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**Active Protection of the Acoustic Field of Wanted Sources
from External Noise in Real Time**

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Активная защита акустического поля желательных источников от внешнего шума в реальном времени

Дается обзор принципиальных основ и современного состояния нового направления в теории математического моделирования устройств активной защиты акустического поля желательных источников звука в заданной подобласти пространства от внешнего шума в ходе однократно в реальном времени протекающего процесса распространения звука. Обзор сделан в связи с критическим анализом статьи С.В. Утюжникова (S.V. Utyuzhnikov. Real-time active wave control with preservation of wanted field // IMA J. Appl. Mathematics, 2014, 79, 1126–1138).

Ключевые слова: активная защита от шума, метод разностных потенциалов (МРП), текущая акустическая разведка (ТАР)

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Active protection of acoustic field of wanted sources from external noise in real time

An overview is made of the fundamental principles, the state of the art, and available applications in a new area concerning the theory of mathematical modeling of active noise control devices intended for the real-time shielding of sound in a given subdomain from outside noise on the basis of the difference potential method and current acoustic exploring. This overview is presented to justify our critical analysis of S.V. Utyuzhnikov's paper "Real-time active wave control with preservation of wanted field" published in IMA J. Appl. Math. 79, 1126-1138 (2014).

Key words: active noise shielding, difference potential method (DPM), current acoustic exploring (CAE)

Introduction

This work provides an overview of a new area in the theory of active noise shielding developed by the author on the basis of the difference potential method (DPM) and current acoustic exploring (CAE).

The overview is supplemented with a critical review of Utyuzhnikov's paper "Real-time active wave control with preservation of wanted field" published in IMA J. Appl. Math. 79, 1126-1138 (2014). Hereafter, this work is referred to as [U0].

1. New Direction in the Theory of Active Noise Shielding

1.1. Preliminaries

Suppose that sound propagates in a spatial domain with a distinguished subdomain. The problem of active shielding of the acoustic field in this subdomain from external noise consists in creating additional sound sources (speakers) placed near the subdomain boundary so that they cancel the unwanted outside noise within the subdomain. The classical noise shielding problem deals with the case when there are no internal sources or when the field they produce near the boundary is known. This problem has been addressed in numerous papers and several monographs, so we do not consider it.

1.2. Problem of Real-Time Active Noise Shielding and the Main Results

Suppose that sound sources located within the given subdomain produce the desired sound, while the outside sources create unwanted noise. For illustrative purposes, we consider the problem of shielding a conversation in a room with an open window from street noise by using additional sound sources placed near the boundary of the room, i.e., in the window frame. The main results obtained by solving this problem are presented below.

A mathematical model of devices intended for shielding sound within a given subdomain from unwanted noise in the course of a single real-time acoustic process was constructed in [14, Section 6]. According to this model, the current control of

additional sound sources ensuring that the noise is reduced by a prescribed number of times is based on available current information. Specifically, only the total sound pressure near the boundary (in the window frame) produced by all sound sources (desired, unwanted, and shielding) and the state of the acoustic medium near the boundary of the shielded subdomain (in the window frame) are used in the model. The current pressure is measured with microphones. The properties of the medium are determined by measuring the temperature, humidity, and wind speed in the window frame with the use of thermometers, barometers, and other simple measuring instruments. Due to its availability, this control information can be used to implement active noise control in practice.

All additional data concerning the shape of the domain, the conditions on the outer boundary of the composite domain (in our example, the locations and shapes of the buildings, the shape of the room, and the conditions on their outer boundaries), and the acoustic properties of the medium outside the window frame are automatically obtained and processed by the device in the course of noise exploration, which is performed simultaneously with the process.

In the example under consideration, such a device (if created) would depend only on the size of the rectangular window, so it could be produced industrially in large quantities.

Below, the problem formulated is referred to as problem 1.2.

