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ИНСТИТУТ ПРИКЛАДНОЙ МАТЕМАТИКИ

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On the linear dependence of cuprates magnetoresistance on magnetic field

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The actual problems of Fermi systems theory are considered. A simple explanation is given to the linear dependence of electrical resistance on temperature and the linear dependence of the magnetoresistance on the magnetic field in high-temperature superconducting cuprates, which has been mysterious for many years. It is shown that this dependence stems from the treatment of a gas of translationally invariant polarons as a system with heavy fermions for wave vectors close to nesting. The destruction of such polarons at finite temperature and an external magnetic field leads to a linear dependence of the magnetoresistance on the magnetic field and temperature. It is shown that the relationship between the slopes of the magnetoresistance curves at zero magnetic field and at zero temperature is determined by the universal ratio of Boltzmann constant and Bohr magneton. A relation between the existence of translationally invariant polarons and the "Planck" time of their relaxation is discussed.

Key words: bipolarons, strange metals, nesting, Kohler rule

В.Д. Лахно

О линейной зависимости магнетосопротивления купратов от магнитного поля

Рассматриваются актуальные вопросы теории Ферми систем. Дается простое объяснение в течение многих лет являвшейся загадочной линейной зависимости магнетосопротивления от температуры и линейной зависимости магнетосопротивления от магнитного поля в высокотемпературных сверхпроводящих купратах. Показано, что к такой зависимости приводит представление о газе трансляционно-инвариантных поляронов как о системе с тяжелыми фермионами для волновых векторов близких к нестингу. Разрушение таких поляронов при наличии температуры и внешнего магнитного поля приводит к линейной зависимости магнетосопротивления от магнитного поля и температуры. Показано, что связь между наклонами кривых магнетосопротивления при нулевом магнитном поле и при нулевой температуре определяется универсальным отношением постоянной Больцмана и магнетона Бора. Обсуждается связь между трансляционно-инвариантными поляронами и планковским временем релаксации.

Ключевые слова: биполяроны, странные металлы, нестинг, правило Колера

1. Introduction

Transport properties are the most prominent features of condensed matter which determining their applications. An important part of condensed matter constitutes metals the description of which is based on the Fermi liquid theory. It was quite extraordinary that for bad metals of cuprates in the pseudogap phase and for some strongly correlated systems the well - established laws are in violation. These extraordinary features are usually explained by the fact that these metals are in a special quantum states which defies the known laws.

The well-known example is the so-called “bad“ or “strange“ metals which exhibit the nonstandard linear low temperature dependence observed for many years and yet agreement of reasonable understanding is still lacking. The goal of this paper is to explain these unusual properties.

2. Linear temperature dependence of electrical resistance in cuprates and systems with heavy fermions

One of the most topical problems in the theory of condensed matter is the description of the properties of metals to which the Fermi liquid (FL) approach is inapplicable. In the theory of condensed matter, such systems have received a common name: strongly correlated electronic systems, since the Fermi-liquid approach is based on the opposite limiting case of independent or weakly correlated electronic states. Alternatively, they are also called non-Fermi liquid systems (NFL)

One of the brightest predictions of the FL theory was the conclusion about the quadratic temperature dependence of the electrical resistance in ordinary metals. Indeed, according to the FL approach, near the Fermi surface, at low temperatures $T \ll E_F$, where E_F is the Fermi energy electrons in a metal can be considered as independent quasiparticles, the number of which is $\propto T$. Such electrons can only be scattered into non-occupied states under the Fermi surface, the number of which is also $\propto T$. Thus, the probability of electron scattering will be $\propto T^2$. Accordingly, the electrical resistance of the metal in the limit of low temperatures will be $\rho \propto T^2$. The quadratic law of the dependence of magnetoresistance on the magnitude of the magnetic field *H* also immediately follows from the FL theory.

This picture could be changed by taking into account the scattering of electrons on phonons. Indeed, consideration of such scattering in ordinary metals at low temperatures leads to a temperature dependence of the electrical resistance $\rho(T)$ of the form:

$$\rho(T) = \rho_0 + a_{ee}\left(\frac{T}{E_F}\right)^2 + a_{ep}\left(\frac{T}{\omega_0}\right)^5 \qquad (1)$$
$$T < 0{,}2\hbar\omega_0 \ll E_F,$$

where ρ_0 is the residual resistance at zero temperature, the second term in the right-hand side is the contribution of electron-electron scattering discussed above, the third one is the contribution of electron-phonon scattering, ω_0 is the Debye frequency of phonons [1].

