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Sliding mode control for three-axis magnetic attitude

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Скользящее управление для трехосной магнитной ориентации спутника

Рассматривается малый спутник, оснащенный магнитной системой управления, обеспечивающей его трехосную ориентацию. Управление формируется на основе скользящего режима. Строится алгоритм, реализующий подпространство скользящего режима, что позволяет перевести спутник в требуемую ориентацию по реализуемой фазовой траектории. Конечная ориентация является асимптотически устойчивой. Таким образом, решается проблема отсутствия управляемости при использовании магнитной системы.

Ключевые слова: магнитная система ориентации, трехосная ориентация, скользящее управление

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Sliding mode control for three-axis magnetic attitude

Satellite equipped with magnetic attitude control system is considered. Sliding mode control is used to ensure three-axis satellite attitude. Sliding manifold construction is discussed. This manifold is achievable at any time using only magnetic control system. Necessary attitude is asymptotically stable. This solves the underactuation problem.

Key words: magnetic attitude control system, three-axis attitude, sliding mode control

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Introduction

Present work is devoted to a three-axis satellite attitude provided with magnetorquers only. Control torque direction is restricted in this case. It should be perpendicular to the geomagnetic induction vector. Common control construction techniques (for example, PD-controller) are unavailable. These techniques provide control torque which cannot be implemented by magnetorquers. This results in three major areas of magnetorquers implementation in satellite attitude stabilization. The most widespread magnetorquers task is damping of angular velocity. This task can be flawlessly resolved since necessary torque is perpendicular to the geomagnetic induction vector. After angular velocity is damped control is passed to other actuators. Another task is attitude stabilization of particular satellite configurations and attitude regimes. One-axis attitude of spin-stabilized satellite is the best example. Finally, magnetorquers may be used with other actuators. Complementing magnetorquers with one flywheel makes some specific attitude regimes available. These schemes of magnetorquers implementation are extensively used on large satellites. However they are unsuitable for small satellites, especially CubeSats. Thrusters are better not be used because of restricted fuel capacity. Reaction wheels are also bad choice because of high price and complexity. Available for small satellites wheels are also frequently unreliable or even more expensive. Magnetorquers have no drawbacks of other actuators. They are cheap, reliable, compact, lightweight and require negligible power and no fuel. Underactuation issue is a pay-off for these advantages.

Increasing number of small satellites and CubeSats launches intensifies the demand on simple attitude stabilization system. This justifies current interest in three-axis stabilization using magnetorquers only and vast number of works on this theme. Majority of these works follow one general scheme. First, some necessary control torque is constructed. Then only available (perpendicular to the geomagnetic induction vector) part is implemented. Some assumptions on control parameters [1] or satellite [2]-[3] allow this scheme to work. The underactuation issue remains unsolved and the control is not robust. Some deviations from theoretical control parameters or satellite parameters may lead to non-operational control. Implementing this scheme on real satellite with lots of disturbances and uncertainties is undesirable.

Underactuation issue has one important trait for magnetorquers. Geomagnetic induction vector rotates in the inertial space. No inaccessible direction in inertial or bound reference frames exists. Any inaccessible at a moment direction will become available, but after some time. This feature allows some accessible path to be constructed. This path requires perpendicular to geomagnetic induction vector torque at each control step. Finally satellite acquires necessary attitude with necessary angular velocity. Sliding control [4] is used in this paper to obtain the above mentioned path. Sliding control was already proposed for attitude stabilization of satellites with magnetorquers only [5]. This scheme was also implemented for underactuated formation flying system [6] with inaccessible control direction in bound frame. Constant sliding manifold parameters limit the importance of these works. This paper focuses on acquiring variable manifold parameters. This allows the sliding manifold to change in such a way that satellite path may be achieved with magnetorquers only.

1. Problem statement

Following reference frames are used:

inertial reference frame $O_aY_1Y_2Y_3$ where O_a is the Earth's center, the O_aY_3 axis is directed along with the Earth's axis, O_aY_1 lies in the Earth's equatorial plane and is directed to the ascending node of the satellite's orbit, the O_aY_2 axis is directed so the system is right-handed;

orbital frame $OX_1X_2X_3$ where O is satellite's center of mass, the O_aX_3 axis is directed along the radius-vector, O_aX_2 is directed along the orbital plane normal, O_aX_1 axis is directed so the system is right-handed (this axis is directed along the orbital velocity for the circular orbit);

bound frame $Ox_1x_2x_3$, its axes are directed along the principal axes of inertia of the satellite.