The first step toward the solution of problem 1.2 was taken in [9], where problem 1.2 was solved under an additional assumption, namely, the total sound pressure of the desired and unwanted sources was assumed to be known in the case of no control.

The solution of problem 1.2 without introducing additional assumptions was not obtained until 15 years later (see [14]). This solution is based on the result of [9] supplemented with the CAE algorithm.

1.3. Basics of the DPM

The DPM was introduced in [1] and was substantially developed later. Surface potentials with projectors for linear differential operators were constructed in [4, 6]. The theory of difference and surface potentials with projectors can be found in [8, Sections 1 and 2].

A common prototype of both difference and surface potentials with projectors is a classical Cauchy-type integral from the theory of analytic functions. The Cauchy-type integral can be interpreted as the potential of a differential Cauchy-Riemann operator acting on the space of piecewise regular complex-valued functions in the complex plane with a jump on a closed contour, where the density of the Cauchy integral is given.

The DPM opens up new opportunities for various applications, since it combines some properties of Cauchy integrals with the universality and algorithmic advantages of stable difference schemes.

Remark. It should be emphasized that the projector-involving surface potentials constructed in [4, 6, 8] for general differential operators cannot be directly used in constructive applications. Preliminarily, they have to be approximated by discrete constructions. This comprises the difference between surface and difference potentials and provides a key advantage of the latter over the former when we want to obtain a constructive computer solution in some application.

The key property of difference potentials underlying noise shielding based on the total sound pressure of all sound sources as measured by microphones is that these potentials involve projectors. Under the action of such a potential-projector, whose density is obtained using microphone data, the desired (but unknown) term remains unchanged, while the unwanted (and also unknown) term vanishes.

1.4. New Fact of Mathematical Physics and CAE Based on It

Obviously, the acoustic field to be controlled and the resulting acoustic field depend on the shape of the underlying domain, on the acoustic properties of the medi-

um, which in turn depend on spatial coordinates and time (i.e., on temperature, humidity, wind speed, snow, cloudiness, building locations, and so on), and on the conditions imposed on the boundary of the entire composite domain (on the ground, the building walls, the walls of the shielded room, and the surfaces of the moving sound sources (e.g., people speaking in the room, noise-producing cars). Therefore, it may seem surprising that boundary data (observations made in the window frame) is sufficient to solve problem 1.2, which was first shown in [14, Section 6].

It was also established in [14] that the protected and noise-producing subdomains obey a reciprocity relation, which can be described as follows. If the contribution of the outer sources to the shielded field is reduced by a given number of times as a result of control, then the contribution of the desired sources to the acoustic field at points of the complementary subdomain is increased exactly by the same number of times.

The fact that boundary information is sufficient for the control of additional shielding sources was not previously known and is of interest in itself.

This earlier unknown fact of mathematical acoustics was used in [14] to design an algorithm serving as a mathematical model of a noise-canceling device, which was called current acoustic exploring (CAE).

1.5. Insufficiency of Boundary Data for Total Noise Suppression in a Shielded Subdomain

In [14, pp. 413-415; 19, Section 3.3, pp. 23-25] theorems were proved stating that the current boundary data is insufficient for real-time total noise suppression in a protected subdomain.

1.6. Properties of CAE

The CAE algorithm for processing current boundary data, which can be used as a mathematical model of noise shielding devices, was first described and rigorously justified in [14, Section 6]. This algorithm does not require any information other than current measurements near the boundary (in the window frame) and the fact that

sound propagation in sound control is a linear process. Importantly, the CAE algorithm provides data for generating control signals in a timely manner, i.e., earlier than this mathematical product is used by the device to generate control signals for the additional sources.

The possibility of real-time signal generation is based on two specific properties of acoustic problems.

One is that solutions of the acoustic wave equations (which are hyperbolic differential equations) can be predicted at some forward time relying on currently known data.