At high temperatures, when the condition $T \geq 0{,}2\hbar\omega_0$ is satisfied, scattering on phonons becomes dominant, which, for $T > 0{,}2\hbar\omega_0$, being proportional to the number of phonons, increases linearly with temperature:

$$\rho \propto T/\omega_0\,,\ T > 0{,}2\hbar\omega_0 \qquad (2)$$

Thus, the fact that the electrical resistance of metals increases linearly in temperature with increasing temperature is not surprising. In ordinary metals, it is observed almost always at sufficiently high temperatures $T > 0{,}2\hbar\omega_0$.

Surprising is the fact that in cuprates and systems with heavy fermions a linear dependence of electrical resistance arises even at very low temperatures.

An explanation of this behavior of the electrical resistance in cuprates was given in [2]. According to [2], in cuprates and systems with heavy fermions, due to the Kohn anomaly, the value of $\omega_0$can be very small (for heavy fermions, their polaron mass tends to infinity as $\omega_0 \to 0$). As a result, the temperature dependence of the electrical resistance, according to (2), turns out to be linear even at very low temperatures.

The above explanation of the linear temperature dependence of the electrical resistance does not require any revision of the existing concepts and, in fact, is associated only with the peculiarities of the phonon spectrum of cuprates.

However, surprising are the experiments on measuring the dependence of the magnetoresistance of cuprates on the magnetic field, in which at low temperatures, instead of a quadratic dependence of the magnetoresistance on the magnetic field, its linear dependence is observed. In particular, it was found that: firstly, the linear dependence of the resistance on the magnitude of the external magnetic field *H* takes place only at low temperatures. Secondly, at sufficiently large field values, the resistance is practically independent of temperature. This leads to some symmetry between *H* and *T*: a linear dependence on *H* exists only for $T{\sim}0$ and vice versa, a linear dependence on *T* exists only for $H{\sim}0$.

Next section is devoted to the explanation of this phenomenon.

3. Linear magnetic field dependence of manetoresistanse in cuprates and systems with heavy fermions

The linear in a magnetic field increase of the magnetoresistance of cuprates and a number of other correlated electronic systems in a wide range of magnetic fields, from the lowest to quite high ones, discovered in recent experiments [3]-[7], is one of the biggest mysteries in the physics of high-temperature superconductors (HTSC). The electrical resistance of these compounds linear in temperature at low temperatures, which has been observed since early experiments [8]–[9], also remains no less of a mystery.

In strongly correlated electronic systems, this behavior of the magnetoresistance is attributed to the existence of a strange metal phase (SM) in them, in particular, a fermionic phase in $BaFe_2(As_{1-x}P_x)_2$ [3]-[4], $FeSe_{1-x}S_x$ [5] and a bosonic phase in $YBa_2Cu_3O_{7-x}$ $(YBCO)$ [6], $La_{2-x}Sr_xCuO_4$ [7], the appearance of which in itself is a mysterious phenomenon. It is believed that, in contrast to ordinary metals, in which at low temperatures the magnetoresistance is quadratic in temperature and magnetic field, which corresponds to their description based on quasiparticles, the description of the strange metal phase based on the idea of well-defined quasiparticles is impossible [10]. The purpose of this paper is to point out the possibility of a simple explanation of the strange behavior of magnetoresistance based on the concept of translation-invariant (TI) polarons and bipolarons used to describe HTSC [2].

According to [2], [11], TI polarons are formed in cuprates when their momentum corresponds to nesting, that is, the wave vector of the charge density wave P_{cdw}. Due to the Kohn anomaly, their mass m_{pol} is very large in this case (heavy fermions), and under the Fermi surface they form a nondegenerate fermionic gas. At a temperature $T > \left|E_{pol}(P_{cdw})\right|$, where E_{pol} is the energy of a TI polaron (reckoned from the Fermi level), the decay of TI polarons into free electronic states becomes possible. The lifetime of such free electrons will be determined by scattering on phonons, that is $\tau \propto \omega_0/T$, where ω_0 is the phonon frequency, thereby determining the linear dependence of the electrical resistance on temperature: $\rho = m/e^2 n\tau \,\propto\, T/\omega_0$, where n is the concentration of free electrons, m is the mass of a free electron. In an external magnetic field *H*, the concentration of free electrons depends on H and, according to [12], is equal to