Satellite attitude in inertial space is described using Euler equations and kinematic relations based on quaternions and direction cosines matrices. Satellite state vector comprises of angular velocity $\boldsymbol{\omega}$ and quaternion $\left(\mathbf{q}, q_0\right)$ or direction cosines matrix $\mathbf{A}$ and its components a_{ij}. Dynamical equations of the satellite with inertia tensor $\mathbf{J}$ are written as

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{abs} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} = \mathbf{M}.$$

The torque consists of the control $\mathbf{M}_{cntrl} = \mathbf{m} \times \mathbf{B}$ and disturbing ones. For example gravitational and aerodynamic disturbing torques may be taken into account. In this case $\mathbf{M} = \mathbf{M}_{cntrl} + \mathbf{M}_{gr} + \mathbf{M}_{aer}$. Gravitational torque will be accounted for in control, while aerodynamic one will remain unaccounted for. Dynamical equations are complemented with kinematic relations. Quaternion is used for numerical simulation, in this case

$$\begin{pmatrix} \dot{\mathbf{q}} \\ \dot{q}_0 \end{pmatrix} = \frac{1}{2}\mathbf{\Omega}\begin{pmatrix} \mathbf{q} \\ q_0 \end{pmatrix} \tag{1}$$

where $\mathbf{q}$ is the vector part of the quaternion, q_0 is the scalar part and

$$\mathbf{\Omega} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}.$$

Direction cosine matrix is used for control construction. In this case

$$\dot{\mathbf{A}} = \mathbf{W}\mathbf{A} \tag{2}$$

where

$$\mathbf{W} = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix}.$$

Satellite motion in orbital reference frame is described in the same manner. Matrix $\mathbf{A}$ and quaternion $(\mathbf{q}, q_0)$ represent satellite attitude with respect to the orbital frame, angular velocity is

$$\boldsymbol{\omega}_{abs} = \boldsymbol{\omega}_{rel} + \mathbf{A}\boldsymbol{\omega}_{orb}$$

where $\boldsymbol{\omega}_{orb}$ is orbital reference frame angular velocity in inertial space, $\boldsymbol{\omega}_{rel}$ is satellite angular velocity relative to the orbital frame. Dynamical equations for relative angular velocity are

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{rel} + \boldsymbol{\omega}_{rel} \times \mathbf{J}\boldsymbol{\omega}_{rel} = \mathbf{M}$$

where

$$\mathbf{M} = \mathbf{M}_{cntrl} + \mathbf{M}_{gr} + \mathbf{M}_{aer} + \mathbf{M}_{rel}$$

and $\mathbf{M}_{rel} = -\mathbf{J}\mathbf{W}\mathbf{A}\boldsymbol{\omega}_{orb} - \boldsymbol{\omega}_{rel} \times \mathbf{J}\mathbf{A}\boldsymbol{\omega}_{orb} - \mathbf{A}\boldsymbol{\omega}_{orb} \times \mathbf{J}\left(\boldsymbol{\omega}_{rel} + \mathbf{A}\boldsymbol{\omega}_{orb}\right)$.

The last expression is accounted for in the control in the same way as $\mathbf{M}_{gr}$. We will use general equations of motion in inertial and orbital frames,

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M} + \mathbf{M}_{cntrl} + \mathbf{M}_{aer}, \tag{3}$$

where $\boldsymbol{\omega}$ is relative or absolute angular velocity, $\mathbf{M}$ is accordingly defined as $\mathbf{M} = \mathbf{M}_{gr} + \mathbf{M}_{rel}$ or $\mathbf{M} = \mathbf{M}_{gr}$.