The other property is that CAE data can be nearly instantaneously processed on a computer by applying simple formulas based on a stable difference scheme approximating the acoustic wave equations near the boundary (in the window frame).

The preliminary computation relies on the fact that the speed of sound is much lower than the speed of light, i.e., it is similar to a telephone call from a visitor who is going to arrive. Since the visitor is coming at transport speed ("speed of sound"), while the phone call arrives at the "speed of light," the host has enough time to get ready for the visitor arrival.

In the CAE-based sound protection problem under consideration, the role of a phone call concerning a forthcoming sound pressure signal to be experienced by the room boundary is played by data received from microphones placed on both sides of the boundary at some distance from it (in the window frame). This data is fed to a computer through wires or radio, i.e., at the speed of light. The role of the host is played by the computer, which generates control signals for the speakers. The speed of sound near the boundary of the shielded subdomain is interpreted as the transportation speed in the visitor case.

1.7. CAE Algorithm for Active Noise Reduction Based on Current Boundary Data Cannot Be Further Improved

Taken collectively, the following three properties of the CAE algorithm mean that it cannot be significantly improved.

Property 1. Generally speaking, the boundary data used as input information in problem 1.2 cannot be replaced by more efficiently received data, for example, by measurements made only on one side of the shielded subdomain boundary, since this information is not sufficient for generating control commands. An algorithm for active sound control in this case is not only unknown, but does not exist at all.

Property 2. By applying the CAE algorithm, noise in the shielded subdomain can be reduced by any desired number of times. It was shown in Section 1.5 that a better algorithm based on boundary information that leads to total noise suppression in the shielded subdomain does not exist.

Property 3. In the CAE algorithm, control commands for noise-cancelling sound sources arrive some time before they have to be executed. This property is an important condition for the technical possibility of using CAE-generated commands based on current boundary data. If commands were not generated in advance, there would be an inevitable delay in their technical implementation.

2. On Utyuzhnikov's Paper

Returning to work after an extended break, I have read several of Utyuzhnikov's papers that misrepresent the fundamental principles, the history of creation, and the state of the art of the above-described new area I founded in the theory of active noise suppression. A detailed analysis of Utyuzhnikov's most recent publication [U0] reveals no scientific results, but contains ill-defined and incorrect assertions.

This analysis is presented in part below.

2.1. Utyuzhnikov's publication [U0] does not contain scientific results.

In [U0] Utyuzhnikov proposes a solution of the active sound control problem aimed at real-time total noise suppression in a shielded subdomain as based on cur-

rent boundary data. However, it was shown in Section 1.5 that this problem has no solution, so Utyuzhnikov's constructions in [U0] involve an irremovable error and of no scientific interest.

2.2. [U0] contains ill-defined and incorrect assertions. Some of them are quoted and commented on below.

Citation no. 1. The last phrase in the abstract on p. 1126 reads "Some limitations of the pro-posed approach are also discussed."

Comment no. 1.

The author of the citation mistakenly takes the total information impossibility of creating a device based on his approach as "some limitations." This impossibility follows directly from Section 1.5 and is similar to that of creating a perpetual motion machine, which violates the energy conservation law.

Citation no. 2. In the second paragraph from the top on p. 1127, we read "For the first time, the non-stationary AC problem in question was tackled in Utyuzhnikov ("Nonstationary problem of active sound control in bounded domains," J. Comput. Appl. Math. 43, 101-112 (2009) where it was proved that the constructed solution is even applicable to resonance regimes."

Comment no. 2.

Actually, in an abstract, i.e., more general formulation, the problem was first set up in [9, 10], where formulas were presented for its solution assuming that the total sound pressure of the desired and unwanted sources producing the field to be controlled is known near the boundary of the shielded subdomain.