$n_H = n \cdot (k_B T/\mu_B H) \cdot th(\mu_B H/k_B T)$ (see Supplement). Hence, the magnetoresistance $\rho_H = m/n_H e^2 \tau$ of the fermion bulk system is written as:

$$\rho_H(T, H) \propto \mu_B H \, cth \frac{\mu_B H}{k_B T} \tag{3}$$

It follows that for H=0: $\rho \propto T$, which is the well-known linear temperature law for the electrical resistance of fermionic compounds, and for $\mu_B H > k_B T$: $\rho \propto H$-linear in a magnetic field law for their magnetoresistance.

It is important to emphasize that expression (1) obtained for the magnetoresistance satisfies the Kohler rule [13]:

$$\rho_H(T, H) - \rho(T, 0) = \rho(T, 0) \, f[H/\rho(T, 0)]$$

According to [3]-[5], the experimental dependence of the magnetoresistance of fermionic correlated systems $\rho(T, H)$ in the case of a strong magnetic field is well approximated by the hyperbolic expression:

$$\rho(T, H) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B H)^2} \tag{4}$$

where α and γ are constants satisfying the relation $\gamma/\alpha = 1{,}01 \pm 0{,}07$.

Figure 1 shows a comparison of ρ_{exp} which is the experimental dependence $\rho(T, H)$ determined by the right side of expression (4) with the theoretical dependence ρ_{theor} determined by the right side of expression (3), for the case $\gamma = \alpha$. It follows from Fig. 1 that both the approximations are in agreement with the experimental dependance within the accuracy of the experiment [3].

The fact that, according to [3], the linear dependence for the magnetoresistance is most clearly expressed near the optimal doping is apparently due to the fact that under these conditions the concentration of TI polarons reaches maximum [2].

We note that the presence of a strong temperature dependence of the Hall effect in fermionic compounds has little to do with the presence of TI polarons in them, since the concentration of the latter is much lower than the concentration of ordinary current carriers.