2. Control construction

Sliding control is constructed in two steps. First sliding manifold $\mathbf{x}(\boldsymbol{\omega}, \mathbf{A}, t) = 0$ is constructed in phase space. Satellite motion should satisfy this relation: the satellite moves on the manifold. The manifold is constructed in such a way that necessary attitude is asymptotically stable. Second step is control torque construction. The torque should ensure motion on the sliding manifold. Common sliding manifold for satellite angular motion is

$$\mathbf{x} = \boldsymbol{\omega} + \boldsymbol{\Lambda}\mathbf{S}(\mathbf{A}) = 0$$

where $\boldsymbol{\Lambda}$ is a positive-defined constant matrix, vector $\mathbf{S}$ characterizes deviation from necessary attitude. This vector $\mathbf{S}$ has is the same as the one used in PD-controller [1],

$$\mathbf{S} = \begin{pmatrix} a_{23} - a_{32} \\ a_{31} - a_{13} \\ a_{12} - a_{21} \end{pmatrix}. \qquad (4)$$

Position $\mathbf{S} = 0$ corresponds to diagonal direction cosines matrix. Sliding manifold equation and matrix $\boldsymbol{\Lambda}$ are independent on time and satellite attitude and velocity. This restriction should be lifted if satellite moves in the geomagnetic field. Since the induction vector rotates sliding manifold should rotate also ensuring the necessary control torque to be perpendicular to the geomagnetic induction vector for each time and attitude. General sliding manifold equation is

$$\mathbf{x} = \lambda(\boldsymbol{\omega}, \mathbf{S}, t)\boldsymbol{\omega} + \boldsymbol{\Lambda}(\boldsymbol{\omega}, \mathbf{S}, t)\mathbf{S} = 0 \qquad (5)$$

where $\boldsymbol{\Lambda}$ is a positive-defined variable matrix and λ is a positive value (it could be a positive-defined matrix also). Satellite motion on the sliding manifold is represented by $\mathbf{x} = 0$. In this case attitude $\boldsymbol{\omega} = 0$, $\mathbf{A} = \mathbf{E}$ is asymptotically stable ($\mathbf{E}$ is a unity matrix). This could be proven with the following reasoning. It may be shown that

$$\mathbf{S} = 4q_0\mathbf{q}.$$

In this case (5) will be

$$\mathbf{x} = \lambda\boldsymbol{\omega} + 4q_0\boldsymbol{\Lambda}\mathbf{q} = 0. \qquad (6)$$

Using (1) we obtain

$$\dot{\mathbf{q}} = \mathbf{Q}\boldsymbol{\omega} \qquad (7)$$

where

$$\mathbf{Q} = q_0\mathbf{E} + \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}.$$

Taking into account (6) equation (7) is rewritten as

$$\dot{\mathbf{q}} = -4\lambda^{-1}q_0\mathbf{Q}\boldsymbol{\Lambda}\mathbf{q}$$

or

$$\dot{\mathbf{q}} = -4\lambda^{-1}q_0^2\boldsymbol{\Lambda}\mathbf{q} - 4\lambda^{-1}q_0\mathbf{q} \times \boldsymbol{\Lambda}\mathbf{q}.$$

Product of this equation by $\mathbf{q}$ will be

$$\mathbf{q}^T\dot{\mathbf{q}} = -4q_0^2\lambda^{-1}\mathbf{q}^T\mathbf{\Lambda}\mathbf{q}$$

or

$$\frac{1}{2}\frac{d}{dt}\left(\mathbf{q}^T\mathbf{q}\right) = -4\lambda^{-1}q_0^2\mathbf{q}^T\mathbf{\Lambda}\mathbf{q} \le 0.$$

Magnitude of the quaternion vector part decreases while moving along the sliding manifold. This means that motion of the satellite on the sliding manifold leads to the necessary attitude. Control torque construction problem is transformed. Instead of maintaining necessary attitude it should ensure motion on the sliding manifold. Sliding manifold should be chosen in such a way that control torque is perpendicular to the geomagnetic induction vector.