During my stay at Manchester University in 2005, Utyuzhnikov got acquainted with the results of [9] from my two talks and our numerous conversations. I asked Utyuzhnikov and his graduate student H. Lim to create an experimental setup for implementing the general algorithm of [9] in the special case of a stable difference scheme approximating the one-dimensional nonstationary acoustic wave equations. Such a setup was created at Manchester University. The experiments carried out with it confirmed that the abstract formula from [9] can be implemented as a physical de-

vice and validated the general theoretical conclusions of [9] in the case of the 1D wave equation. This experimental setup and experiments were described in [20] and were also reported by Lim, Utyuzhnikov, et al. at the 40th International Congress and Exposition on Noise Control Engineering held in Osaka (Japan) in 2011. So the assertion in citation no. 2 is not correct.

Citation no. 3. In the second paragraph from the bottom on p. 1127, Utyuzhnikov says: "It should be noted that a local solution has recently been obtained in Ryaben'kii ("Model of real-time active noise shielding of a given subdomain subject to external noise sources," *Comput. Math. Math. Phys.* 51, 444-454 (2011)) in a finite-difference formulation. However, it requires additional one-dimensional solutions of the wave equation and the implementation of two layers of secondary sources situated on both sides of the boundary of the protected domain."

Comment no. 3.

The author of the citation makes errors when describing the solution algorithm for problem 1.2. For example, Ryaben'kii introduces two layers of microphones, rather than secondary sources, as stated by Utyuzhnikov, who presumably failed to understand the principles of the CAE algorithm for solving problem 1.2. Furthermore, the CAE algorithm was first published in [14, Section 6], which appeared in 2010, rather than "recently in 2011", as the citation reads. Note that [14] is not mentioned at all in [U0].

Citation no. 4. In Section 3 on p. 1129, one can read: "Following Utyuzhnikov (2009c), we introduce an operator P ..." and "The authors of Ryaben'kii (2002) and Utyuzhnikov (2009c) introduced the notion of a clear trace ..."

Comment no. 4.

The concepts of a clear trace and of difference and surface potentials with projectors P were introduced for linear difference and differential operators about 30 years ago in [3], [4], and [6], respectively.

Sharp results concerning the approximation of surface potentials by difference ones in the case of the Laplacian and its difference analogue were obtained by Reznik [5].

In Lazarev's work [7], the concept of projector-involving potentials with densities from the space of clear traces was extended to abstract linear operators in Banach spaces.

All these results were presented in [8, Sections 1, 2]. However, in [U0] Utyuzhnikov does not refer to them or [8].

The fault is obvious.

Citation no. 5. In the Conclusion on p. 1137, the author states "The general solution to a nonstationary AC problem has been obtained using the theory of the CR potentials."

Comment no. 5.

It was shown in Section 2.1 above that [U0] does not contain scientific results at all. Note additionally that the citation makes a wrong impression that surface CR-potentials introduced by Utyuzhnikov in "Generalized Calderon-Ryaben'kii potentials," IMA J. Appl. Math. 74, 128-148 (2009) are a new tool for noise shielding problems. Actually, surface potentials with projectors are addressed in [8, Sections 1 and 2]. Surface potentials are not used by Ryaben'kii in noise suppression problems, because DPM and CAE provide a self-contained general approach without applying extra tools like surface potentials, which are needless in noise shielding problems.

Citation no. 6 (at the end of [U0]): "The author is grateful to Professor Victor S. Ryaben'kii for fruitful discussions."

Comment no. 6.

The acknowledgment makes an impression on the manuscript reviewers and readers that Ryaben'kii is aware and approves of Utyuzhnikov's study. Actually, Utyuzhnikov never told me about his written or conceived papers.

3. Concluding Remark

Returning to the above overview of the new direction in the theory of active sound control and noise suppression on which this paper is focused, it should be stressed that the mathematical apparatus of DPM and CAE can be applied not only to a wide variety of noise shielding problems in air and water, but also to the control of sound propagation in composite materials. The DPM can also be used in conjunction with additional information for a posteriori analysis of various audio records.

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