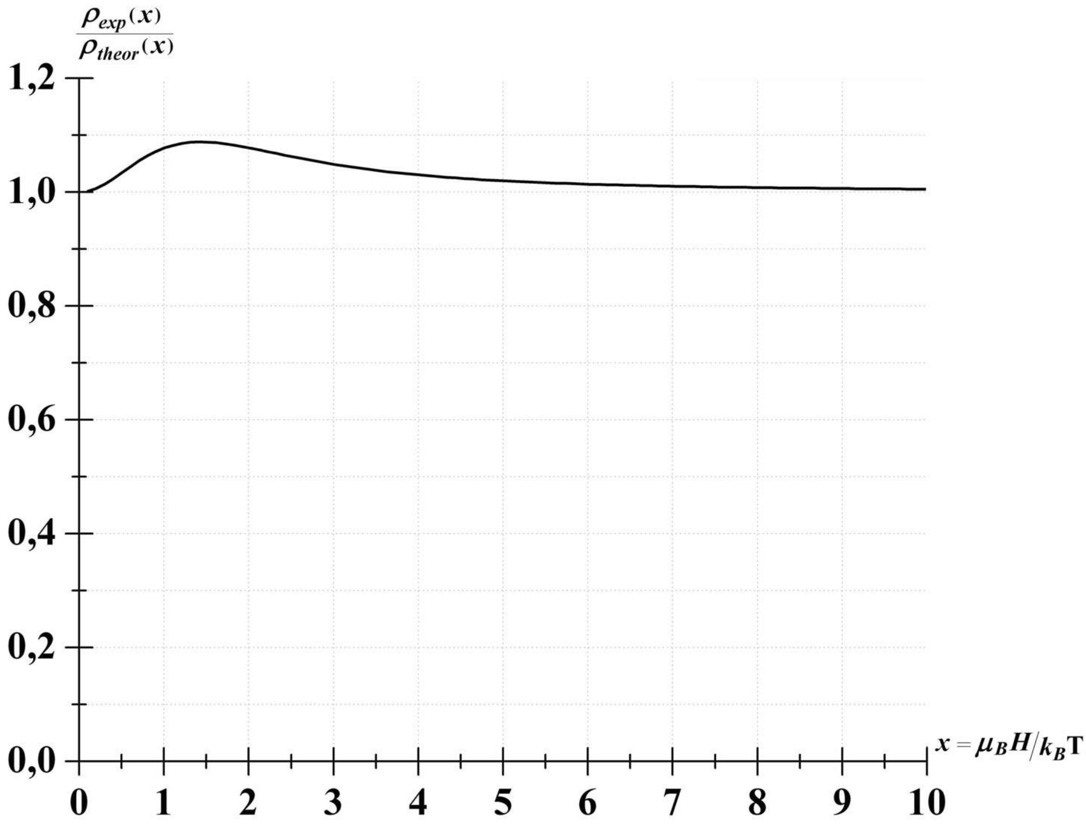

Fig.1 Comparison of the theoretical dependence $\rho(x) = \rho_{theor}(x)$, determined by (3) with the approximation of the experimental dependence $\rho(x) = \rho_{exp}(x)$, determined by (4). The maximum difference for $x = \mu_B H / k_B T$ = 1.432 is 8.8 percent.

In the case of bosonic cuprates, TI bipolarons are the dominant current carriers. Each decaying TI - bipolaron will now lead to the appearance of two free electrons. Accordingly, in the expression for n_H , μ_B should be replaced by $2\mu_B$. As a result, for the magnetoresistance, instead of (3), we obtain:

$$\rho \propto 2\mu_B H \, cth \, 2\mu_B H / k_B T \qquad (5)$$

According to [6], the experimental dependence of the magnetoresistance of bosonic thin cuprate films is well approximated by the expression:

$$\rho \propto k_B T + 2\mu_B H \qquad (6)$$

The proof of the bosonic nature of current carriers in YBCO and LSCO in [6] was based on measurements of the Little and Parks effect near the critical superconducting transition temperature Tc, which, generally speaking, is not a strong argument in favor of the bosonic nature of current carriers at temperatures above T_c . A stronger argument is the validity of the relations:

$$\rho'_T(T,0)/\rho'_H(0,H) = k_B/\mu_B \quad (7)$$

for fermionic systems and:

$$\rho'_T(T,0)/\rho'_H(0,H) = k_B/2\mu_B \quad (8)$$

for bosonic ones which follow from (3) – (6), and are confirmed by experiments [3],[6].

As noted above, there is currently not a generally accepted explanation for the dependence of the magnetoresistance linear in T and H. In particular, such behavior is often observed for systems in the vicinity of their quantum critical points. In this regard, in [14] a concept of the universal "Planck" relaxation time $1/\tau_P \sim k_B T/\hbar$ was introduced. In the case of metallic systems, this means that the Fermi energy ceases to play the role of a scaling quantity in describing the electronic properties near the quantum critical point and, accordingly, the temperature takes on the role of a scaling energy. In the case of a TI polaron (bipolaron) gas under consideration, the characteristic frequency of fluctuations near the critical point ω_0, according to [11], tends to zero due to the Kohn anomaly, i.e for $T > 0$, phase transitions can be described in terms of classical statistical mechanics, since in this case the inequality $T > \omega_0$ always takes place, which explains the nature of the occurrence of the "Planck" time in strongly correlated electronic systems near their quantum phase transitions. In this case, the participation of the magnetic field in the scaling relations of quantum phase transitions also becomes clear, since it plays the role of an external control parameter in them.

4. Discussion

It is now generally recognized that polarons, which are electrons or holes dressed by the lattice deformations or perturbations caused by them, are the key objects in explaining the properties of transition metal oxides, such as high-temperature superconductors, nickelates, manganites, and systems with heavy fermions [15-19]. In particular, according to [2], the microscopic description of HTSC is based on the bipolar mechanism.

Since most HTSC materials have a magnetic order, the question of the type of polarons (phonon or magnon) responsible for HTSC remains open.

It was shown in [2], [20] that numerous properties of HTSC in the pseudogap phase can be explained on the basis of the concept of phonon translationally invariant polarons and bipolarons. In the approach considered, the type of interaction between the current carriers and the lattice has not been specified anywhere. For this reason, the above consideration is also applicable in the case of magnetic polarons. Its universality is demonstrated by experimental observations in a wide variety of types of compounds.