3. Iterative sliding manifold construction

Control should ensure motion on the sliding manifold according to the equation

$$\dot{\mathbf{x}} = -\mathbf{J}^{-1}\mathbf{P}\mathbf{x} \tag{8}$$

where $\mathbf{P}$ a is positive-defined matrix. Inertia tensor is introduced to simplify further reasoning. Taking into account (5) we rewrite (8) as

$$\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda\mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{J}\dot{\mathbf{\Lambda}}\mathbf{S} + \mathbf{J}\mathbf{\Lambda}\dot{\mathbf{S}} = -\lambda\mathbf{P}\boldsymbol{\omega} - \mathbf{P}\mathbf{\Lambda}\mathbf{S}.$$

Scalar function λ characterizes damping part in control. Matrix $\boldsymbol{\lambda}$ allows different gains for each bound frame axis. This may be useful for satellite with particular dynamical configuration, for example long cylinder or flat disk. These cases are outside of frame of this paper. Matrix $\mathbf{\Lambda}$ characterizes positional control part. Matrix $\mathbf{P}$ represents the time-response of sliding manifold acquiring. $\dot{\mathbf{S}}$ is found using (4) and (2),

$$\dot{\mathbf{S}} = \begin{pmatrix} -\omega_3 a_{13} + \omega_1 a_{33} - \omega_2 a_{12} + \omega_1 a_{22} \\ \omega_2 a_{11} - \omega_1 a_{21} - \omega_3 a_{23} + \omega_2 a_{33} \\ \omega_3 a_{22} - \omega_2 a_{32} - \omega_1 a_{31} + \omega_3 a_{11} \end{pmatrix}.$$

Taking into account dynamical equations (3) we obtain

$$\lambda\mathbf{m}\times\mathbf{B} = -\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda\left(\boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega} - \mathbf{M}\right) - \dot{\mathbf{\Lambda}}\mathbf{J}\mathbf{S} - \mathbf{\Lambda}\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{P}\mathbf{S}\right) - \lambda\mathbf{P}\boldsymbol{\omega}. \tag{9}$$

Expression (9) governs magnetorquers dipole moment. Altering values of λ and $\mathbf{\Lambda}$ should allow the latter relation for each time and satellite attitude and velocity. Damping coefficient λ will be considered constant and known. The main problem is to find matrix $\mathbf{\Lambda}$ and its derivative. Iterative approach is considered below.

Matrix $\mathbf{\Lambda}$ derivative is written as

$$\dot{\mathbf{\Lambda}} = \frac{\mathbf{\Lambda}(k+1) - \mathbf{\Lambda}(k)}{\Delta t}$$

where Δt is control implementation step. Suppose we know satellite attitude, angular velocity and geomagnetic induction vector for $k+1$ step and previous matrix $\mathbf{\Lambda}(k)$. Our purpose is to find $\mathbf{\Lambda}(k+1)$. Substituting approximation into (9) we obtain

$$\lambda\Delta t\mathbf{m}\times\mathbf{B} = \left(-\dot{\lambda}\mathbf{J\omega} + \lambda(\mathbf{\omega}\times\mathbf{J\omega} - \mathbf{M}) - \mathbf{\Lambda}\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{PS}\right) - \lambda\mathbf{P\omega}\right)\Delta t - \mathbf{\Lambda}(k+1)\mathbf{JS} + \mathbf{\Lambda JS}. \quad (10)$$

Here all indices except in $\mathbf{\Lambda}(k+1)$ are omitted.

Introduce notations

$$\mathbf{a} = \left(-\dot{\lambda}\mathbf{J\omega} + \lambda(\mathbf{\omega}\times\mathbf{J\omega} - \mathbf{M}) - \mathbf{\Lambda}\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{PS}\right) - \lambda\mathbf{P\omega}\right)\Delta t + \mathbf{\Lambda JS},$$

$$\mathbf{b} = -\mathbf{JS},$$

$$\mathbf{d} = \lambda\Delta t\mathbf{B}$$

and rewrite (10) as

$$\mathbf{a} + \mathbf{\Lambda}(k+1)\mathbf{b} = \mathbf{m}\times\mathbf{d}. \quad (11)$$

Set new reference frame using basis vectors

$$\mathbf{e}_1 = \frac{\mathbf{d}}{|\mathbf{d}|},\ \mathbf{e}_3 = \frac{\mathbf{d}\times\mathbf{b}}{|\mathbf{d}\times\mathbf{b}|},\ \mathbf{e}_2 = \mathbf{e}_3\times\mathbf{e}_1.$$

