → LazyCograph Conf
build-cograph c = build-cograph' [] c
```

The definition of `build-cograph'` uses auxiliary functions `build-cograph $\Rightarrow$`  and `build-cograph*`, while the definition of `lazy-mrsc` just calls `map` at corresponding places. This is necessary in order for `build-cograph'` to pass Agda's "productivity" check.

```
build-cograph $\Rightarrow$  : (h : History) (c : Conf)
  (css : List (List Conf))  $\rightarrow$  List (List (LazyCograph Conf))
```

```
build-cograph* : (h : History)
  (cs : List Conf)  $\rightarrow$  List (LazyCograph Conf)
```

```
build-cograph' h c with foldable? h c
... | yes f = stop c
... | no  $\neg$ f =
  build c ( $\#$  build-cograph $\Rightarrow$  h c (c  $\Rightarrow$ ))
```

```
build-cograph $\Rightarrow$  h c [] = []
build-cograph $\Rightarrow$  h c (cs :: css) =
  build-cograph* (c :: h) cs :: build-cograph $\Rightarrow$  h c css
```

```
build-cograph* h [] = []
build-cograph* h (c :: cs) =
  build-cograph' h c :: build-cograph* h cs
```

### 6.3 Pruning lazy cographs

A lazy cograph can be pruned by means of the function `prune-cograph` to obtain a finite lazy graph.

```
prune-cograph : (l : LazyCograph Conf)  $\rightarrow$  LazyGraph Conf
```

which can be derived from the function `lazy-mrsc` by removing the machinery related to generation of nodes (since it only consumes nodes that have been generated by `build-cograph`).

`prune-cograph` is defined in terms of a more general function `prune-cograph'`:

```
prune-cograph l = prune-cograph' [] bar [] l
```

The definition of `prune-cograph'` uses the auxiliary function `prune-cograph*`.

```
prune-cograph* : (h : History) (b : Bar  $\zeta$  h)
  (ls : List (LazyCograph Conf))  $\rightarrow$  List (LazyGraph Conf)
```

```
prune-cograph' h b  $\emptyset$  =  $\emptyset$ 
prune-cograph' h b (stop c) = stop c
prune-cograph' h b (build c lss) with  $\zeta?$  h
```

```

... | yes w = ∅
... | no ¬w with b
... | now bz with ¬w bz
... | ()
prune-cograph' h b (build c lss) | no ¬w | later bs =
  build c (map (prune-cograph* (c :: h) (bs c)) (b lss))

```

```

prune-cograph* h b [] = []
prune-cograph* h b (l :: ls) =
  prune-cograph' h b l :: (prune-cograph* h b ls)

```

Note that, when processing a node `build c lss`, the evaluation of `lss` has to be explicitly forced by `b`.

`prune-cograph` and `build-cograph` are correct with respect to `lazy-mrsc`:

```

prune◦build-correct :
  prune-cograph ◦ build-cograph ≐ lazy-mrsc

```

A proof of this theorem can be found in `Cographs.agda`.

## 6.4 Promoting some cleaners over the whistle

Suppose `clean∞` is a cograph cleaner such that

$$\text{clean} \circ \text{prune-cograph} \doteq \text{prune-cograph} \circ \text{clean}\infty$$

then

$$\begin{aligned} \text{clean} \circ \text{lazy-mrsc} &\doteq \\ \text{clean} \circ \text{prune-cograph} \circ \text{build-cograph} &\doteq \\ \text{prune-cograph} \circ \text{clean}\infty \circ \text{build-cograph} & \end{aligned}$$

The good thing about `build-cograph` and `clean∞` is that they work in a lazy way, generating subtrees by demand. Hence, evaluating

$$\ll \text{prune-cograph} \circ (\text{clean}\infty (\text{build-cograph } c)) \gg$$

may be less time and space consuming than evaluating

$$\ll \text{clean} (\text{lazy-mrsc } c) \gg$$

In `Cographs.agda` there is defined a cograph cleaner `cl-bad-conf∞` that takes a lazy cograph and prunes subtrees containing bad configurations, returning a lazy subgraph (which can be infinite):