5. Supplement

According to [2], the conduction pattern in the bad metal phase looks as follows. As distinct from the pseudogap phase, which is determined by the TI bipolarons existing in this phase, the TI polaron states dominate in the higher temperature phase of a strange metal. Formed near the momentum corresponding to the momentum of the charge density wave P_{CDW}, TI polarons turn out to be almost immobile (bad metal). Hence, in the strange metal phase, the current carriers are "bare" electrons which are formed as a result of the decay of TI polarons under the influence of temperature and an external magnetic field. Their concentration can be found from the condition of instability of the TI-polaron gas:

$$Z_{el} Z_{ph} \geq Z_p. \tag{S.1}$$

Where Z_{el} is the statistical sum of electrons, Z_{ph} -of phonons, Z_p - of TI polarons. Relation (S.1) is a criterion when the free energy of the TI polaron gas is lower than the free energy of the electron and phonon gases formed as a result of the decay of TI polarons. For Z_{el}, Z_{ph}, Z_p, in [2] it was obtained:

$$\begin{gathered} Z_p = \left[e^{-(\omega_0 + E_p)/T} \left(\frac{2\pi m^* T}{\mathrm{h}^2} \right)^{3/2} \frac{eV}{N_p} \right]^{N_p}, \\ Z_{el} = \left[\left(\frac{2\pi m T}{\mathrm{h}^2} \right)^{3/2} \frac{eV}{N_p} \right]^{N_p} \\ Z_{ph} = \left[\frac{e^{-\omega_0/2T}}{1 - e^{-\omega_0/T}} \right]^{N_p} \end{gathered} \tag{S.2}$$

where N_p is the number of TI polarons, $E_p = E_p(P_{CDW})$ is the energy of a TI polaron with the wave vector P_{CDW}, ω_0 is the frequency of an optical phonon,

$m^*$is the effective mass of an electron, m is the mass of a free electron, T is the temperature, h is Planck's constant (h=$2\pi\hbar$), e = 2.718 is the Napierian base.

With the use of (S.2), criterion (S.1) takes on the form:

$$T ln[\left(\frac{m^*}{m}\right)^{\frac{3}{2}}\frac{T}{\omega_0}] \geq |E_p| \tag{S.3}$$

It follows from (S.3) that, with logarithmic accuracy, the strange metal phase is stable up to temperatures of the order of $|E_p|$.

In the presence of an external magnetic field, the statistical sum for the electron gas Z_{el}, according to [12], should be replaced by the expression:

$$Z_{el}(H) = Z_{el}\left[\frac{\mu_B H/T}{\mathrm{sh}(\mu_B H/T)}ch(\mu_B H/T)\right]^{N_p} \tag{S.4}$$

where Z_{el} is determined by (S.2).

From (S.2) and (S.4) it follows that for a given value of the magnetic field *H*, the statistical equilibrium with the polaron gas corresponds to the concentration of the electron gas determined by the expression:

$$n_H = n_P\frac{T}{\mu_B H}th\frac{\mu_B H}{T},\ n_P = N_P/V \tag{S.5}$$

Hence, the magnetoresistance $\rho(H)$ in the strange metal phase will be determined by the expression:

$$\rho(H) = \frac{m}{en_H\tau}, \tag{S.6}$$

where e is the electron charge, τ is the relaxation time of current carriers in the strange metal phase.

References

[1] Ashcroft N., Mermin N., Solid state Physics // Holt, Rinehart and Winston, New York, 1976

[2] Lakhno V.D., High-Temperature Superconductivity. Bipolaron Mechanism // De Gruyter, Germany, Berlin/Boston, 2022, 244p

[3] Hayes I.M., McDonald R.D., Breznay N.P., Helm T., Moll P.J.W., Wartenbe M., Shekhter A., Analytis J.G., Scaling between magnetic field and temperature in the high-temperature superconductor $BaFe_2(As_{1-x}P_x)_2$ // Nature Physics, v.12, p.916-919, (2016); doi: 10.1038/NPHYS3773

[4] Maksimovic N., Hayes I.M., Nagarajan V., Analytis J.G., Koshelev A., Singleton J., Lee Y., Schenkel T., Magnetoresistance Scaling and the Origin of H-Linear Resistivity in $BaFe_2(As_{1-x})_2$ // Phys.Rev.X, v.10, p.041062, (2020); doi: 10.1103/PhysRevX.10.041062