Transition matrix from bound frame to the new one is $\mathbf{D} = [\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3]$. Scalar product of (11) and $\mathbf{d}$ is

$$\left(\mathbf{\Lambda}(k+1)\mathbf{b}\right)\mathbf{d} = -\mathbf{ad}.$$

Taking into account $\mathbf{d} = (d_1, 0, 0)^T$ and $\mathbf{b} = (b_1, b_2, 0)^T$ we get

$$\Lambda_{11}(k+1)b_1 + \Lambda_{12}(k+1)b_2 = -a_1. \quad (12)$$

Matrix $\mathbf{\Lambda}(k+1)$ construction is performed in a few steps. First $\Lambda_{11}(k+1) > 0$ should be chosen. For example, $\Lambda_{11}(k+1) = \Lambda_{11}(k)$. This allows using (12) to find $\Lambda_{12}(k+1)$ and $\Lambda_{21}(k+1)$,

$$\Lambda_{12}(k+1) = \Lambda_{21}(k+1) = \left(-a_1 - \Lambda_{11}(k+1)b_1\right)/b_2.$$

$\Lambda_{22}(k+1)$ should satisfy

$$\Lambda_{11}(k+1)\Lambda_{22}(k+1) - \Lambda_{12}^2(k+1) > 0. \quad (13)$$

For example $\Lambda_{22}(k+1) = \Lambda_0 + \dfrac{\Lambda_{12}^2(k+1)}{\Lambda_{11}(k+1)}$, Λ_0 is some constant value. It characterizes overall positional control part. However if $\Lambda_{22}(k)$ satisfies (13) previous step value may be used. Finally $\Lambda_{33}(k+1)=\Lambda_{33}(k)$. Matrix $\mathbf{\Lambda}(k+1)$ is then transformed to the bound frame. Expression (11) is used to find control torque and dipole moment. First step values may be set as $\mathbf{\Lambda}(k+1)=\mathbf{\Lambda}(k)=\Lambda_0\mathbf{E}$.

This reasoning cannot be used in the vicinity of necessary attitude since b_1 and b_2 are close to zero. To mitigate this problem element $\Lambda_{12}(k+1)$ is constructed according to

$$\Lambda_{12}(k+1)=-\frac{a_1+\Lambda_{11}(k+1)b_1}{b_2+\delta b_2}$$

where δb_2 is small positive constant. This artificial error leads to slight discrepancy between control torque direction and possible plane perpendicular to the geomagnetic induction vector. Control torque is projected on this plane to construct dipole moment.

This section provided iterative approach to sliding manifold construction. The procedure involves simple calculations using end formulae. However this approach results in some errors in comparison with theoretical representation of matrix $\mathbf{\Lambda}(\boldsymbol{\omega},\mathbf{S},t)$. This, in turn, leads to errors in satellite stabilization.

4. Numerical examples

Control implementation involves choosing coefficient or function λ, approximate positional control contribution Λ_0 and matrix $\mathbf{P}$. These values depend on expected magnetorquers dipole moment value and relation between damping and positional control parts. Fig. 1 brings numerical simulations result for “Chibis-M”. This is a typical microsatellite with a mass of several tens kilo. Its inertia tensor is $\mathbf{J}=diag(1.0255, 1.5393, 1.8172)$ kg·m^2. Control parameters are $\Lambda_0=10^{-4}$, $\delta b_2=0.001$, $\lambda=0.07$, $\mathbf{P}=10^{-3}\mathbf{E}$, maximum dipole moment is 3.2 Am^2, control step $\Delta t=0.1$ s, circular orbit altitude 400 km, inclination 60 degrees, geomagnetic field is modelled by tilted dipole. Gravitational torque is taken into account in control construction, aerodynamic torque is a disturbing one.