```

cl-bad-conf∞ : {C : Set} (bad : C → Bool) (l : LazyCograph C) →
  LazyCograph C

```

`cl-bad-conf∞` is correct with respect to `cl-bad-conf`:

$$\begin{aligned} \text{cl-bad-conf}\infty\text{-correct} &: (\text{bad} : \text{Conf} \rightarrow \text{Bool}) \rightarrow \\ &\text{cl-bad-conf bad} \circ \text{prune-cograph} \stackrel{\circ}{=} \\ &\text{prune-cograph} \circ \text{cl-bad-conf}\infty \text{ bad} \end{aligned}$$

A proof of this theorem can be found in `Cographs.agda`.

## 7 Related work

The idea that supercompilation can produce a compact representation of a collection of residual graphs is due to Grechanik [7, 8]. In particular, the data structure ‘LazyGraph C’ we use for representing the results of the first phase of the staged multi-result supercompiler can be considered as a representation of "overtrees", which was informally described in [7].

Big-step supercompilation was studied and implemented by Bolingbroke and Peyton Jones [4]. Our approach differs in that we are interested in applying supercompilation to problem solving. Thus

- We consider multi-result supercompilation, rather than single-result supercompilation.
- Our big-step supercompilation constructs graphs of configurations in an explicit way, because the graphs are going to be filtered and/or analyzed at a later stage.
- Bolingbroke and Peyton Jones considered big-step supercompilation in functional form, while we have studied both a relational specification of big-step supercompilation and the functional one and have proved the correctness of the functional specification with respect to the relational one.

A relational specification of single-result supercompilation was suggested by Klimov [11], who argued that supercompilation relations can be used for simplifying proofs of correctness of supercompilers. Later, Klyuchnikov [20] used a supercompilation relation for proving the correctness of a small-step single-result supercompiler for a higher-order functional language. In the present work we consider a supercompilation relation for a *big-step multi-result* supercompilation.

We have developed an abstract model of big-step multi-result supercompilation in the language Agda and have proved a number of properties of this model. This model, in some respects, differs from the other models of supercompilation.

The MRSC Toolkit by Klyuchnikov and Romanenko [23] abstracts away some aspects of supercompilation, such as the structure of configurations and the details of the subject language. However, the MRSC Toolkit is a framework for implementing small-step supercompilers, while our model in Agda [2] formalizes big-step supercompilation. Besides, the MRSC Toolkit is implemented in Scala, for which reason it currently provides no means for neither formulating nor proving theorems about supercompilers implemented with the MRSC Toolkit.

Krustev was the first to formally verify a simple supercompiler by means of a proof assistant [26]. Unlike the MRSC Toolkit and our model of supercompilation, Krustev deals with a specific supercompiler for a concrete subject language. (Note, however, that also the subject language is simple, it is still Turing complete.)

In another paper Krustev presents a framework for building formally verifiable supercompilers [27]. It is similar to the MRSC in that it abstracts away some details of supercompilation, such as the subject language and the structure of configurations, providing, unlike the MRSC Toolkit, means for formulating and proving theorems about supercompilers.

However, in both cases Krustev deals with single-result supercompilation, while the primary goal of our model of supercompilation is to formalize and investigate some subtle aspects of multi-result supercompilation.

## 8 Conclusions

When using supercompilation for problem solving, it seems natural to produce a collection of residual graphs of configurations by multi-result supercompilation and then to filter this collection according to some criteria. Unfortunately, this may lead to combinatorial explosion.

We have suggested the following solution.

- Instead of generating and filtering a collection of residual graphs of configurations, we can produce a compact representation for the collection of graphs (a "lazy graph"), and then analyze this representation.
- This compact representation can be derived from a (big-step) multi-result supercompiler in a systematic way by (manually) staging this supercompiler to represent it as a composition of two stages. At the first stage, some graph-building operations are delayed to be later performed at the second stage.
- The result produced by the first stage is a "lazy graph", which is, essentially, a program to be interpreted at the second stage, to actually generate a collection of residual graphs.
- The key point of our approach is that a number of problems can be solved by directly analyzing the lazy graphs, rather than by actually generating and analyzing the collections of graphs they represent.
- In some cases of practical importance, the analysis of a lazy graph can be performed in linear time.

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