[5] Licciardello S., Maksimovic N., Ayres J., Buhot J., Gulo M., Bryant B., Kasahara S., Matsuda Y., Shibauchi T., Nagarajan V., Analytis J.G., Hussey N.E., Coexistence of orbital and quantum critical magnetoresistance in FeSe1-x // Phys.Rev.Research, v.1, p.023011, (2019); doi: 10.1103/PhysRevResearch.1.023011

[6] Yang C., Liu H., Liu Yi., Wang J., Oiu D., Wang S., Wang Y., He Q., Li X., Li P., Tang Y., Wang Jian, Xie X.C., Valles J.M. Jr, Xiong J., Li Y., Signatures of strange metal in a bosonic system // Nature, v.601, p.205-210, (2022); doi: 10.1038/s41586-021-04239-y

[7] Giraldo-Gallo P., Galvis J.A., Stegen Z., Modic K.A., Balakirev F.F., Betts J.B., Lian X., Moir C., Riggs S.C., Shekhter A., Scale-invariant magnetoresistance in a cuprate superconductor // Science, v.361, p. 479-481, (2018); doi: 10.1126/science.aan3178

[8] Batlogg B., Physical Properties of High-Tc Superconductors // Phys. Today, v.44(6), p.44, (1991); doi: 10.1063/1.881260

[9] Iye Y. // In: Physical Properties of High-Temperature Superconductors III, (Ed. D.M. Ginsberg), Singapore: World Scientific, 1992, p.285

[10] Green R.L., Mandal P.R., Poniatowski N.R., Sarkar T., The strange Metal State of the Electron-Doped Cuprates // Annual reviews of Condensed Matter Physics, v.11, p. 213-229, (2020); doi: 10.1146/annurev-conmatphys-031119-050558

[11] Lakhno V.D., Translation-Invariant Bipolarons and Charge Density Waves in High-Temperature Superconductors // Frontiers in Physics, v.9, 662926, (2021); doi: 10:3389/fphy.2021.662926

[12] Kubo R., Statistical Mechanics. An advanced course with problems and solutions // 1965, North-Holland Publishing Company-Amsterdam

[13] Pippard A.B., Magnetoresistance in Metals // 1989, Cambridge University Press, Cambridge, England

[14] Zaanen J., Why the temperature is high, Nature, 430, 512-513, (2004). doi: 10.1038/430512a

[15] Mueller K.A., Handbook of high Temperature Superconductivity. Theory and experiment // 2007Springer Verlag, Berlin

[16] Madan I., Kurosawa T., Toda Y., Oda M., Mertelj T., Mihailovic D., Evidence for carrier localization in the pseudogap state of cuprate superconductors from coherent quench experiment // Nature Communications, V.6, p.6958, (2015); doi: 10.1038/ncomms7958|www.nature.com/naturecommunicaations

[17] Weber F., Aliouane N., Zheng H., Mitchell J.F., Argyrion D.N., Reznik D., Signature of checkerboard fluctuations in the phonon spectra of a possible polaronic metal $La_{1.2}Sr_{1.8}Mn_2O_7$ // Nat Mater, v.8(10), p.798-802, (2009); doi: 10.1038/nmat2513

[18] Massee F., de Jong S., Huang Y., Siu W.K., Santoso I., Mans A., Boothroyd A.T., Prabhakaran D., Follath R., Varykhalov A., Patthey L., Shi M., Goedkoop J.B., Golden M.S., Bilayer manganites reveal polarons in the midst of a metallic breakdown // Nature Physics, v.7, p.978-982, (2011); doi: 10.1038/nphys2089

[19] Li J., Wei-Guo Yin, Wu L., Zhu P., Konstantinova T., Tao J., Yang J., Cheong, F. Carbone S.-W., Misewich J.A., Hill J.P., Wang X., Cava R.J., Zhu Y., Dichotomy in ultrafast atomic dynamics as direct evidence of polaron formation in manganites // npj Quantum Materials, v.1, p.16026, (2016); doi: 10.1038/npjquantmats.2016.26

[20] Lakhno V.D., Pseudogap Isotope Effect as a Probe of Bipolaron Mechanism in High Temperature Superconductors // Materials, v.14, p.4973, (2021); doi: 10.3390/ma14174973

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