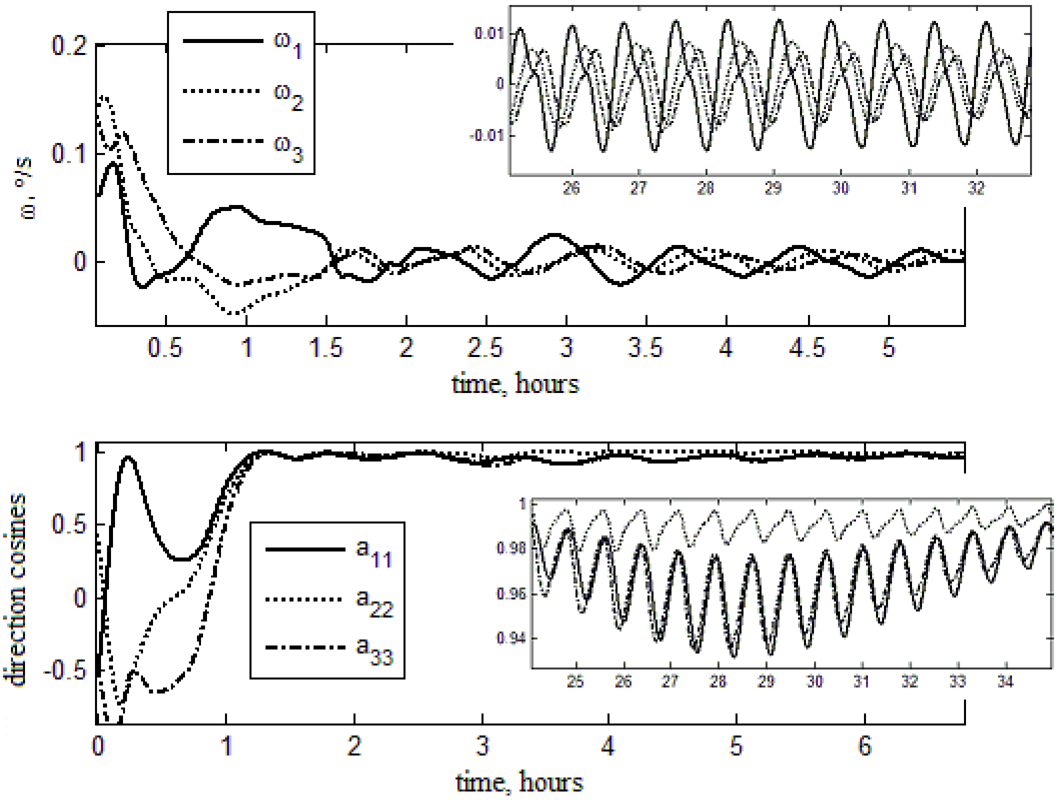

Fig. 1. Microsatellite inertial attitude

Iterative approach to sliding manifold restricts microsatellite attitude accuracy to 15-20 degrees with 0.1 s control step. Nanosatellites can be controlled with greater accuracy. Fig. 2 brings numerical simulation for "TabletSat-Aurora" with inertia tensor $\mathbf{J} = diag(0.52, 0.58, 0.705)$ kg·m^2. Control parameters are $\mathbf{P} = 5\cdot 10^{-4}\mathbf{E}$, $\lambda = 0.15$, step $\Delta t = 1$ s. Other parameters are the same. Dipole moment of magnetorquers is bounded by 1 Am^2. Accuracy is better than 5 degrees.

Finally CubeSat with inertia tensor $\mathbf{J} = diag(0.09, 0.011, 0.007)$ kg·m^2 and control parameters $\mathbf{P} = 10^{-5}\mathbf{E}$, $\lambda = 0.1$, $\Delta t = 1$ s and magnetorquers producing no more than 0.1 Am^2 is again stabilized with accuracy of few degrees (Fig. 3).

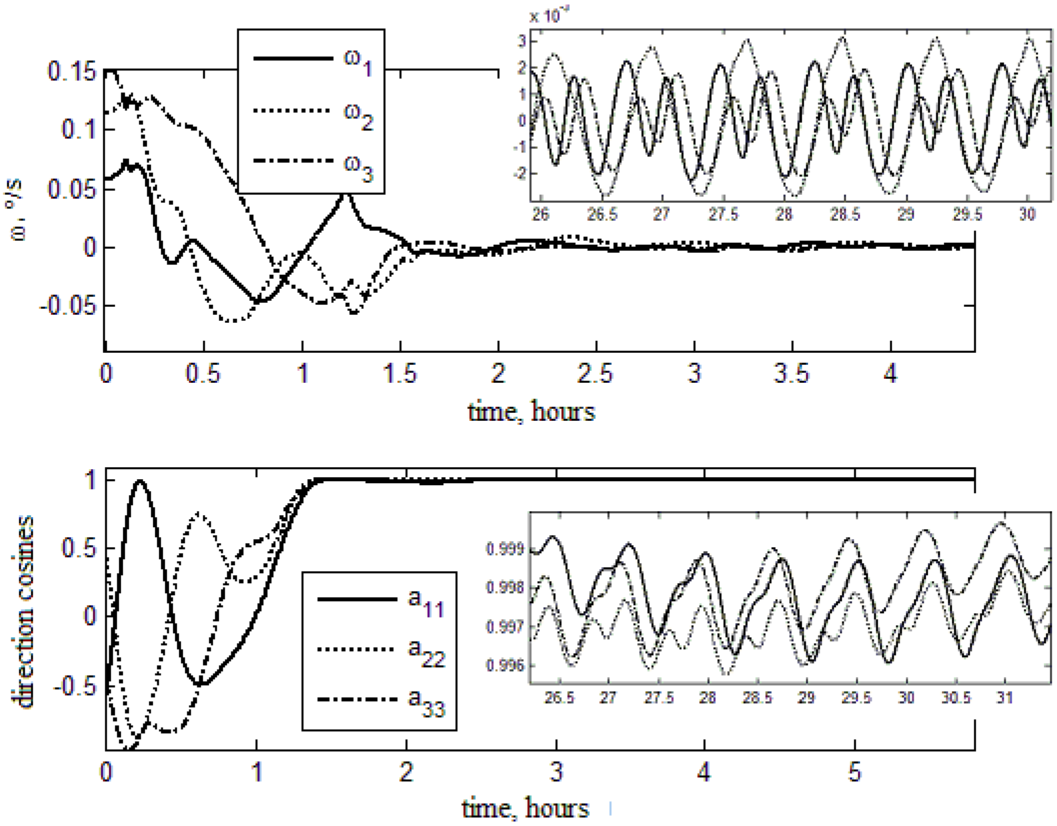

Fig. 2. Nanosatellite inertial attitude

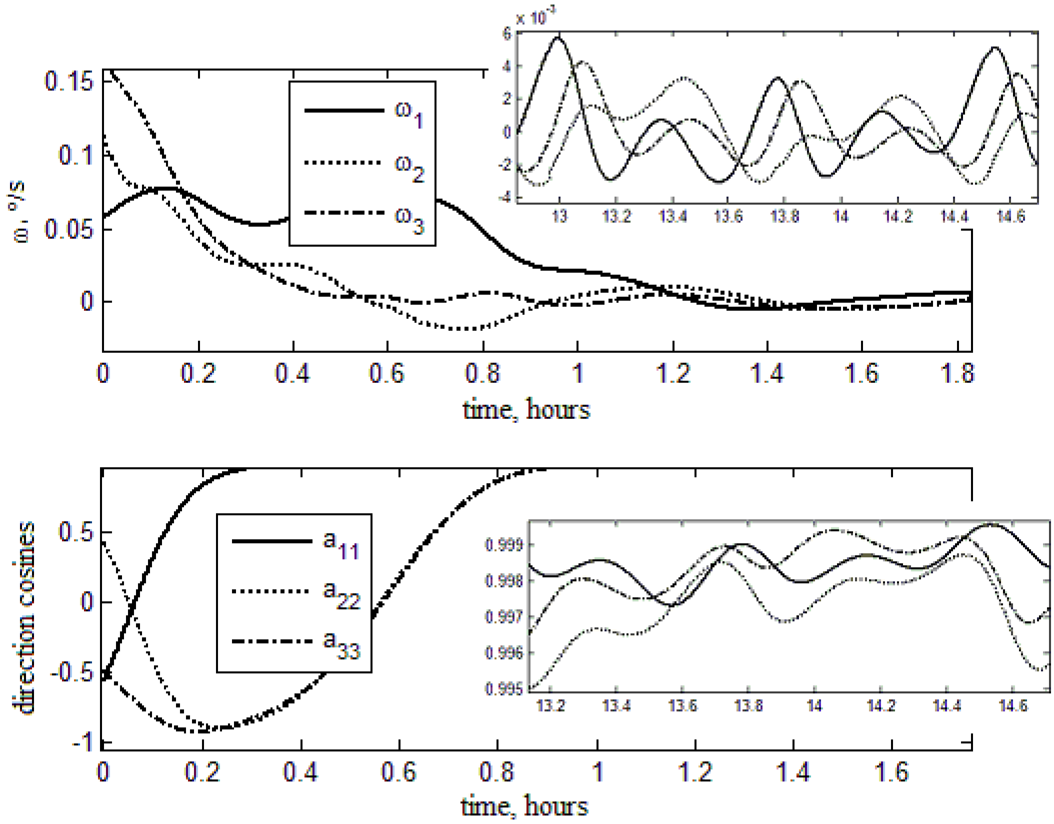

Fig. 3. CubeSat inertial attitude

Fig. 4. Brings CubeSat stabilization in orbital reference frame.

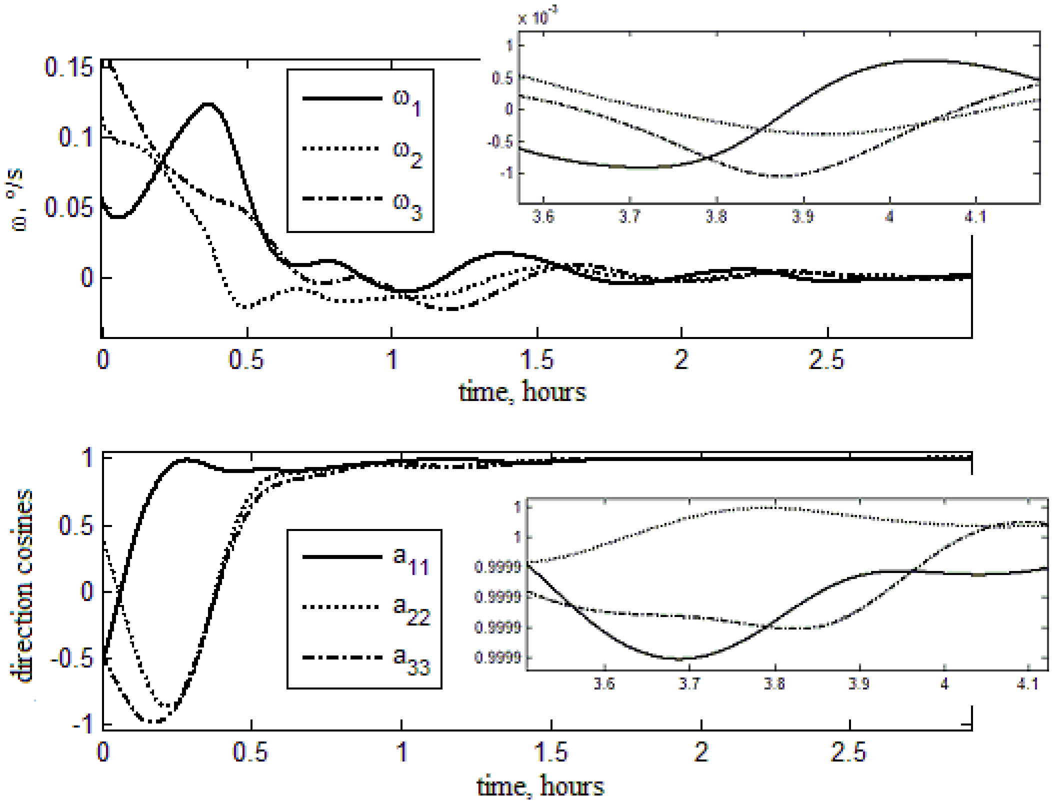

Fig. 4. Cubesat orbital attitude

Attitude accuracy is better than one degree. This is due to gravitational torque being close to zero in necessary attitude (this position is not stable in gravitational field however).

Conclusion

Sliding control for three-axis magnetic attitude is considered. Control system consists of magnetorquers only. Sliding manifold is constructed with varying parameters. This allows constructing accessible angular motion path leading the satellite to the necessary attitude. Control torque employing this path is perpendicular to geomagnetic induction vector for each time and path point. Iterative approach for varying sliding manifold construction is proposed. CubeSat attitude accuracy is few degrees in inertial space and less than one degree in orbital frame.

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